Study Title ______________________ Date ____________

※ 2.4 Mathematical Induction

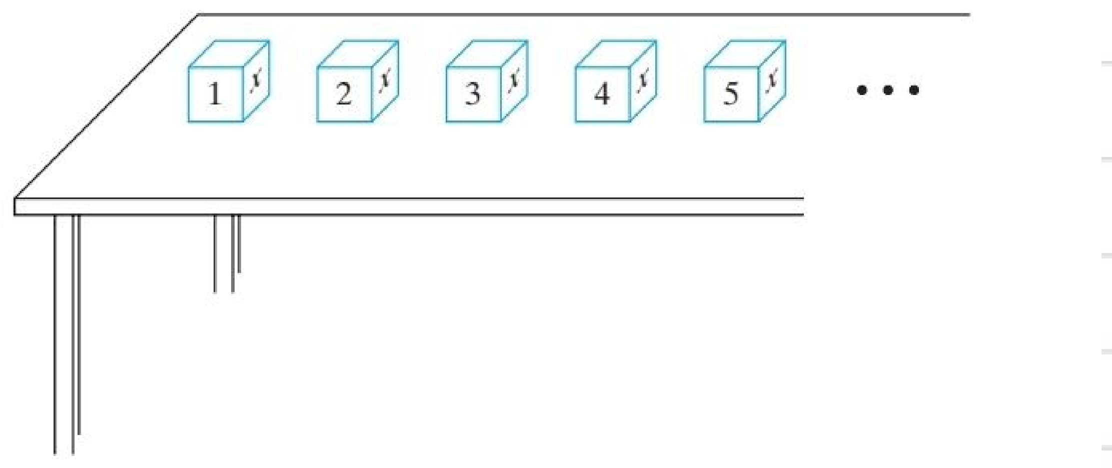

Figure 2.4.1 Numbered blocks on a table.

· Suppose that a sequence of blocks numbered 1, 2, ⋯, sits on an long table and some blocks are marked " X "

① The first block is marked

② For all n, if block n is marked, then block n+1 is also marked.

→ Thus block 2, 3, ⋯ is also marked

· This preceding example illustrate the Principle of Mathematical Induction.

↳ How to prove the proposition that all natural number satisfy the given proposition.

→ using the axioms, $1 \in X$, $n \in X \rightarrow n+1 \in X$

· $S_n = 1+2+\cdots+n = \frac{n(n+1)}{2}$

$S_{n+1} = ?$

$$\rightarrow S_{n+1} = 1+2+\cdots+n+n+1$$
$$= \frac{n(n+1)}{2} + n+1$$
$$= \frac{n^2+n+2n+2}{2} = \frac{(n+1)(n+2)}{2}$$

· Proof using induction consisted of two steps.

① verify that the statement corresponding to $n=1$ is true // Basic Step

② n is true, and then proves that statement n+1 is also true. // Inductive Step

※ 2.5 Strong Form of Induction and the Well-Ordering Property

• Strong Form of Induction

∘ Def : Suppose that we have a propositional function $S(n)$ whose domain of discourse is the set of integer greater than or equal to n_0.

1. $S(n_0)$ is true
2. for all $n > n_0$, if $S(k)$ is true for all k, $(n_0 \le k < n)$, then $S(n)$ is true.

→ $P(1)$ is true, then for all integer k, $P(1), P(2), \dots, P(k)$ is true, then $P(k+1)$ is true.

∘ When ?

▫ Strong form : you have to know the result before the k_{th}

▫ Normal form : when the $k+1$ can be proved only with the k expression.

• Well-Ordering Property

∘ def : every nonempty set of non-negative integer has a least element.

→ It is equivalent to the axioms of induction.

∘ Quotient-Remainder Theorem

▫ If d and n are integer $(d>0)$, there exist integer q and r satisfying $n = dq + r$ (q: quotient, r: remainder)

ex) $n=74$, $d=13$

$$d \cdots 13\overline{)74} \quad (5 \cdots q), \quad 65, \quad 9 \cdots r$$

$n = dq + r$

$74 = 13 \cdot 5 + 9$