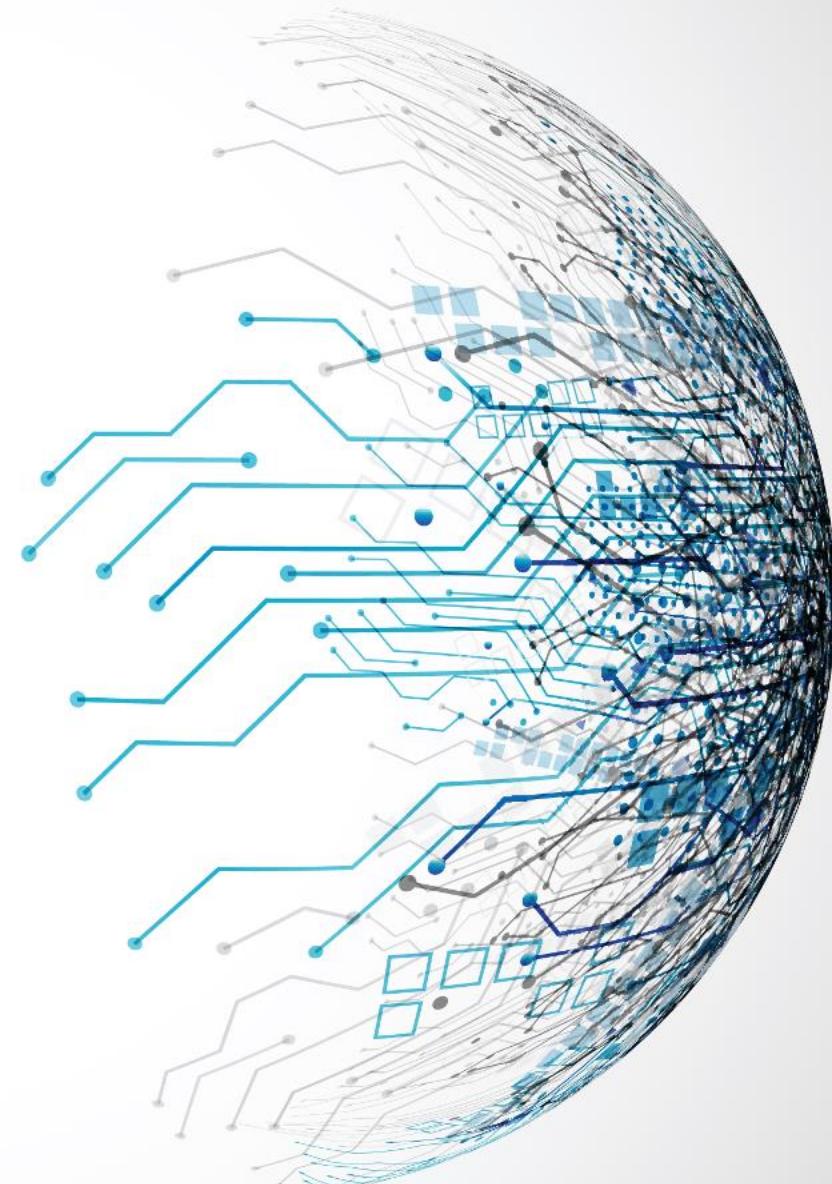


Deep Learning

Backpropagation

Dr. Mohammed Salah Al-Radhi
(slides by: Dr. Bálint Gyires-Tóth)



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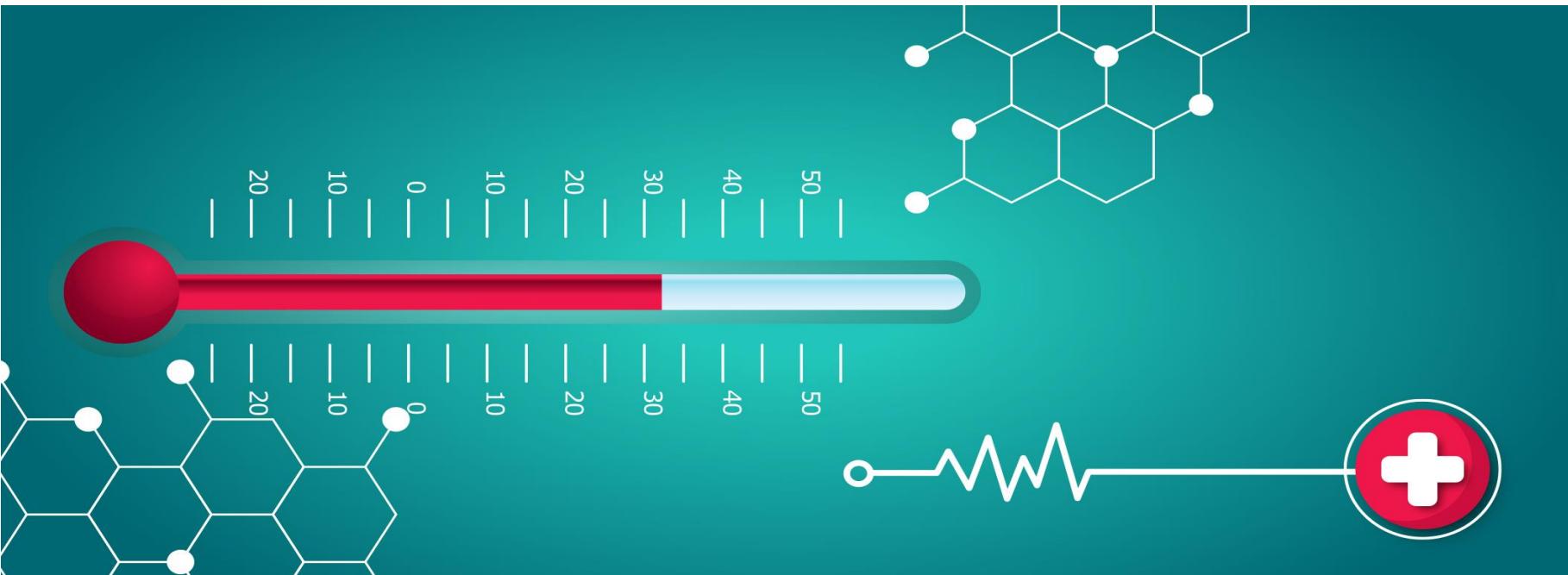
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Task



- Input: [temperature ($^{\circ}\text{C}$), medicine (mg)] $= X$
- Output:
 - Regression: [temperature in 2 hours]
 - Classification: [fever/normal within 2 hours] $= y$

Train and test datasets

$$X = \begin{bmatrix} 38.6 & 25 \\ 37.8 & 25 \\ 37.9 & 50 \\ 38.2 & 50 \end{bmatrix}$$

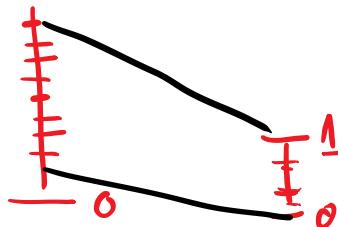
$$y = \begin{bmatrix} 37.2 \\ 37.3 \\ 36.6 \\ 36.9 \end{bmatrix}$$

$$X_{\text{test}} = \begin{bmatrix} 38.3 & 35 \end{bmatrix}$$
$$\hat{y} = ?$$

$$x = (x - \text{mean } X) / \text{var } X$$

↳ variance
↳ mean

$$y = \text{minmax}(y, 0, 1)$$

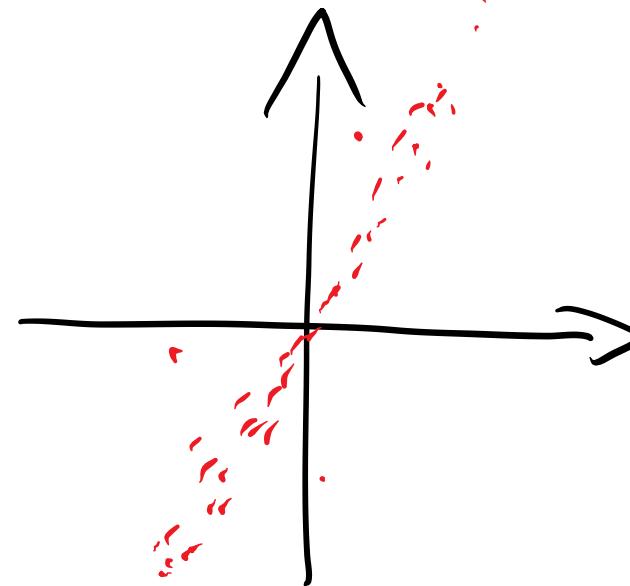
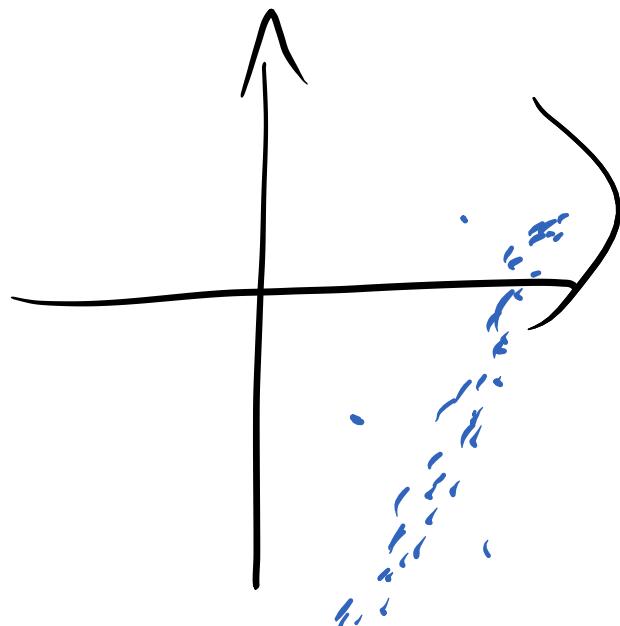


Standardization

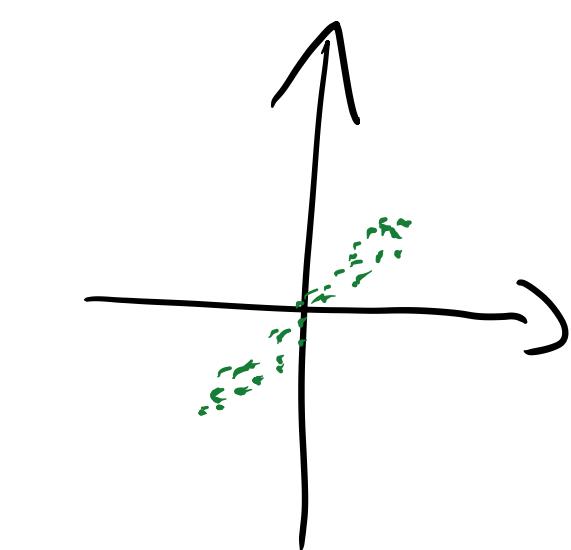
To have 0 mean, 1 variance

In practice: subtracting the mean and dividing by the variance

$$X = (x - \text{mean}(x)) / \text{std}(x)$$



\emptyset mean



\emptyset mean, 1 variance

Standardization

Mean and variance should be calculated on training data only

The distribution of the data should be inspected first

Why we need it:

- To avoid large biases and slow convergence
- To have different features the same scale
- To match data to initial model (see random initialization methods)

Min-max scaler

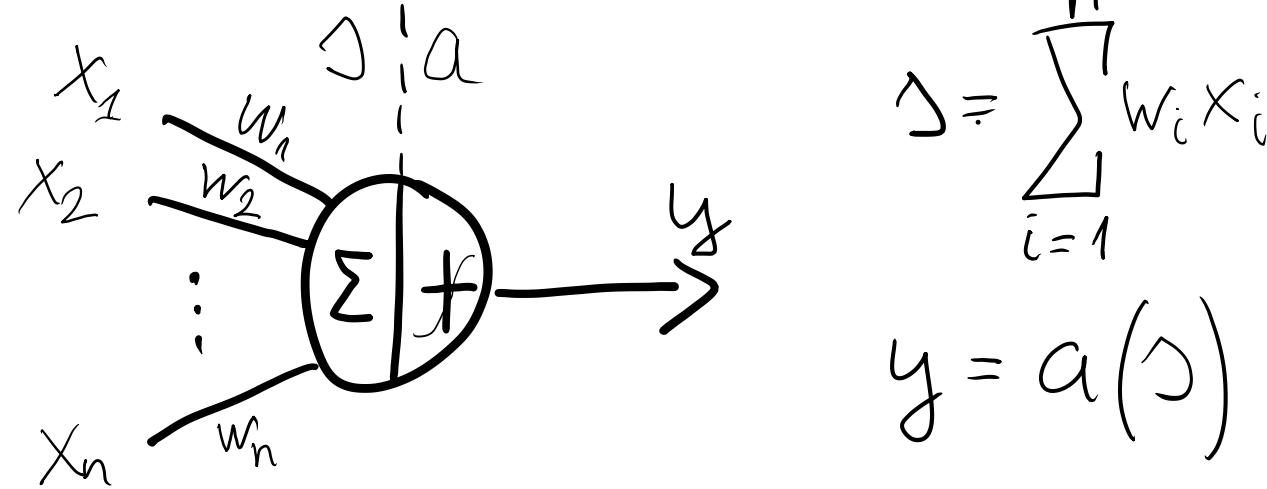
The range is fixed.

Values are not centered around 0.

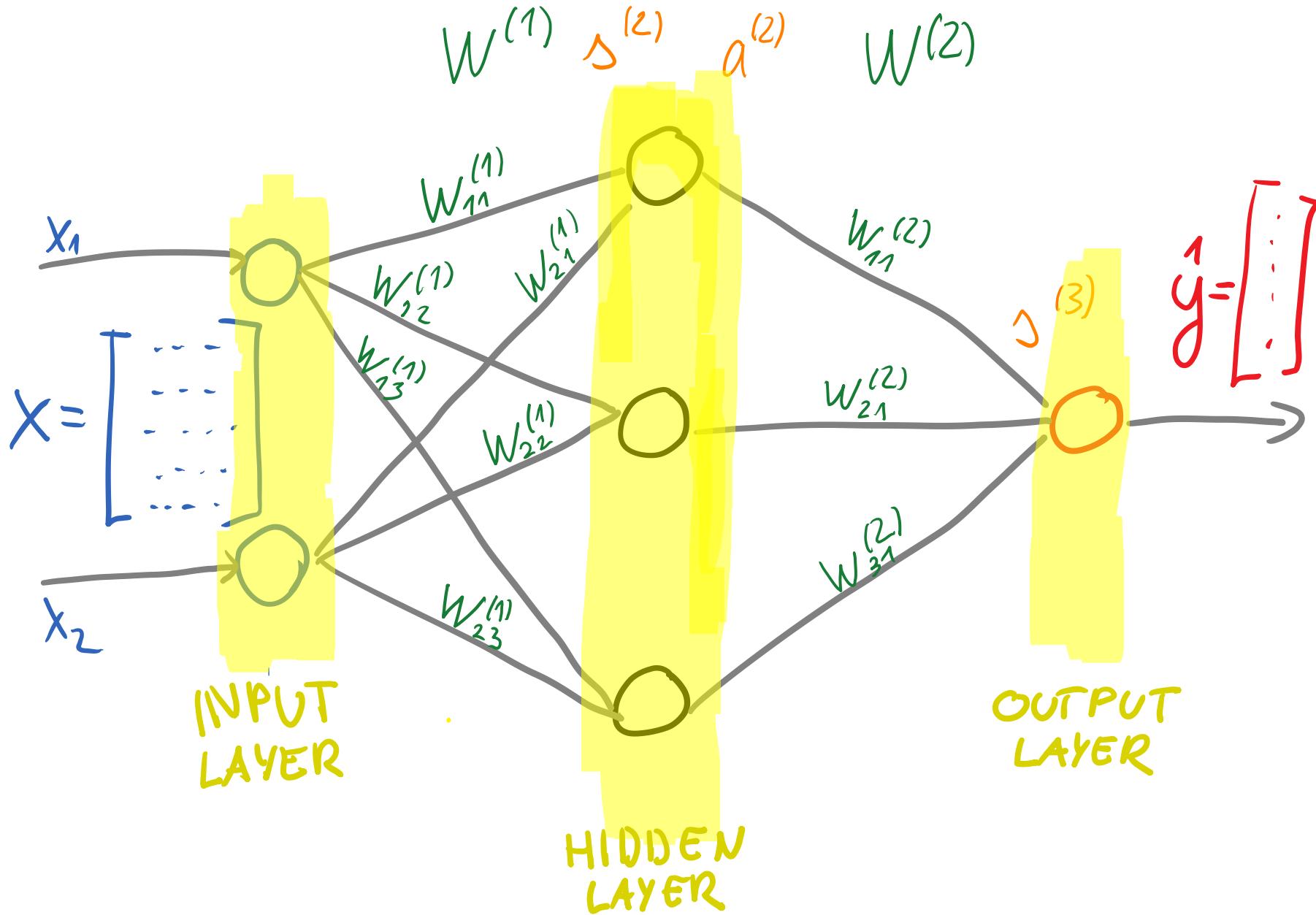
$$y_{\text{-std}} = (y - \min y) / (\max y - \min y)$$
$$y_{\text{-scaled}} = y_{\text{-std}} \cdot (\max - \min) + \min \quad \left| \begin{array}{l} \max = 1 \\ \min = 0 \end{array} \right.$$

Backpropagation algorithm

Single neuron with non-linear activation

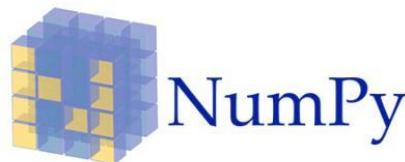


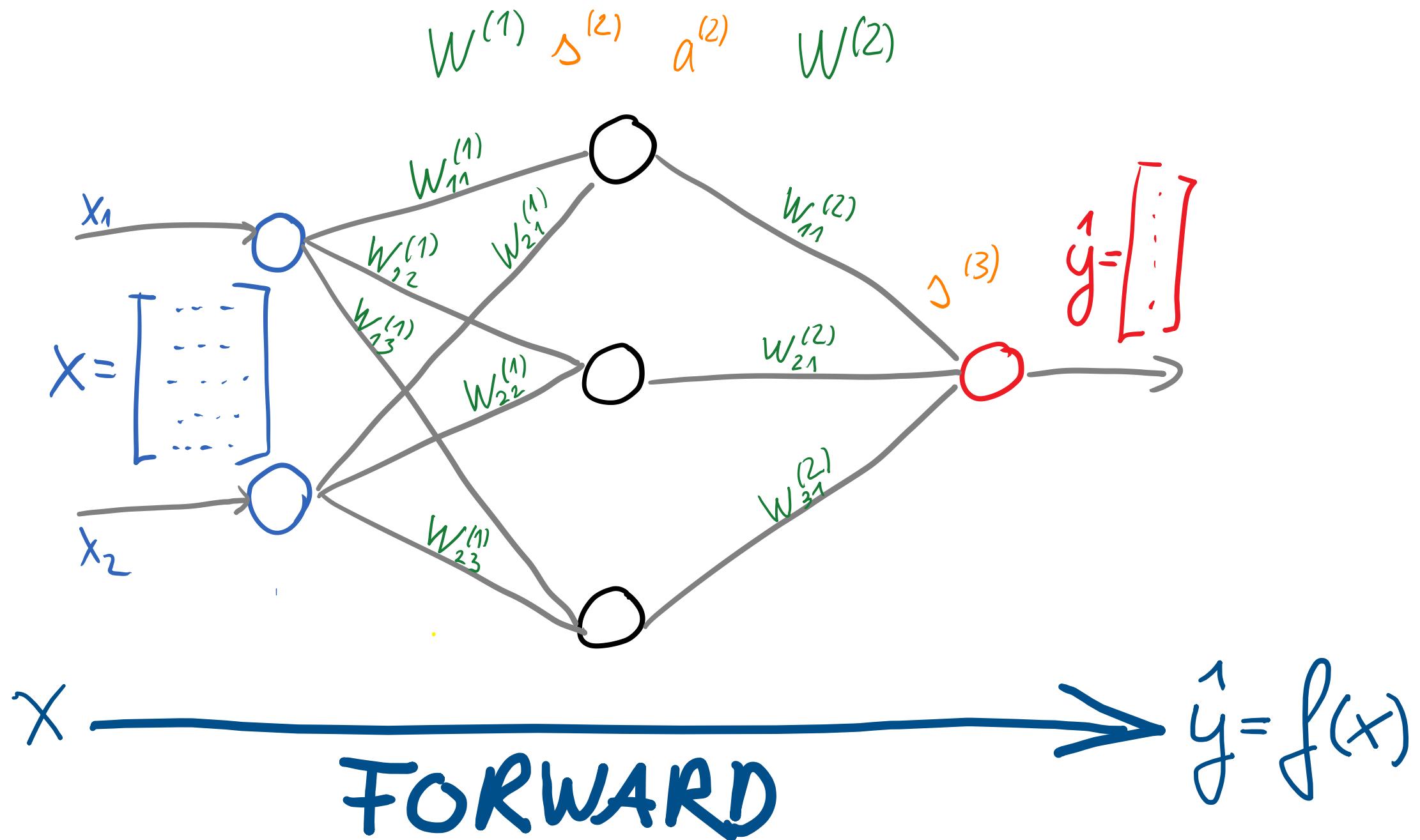
Fully connected feedforward neural network



Matrix algebra

- Matrix multiplication
- Transpose
- Partial derivative





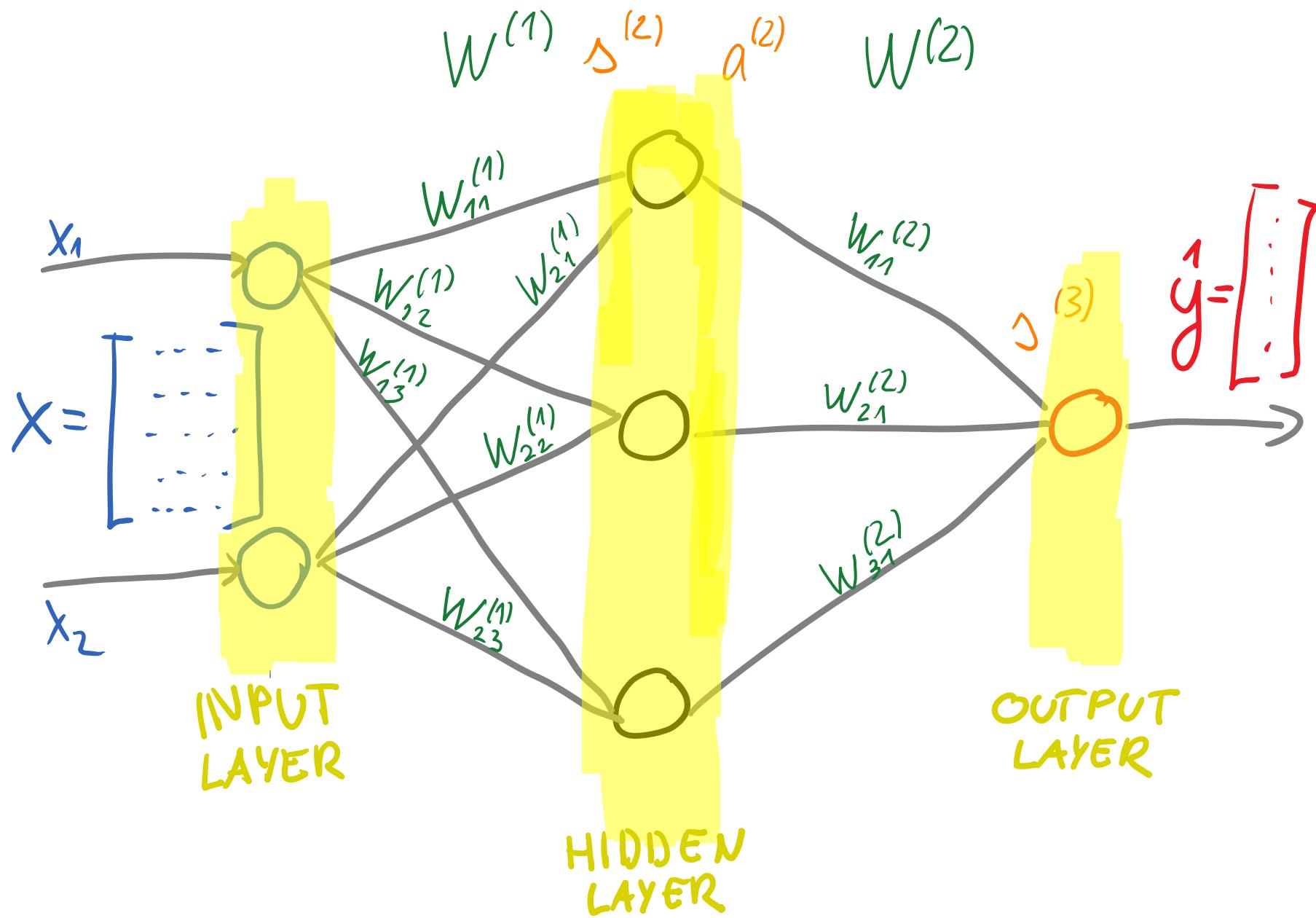
Forward propagation – step 1

$$\textcircled{1} \quad X \overset{(4 \times 2) \quad (2 \times 3)}{W^{(1)}} = \overset{(4 \times 3)}{Z^{(2)}}$$

matrix drawing

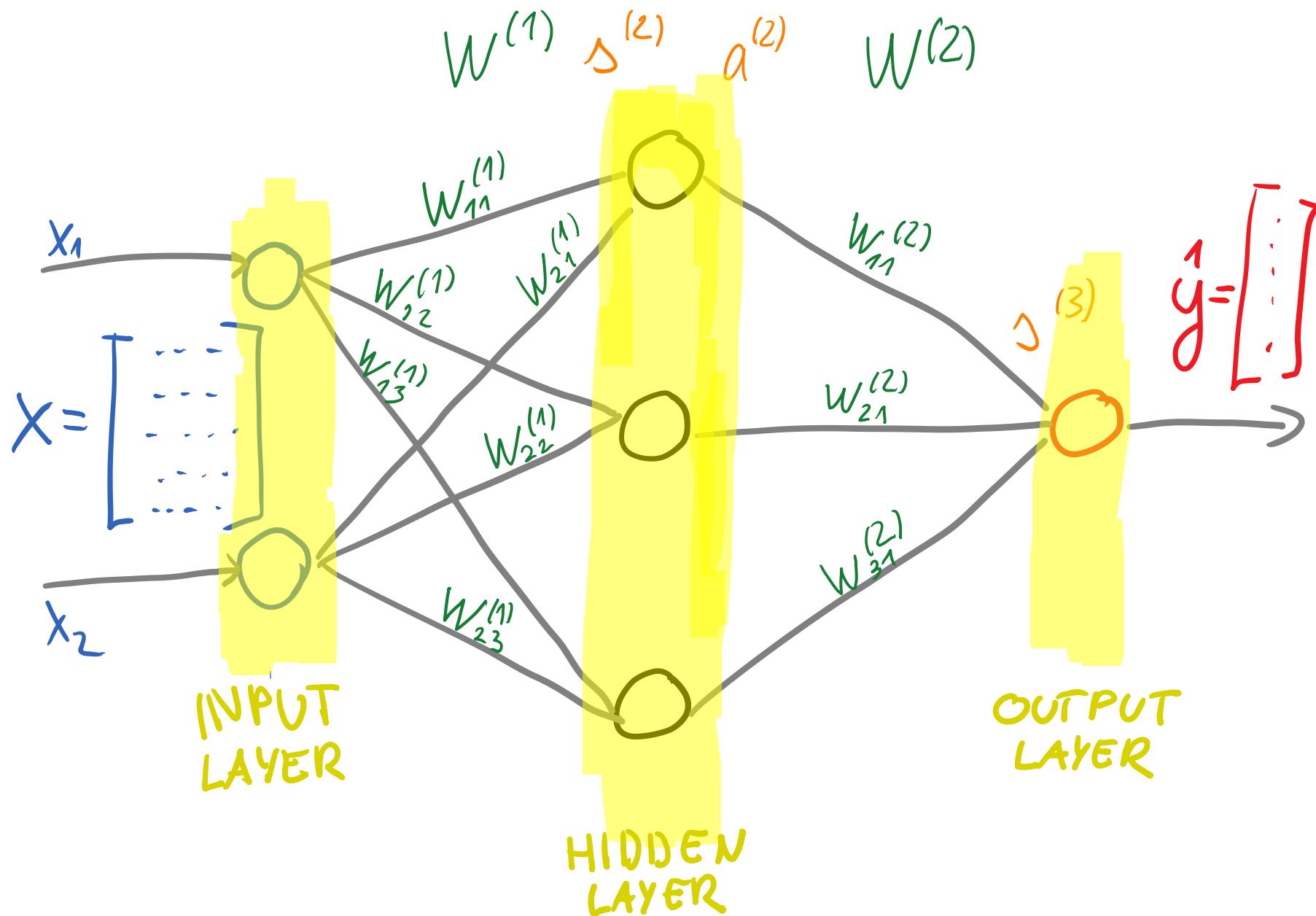
$$X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} \\ x_1^{(2)} & x_2^{(2)} \\ x_1^{(3)} & x_2^{(3)} \\ x_1^{(4)} & x_2^{(4)} \end{bmatrix}$$

tulajdonságok (features)



Forward propagation – step 2

$$\textcircled{2} \quad a^{(2)} = f(\mathcal{Z}^{(2)}) = \text{Sigmoid}\{\mathcal{Z}^{(2)}\} \quad (\tanh / \text{ReLU} / \text{PReLU} \dots)$$



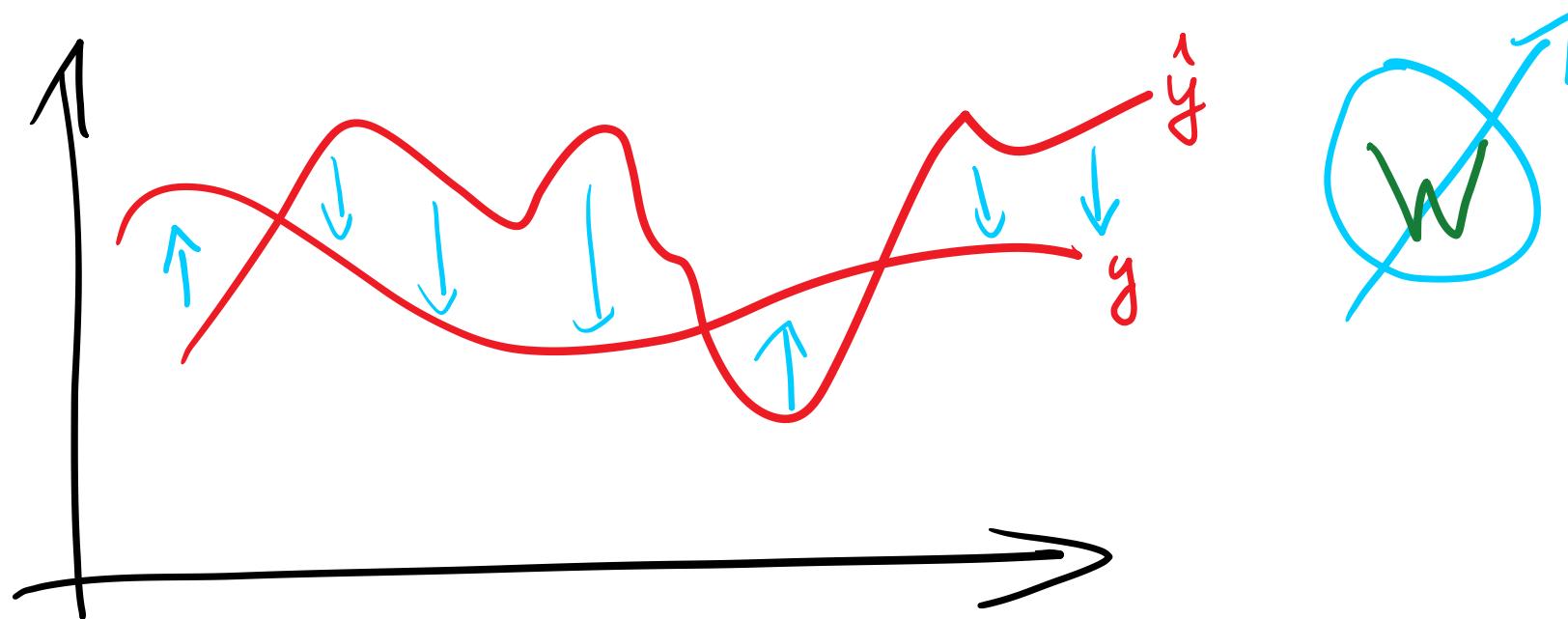
Forward – step 3 Loss/cost/error function

For example:

- Mean squared error
- Cross entropy

⑤

$$C = \sum \frac{1}{2} (y - \hat{y})^2$$



Loss function optimization

- Random search 
- Numerical differentiation

$$g(\theta) \approx \frac{C(\theta + \epsilon) - C(\theta - \epsilon)}{2\epsilon}$$

- Gradient descent

$$-\frac{\partial C}{\partial \theta}$$

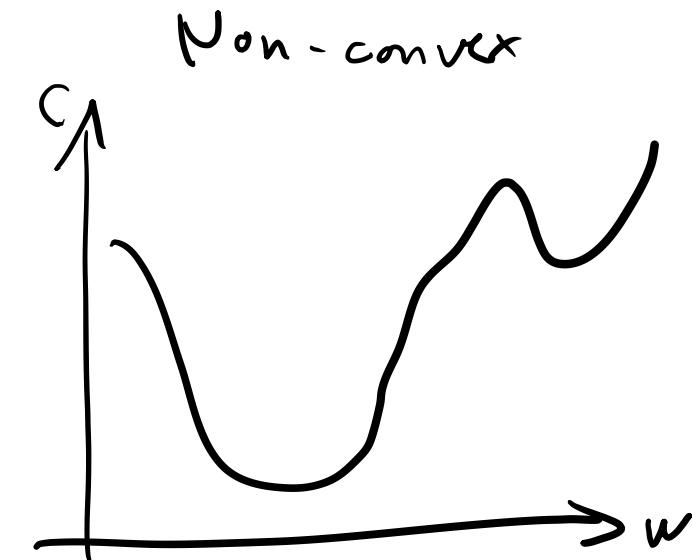
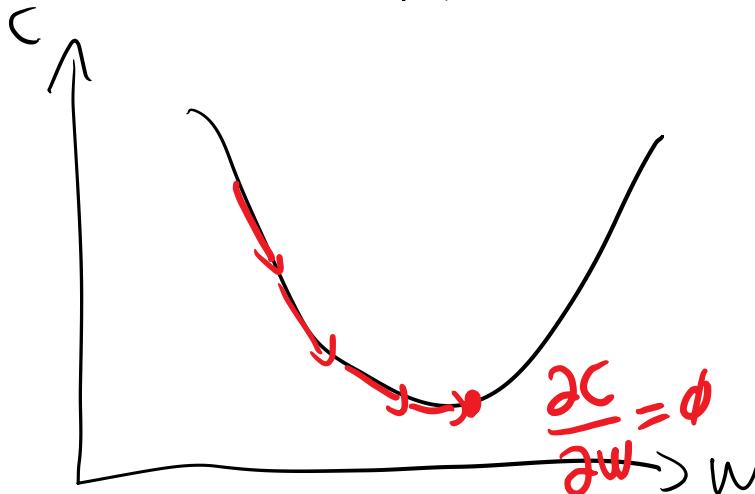


Gradient descent

$$⑥ C = \sum \left\{ \frac{1}{2} (y - f(f(XW^{(1)})W^{(2)}))^2 \right\}$$

$$\frac{\partial C}{\partial w} > 0 \uparrow \quad < 0 \downarrow$$

GOAL: approach 0



Matrix calculus – cheetsheet

y scalar, x vector

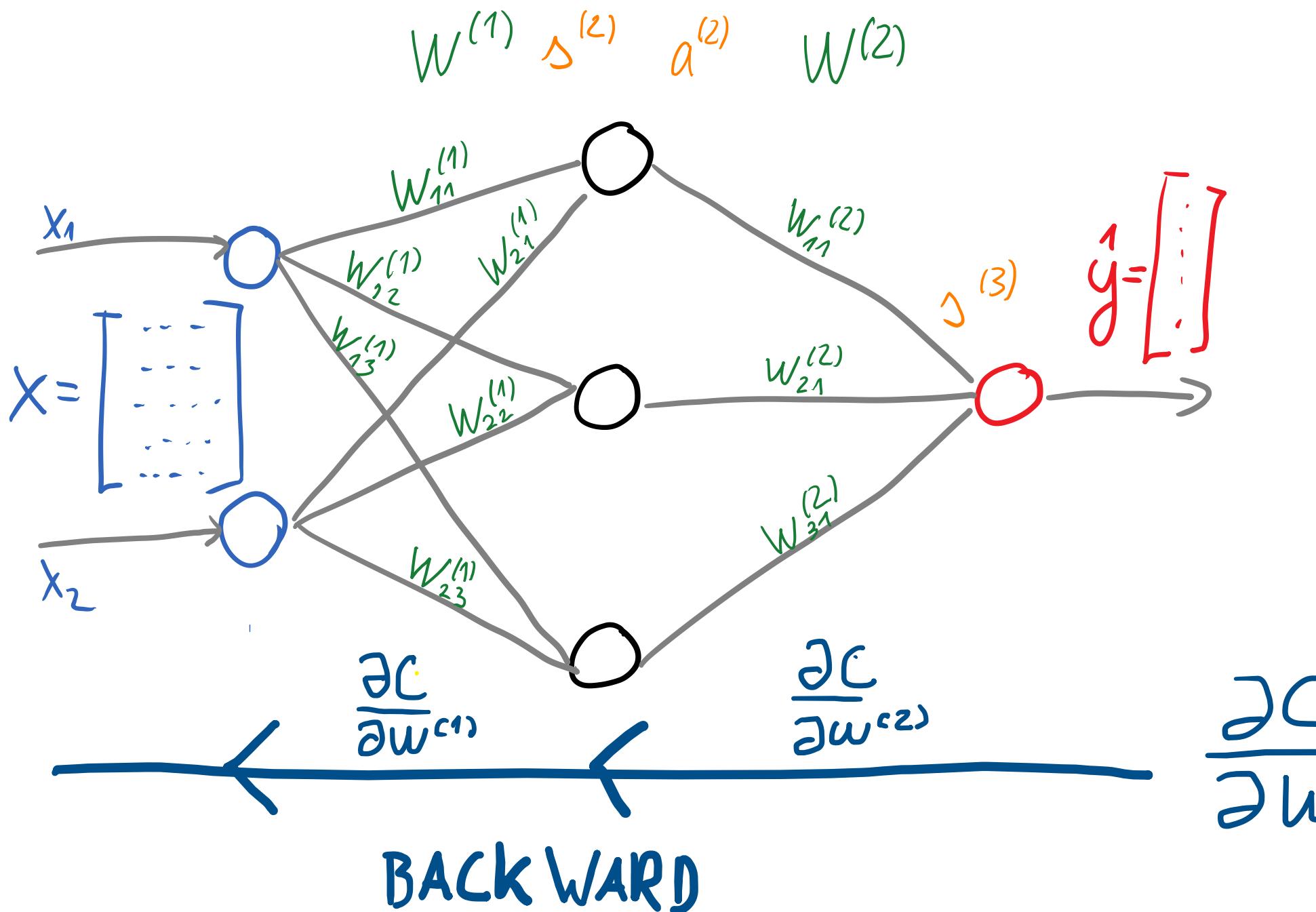
$$\frac{\partial y}{\partial x} = \begin{bmatrix} \frac{\partial y}{\partial x_1} & \frac{\partial y}{\partial x_2} & \dots & \frac{\partial y}{\partial x_n} \end{bmatrix}$$

y vector, x vector

$$\frac{\partial y}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \dots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}$$

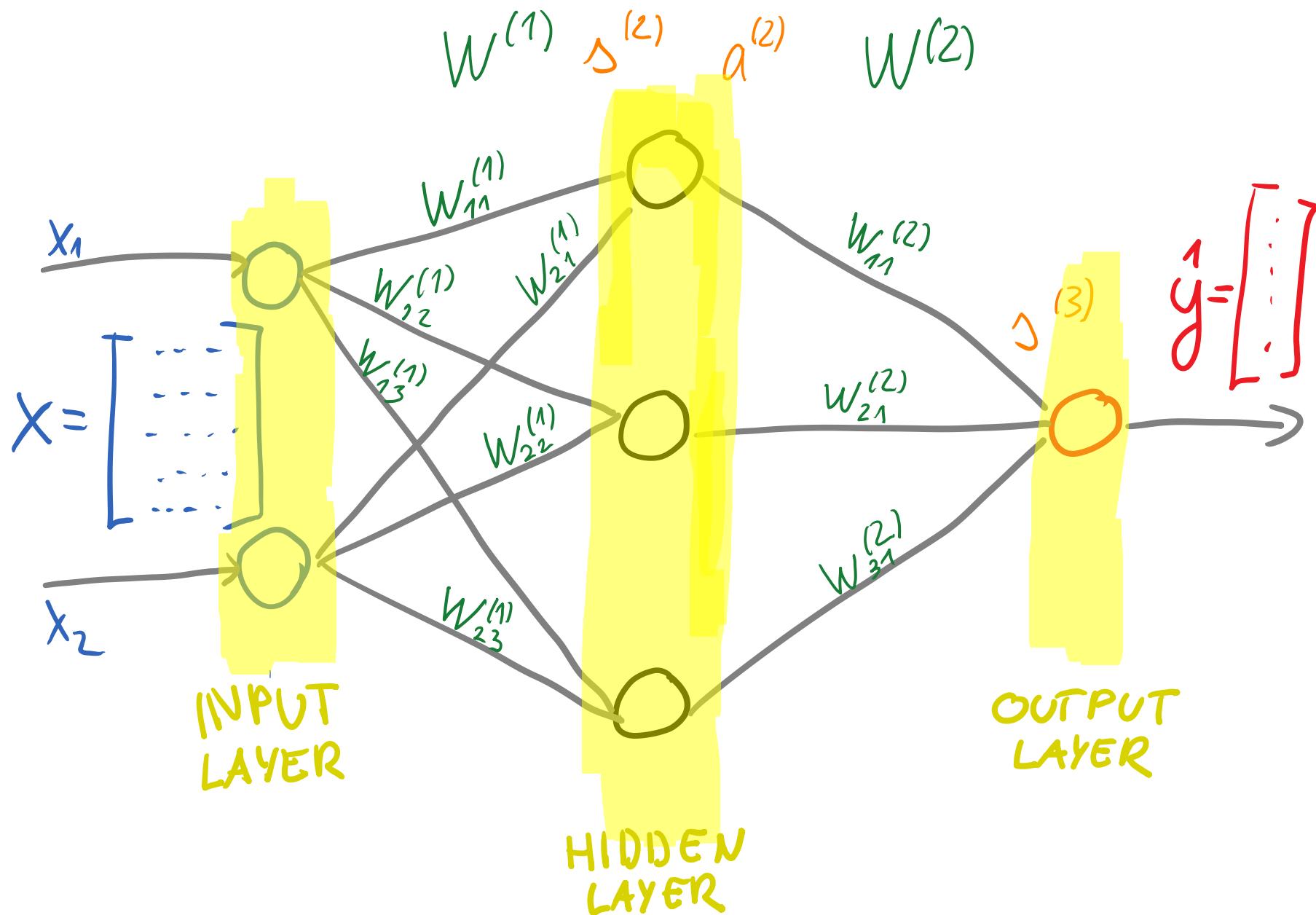
y scalar, W matrix

$$\frac{\partial y}{\partial W} = \begin{bmatrix} \frac{\partial y}{\partial w_{11}} & \dots & \frac{\partial y}{\partial w_{1n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y}{\partial w_{m1}} & \dots & \frac{\partial y}{\partial w_{mn}} \end{bmatrix}$$



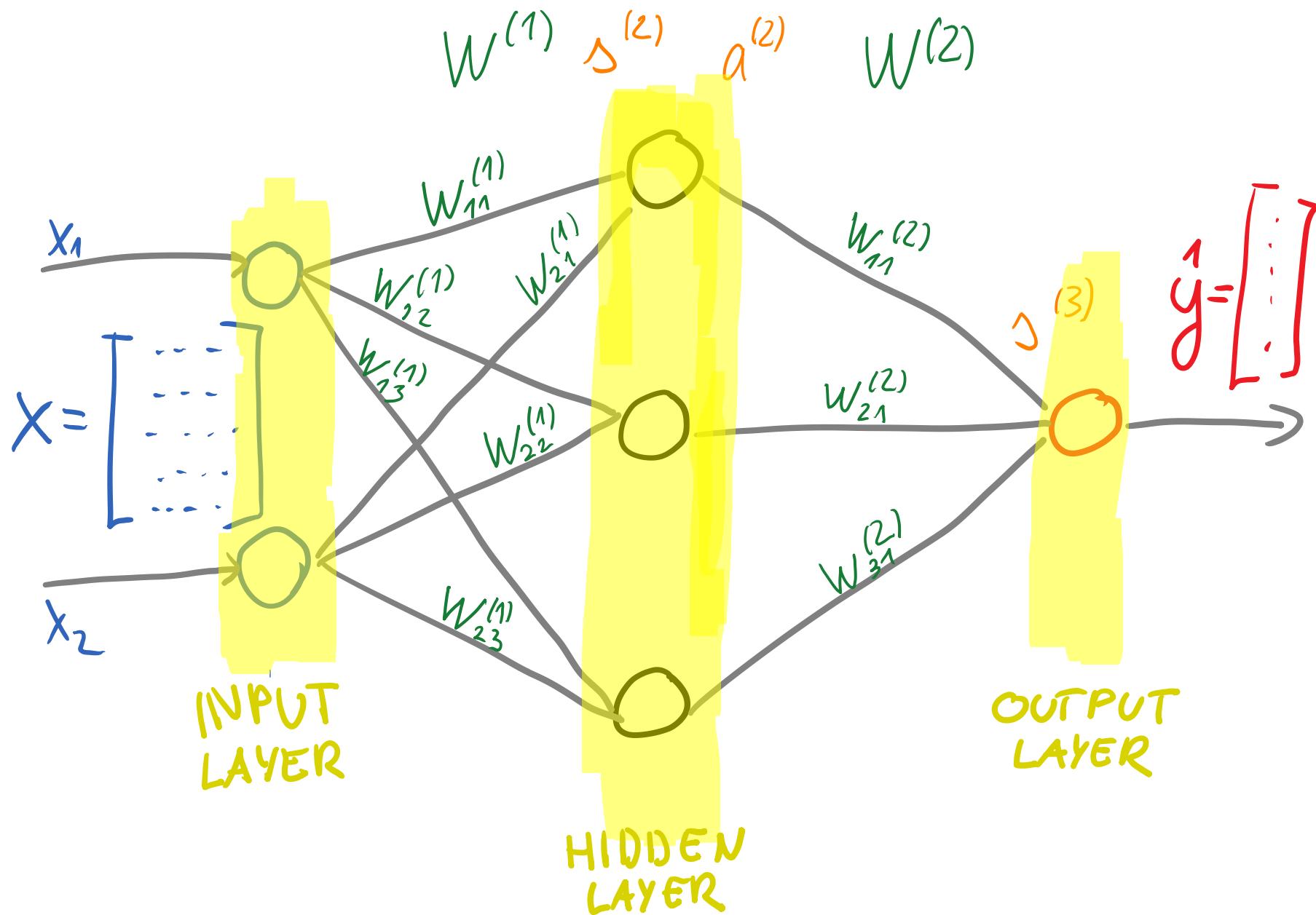
Backward propagation – step 1

$$\frac{\partial C}{\partial w^{(2)}} = \frac{\partial \sum \frac{1}{2} (y - \hat{y})^2}{\partial w^{(2)}}$$



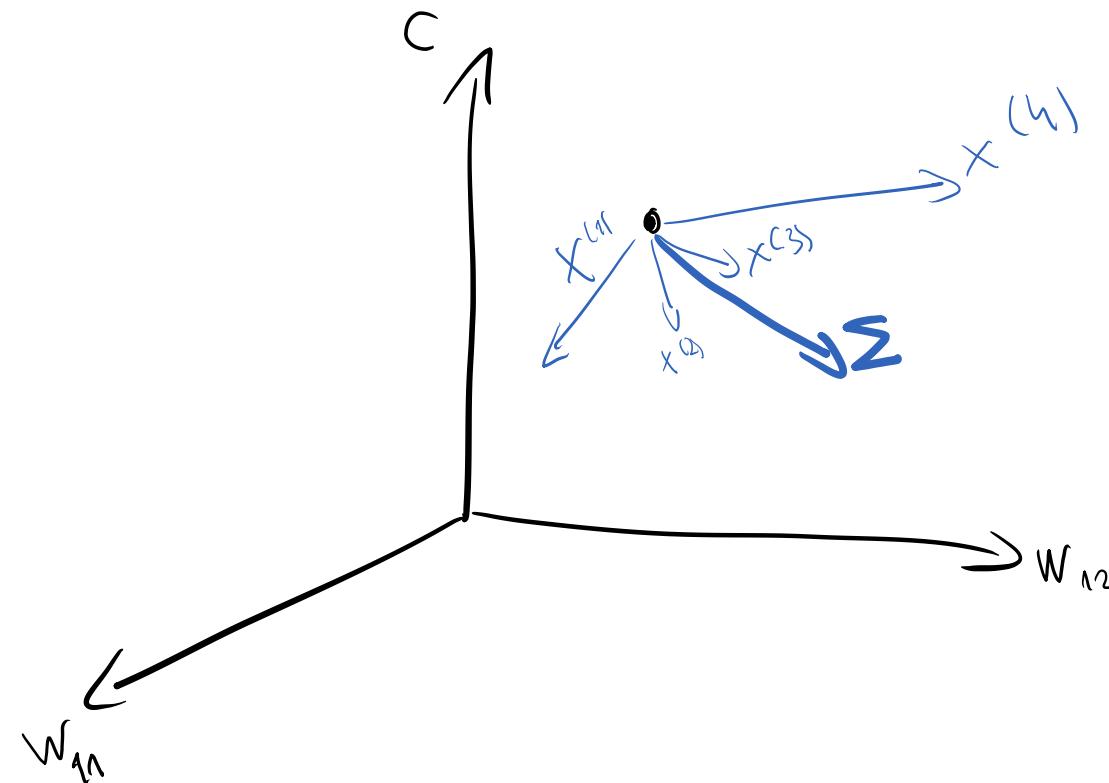
Backward – step 2: batch gradient descent

$$\textcircled{7} \quad \frac{\partial C}{\partial w^{(2)}} = \sum \frac{\partial \frac{1}{2} (y - \hat{y})^2}{\partial w^{(2)}} = (a^{(2)})^T \delta^{(3)} =$$



Backward – step 2: batch gradient descent

Calculate the gradient for all the training samples and calculate the sum or mean of it.



Backward – step 3

$$\frac{\partial C}{\partial w^{(1)}} = \frac{\partial \frac{1}{2}(y - \hat{y})^2}{\partial w^{(1)}}$$

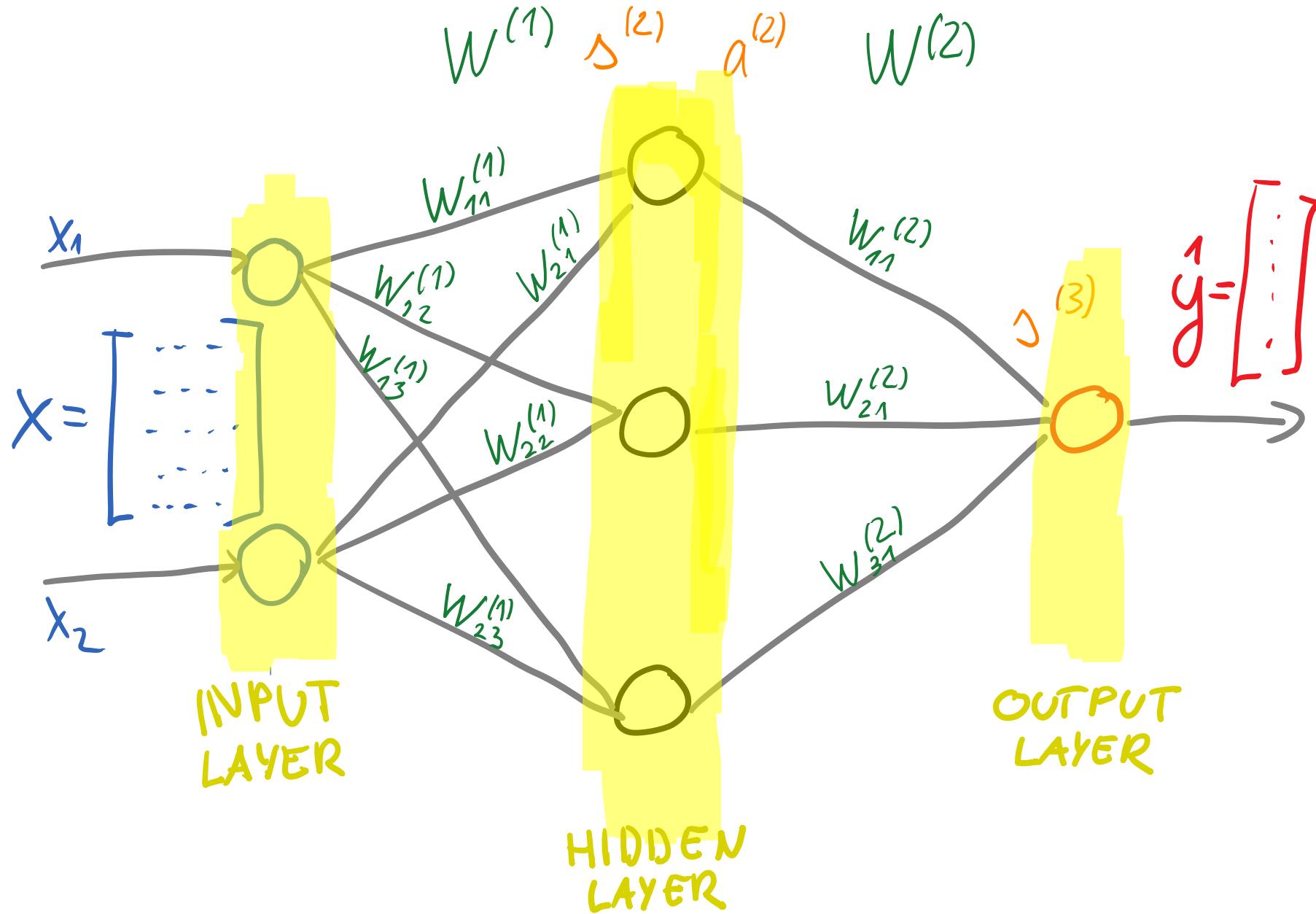
TRAINING

$$W^{(1)} = W^{(1)} - \mu \frac{\partial C}{\partial w^{(1)}}$$

μ : learning rate

$$W^{(2)} = W^{(2)} - \mu \frac{\partial C}{\partial w^{(2)}}$$

Fully connected feedforward neural network

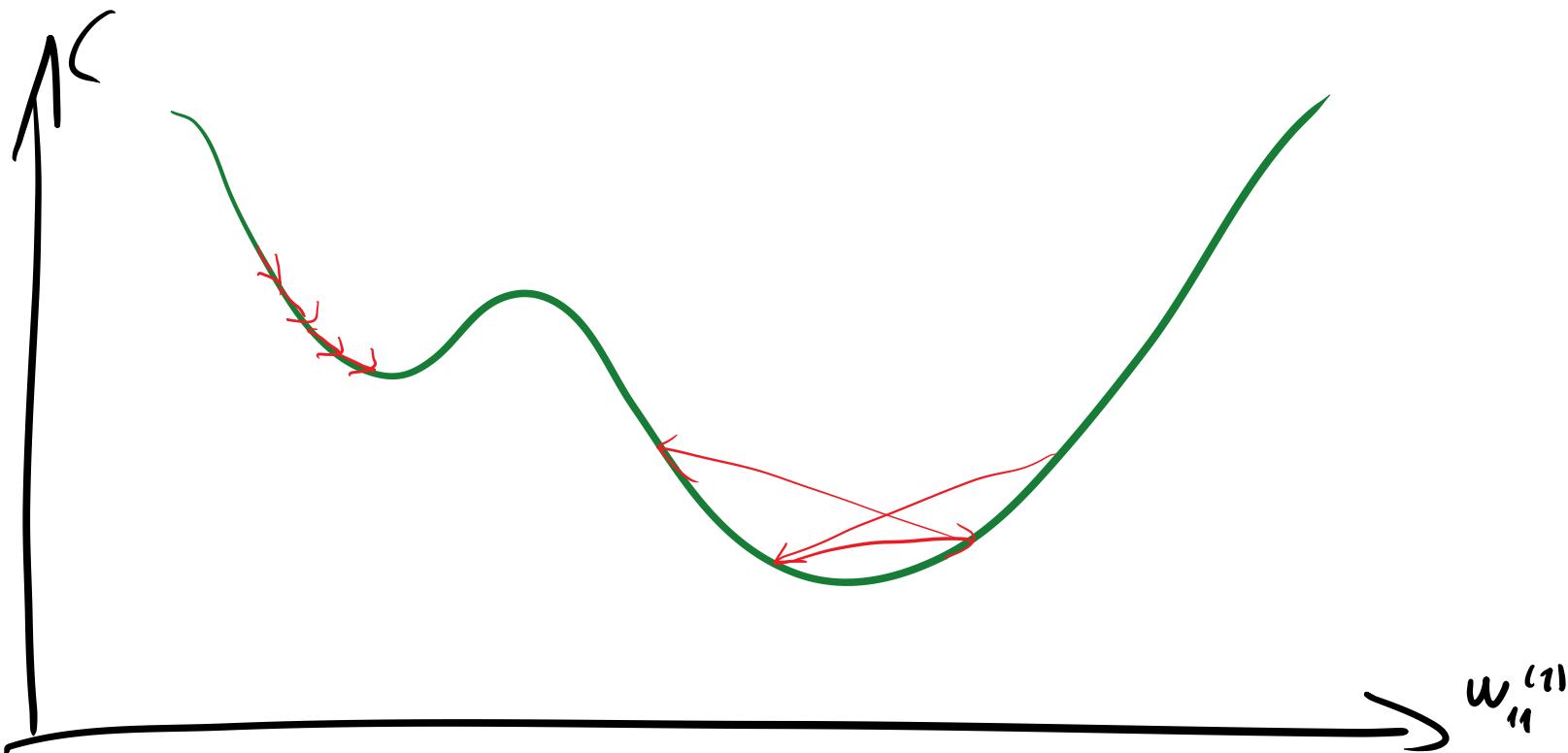


Gradient descent

- Batch gradient descent: using all the training examples
- Stochastic Gradient Descent (SGD): using one sample or a subset (called mini-batch)

Gradient descent

Local vs. global minimum





More information

- Yann LeCunn: Efficient Backprop (1998)
<http://yann.lecun.com/exdb/publis/pdf/lecun-98b.pdf>
- Description and optimization on artificial error surfaces:
<https://www.deeplearning.ai/ai-notes/optimization/>
- Andrej Karpathy: Yes you should understand backprop
<https://medium.com/@karpathy/yes-you-should-understand-backprop-e2f06eab496b>

Please, don't forget
to send feedback:

<https://bit.ly/bme-dl>



Thank you for your attention

Dr. Mohammed Salah Al-Radhi

malradhi@tmit.bme.hu

(slides by: Dr. Bálint Gyires-Tóth)

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