

X: 3.5 Matrices of Relation

• Matrix of Relation

• Def: Set the entry in row x and column y to 1 if xRy and to 0 otherwise.

□ Label the rows with the element of X , the columns with the element of Y (relative to the orderings of X, Y)

• Features

- Always a square matrix
- Convenient way to represent a relation.

• Determine Property

□ Reflexive: If and only if matrix has 1's on the main diagonal

□ Main diagonal consists of the entries on a line from the upper left to lower right

ex)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \text{main diagonal}$$

□ Symmetric: If and only if A is symmetric about the main diagonal

ex)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \text{symmetric}$$

□ Transitive: If and only if whenever entry i, j in A^2 is non-zero, entry i, j in A is also non-zero

ex) $P = \{ (a, a), (c, b), (c, c), (c, d), (c, c), (c, b) \}$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow A^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

non-zero non-zero