Deep Reinforcement Learning

7 - Model-Free Control

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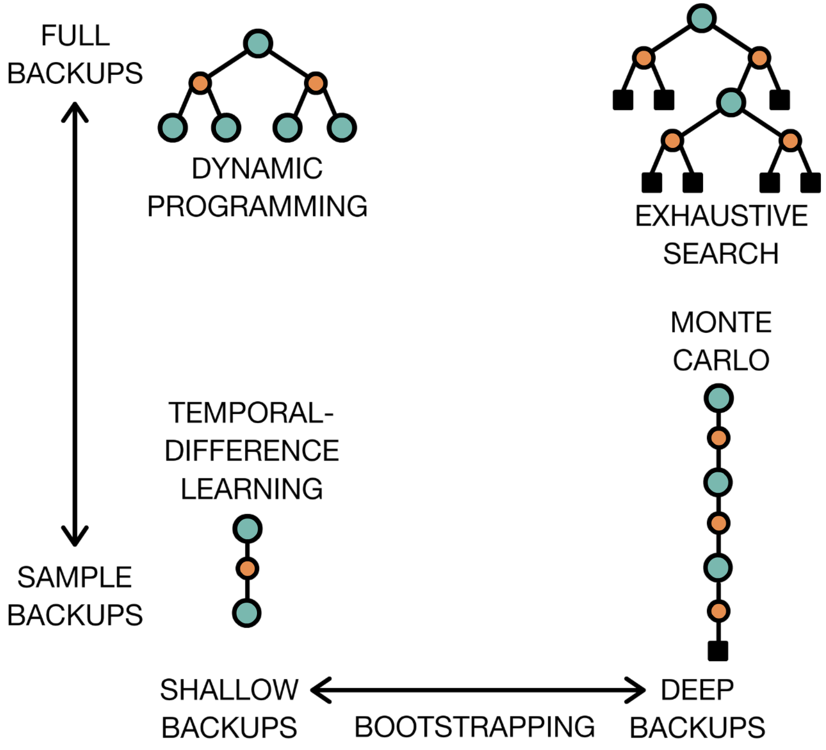
Recap Overview – Bootstrapping and Sampling

- Bootstrapping: update involves an estimate
  - MC does not bootstrap
  - DP bootstraps
  - TD bootstraps
- Sampling: update samples an expectation
  - MC samples
  - DP does not sample
  - TD samples



(Arulkumaran u. a. 2017)

Overview Lecture

- Prediction: Estimate the value function of an unknown MDP
  - Monte-Carlo Method
  - Temporal Difference Learning
- Model-free control (today):
  - **Optimise the policy**: General Policy Improvement
    - On-Policy Approach
    - Off-Policy Approach

Recap - From MC to Temporal Difference Learning

In both: Goal is to learn v_π online from experience under policy π

Incremental every-visit Monte-Carlo:

- Update value $v(S_t)$ towards **actual** return G_t:

$$v(S_t) \leftarrow v(S_t) + \alpha(G_t - v(S_t))$$

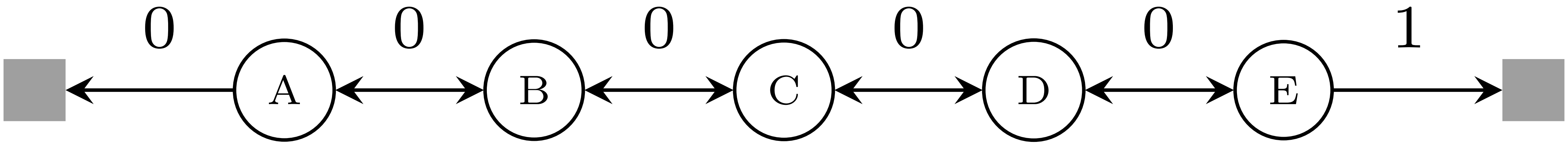
Simple temporal-difference learning algorithm TD(0):

- Update value $v(S_t)$ towards **estimated** return $R_{t+1} + \gamma v(S_{t+1})$:

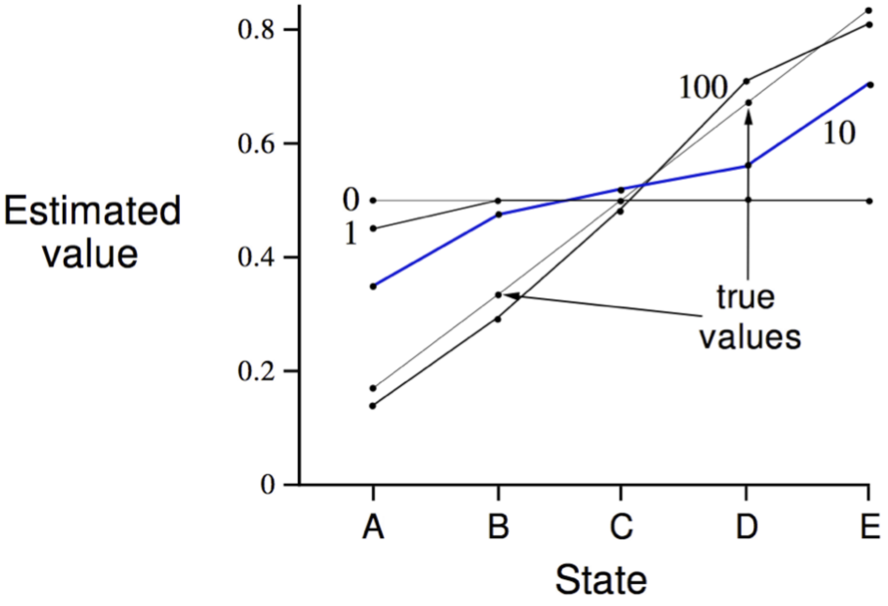
$$v(S_t) \leftarrow v(S_t) + \alpha(R_{t+1} + \gamma v(S_t + 1) - v(S_t))$$

(Silver 2015)

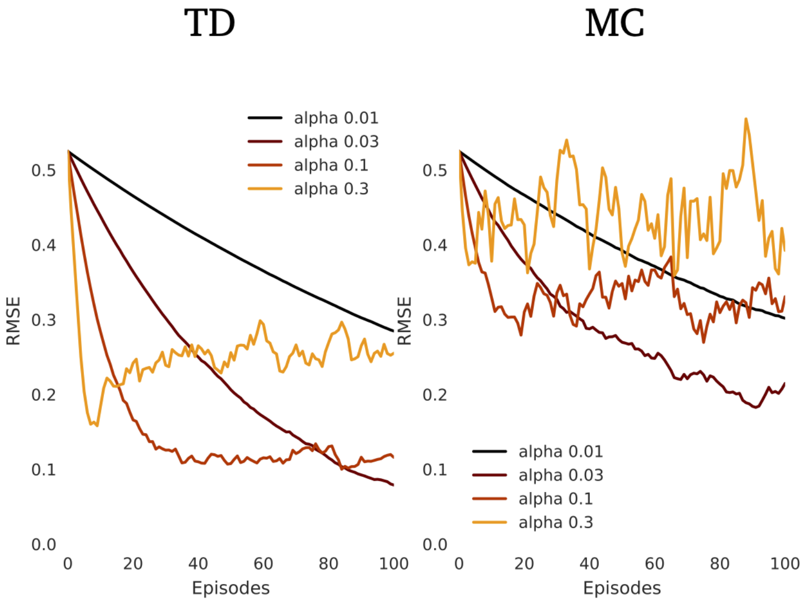
Last week – Example: Compare MC and TD empirically



TD(0) Estimates for v_π



Learning Curves



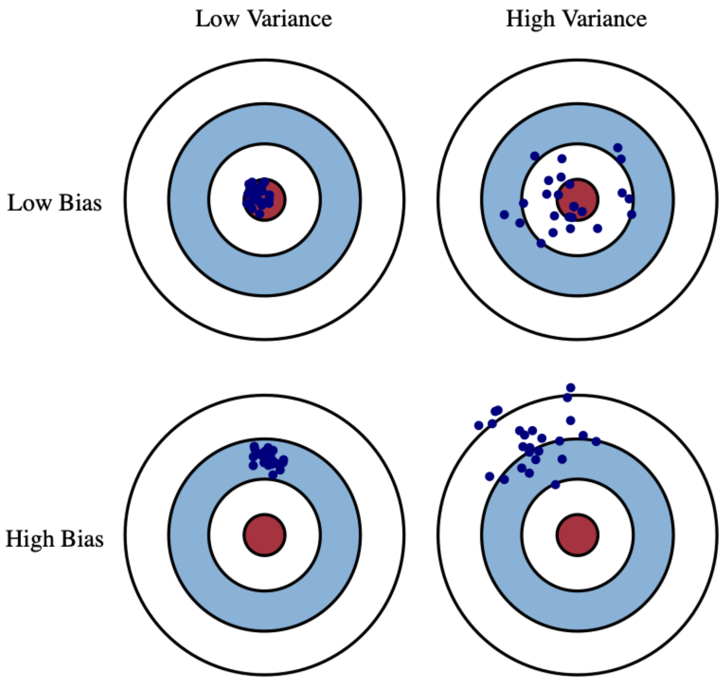
(Sutton und Barto 2018; Hasselt und Borsa 2021)

Recap – Bias-Variance Trade-Off

MC has high variance, zero bias

- Good convergence properties
- Even with function approximation
- Not very sensitive to initial value
- Very simple to understand and use

Bias and Variance



TD has low variance, some bias

- Usually more efficient than MC
- TD(0) converges to $v_\pi(s)$
- More sensitive to initial value

(Satakhutdinov 2018), (Fortmann-Roe 2012)

Recap – Temporal difference learning – q-function

- We can apply the same idea to action values – when dynamics are unknown, this is much more important
- Temporal-difference learning for action values:
  - Update value $q_t(S_t, A_t)$ towards estimated return $R_{t+1} + \gamma q(S_{t+1}, A_{t+1})$

$$q(S_t, A_t) \leftarrow q(S_t, A_t) + \alpha \Big(\underbrace{R_{t+1} + \gamma q(S_{t+1}, A_{t+1} - q_t(S_t, A_t)}_{\text{TD Error}} \Big)$$

This algorithm is known as SARSA, because it uses $(S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1})$.

(Hasselt und Borsa 2021)

Advantages and Disadvantages of MC vs. TD

TD can learn before knowing the final outcome

- TD can learn online after every step
- MC must wait until end of episode before return is known

TD can learn without the final outcome

- TD can learn from incomplete sequences
- MC can only learn from complete sequences
- TD works in continuing (non-terminating) environments
- MC only works for episodic (terminating) environments

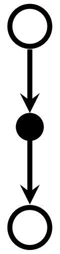
Advantages and Disadvantages of MC vs. TD (2)

- TD is independent of the temporal span of the prediction
  - TD can learn from single transitions
  - MC must store all predictions (or states) to update at the end of an episode
- TD needs reasonable value estimates
- TD exploits Markov property
  - Usually more efficient in Markov environments
- MC does not exploit Markov property
  - Usually more effective in non-Markov environments
- With finite data (or with function approximation) the solutions may differ

(Silver 2015)

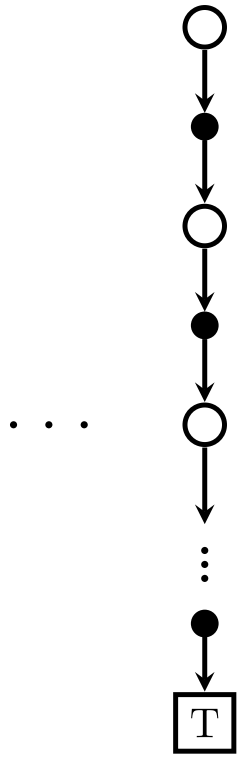
Combining the two approaches – a unifying perspective

TD (1-step)

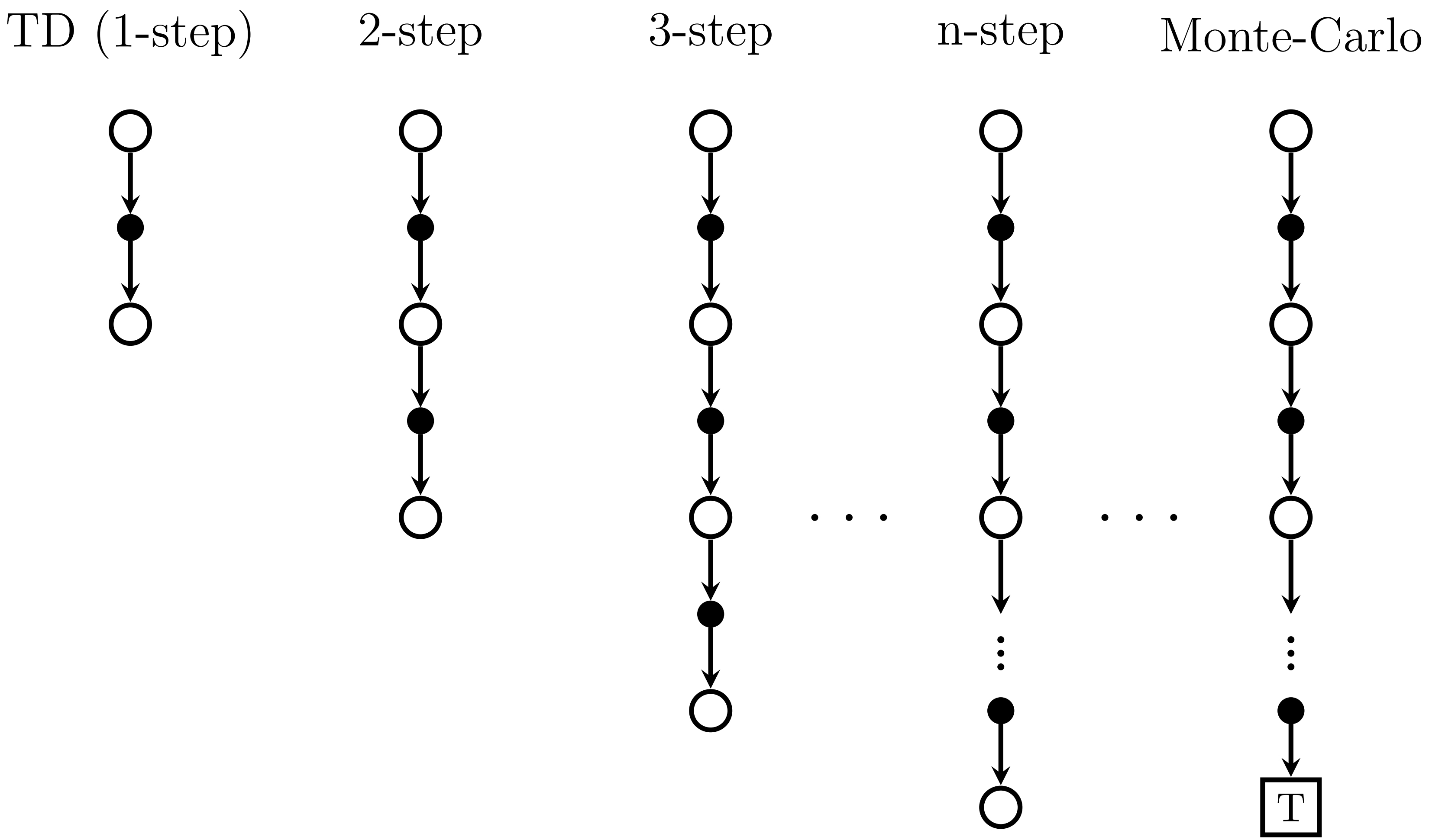


- TD uses value estimates which might be inaccurate
- In addition, information can propagate back quite slowly (possible bias)
- In MC information propagates faster, but the updates are noisier (high variance)
- We can go in between TD and MC

Monte-Carlo



Multi-step Predictions



Make n steps and then use TD target for prediction.

n-step Return

Consider the following n-step returns for $n = 1, 2, \infty$:

steps	Approach	Return
$n = 1$	TD	$G_t^{(1)} = R_{t+1} + \gamma v(S_{t+1})$
$n = 2$		$G_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 v(S_{t+2})$
$\vdots$		$\vdots$
$n = \infty$	MC	$G_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{T-1} R_T$

Define the n-step return

$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n v(S_{t+n})$$

n-step Return

n-step return

$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n v(S_{t+n})$$

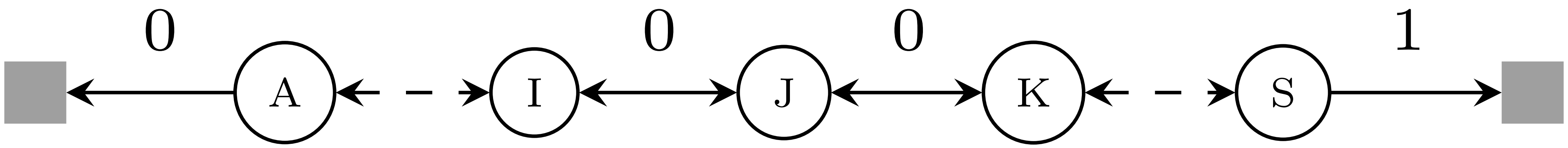
n-step Temporal Difference learning

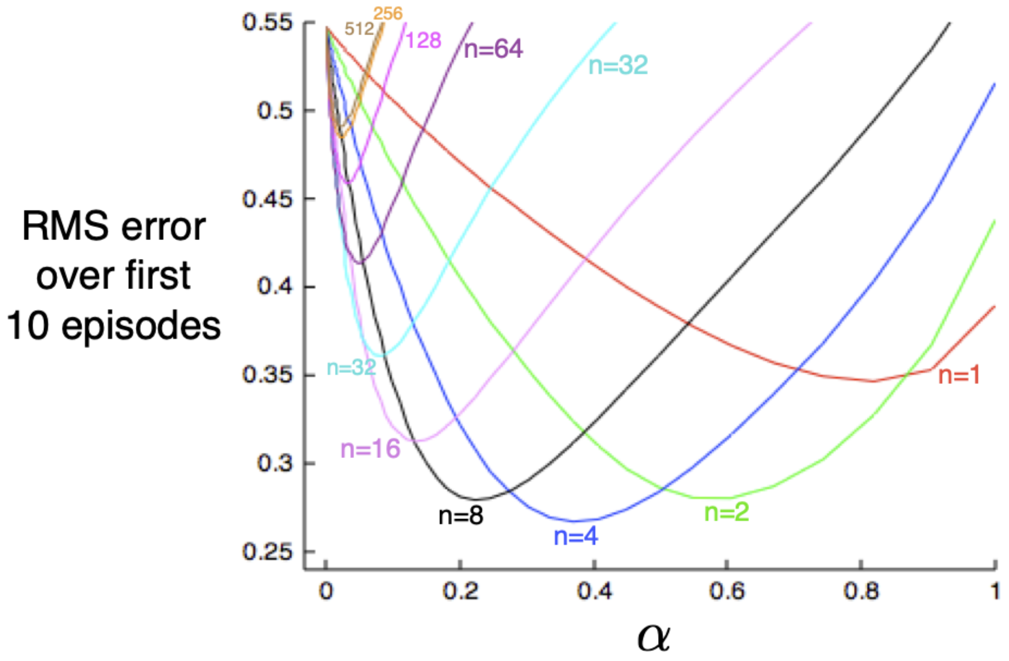
$$v(S_t) \leftarrow v(S_t) + \alpha\left(G_t^{(n)} - v(S_t)\right)$$

(Silver 2015)

(Large) Random Walk Example – Error for different n-steps

MDP: Chain of 19 Nodes; Start in J; random policy; $r = 0$ unless terminating right: $r = 1$





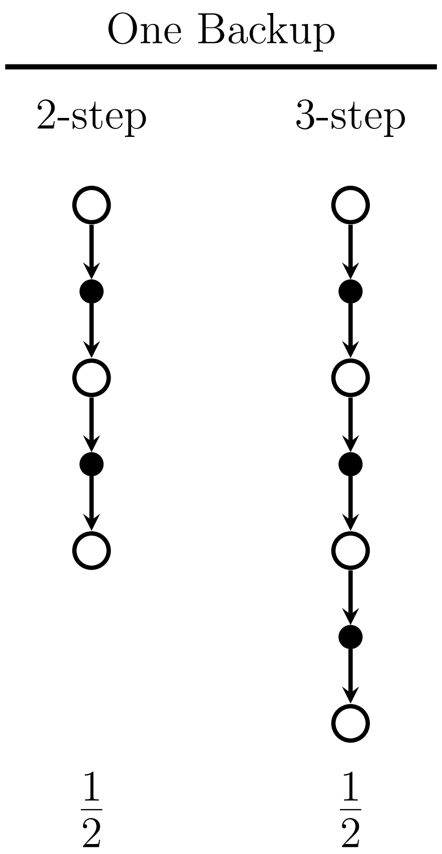
(Hasselt und Borsa 2021) following (Sutton und Barto 2018)

Averaging n-step Return

- We can average n-step returns over different n.
- For example: average the 2-step and 3-step returns as

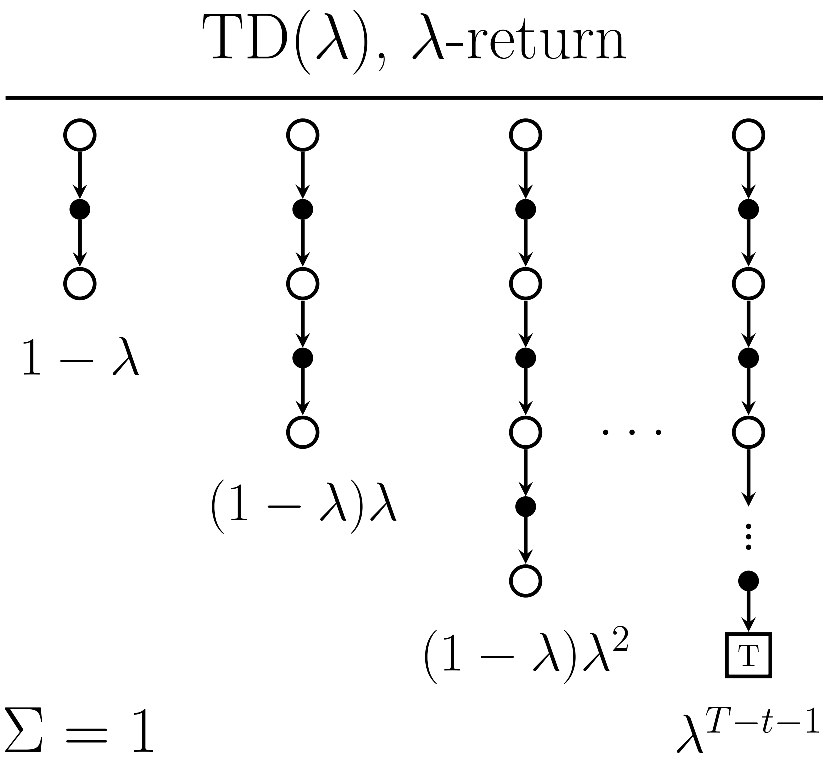
$$\frac{1}{2}G^{(2)} + \frac{1}{2}G^{(3)}$$

- Combines information from two different time-steps



(Silver 2015)

λ-Return



The λ-return G_t^λ combines all n-step returns $G_t^{(n)}$ using as a weight $(1-\lambda)\lambda^{n-1}$

$$G_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty}\lambda^{n-1}G_t^{(n)}$$

Forward-view TD(λ)

$$v(S_t) \leftarrow v(S_t) + \alpha\Big(G_t^\lambda - v(S_t)\Big)$$

(Silver 2015)

Computation of TD(λ)

Forward-view TD(λ)

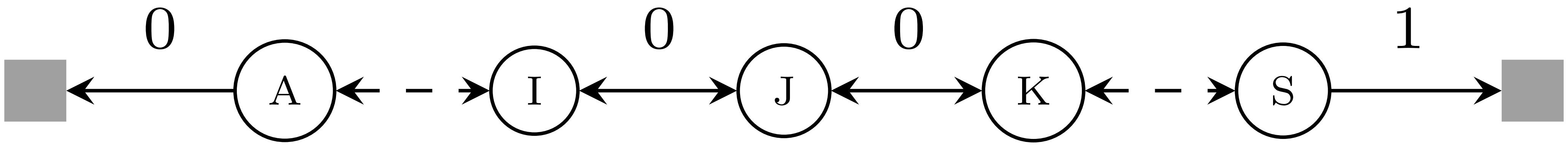
- Update value function towards the λ-return
- Forward-view looks into the future to compute G_t^λ
- Drawback: Like Monte-Carlo, can only be computed from complete episodes

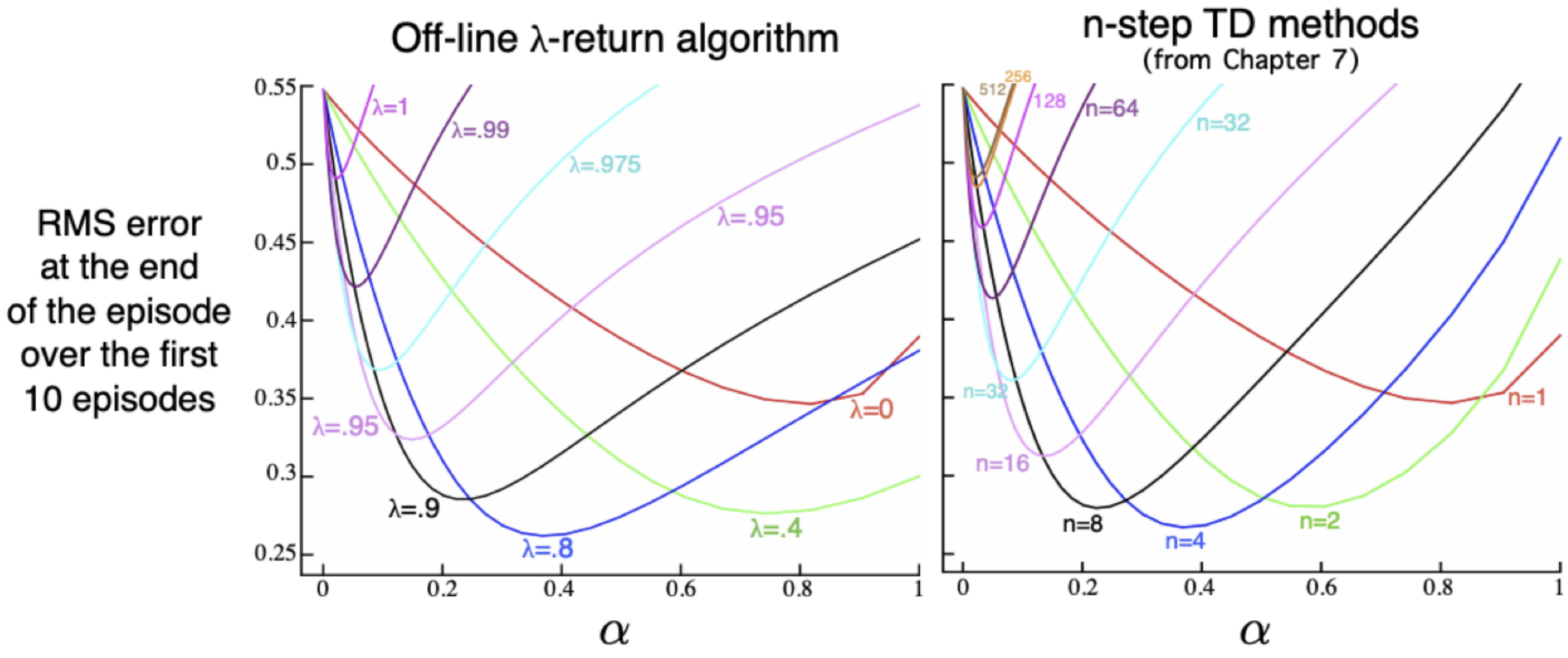
Backward View TD(λ)

- While Forward view provides theory
- Backward view provides mechanism
- Update online, every step, from incomplete sequences – and update for previous steps going backwards into the past

(Silver 2015)

(Large) Random Walk Example – Error for TD(λ)





Intuition: $\frac{1}{1-\lambda}$ is the `horizon', e.g., $\lambda = 0.9 \rightsquigarrow n = 10$ steps.

(Hasselt und Borsa 2021) following (Sutton und Barto 2018)

Benefits of multi-step returns

Multi-step returns have benefits from both TD and MC

- Bootstrapping can have issues with bias
- Monte-Carlo can have issues with variance
- Typically, intermediate values of n or λ are good (e.g., $n = 10, \lambda = 0.9$)

(Hasselt und Borsa 2021)

Model-free Control – Monte-Carlo

Example: Learning Walking on a Robot

See Video of robot learning walking from scratch (exploration directly on robot)

Details see (Haarnoja u. a. 2018)

Uses of Model-Free Control

Example problems that can be modelled as MDPs

- Elevator control
- Parallel Parking
- Helicopter
- Robocup Soccer
- Portfolio management
- Robot walking

For most of these problems, either:

- MDP model is unknown, but experience can be sampled
- MDP model is known, but is too big to use, except by samples

Model-free control can solve these problems

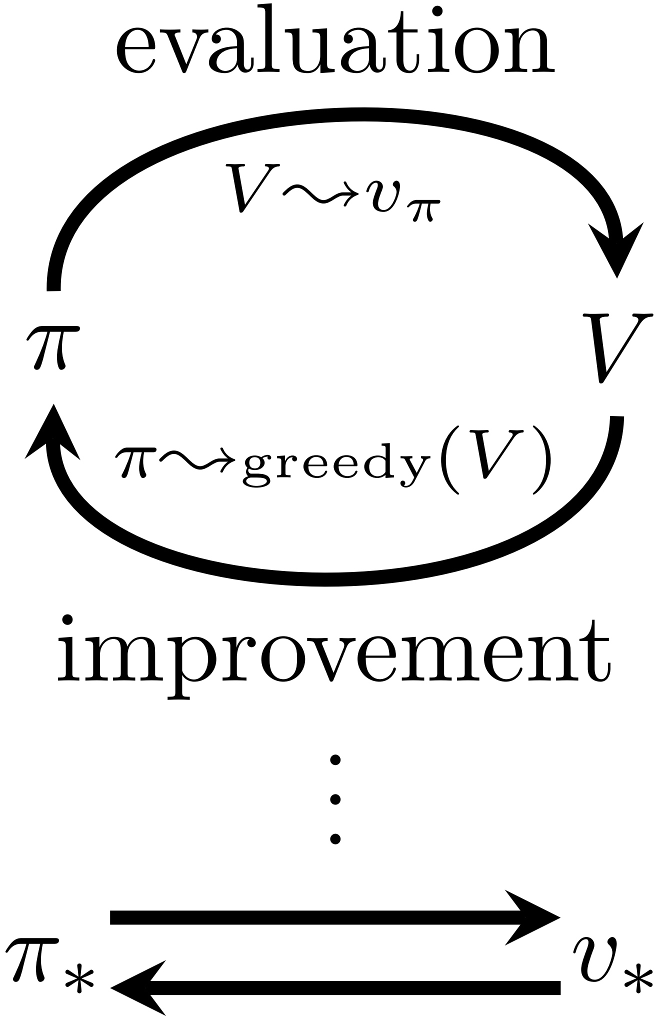
(Silver 2015)

- policy improvement: derive new policy π', $v_{\pi'}(s) \geq v_{\pi}(s), \forall s$

in interaction.

Most RL learning methods can be described as GPI: they consist of a policy and value function.

When evaluation and improvement process each converge, then value function and policy are optimal – the policy is greedy wrt. the stable value function. This implies: the Bellman optimality equation holds.



Summary: Model-Free Policy Evaluation Approaches

Iterative approximation of value function for given policy π:

$$v_{n+1}(S_t) = v_n(S_t) + \alpha\Big(G_t - v_n(S_t)\Big)$$

Different Methods:

Approach	Target computation
Monte-Carlo	$G_t^{\text{MC}} = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots$
TD(0)	$G_t^{(1)} = R_{t+1} + \gamma v_t(S_{t+1})$
n-step TD	$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{n-1} R_{t+n} + \gamma^n v_t(S_{t+n})$
TD(λ)	$G_t^{(\lambda)} = R_{t+1} + \lambda\Big((1-\gamma)v_t(S_{t+1}) + \lambda G_{t+1}^{\lambda}\Big)$

Recap – Policy Iteration (Control)

$$\pi_0 \xrightarrow{\mathrm{E}} v_{\pi_0} \xrightarrow{\mathrm{I}} \pi_1 \xrightarrow{\mathrm{E}} v_{\pi_1} \xrightarrow{\mathrm{I}} \pi_2 \xrightarrow{\mathrm{E}} \cdots \xrightarrow{\mathrm{I}} \pi_* \xrightarrow{\mathrm{E}} v_*$$

Policy evaluation: $\xrightarrow{\text{E}}$

Policy improvement $\xrightarrow{\text{I}}$

For deterministic policies: each policy is guaranteed to be strictly better until we reach the optimal policy.

For finite MDP: $\exists$ only a finite number of deterministic policies; therefore this converges to an optimal policy and an optimal value function in a finite number of iterations.

Model-Free Policy Iteration Using Action-Value Function

Using Value Function

Using Value Function for improvement: We still need a model (which state do we arrive when using an action) for greedy policy improvement:

$$\pi'(s) = \arg\max_a \mathbb{E}(R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s, A_t = a)$$

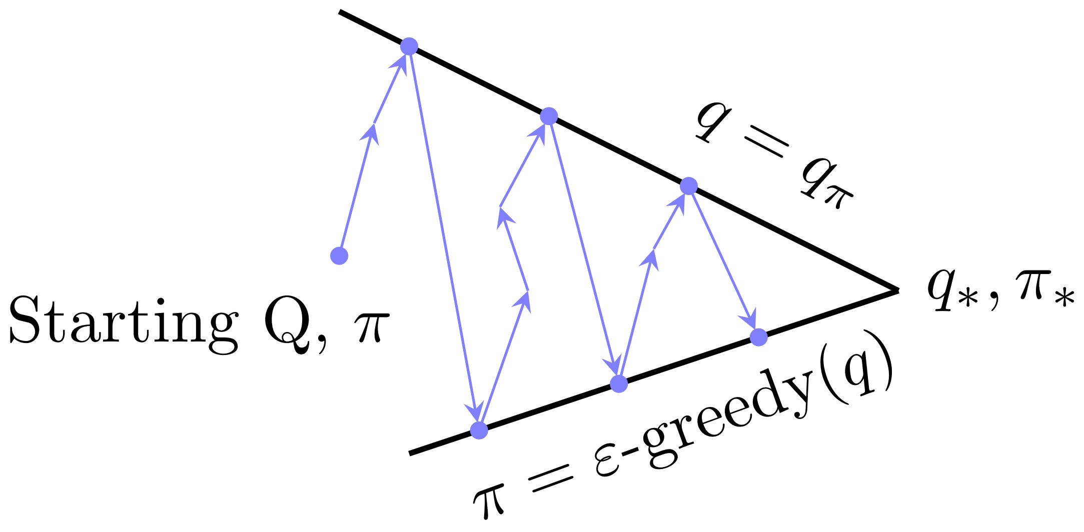
Using Action-Value Function

In contrast: Greedy policy improvement over $q(s,a)$ is model-free which makes it directly applicable

$$\pi'(s) = \arg\max_a q_\pi(s,a)$$

Monte-Carlo Policy Improvement

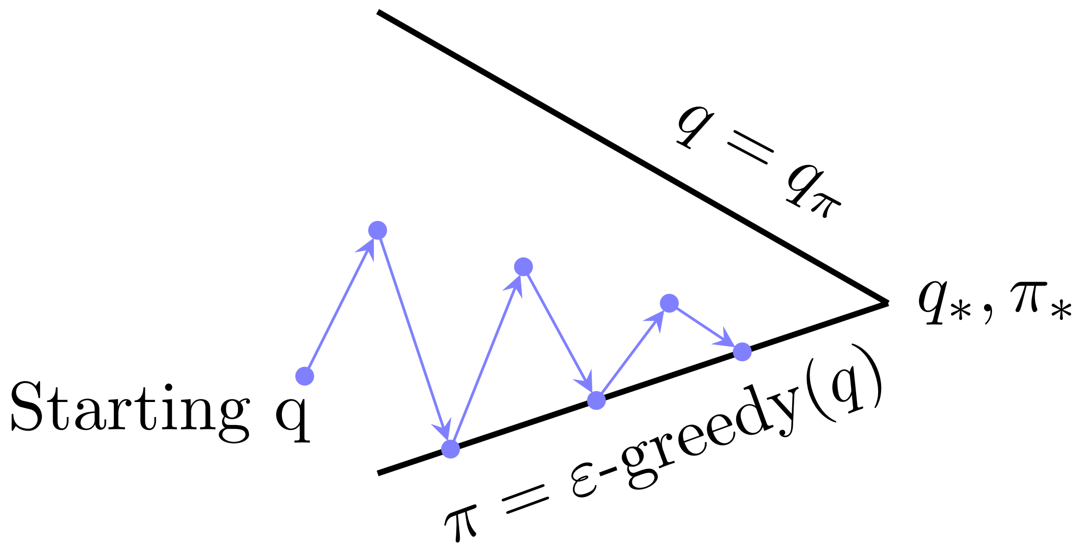
Monte-Carlo Policy Iteration



Policy evaluation: MC policy evaluation, $q = q_\pi$

Policy improvement: ε-greedy policy improvement

Monte-Carlo Control



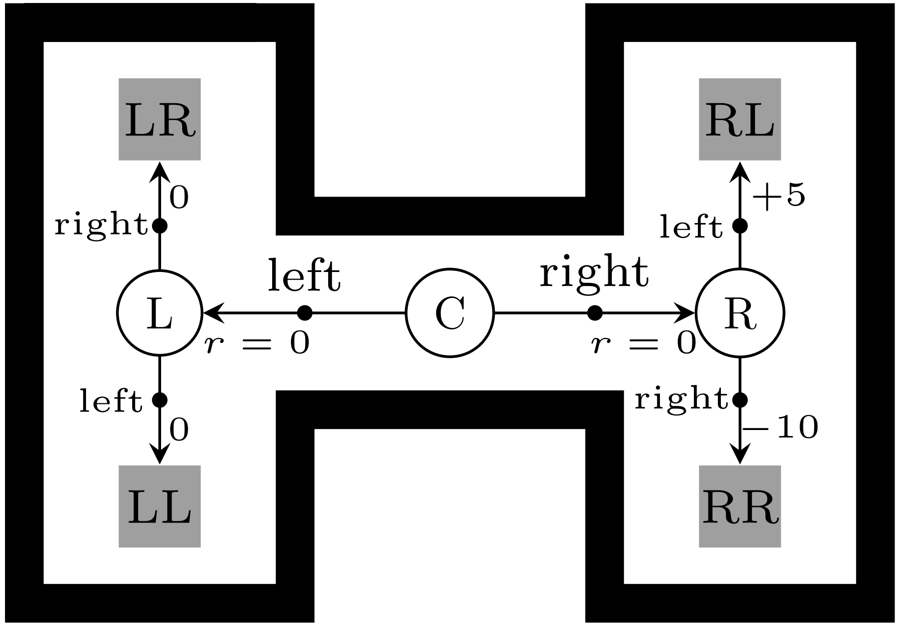
Every episode:

MC policy evaluation, $q \approx q_\pi$

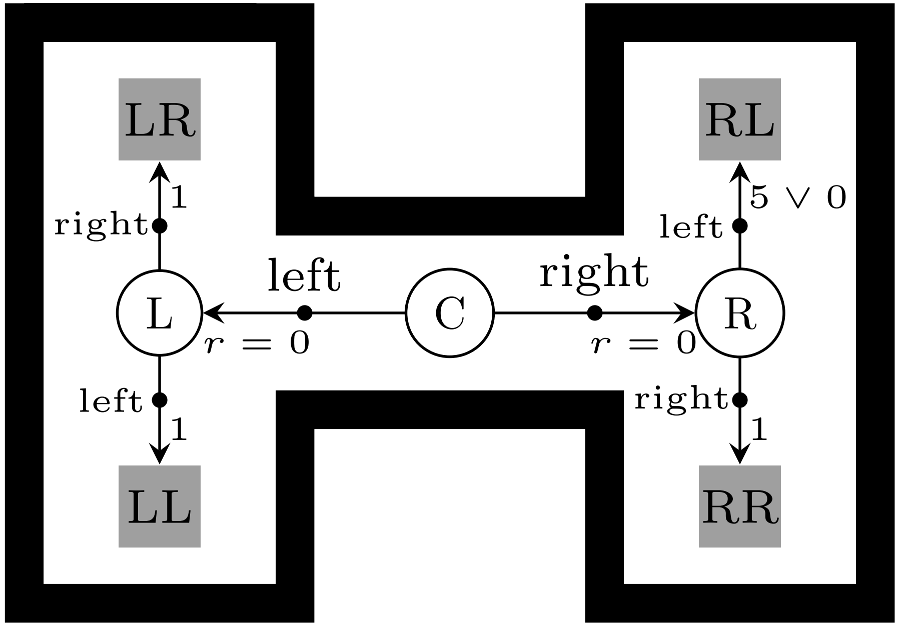
ε-greedy policy improvement

Explore a maze – but structure and rewards are unknown (visualized here as a MDP). Use MC to find an action-value function for a random policy, and improve the policy. What are your observations and takeaways for GPI?

Deterministic Rewards

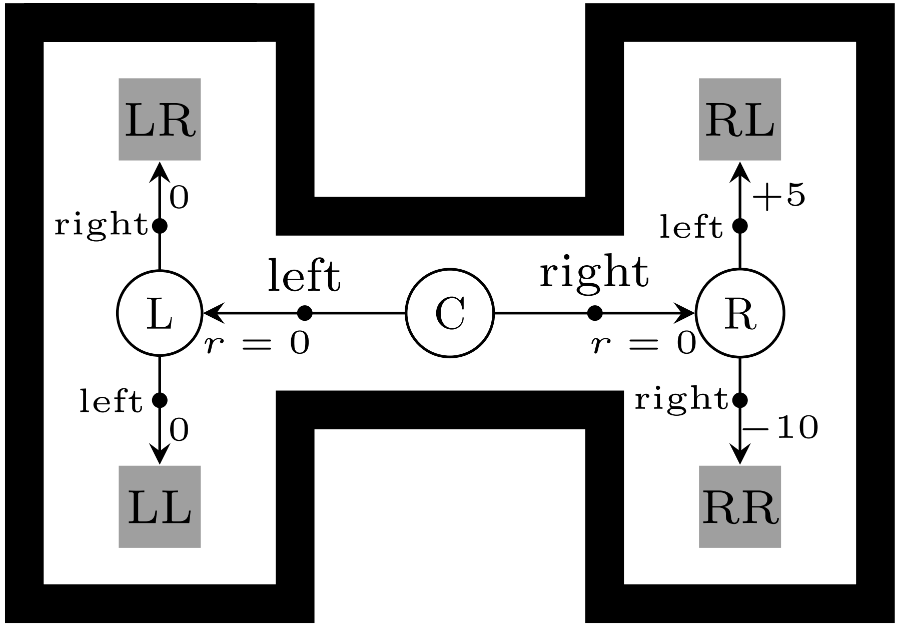


Probabilistic rewards



Left Task – Value Function Considerations

Deterministic Rewards



Action-Value Function

s	a	$q(s, a)$
C	left	0
C	right	-2.5

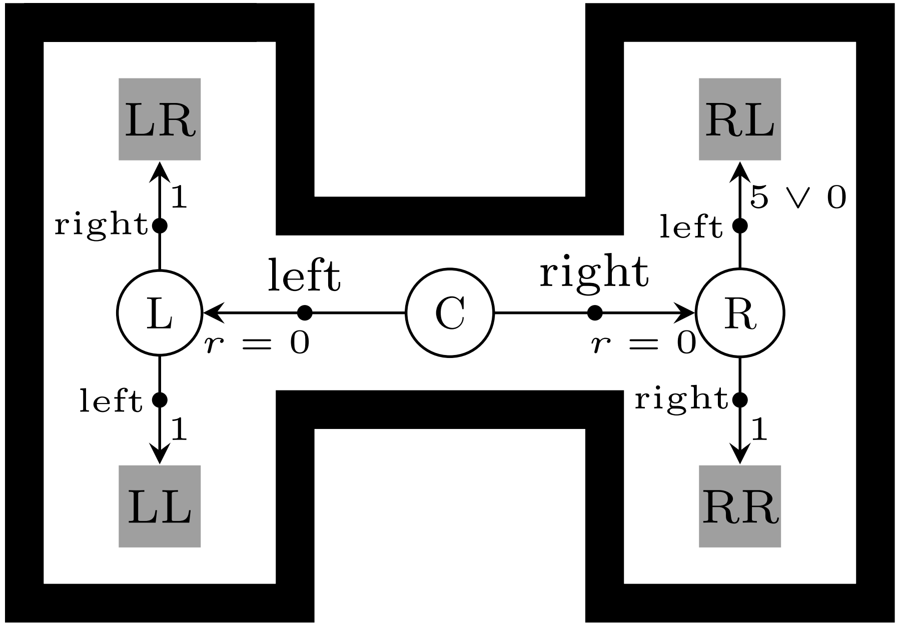
with discount factor $\gamma = 1$

Observation: After first iteration, action-value-function for moving right – $q(C, right)$ – is negative and an improved policy will pick the (overall) suboptimal policy.

Important: Action-values are always with respect to a given policy. As this is still

Right Task – Probabilistic Rewards

Probabilistic rewards



Action-Value Function

s	a	$q(s,a)$	single episode
C	left	1	1
C	right	1.75	$0 \vee 1 \vee 5$

with discount factor $\gamma = 1$, but converges only for enough episodes.

Observation: As reward is probabilistic, we have to sample enough to get a good estimate. If we only sample once we might improve our policy in a way that excludes the optimal action and never recovers.

Important: We still have to guarantee sufficient exploration in order to (guaranteed!)

Convergence of Policy Improvement with Greedy Selection

Policy improvement assures us that π_{k+1} is uniformly better than π_k – unless π already converged and is then (guaranteed) the optimal policy.

But: this requires, that our estimates converges for each action, state pair for which we have to test these an infinite number of times.

Approach: Exploring Starts

In, e.g., simulated settings we can enforce this using Exploring Starts – and can assure starting from each possible state and selecting in that state each possible action.

Example: Blackjack scenario for which we can initialize states and select the actions.

We use a single policy for evaluation and directly improve this. This is called **on-policy**.

Problem: When exploring starts is not possible

Monte-Carlo Exploring Starts, for estimating $\pi \approx \pi_*$

$\pi(s) \in \mathcal{A}(s)$ (arbitrarily), for all $s \in \mathcal{S}$
$Q(s, a) \in \mathbb{R}$ (arbitrarily), for all $s in \mathcal{S}, a \in \mathcal{A}(s)$
$\texttt{Returns}(s, a) \leftarrow$ empty list, for all $s in \mathcal{S}, a \in \mathcal{A}(s)$
for *each episode (without termination)* **do**
  Choose $S_0 \in \mathcal{S}$, $A_0 \in \mathcal{A}(S_0)$ randomly such that all pairs have probability > 0.
  Generate an episode from S_0, A_0 following π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$.
  $G \leftarrow 0$.
  for *each step of the episode* $t = T-1, T-2, \ldots, 0$ **do**
    $G \leftarrow \gamma G + R_{t+1}$
    if S_t, A_t *does not appear in* $S_0, A_0, S_1, A_1, \ldots, S_{t-1}, A_{t-1}$ **then**
      Append G to $\texttt{Returns}(S_t, A_t)$
      $Q(S_t, A_t) \leftarrow \texttt{average}(\texttt{Returns}(S_t, A_t))$ `// Policy Evaluat.`
      $\pi(S_t) \leftarrow \arg\max_a Q(S_t, a)$ `// Policy Improvement`
    end
  end
end

Agent part of the algorithm; Environment interaction

Recap Example: Blackjack

Game: Play only against a dealer.

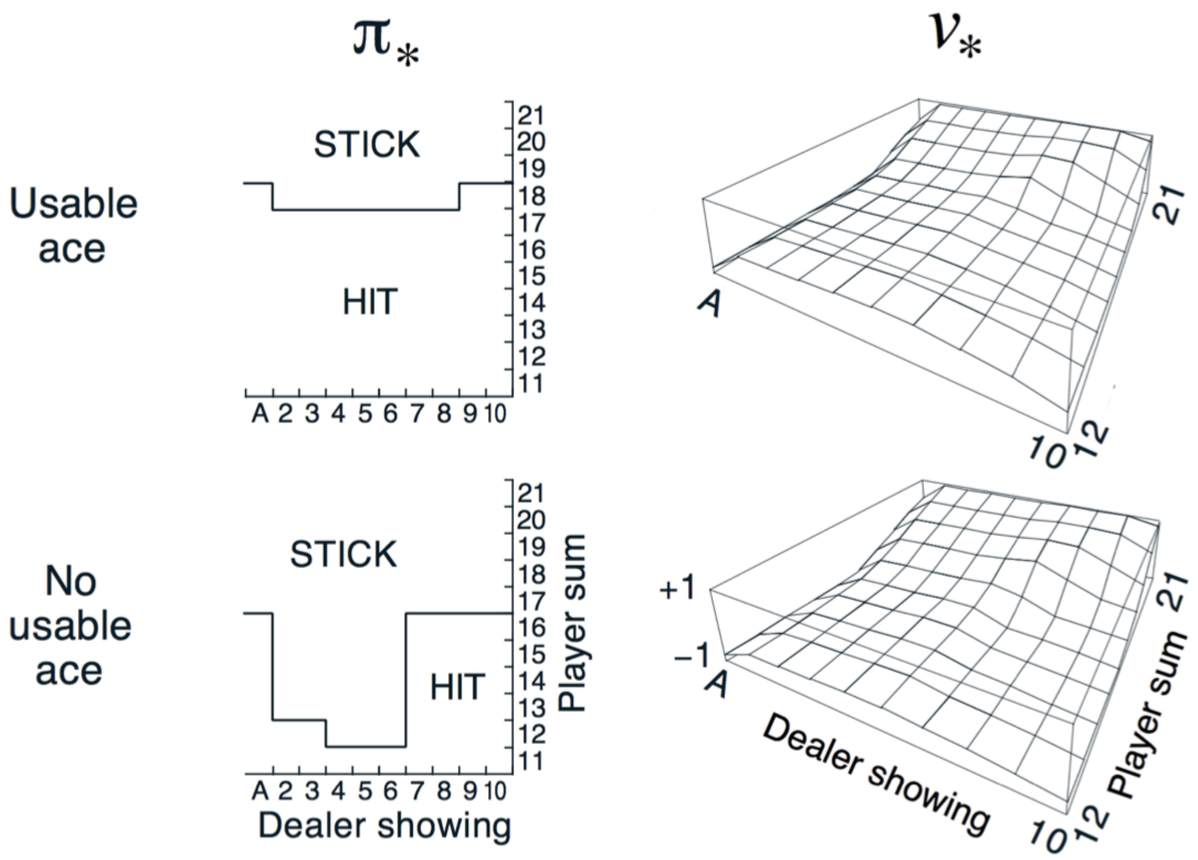
Goal: sum of cards is as great as possible without exceeding 21.

Counting:

- Number cards equal their number,
- all face cards count as 10,
- an ace can count as either 1 or 11.

MC Converged Policy

Optimal policy and state-value function for blackjack, found by Monte Carlo (using exploring starts) (Sutton und Barto 2018).
This direct improvement of a policy while running the environment interaction is called **on-policy**.



On and Off-Policy Learning

On-policy learning

- "Learn on the job"
- Learn about policy π from experience sampled from π
- e.g., using MC with exploring starts

Off-policy learning

- "Look over someone's shoulder"
- Learn about policy π from experience sampled from different policy b

(Silver 2015)

ε-Greedy Exploration

Simplest idea for ensuring continual exploration: Continue to sample randomly (for a small fraction).

- All m actions are tried with non-zero probability
- With probability $1 - \varepsilon$ choose the greedy action
- With probability ε choose an action at random

$$\pi(a|s) = \begin{cases} \frac{\varepsilon}{m+1-\varepsilon} & \text{if } a^* = \arg\max_{a \in \mathcal{A}} q(s,a) \\ \frac{\varepsilon}{m} & \text{otherwise} \end{cases}$$

(Silver 2015)

On-Policy Characteristics

The policy ...

- is generally *soft*: $\pi(a|s) > 0, \forall s \in \mathcal{S}$ and $a \in \mathcal{A}$,
- gradually shifts closer and closer to a deterministic optimal policy.

We can use an ε-greedy policy.

ε-soft policy

A policy, for which

$$\pi(a|s) \geq \frac{\varepsilon}{|\mathcal{A}(s)|}, \forall \text{ states and actions for some } \varepsilon > 0$$

Among ε-soft policies: ε-greedy policies are closest to greedy.

Overall: Idea of on-policy Monte Carlo methods is General Policy Improvement

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