Study Title Date

※ 2.2 More Methods of Proof

- Proof by Contradiction

  - def: establishes $p \to q$ by assumming that p is true, q is false and then, derives contradiction

    ↓

    proposition of the form $r \wedge \neg r$ (r may be any propos)

  - Point 1. this proof assumes that conclusion is negated.

    2. Justified by noticing $p \to q$ and $(p \wedge \neg q) \to (r \wedge \neg r)$ are eqnivalence.

- Proof by Contrapositive

  - def: using that $p \to q$ and $\neg q \to \neg p$ are equivalent

- Proof by Cases (exhaustive proof)

  - def: proof by using when the original hypothesis naturally divides itself into various cases.

    ↓

    to prove $p \to q$ ⇒ $p_1 \vee p_2 \vee \cdots \vee p_n$ is equal to p

  - we prove $(p_1 \to q) \wedge (p_2 \to q) \wedge \cdots \wedge (p_n \to q)$ ⋯ ⓐ

    equals to $(p_1 \vee p_2 \vee \cdots \vee p_n) \to q$

  - Process

    1. Suppose that q is true.

       ▫ If q is true, then all implications in ⓐ are true. regardless of the truth value of the hyphothesis.

    2. Suppose that q is false.

       ▫ If q is false, and all p_i are false, then ⓐ are true.

       ↓

       ▫ 〃 , and for some j, p_j is true, $p_1 \vee p_2 \vee \cdots \vee p_n$ is true. so $(p_1 \vee p_2 \vee \cdots \vee p_n) \to q$ is false.

       Since $p_j \to q$ is false, ⓐ is false.

- Proofs of Equivalence

  - def: prove by using the equivalence

    $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

  - It can use the proof by cases

    ↓

    $(p_1 \to p_2) \wedge (p_2 \to p_3) \wedge \cdots \wedge (p_{n-1} \to p_n) \wedge (p_n \to p_1)$

- Extensive Proof

  - Def: A proof of $\exists x P(x)$