

```
restart  
with(LinearAlgebra) :  
interface(rtablesize = 20) :  
with(plots) :
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1 bezier := proc(P::list(Vector)) :: algebraic;
2   local n;
3   n := numelems(P) - 1;
4   t -> sum(binomial(n, i) * (1-t)^(n-i) * t^i * P[i+1], i = 0..n);
5 end proc;
6
7 bezierfitting := proc(B::procedure) local t, n, i, s, p, poly, rows, M, S;
8   n := nops(B(t)[1]) - 1;
9   s := sum(i, i = 1..(n+1));
10  poly := [seq(seq(x^(i-j) * y^j, j = 0..i), i = 0..n)];
11
12  rows := [];
13  for i from 1 to s do;
14    p := eval(B(t), t = i / s);
15    rows := [op(rows), eval(poly, {x = p[1], y = p[2]})];
16    rows[i] := [op(rows[i]), 0];
17  end do;
18
19  M := Matrix(s, s + 1, rows);
20  print(M);
21
22  S := eval(LinearSolve(M, free = 'k'), k = Vector(s, 1));
23  add(S *~ poly) = 0;
24 end proc;
25
26 parametric2cartesian := proc(E::algebraic);
27   collect(expand(eliminate(convert(<x, y> = E(t), setofequations), t)), {x, y}, distributed)[2][1] = 0;
28 end proc;
29
30 ## Plots
31 labeledpointplot := proc(P::list(Vector)) local points, labels;
32   points := pointplot(P, _rest);
33   labels := textplot([seq([P[i][1], P[i][2], cat("P", i-1)], i = 1 .. numelems(P))], align = RIGHT);
34   display({points, labels});
35 end proc;
36
37 connectedlinesplot := proc(P::list(Vector));
38   display({seq(plot([op(convert((1-t)*P[i] + t*P[i + 1], list)), t = 0..1], _rest), i = 1..(numelems(P)-1))});
39 end proc;
40
41 bezierplot := proc(P::list(Vector), r::range)
42   local pointsPlot, linesPlot, bezierPlot;
43   pointsPlot := labeledpointplot(P, symbol = circle, symbolsize = 10);
44   linesPlot := connectedlinesplot(P, color=blue, transparency = 0.7);
45   bezierPlot := plot([op(convert(bezier(P)(t), list)), t = 0 .. 1], x = r, y = r, numpoints = 10000, _rest);
46   display({bezierPlot, linesPlot, pointsPlot});
47 end proc;

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bezier := proc(P::(list(Vector)))::algebraic;
  local n;
  n := numelems(P) - 1; t→sum(binomial(n, i) * (1 - t)^(n - i) * t^i * P[i + 1], i=0..n)
end proc
bezierfitting := proc(B::procedure)
  local t, n, i, s, p, poly, rows, M, S;
  n := nops(B(t)[1]) - 1;
  s := sum(i, i=1..n + 1);
  poly := [seq(seq(x^(i - j) * y^j, j=0..i), i=0..n)];
  rows := [ ];
  for i to s do
    p := eval(B(t), t=i/s);
    rows := [op(rows), eval(poly, {x=p[1], y=p[2]})];
    rows[i] := [op(rows[i]), 0]
  end do;
  M := Matrix(s, s + 1, rows);
  print(M);
  S := eval(LinearAlgebra:-LinearSolve(M, free = 'k'), k = Vector(s, 1));
  add( ~`[:-`*`](S, $, poly) ) = 0
end proc
parametric2cartesian := proc(E::algebraic)
  collect(expand(eliminate(convert( < x, y > = E(t), setofequations), t)), {x, y}, distributed)[2][1]
  ] = 0
end proc
labeledpointplot := proc(P::(list(Vector)))
  local points, labels;
  points := plots:-pointplot(P, _rest);
  labels := plots:-textplot([seq([P[i][1], P[i][2], cat("P", i - 1)], i=1..numelems(P)]), align
  = RIGHT);
  plots:-display({points, labels})
end proc
connectedlinesplot := proc(P::(list(Vector)))
  plots:-display({seq(plot([op(convert((1 - t) * P[i] + t * P[i + 1], list)), t=0..1], _rest), i=1
  ..numelems(P) - 1)})
end proc
bezierplot := proc(P::(list(Vector)), r::range)
  local pointsPlot, linesPlot, bezierPlot;
  pointsPlot := labeledpointplot(P, symbol = circle, symbolsize = 10);
  linesPlot := connectedlinesplot(P, color = blue, transparency = 0.7);
  bezierPlot := plot([op(convert(bezier(P)(t), list)), t=0..1], x=r, y=r, numpoints = 10000,
  _rest);
  plots:-display({pointsPlot, linesPlot, bezierPlot})

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end proc

$P := [\langle 90, 110 \rangle, \langle 25, 40 \rangle, \langle 230, 40 \rangle, \langle 150, 240 \rangle]$

$$P := \left[\left[\begin{array}{c} 90 \\ 110 \end{array} \right], \left[\begin{array}{c} 25 \\ 40 \end{array} \right], \left[\begin{array}{c} 230 \\ 40 \end{array} \right], \left[\begin{array}{c} 150 \\ 240 \end{array} \right] \right] \quad (1)$$

$B := \text{bezier}(P)$

$$B := t \mapsto \sum_{i=0}^n \binom{n}{i} (1-t)^{n-i} t^i \left[\left[\begin{array}{c} 90 \\ 110 \end{array} \right], \left[\begin{array}{c} 25 \\ 40 \end{array} \right], \left[\begin{array}{c} 230 \\ 40 \end{array} \right], \left[\begin{array}{c} 150 \\ 240 \end{array} \right] \right]_{i+1} \quad (2)$$

$\text{CartesianFitting} := \text{bezierfitting}(B)$

$$\left[\left[1, \frac{15609}{200}, \frac{9123}{100}, \frac{243640881}{40000}, \frac{142400907}{20000}, \frac{83229129}{10000}, \frac{3802990511529}{8000000}, \frac{2222735757363}{4000000}, \frac{1299123474561}{2000000}, \frac{759299343867}{1000000}, 0 \right], \right. \\ \left. \left[1, \frac{1974}{25}, \frac{1936}{25}, \frac{3896676}{625}, \frac{3821664}{625}, \frac{3748096}{625}, \frac{7692038424}{15625}, \frac{7543964736}{15625}, \frac{7398741504}{15625}, \frac{7256313856}{15625}, 0 \right], \right. \\ \left[1, \frac{17883}{200}, \frac{6941}{100}, \frac{319801689}{40000}, \frac{124125903}{20000}, \frac{48177481}{10000}, \frac{5719013604387}{8000000}, \frac{2219743523349}{4000000}, \frac{861557892723}{2000000}, \right. \\ \left. \frac{334399895621}{1000000}, 0 \right], \left. \left[1, \frac{2652}{25}, \frac{1698}{25}, \frac{7033104}{625}, \frac{4503096}{625}, \frac{2883204}{625}, \frac{18651791808}{15625}, \frac{11942210592}{15625}, \frac{7646257008}{15625}, \frac{4895680392}{15625}, 0 \right], \right. \\ \left[1, \frac{1005}{8}, \frac{295}{4}, \frac{1010025}{64}, \frac{296475}{32}, \frac{87025}{16}, \frac{1015075125}{512}, \frac{297957375}{256}, \frac{87460125}{128}, \frac{25672375}{64}, 0 \right], \left. \left[1, \frac{3618}{25}, \frac{2192}{25}, \frac{13089924}{625}, \frac{7930656}{625}, \frac{4804864}{625}, \frac{47359345032}{15625}, \frac{28693113408}{15625}, \frac{17383997952}{15625}, \frac{10532261888}{15625}, 0 \right], \right. \\ \left[1, \frac{32007}{200}, \frac{11049}{100}, \frac{1024448049}{40000}, \frac{353645343}{20000}, \frac{122080401}{10000}, \frac{32789508704343}{8000000}, \frac{11319126493401}{4000000}, \frac{3907427394807}{2000000}, \right. \\ \left. \frac{1348866350649}{1000000}, 0 \right], \left. \left[1, \frac{4206}{25}, \frac{3574}{25}, \frac{17690436}{625}, \frac{15032244}{625}, \frac{12773476}{625}, \frac{74405973816}{15625}, \frac{63225618264}{15625}, \frac{53725240056}{15625}, \frac{45652403224}{15625}, 0 \right], \right. \\ \left[1, \frac{33201}{200}, \frac{18587}{100}, \frac{1102306401}{40000}, \frac{617106987}{20000}, \frac{345476569}{10000}, \frac{36597674819601}{8000000}, \frac{20488569075387}{4000000}, \frac{11470167567369}{2000000}, \right. \\ \left. \frac{6421372988003}{1000000}, 0 \right], \left. \left[1, 150, 240, 22500, 36000, 57600, 3375000, 5400000, 8640000, 13824000, 0 \right] \right.$$

$$\text{CartesianFitting} := -\frac{649600695000}{2197} + \frac{4815938700}{2197} x + \frac{11568340350}{2197} y - \frac{29458890}{2197} x^2 \quad (3) \\ + \frac{1918080}{2197} xy - \frac{403886655}{8788} y^2 + x^3 + \frac{333}{26} x^2 y + \frac{36963}{676} xy^2 + \frac{1367631}{17576} y^3 = 0$$

$\text{CartesianEliminate} := \text{parametric2cartesian}(B)$

$$\text{CartesianEliminate} := 17576 x^3 + 225108 x^2 y + 961038 x y^2 + 1367631 y^3 - 235671120 x^2 \quad (4) \\ + 15344640 x y - 807773310 y^2 + 38527509600 x + 92546722800 y - 5196805560000 = 0$$

$\text{plotRange} := (\text{min}(P) - 1) .. (\text{max}(P) + 1)$

$$\text{plotRange} := 24..241 \quad (5)$$

$\text{bezierPlot} := \text{bezierplot}(P, \text{plotRange}, \text{color} = \text{red}) :$

$\text{cartesianFittingPlot} := \text{implicitplot}(\text{CartesianFitting}, x = \text{plotRange}, y = \text{plotRange}, \text{numpoints} \\ = 10000, \text{color} = \text{green}) :$

$\text{display}(\{\text{bezierPlot}, \text{cartesianFittingPlot}\})$

