

* 2.1 Mathematical Systems, Direct Proofs, and Counterexample

• Mathematical System

- consist of axioms, definitions, undefined terms.

↓
 Assumed to be true. ↳ used to create new concepts. (existing ones) ↳ Implicitly defined by the axioms.

- theorem: proposition that has been proved to be true.

- lemmas: special kinds of theorem ①

→ usually not too interesting in its own right but useful in proving another theorem.

- Corollary: special kinds of theorem ②

→ follows easily from another theorem.

- Proof: An argument that establishes the truth of a theorem.

→ then, Logic is a tool for the analysis of proofs

• Direct Proof

- Def: assumes that assumption is true, and then shows directly that conclusion is true.

- only consider the case hypothesis is true

→ because of vacuously true

- In constructing a proof, we may find that we need some auxiliary results.

→ subproof: proofs of auxiliary results

• Disproving a universally Quantified Statement.

- Counterexample: value for x in the domain of discourse that makes false

• Some Common Errors

1. The same notation for two possibly distinct quantities.

2. Showing that the propositional function is true for specific values in the domain of discourse is not a proof

→ propositional function is true for all value in the domain of discourse.

3. cannot assume what you are supposed to prove.

→ begging the question or circular reasoning