

# ALGEBRAIC DATA TYPES (ADT)

Malte Neuss

# ALGEBRA

Numbers:

$$2 + 2 + 2 = 3 * 2$$

# ALGEBRA

Numbers:

```
2 + 2 + 2 = 3 * 2
```

Types:

```
class User
  verified: Bool
  email:    String
```

# CONTENT

# CONTENT

- Product Type

# CONTENT

- Product Type
- Sum Type

# CONTENT

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- Sum Type
- Examples

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- Product Type
- Sum Type
- Examples

*Make illegal state unrepresentable*

# BASIC TYPES

```
type Void:                      // 0
type Unit: unit                  // 1
type Bool: true, false           // 2
...
type String: "", "a", "b" ...    // infy
```

# PRODUCT TYPE

```
type ProductType = Type × Type
```

# PRODUCT TYPE

```
type ProductType = Type x Type
```

```
type User = Bool x String
```

# PRODUCT TYPE

```
type ProductType = Type x Type
```

```
type User = Bool x String
```

```
class User
  verified: Bool
  email:    String
```

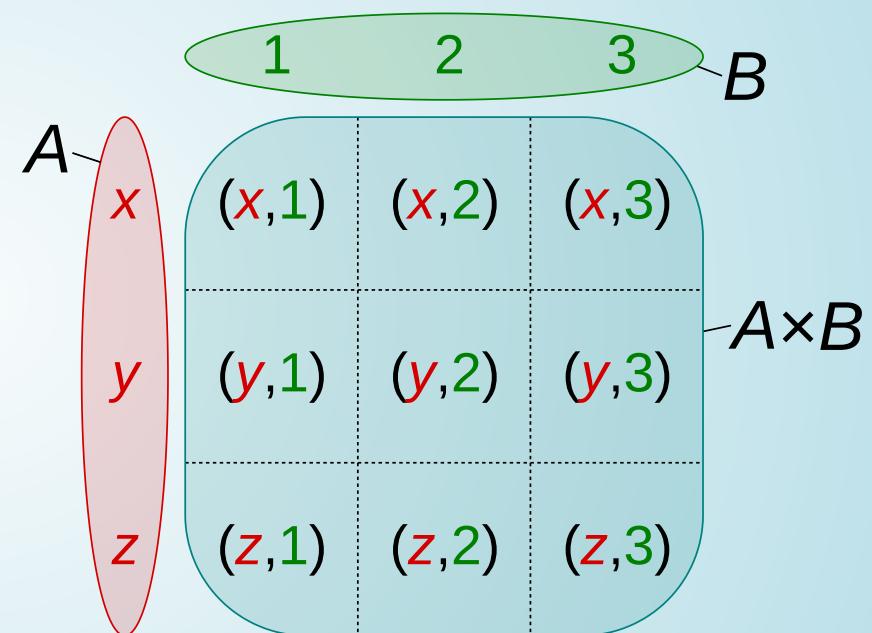
# PRODUCT TYPE

Type:

```
type User = Bool × String
```

Values:

```
User(true, "no@reply.com")
User(false, "no@reply.com")
User(true, "ok@reply.com")
User(false, "ok@reply.com")
...
```



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# ALGEBRA

Bool x Bool: (true,true) (false,false) (true,false) (false,true)  
2 x 2 = 4

# ALGEBRA

Bool x Bool: (true,true) (false,false) (true,false) (false,true)  
 $2 \times 2 = 4$

Bool x Unit: (true,unit) (false,unit) ~ Boolean  
 $2 \times 1 = 2$

# ALGEBRA

Bool x Bool: (true,true) (false,false) (true,false) (false,true)  
 $2 \times 2 = 4$

Bool x Unit: (true,unit) (false,unit) ~ Boolean  
 $2 \times 1 = 2$

Bool x Void: (true,???) ~ Void  
 $2 \times 0 = 0$

# SUM TYPE

```
type SumType = Type + Type
```

# SUM TYPE

```
type SumType = Type + Type
```

```
type ID = Int + String
```

# SUM TYPE

```
type SumType = Type + Type
```

```
type ID = Int + String
```

```
type ID = Int | String
```

TypeScript

# SUM TYPE

```
type SumType = Type + Type
```

```
type ID = Int + String
```

```
type ID = Int | String
```

TypeScript

```
ID = Union[int, str]
```

Python

# SUM TYPE

```
type SumType = Type + Type
```

```
type ID = Int + String
```

```
type ID = Int | String
```

TypeScript

```
ID = Union[int, str]
```

Python

```
sealed trait ID
```

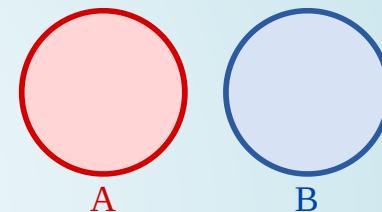
Scala

```
case class IntID(Int)      extends ID
case class StringID(String) extends ID
```

# SUM TYPE

Type:

```
type ID = Int | String
```



Values:

```
myID: ID = 1
      ✓
myID: ID = "123e4567-e89b..."  
      ✓
...
```

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<https://commons.wikimedia.org/w/index.php?curid=14978640>

# ALGEBRA

```
Bool | Unit: true, false, unit  
2 + 1 = 3
```

# ALGEBRA

Bool | Unit: true, false, unit  
2 + 1 = 3

Bool | Void: true, false ~ Bool  
2 + 0 = 2

# ALGEBRA

```
Bool | Unit: true, false, unit  
2 + 1 = 3
```

```
Bool | Void: true, false           ~ Bool  
2 + 0 = 2
```

```
Bool | Int: true, false, 1, 2, 3, ...  
2 + 2^64
```

# UNIT TYPE

```
type BoolOpt      = Bool | Unit
```

```
type Optional[T] = T | Unit
```

# UNIT TYPE

```
type BoolOpt      = Bool | Unit
```

```
type Optional[T] = T | Unit
```

```
type Optional[T] = T | undefined
```

TypeScript

# UNIT TYPE

```
type BoolOpt      = Bool | Unit
```

```
type Optional[T] = T | Unit
```

```
type Optional[T] = T | undefined
```

TypeScript

```
Optional[T] = Union[T, NoneType]
```

Python

# UNIT TYPE

```
type BoolOpt      = Bool | Unit
```

```
type Optional[T] = T | Unit
```

```
type Optional[T] = T | undefined
```

TypeScript

```
Optional[T] = Union[T, NoneType]
```

Python

```
sealed trait Option[A]
```

Scala

```
case class Some[A] extends Option[A]
case object None   extends Option[A]
```

# EXAMPLES

## Modelling Data

*Make illegal state unrepresentable*

# NO ADT: ERROR CODES

```
class UserResult:  
    error_code: int          # 0 means ok  
    user:       User          # contains dummy data on error  
  
def fetchUser() -> UserResult:  
    # network call
```

# NO ADT: ERROR CODES

```
class UserResult:  
    error_code: int          # 0 means ok  
    user:       User          # contains dummy data on error  
  
def fetchUser() -> UserResult:  
    # network call
```

```
myUser = fetchUser()  
  
if myUser.error_code == 0:  
    # do sth.  
    # what if myUser.user still has dummy data?  
else  
    # do fallback
```

# NO ADT: ERROR CODES

```
class UserResult:  
    error_code: int          # 0 means ok  
    user:       User          # contains dummy data on error
```

# NO ADT: ERROR CODES

```
class UserResult:  
    error_code: int          # 0 means ok  
    user:      User          # contains dummy data on error
```

Representable valid values:

```
UserResult(0, User("John"))  
UserResult(0, User("Jane"))  
UserResult(1, User("dummy"))  
...
```

# NO ADT: ERROR CODES

```
class UserResult:  
    error_code: int          # 0 means ok  
    user:      User           # contains dummy data on error
```

Representable valid values:

```
UserResult(0, User("John"))  
UserResult(0, User("Jane"))  
UserResult(1, User("dummy"))  
...
```

Representable invalid values:

```
UserResult(1, User("John"))  
UserResult(0, User("dummy"))
```

# NO ADT: EXCEPTIONS

```
def fetchUser() -> User:  
    # raise Exception on error
```

# NO ADT: EXCEPTIONS

```
def fetchUser() -> User:  
    # raise Exception on error
```

```
myUser = fetchUser()  
# do sth.
```

# NO ADT: EXCEPTIONS

```
def fetchUser() -> User:  
    # raise Exception on error
```

```
myUser = fetchUser()  
# do sth.
```

```
try:  
    myUser = fetchUser()  
    # do sth.  
except ...
```

# ADT: OPTIONAL

```
type Optional[T] = NoneType | T

def fetchUser() -> Optional[User]:
    # return None on error
```

# ADT: OPTIONAL

```
type Optional[T] = NoneType | T

def fetchUser() -> Optional[User]:
    # return None on error
```

```
myUser = fetchUser()

if isinstance(myUser, User):
    # do sth.
elif isinstance(myUser, NoneType):
    # do fallback
```

# ADT: EITHER

```
type Either[E, T] = E | T

def fetchUser() -> Either[Error, User]:
    # return Error value on error
```

# ADT: EITHER

```
type Either[E, T] = E | T

def fetchUser() -> Either[Error, User]:
    # return Error value on error
```

```
myUser = fetchUser()

if isinstance(myUser, User):
    # do sth.
elif isinstance(myUser, Error):
    # analyze reason
```

# ADT: CUSTOM

```
type User = Anonymous | LoggedIn

def fetchUser() -> User:
```

# ADT: CUSTOM

```
type User = Anonymous | LoggedIn

def fetchUser() -> User:

myUser = fetchUser()

if isinstance(myUser, Anonymous):
    # do anonymous stuff.
elif isinstance(myUser, LoggedIn):
    # do logged-in stuff.
```

# ADT: CUSTOM EXTENDED

```
type User = Anonymous | LoggedIn | Admin
           # !new!
def fetchUser() -> User:
```

# ADT: CUSTOM EXTENDED

```
type User = Anonymous | LoggedIn | Admin
           # !new!
def fetchUser() -> User:
```

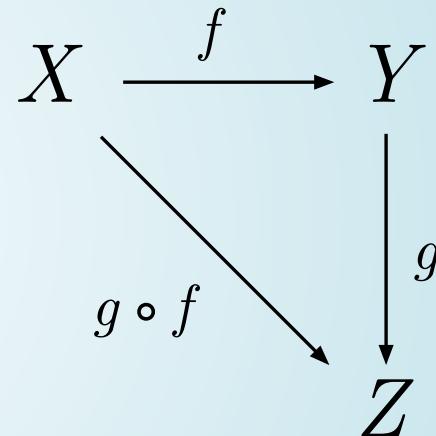
```
myUser = fetchUser()

if isinstance(myUser, Anonymous):
    # do anonymous stuff.
elif isinstance(myUser, LoggedIn):
    # do logged-in stuff.
!!! Compiler error: forgot Admin case !!!
```

# FURTHER TOPICS

- Scala
- Haskell
- Rust
- Dependent Types
- Linear Types

Category Theory



- Functor (map)
- Monad (flatMap)
- ...

Thanks

