

Chapter 1

Case study: Zipf's Law and Music

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1.1 Music as Language

Sugarscape is an agent-based model developed by Joshua M. Epstein and Robert Axtell to investigate the economics of wealth distribution. They presented the original model in their book, *Growing Artificial Societies*.

The sugarscape is a virtual 2D grid where each cell has a certain amount of abstract wealth, called sugar. Agents roam the grid and accumulate sugar.

1.1.1 Pink Noise

In the simplest Sugarscape, each agent has a sugar reserve, a metabolism at which rate it consumes its sugar, and a range of nearby cells that it can observe. At each time step, the agent observes its nearby cells and moves to the cell with the most sugar. These rules can be expanded to include topics as varied as reproduction, death, disease, loans, and warfare. For example, a disease can be introduced to the system where a sick agent can infect nearby healthy agents.

Despite its simplicity, the model generates outcomes that resemble the real world. When modeling wealth with Sugarscape, a long-tailed distribution appears where some agents are vastly richer than others. Similar behavior is seen in most real economies, where a small part of the population holds a large fraction of the wealth. Extreme wealth inequality is generally considered a problem, because it means there are many people barely surviving while others are fabulously rich.

1.2 MIDI files

Wealth inequality has partly fueled a modern social movement known as the Occupy movement. The first significant Occupy protest was on Wall Street in New York City, where thousands of protesters gathered to express their dismay with the distribution of wealth, among other things. The movement's motto is "We are the 99%", reminding politicians to

serve the majority, not the 1% who control more than a third of the nation's wealth. A major goal of the movement is to achieve a more equal distribution of income, which protesters hope to accomplish by implementing a more progressive tax policy.

One of the effects of taxation is to redistribute wealth from the rich to the poor. But opponents of the Occupy movement (and many fiscal conservatives) claim that high tax rates for the rich actually hurt the population as a whole. The logic is that wealthy people employ the poor, redistributing the wealth without the need for tax levies.

1.3 Words and Music

Our implementation of Sugarscape aims to study the effect of taxation on the wealth of a society. We want to show how extreme under- or over-taxation can affect the society and its individual agents, and what happens in between these two extremes. The model tests a "flat tax" system where every agent gets taxed a constant rate (say 10% of its total wealth) and the tax pool is redistributed evenly among all the agents. We recreate the original Sugarscape and expand on it with the end goal of determining whether it is possible to shrink the wealth gap without crippling the society.

1.3.1 Pygame

In the process of implementing Sugarscape, we made a GUI to better understand what was happening on the grid. The visualization of the Sugarscape is done with Pygame, a set of Python modules that allows easy graphic drawing. Pygame can blit images onto the screen (see http://en.wikipedia.org/wiki/Bit_blit) and it has built-in methods for handling user input like mouse clicks and button presses, making it ideal for designing games or other programs that receive a lot of input.

Below is an abbreviated version of our event loop that draws the cells in the GUI at each time step. `Sugarscape.nextstep` moves every agent forward by one time step and the rest of the code redraws the update. Redrawing the entire grid is slightly less efficient than changing existing rectangle objects but is a common convention for pygame. A square is drawn for each location, and the color of the square changes based on the amount of sugar contained there. Agents are represented by circles drawn on top of their current location.

```
def event_loop(self,sugarscape):
    while True:
        sugarscape.nextstep()
        for i in range(sugarscape.length):
            for j in range(sugarscape.width):
                loc = sugarscape.get_location(i,j)
                health_color = (0, 0, loc.get_sugar_amt()/loc.get_max_sugar())
                pygame.draw.rect(self.window, healthColor,(12*i,12*j,10,10))
            pygame.display.update()
```

Users can control certain attributes of the Sugarscape by moving sliders underneath the grid. A histogram, implemented using the matplotlib library, shows the current distribution of wealth, and text fields show certain characteristics of the distribution.

1.4 Scale-free Music

Taxation in our implementation of Sugarscape is handled with a Government object. Every ten time steps, the Government object collects a fraction of each agent's sugar reserve, then distributes the collected sugar to each agent equally. This transfer represents services provided by the government as well as explicit redistribution of wealth.

But if opponents of the Occupy movement are correct, transferring wealth from rich to poor makes society as a whole less productive. According to this theory, the rich create more wealth than the poor because they can open factories, fund research, and generally make investments into the economy.

In order to simulate this effect, we need to augment the model with a mechanism of wealth creation. We implement a simple "leave behind" feature, where agents leave some sugar behind as they leave a location:

$$leave_behind = \frac{1}{5} \left(\frac{wealth \times N}{total_wealth} \right)^{1.1}$$

where N is the total number of agents, $wealth$ is the amount of sugar the agent has, and $total_wealth$ is the total sugar owned by all the agents. Agents who own a large proportion of the total wealth leave behind larger amounts of sugar, making an investment into the Sugarscape, and increasing the total wealth.

1.5 Zipf's Law and Beauty

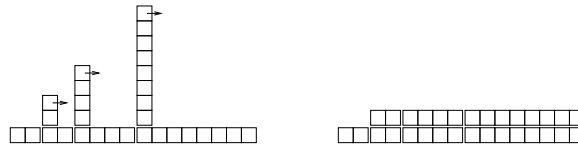


Figure 1.1: Histogram of wealth with no tax.

To compare the effect of taxation on wealth distribution, we need a metric that measures how distributed or flat a certain wealth distribution is. We use the Gini coefficient, which is often used in economics to measure the wealth gap (see http://en.wikipedia.org/wiki/Gini_coefficient). The Gini coefficient is between 0 and 1, with 0 the measurement of a perfectly uniform distribution, and 1 the measurement of a distribution with complete inequality.

Figure 1.1 shows a histogram describing the wealth distribution when there is no tax system in place. For most initial conditions without taxation, the Sugarscape quickly develops a long-tailed distribution of wealth, skewed to the right. In these cases, some agents die quickly, particularly in an environment with many agents or one with low sugar regrowth rate. The separation between the rich and the poor is significant, and there aren't many agents occupying the middle ground. This is seen in real life in societies where there is no tax structure and there isn't much of a middle class.

1.6 Reductionism or Holism?



Figure 1.2: Histogram of wealth, with tax.

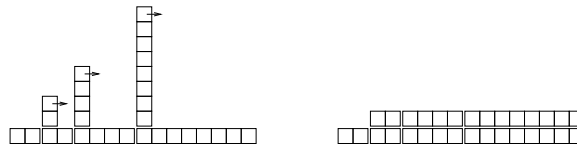


Figure 1.3: The Gini coefficient versus the tax rate.

Figure 1.2 shows the effect of a relatively high tax rate. The agents have a similar amount of sugar, and the economy has a low Gini coefficient, 0.02.

Figure 1.3 shows that higher taxes in general result in lower Gini coefficients. This makes sense, since the point of our tax system is to redistribute wealth.



Figure 1.4: Mean wealth versus tax rate.

In this model, perfect equality comes at a price. With no taxation the mean wealth was 358; with a 20% tax rate it drops to 157. Figure 1.4 shows the effect of tax rate on wealth; mean wealth gets smaller as taxes get higher.

1.7 Instrumentalism and Music

It's up to a society to determine its ideal wealth distribution. In our model, there is a conflict between the goals of maximizing total wealth and minimizing inequality.

One way to reconcile this conflict is to maximize the wealth of the bottom quartile. Figure 1.5 shows the mean wealth of the poorest 25% for a range of tax rates. The optimal tax rate is around 4%. At lower rates, there is more total wealth, but the poor do not share it. At higher rates, the poor have a bigger share of a smaller pie.

Of course, this result depends on the details of our Sugarscape, especially the model of productivity. But this simple model provides a way to explore relationships between wealth creation, taxation and inequality.

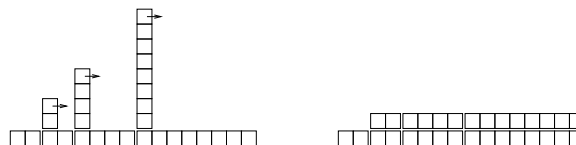


Figure 1.5: Bottom quartile value versus tax rate. At 4% the average wealth of the bottom quartile is maximized.

Exercise 1.1. You can download our implementation of Sugarscape from thinkcomplex.com/Sugarscape.zip. Launch it by running `Gui.py`. The sliders allow you to control the parameters of the simulation. Experiment with these parameters to see what effect they have on the distribution of wealth.

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