Problem Set - VII

Undergraduate Directed Group Reading Program 2024

Geometric Group Theory

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Instructions

- This is the seventh problem set—please attempt it sincerely, as it will greatly enhance your understanding over time. While it is not graded, we strongly encourage you to approach it diligently.
- If you face challenges with the readings or specific problems, feel free to reach out to either of us for guidance.
- References are provided for each problem, allowing curious readers to explore the solutions in the cited sources and delve deeper into the applications of the ideas. Download links for the referenced books can be found in the Reference section.
- **If possible**, we encourage you to write up your solutions in LaTeX and share them with both instructors via email. Effort will be duly acknowledged. If you cannot solve all the problems, simply share the LaTeX solutions for the ones you manage to solve.
- If you would like your solutions checked, please send them to us via WhatsApp. For any progress on the Chocolate Problem, kindly **email** both the instructors (with subject **"Chocolate Problem"**). Ofcourse, you will receive proper credits (even for partial solutions) and chocolates—after the semester break! Partial solutions to the Chocolate Problem are particularly encouraged. Don't hesitate to message us with any ideas for that problem, but please avoid submitting solutions copied from books or platforms like Math Stack Exchange.
- For any problem where you rely on ideas or theorems not covered in class, include a justification or a rough proof of the theorem you are using. This is especially important for the **"Chocolate Problem"**.
- Collaboration with peers is encouraged, as long as it promotes genuine learning. Best of luck, and happy problem-solving!

Problems

Note: This problem set is brief since several exercises were already assigned in class. Please ensure that you attempt all the exercises given in class as well. If you missed Lecture 7, be sure to watch the recording, as the following problems are based on the discussions held in class.

Problem I
In the proof of the Nielsen–Schreier Theorem presented in class, identify the step where we used the action of G on T to be "free"?

Problem II (Chocolate Problem)
We proved the Nielsen–Schreier Theorem in a specific case and demonstrated it for $\mathbb{F}_2$. Try to mimic the same idea for $\mathbb{F}_n$.

Hint: You may look at OHGGT for this question.