Study Title <2> PROOFS Date

※ 2.1 Mathematical Systems, Direct Proofs, and Counterexample

- Mathemetical system
  - consist of axioms, definitions, undefined terms.
    - axioms: Assumed to be true.
    - definitions: used to create new concepts. (existing ones)
    - undefined terms: Implicitly defined by the axioms.
  - theorem: proposition that has been proved to be true.
    - lemmas: special kinds of theorem ①
      → usually not too interesting in its own right but useful in proving another theorem.
    - Corollary: special kinds of theorem ②
      → follows easily from another theorem.
  - Proof: An argument that establishes the truth of a theorem.
    → then. Logic is a tool for the analysis of proofs

- Direct Proof
  - Def: assumes that assumption is true, and then shows directly that conclusion is true.
  - only consider the case hypothesis is true
    → because of vacuously true
  - In constructing a proof, we may find that we need some auxiliary results.
    → Subproof: proofs of auxiliary results

- Disproving a universally Quantified Statement.
  - Counterexample: value for x in the domain of discourse that makes false

- Some Common Errors
  1. The same notation for two possibly distinct quantities.
  2. Showing that the propositional function is true for specific values in the domain of discourse is not a proof
     → propositional function is true for all value in the domain of discourse.
  3. Cannot assume what you are supposed to prove.
     → begging the question or Circular reasoning