Problem Set - VI

Undergraduate Directed Group Reading Program 2024

Geometric Group Theory

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Instructions

- This is the sixth problem set—please attempt it sincerely, as it will greatly enhance your understanding over time. While it is not graded, we strongly encourage you to approach it diligently.
- If you face challenges with the readings or specific problems, feel free to reach out to either of us for guidance.
- References are provided for each problem, allowing curious readers to explore the solutions in the cited sources and delve deeper into the applications of the ideas. Download links for the referenced books can be found in the Reference section.
- **If possible**, we encourage you to write up your solutions in LaTeX and share them with both instructors via email. Effort will be duly acknowledged. If you cannot solve all the problems, simply share the LaTeX solutions for the ones you manage to solve.
- If you would like your solutions checked, please send them to us via WhatsApp. For any progress on the Chocolate Problem, kindly **email** both the instructors (with subject **"Chocolate Problem"**). Ofcourse, you will receive proper credits (even for partial solutions) and chocolates—after the semester break! Partial solutions to the Chocolate Problem are particularly encouraged. Don't hesitate to message us with any ideas for that problem, but please avoid submitting solutions copied from books or platforms like Math Stack Exchange.
- For any problem where you rely on ideas or theorems not covered in class, include a justification or a rough proof of the theorem you are using. This is especially important for the **"Chocolate Problem"**.
- Collaboration with peers is encouraged, as long as it promotes genuine learning. Best of luck, and happy problem-solving!

Problems

Problem - I
Let X and Y be metric spaces. Y is bounded. Prove that $X \times Y$ is quasi-isometric to X.

Hint: Use the natural map.

Problem - II
Consider $X = \{0, 1, 2, \cdots\}$ with the usual metric. Prove that $\mathrm{Isom}(X)$ is trivial and $\mathrm{QI}(X)$ is infinite.

Remark: $\mathsf{Isom}(X)$ is the set of isometries of X.

Problem - III
Find a metric space such that $\mathrm{Isom}(X)$ is ∞ but $\mathrm{QI}(X)$ is finite.

Hint: Try to use the discrete metric.

Problem - IV
Prove that $\mathbb{Z}$ is not quasi-isometric to F_2.

Remark: F_2 is the free group generated by two elements.

Problem - V (Chocolate Problem)
If Γ and G are quasi isometric groups, then show that $f_\Gamma \approx f_G$.

Remark: Here f denotes the growth function.