Study Title ______ Date ______

※ 3.3 Relations

- Relation
  - Def: be thought of as a table that lists which elements of the first set relate to which elements of the second set. R
    - Define a relation to be a set of ordered pairs.
    - Subset of the Cartesian product $X \times Y$

ex) Relation of students to courses

| *Student* | *Course* |
|---|---|
| Bill | CompSci |
| Mary | Math |
| Bill | Art |
| Beth | History |
| Beth | CompSci |
| Dave | Math |

Let $X = \{$Bill, Mary, Beth, Dave$\}$, $Y = \{$CompSci, Math, Art, History$\}$, then $R = \{$(Bill, CompSci), (Mary, Math), (Bill, Art), … (Dave, Math)$\}$

→ since (Beth, History) $\in R$, Beth R History

- Digraph
  - Informative way to picture a relation on a set is to draw
  - Draw dots or vertices to elements of X

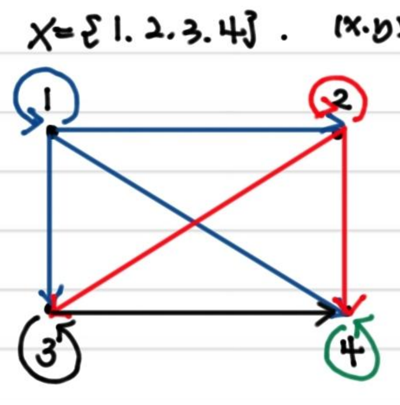
    ↓

    If the element $(x, y)$ is in the relation, we draw an arrow directed edge from x to y

    ( $(x, x)$ is from x to x, loop )

ex) $X = \{1, 2, 3, 4\}$ defined by $(x, y) \in R$ if $x \le y$, $x, y \in X$

$X = \{1, 2, 3, 4\}$. $(x, y) \in R$ if $x \le y$. $x, y \in X$




- Properties
  - Reflective: If $(x, x) \in R$ for every $x \in X$

    ex) $X = \{1, 2, 3, 4\}$ is reflective because $(1,1)$ $(2,2)$, $(3,3)$ $(4,4)$ $\in R$

  - Symmetric: If for all $x, y \in X$, if $(x, y) \in R$, then $(y, x) \in R$

    ex) $X = \{a, b, c, d\}$, $R = \{(a,a), (b,c), (c,b), (d,d)\}$ is symmetric.

  - Antisymmetric: If for all $x, y \in X$, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$

    → so, If there is no symmetric other than $xRx$, it is an antisymmetric.

    ex) $X = \{1, 2, 3\}$, $R = \{(2,1), (2,3), (3,3)\}$ then, it is antisymmetric because there is only $xRx$ of $(3,3)$

  - Transitive: If for all $x, y, z \in X$, if $(x, y)$ and $(y, z) \in R$, then $(x, z) \in R$

    ex) $X = \{1, 2, 3, 4\}$, $(x, y) \in R$ if $x \le y$ is transitive

- Order
  - Partial order: R is reflexive, antisymmetric, transitive

    → R is partial order

  - Comparable: If $x, y \in X$ and either $x \le y$ or $y \le x$, then x and y are comparable

    ↔ Incomparable

  - Total order: If every pair of elements in X is comparable, then R is total order.

    → so, every $(x, y)$ must be included in R, the order should not be reversed.

- other terms
  - Inverse: $R^{-1} = \{(y, x) \mid (x, y) \in R\}$
  - Composition: $R_2 \circ R_1 = \{(x, z) \mid (x, y) \in R_1$ and $(y, z) \in R_2$ for some $y \in Y\}$