$\mathbb{R}/\mathbb{Z} \cong S^1 = \{ z \in \mathbb{C} : |z| = 1 \}$

"0"

$\mathbb{R}/\mathbb{Z}$

$+$

$\{1, 2, 3, \ldots\}$

$3.14 = \underline{\underline{3}} + 0.14$

~ 0.14

$x = \underline{\underline{[x]}} + \{x\}$

$x \sim \{x\} \in [0,1)$

$\underline{\underline{a}}\mathbb{Z} \cong \mathbb{Z}$

$\underline{\underline{[0,1)}}$ $7\mathbb{Z}$

$\mathbb{R}/\mathbb{Z} \xrightarrow[\cancel{\cong}]{\sim} \underline{\underline{[0,1)}} \cong S^1$

$\theta \longrightarrow e^{2\pi\theta i}$

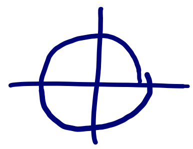
$$\varphi : \mathbb{R} \longrightarrow S^1.$$

$$e^{i\theta_1}, \; e^{i\theta_2} \xrightarrow{(*)} e^{i(\theta_1+\theta_2)}$$

$$x \longmapsto e^{2\pi x i}$$

$$x+y \longmapsto e^{2\pi(x+y)i}$$

$$f(x+y) = \underbrace{e^{2\pi x i}}_{f(x)} * \underbrace{e^{2\pi y i}}_{f(y)}$$



$$\ker \varphi = \{x \in \mathbb{R} : \varphi(x) = 1\}$$

$$= \{x \in \mathbb{R} : \underbrace{e^{2\pi x i}}_{\underbrace{\cos(2\pi x)}_{1} + i\underbrace{\sin(2\pi x)}_{0}} = 1\}$$

$$= \mathbb{Z}.$$

$$G/_{\ker \varphi} \cong \operatorname{Im} \varphi.$$

$\mathbb{R}/\mathbb{Z} \cong S^1.$

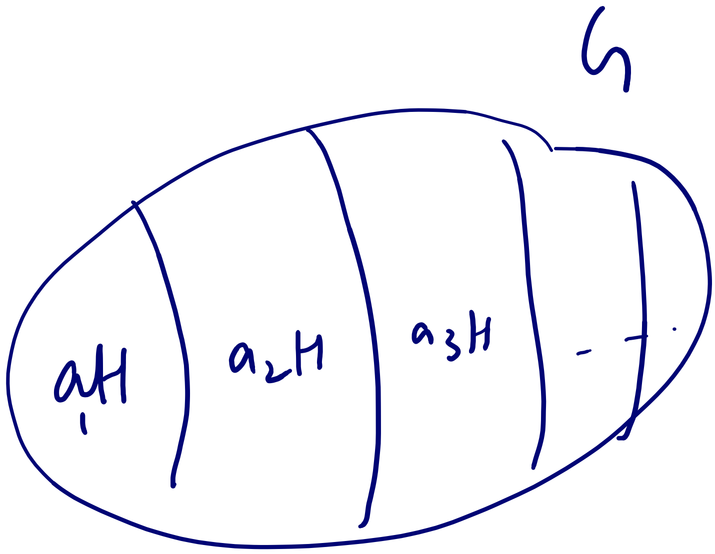
3.

$= \{(x,0) : x \in \mathbb{R}\}.$

$G = \mathbb{R}^2 \qquad H = \mathbb{R}e_1 \cong \mathbb{R}.$

$H \leq G.$

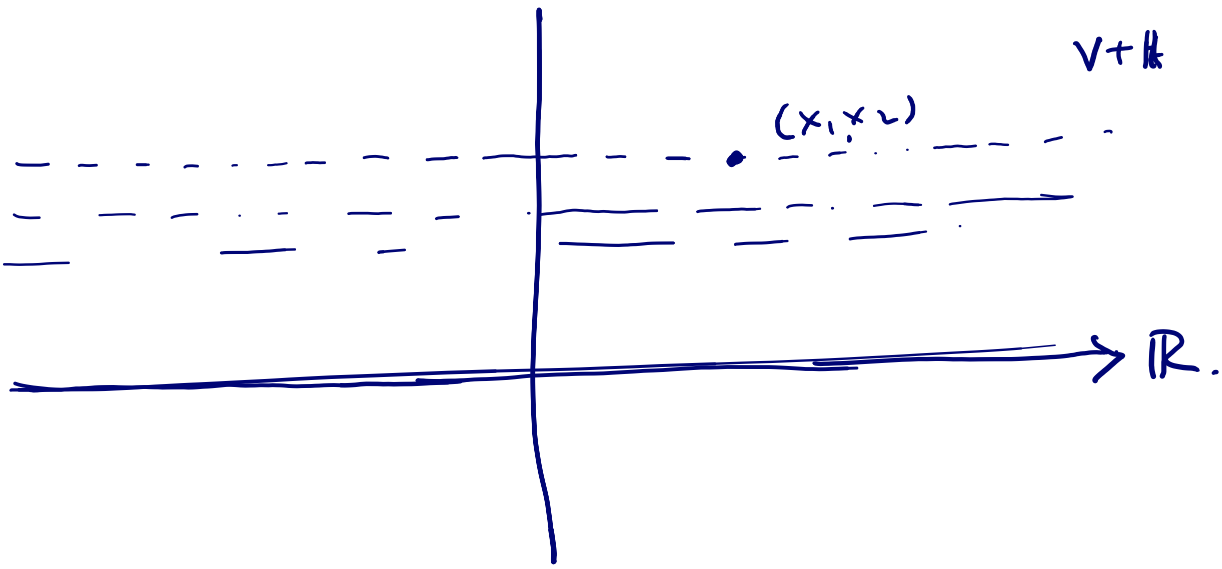
subgrp.



$G/H = \{aH : a \in G\}.$

$\mathbb{R}^2/\mathbb{R} \cong \mathbb{R}.$

$H \cong \mathbb{R} \qquad \mathbb{R}^2$



$\in (x_1, x_2)$

$g + H$

$G = \coprod_{a \in G} aH$

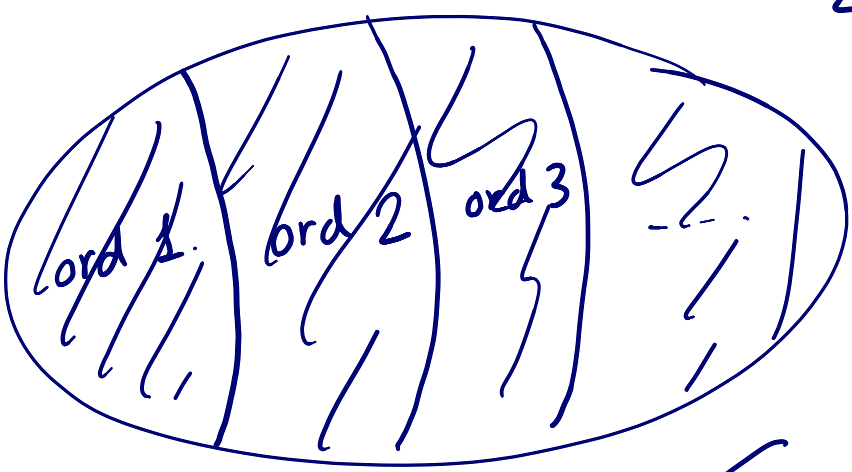
no. of elements having order 'd'

4. $\boxed{\sum_{d|n} \varphi(d) = n}$

$\mathbb{Z}_n$

$(\mathbb{Z}_n, +)$

$\mathbb{Z}_n$.



n $\quad a \in \mathbb{Z}_n$

$a^m, \; m|n$

$$\langle a \rangle = \underline{\underline{\{a^1, a^2, \ldots a^m\}}} \subseteq \mathbb{Z}_n$$

$$a^{m} = e$$

$\underline{\text{minimal}}$

$$\operatorname{ord}(a) = m.$$

$$\langle a \rangle = \{a^1, \ldots a^m\} \leq \mathbb{Z}_n$$

$$\underline{\underline{m \mid n}}.$$

$$a \leftarrow \operatorname{ord}(m).$$

$a^k, \quad k \in \{1, \ldots n\}$

$\operatorname{ord}(m)$

$$\operatorname{ord}(a^k) = ?$$

Exc:

$$\underline{\underline{\operatorname{ord}(a^k)}} = \frac{\operatorname{ord}(a)}{\underbrace{\gcd(\underline{\underline{k}}, m)}_{=1}} = \underline{\underline{\operatorname{ord}(a)}}.$$

$$\| \qquad\qquad\qquad\qquad\qquad m$$

$$\operatorname{ord}(a)$$

$\underline{\underline{\varphi(m)}}$ possible choices for 'k'.

$\gcd(k, m) = 1$.

$\mathbb{Z}_n = \langle x \rangle$

$$\left\{ \begin{array}{l} \mathrm{ord}(a) = m \\ \mathrm{ord} \\ \\ \\ \end{array} \right.$$

$\varphi(m)$

m

$\underline{\underline{a = x^{b}}}$,

$b = x^{r}$
$\in \mathbb{Z}$.

x

$\mathrm{ord}(x) = m$

$\varphi(m)$

$$\left\{ \begin{array}{l} (x^{k_1}) = m \\ (x^{k_2}) = m \\ \vdots \\ (\quad) = m \end{array} \right.$$

$\mathrm{ord}(a) = m.$

$$\mathrm{ord}(a^k) = \frac{\mathrm{ord}(a)}{\gcd(k,m)}$$

$$= \mathrm{ord}(a)$$

$$\mathbb{Z}_n = \{\underbrace{a^1, a^2, \ldots\ldots a^n}\}$$

Exc: If G is cyclic then for any $d|n$, $\exists!$ subgroup of G of size d. ($H \leq G$, is also cyclic).

$$\mathrm{ord}(a) = m$$

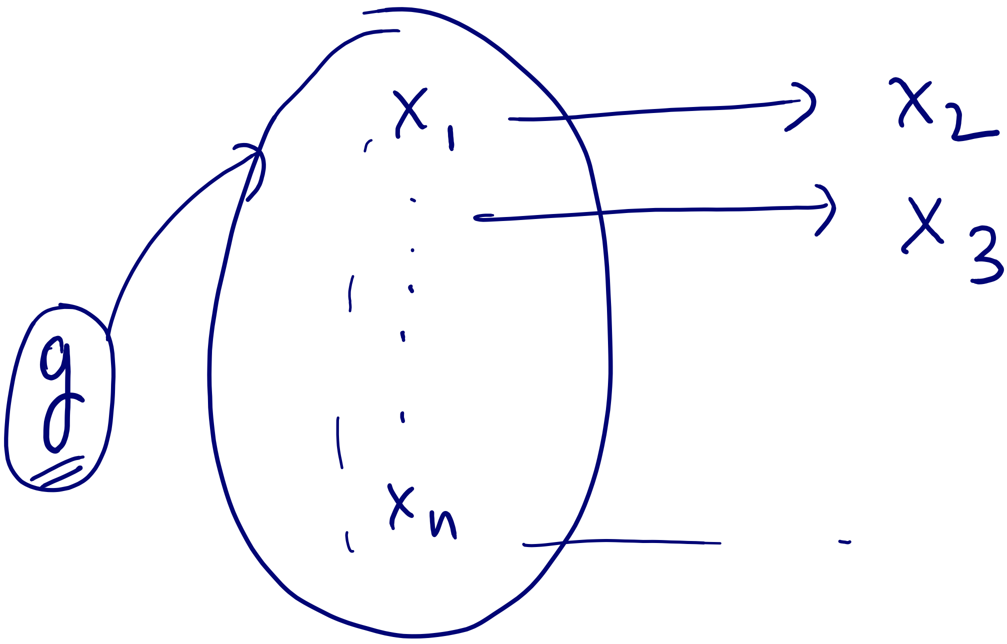
$$\mathrm{ord}(a^k) = t$$

$$(a^k)^t = e$$

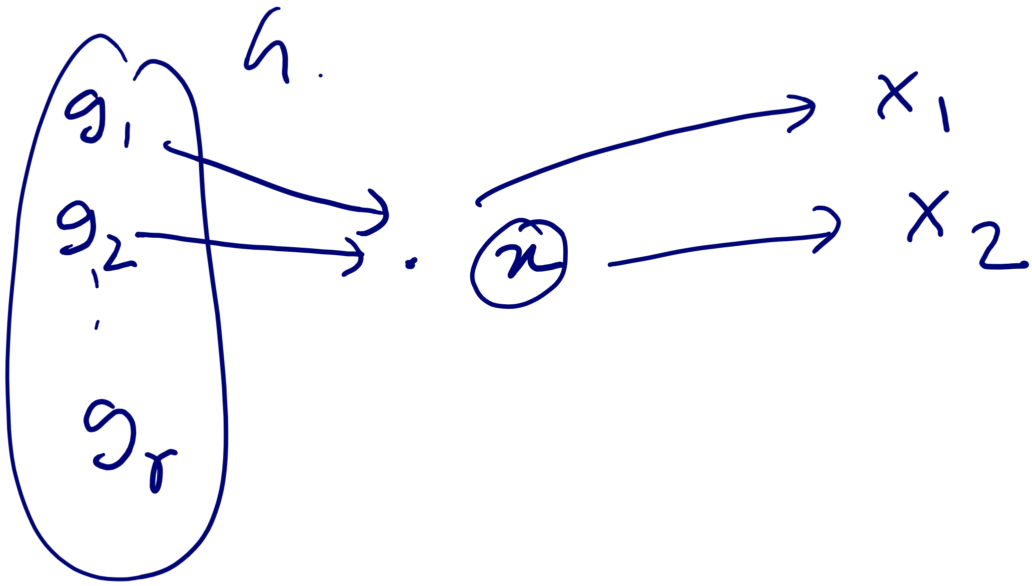
$G \cong K \leq S_n$ (Cayley's thm).

$f(n) \leftarrow$ denotes no of non-iso. grps of order n, find some bounds for $f(n)$.

$G \curvearrowright X$.



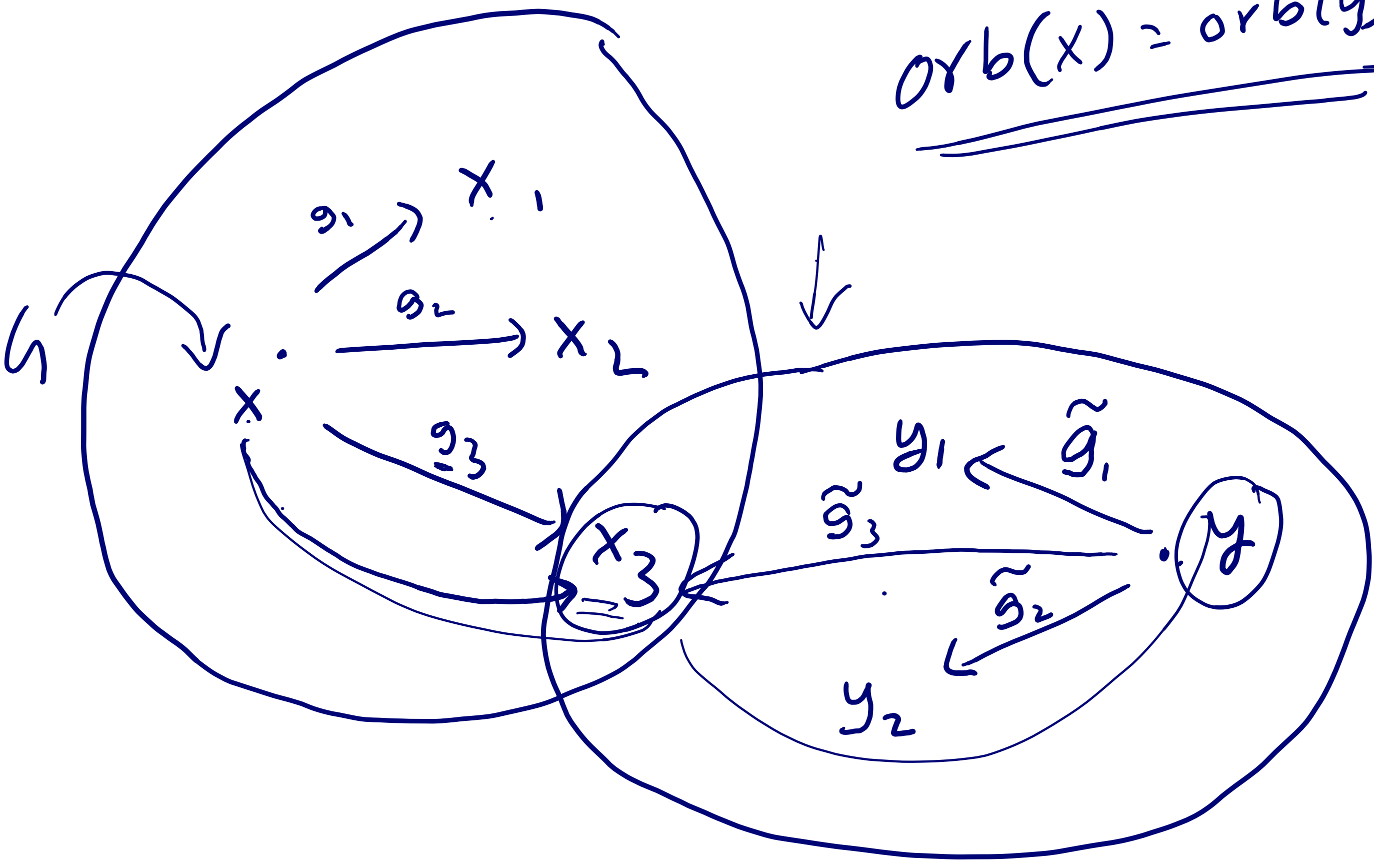
$\underline{\underline{orb(x)}} = \{ g \cdot x : \underline{\underline{g \in G}} \}$



X

$$\boxed{a \sim b \iff a \in orb(b)}$$

$orb(x) = orb(y)$



$H \leq G, \quad |H| \,\big|\, |G|.$

$X \leftarrow$ acting space.

$H \curvearrowright G$

$G \curvearrowright X$

$(h, g) \longmapsto hg$

$$\mathrm{orb}(g) = \{hg : h \in H\}$$

$$= Hg$$

for any g_1, g_2

$$|Hg_1| = |Hg_2|$$

$$f : H \longrightarrow Hg_1$$
$$h \longmapsto hg_1$$

$$G = \coprod_{g_i} g_i H$$

$$|G| = \sum \underbrace{|gH|}_{|H|}$$

$$= k \cdot |H| .$$

$\boxed{G} \Leftarrow \underline{\underline{231}}$

$\uparrow$

ord $= p$

$p \mid |G|$, $a \neq e$. $p \mid |G|$

$X = \{ \underline{\underline{(x_1, \dots x_p)}} \in \underset{=}{G}^p : \underline{\underline{x_1 \dots x_p}} = e \}$

$\uparrow$

$\Downarrow$

$(e, e, \dots e)$ $\quad G \times G \times \dots G$

$|X| = |G|^{p-1}$ $\qquad (x, \dots \dots x) =$

$2, \ldots$

$\mathbb{Z}_p \curvearrowright X.$ $\qquad (x_1, x_2 x_3) \xrightarrow{1} (x_2, x_3, x_1)$

$(\underset{=}{a}, (x_1 \dots x_p)) \longmapsto (x_{1+a}, x_{2+a} \dots x_{p+a})$

$\mathrm{orb}((x_1, \dots, x_p)) = \{$

$$|G| = |\mathrm{orb}(x)| \cdot |\mathrm{stab}(x)|.$$

$$p = \underbrace{|\mathrm{orb}(x)|} \cdot |\mathrm{stab}(x)|.$$

$|\mathrm{orb}(x)| = \cancel{A}$ or p.

$(\underbrace{a, a, \dots, a}) \in X$

$x_i \neq x_j$

$$\mathbb{Z}_p \curvearrowright \underline{X} \qquad |p|^{p-1}$$

$$X = \underline{\underline{pk}} \qquad |a|^{p-1}$$

a

$$\operatorname{orb}((e, \underline{\underline{e, \ldots e}})) = 1.$$

$$\underbrace{e \ldots e} = e$$

$$\underbrace{X} = \underbrace{\operatorname{orb}((e, \ldots e))} + |\operatorname{orb}(\underset{\neq e}{a})|$$

$$\underbrace{|a|^{p-1}} = 1 + \underbrace{p}_{pk}$$

$p \mid$

$G \cong K \leq GL_n(\mathbb{R})$.

$S_n \curvearrowright \mathbb{R}^n$

$v_{1} \longrightarrow v_{f(1)}$

$\Downarrow$

f

$\vec{v} \in \mathbb{R}^n$

$f \cdot (v_1, \dots v_n) \longmapsto (v_{f(1)}, \dots v_{f(n)})$

$f : \{1, \dots n\} \to \{1, \dots n\}$

$\sigma \cdot (e_1 \dots e_n) \longrightarrow (e_{\sigma(1)}, e_{\sigma(2)} \dots e_{\sigma(n)})$

$\mathbb{R}^n$

$G \curvearrowright X \qquad \Phi : G \times X \to X.$

$\downarrow$

$$\varphi : G \longrightarrow Sym(X)$$
$$g \longmapsto \Phi_g$$

$$\varphi_2 : \underset{S_n}{\underset{=}{G}} \longrightarrow GL_n(\mathbb{R}).$$

$|G|$

$$\begin{tikzcd} G \arrow[r, "\varphi_1", "\text{Cayley's thm}"'] \arrow[rr, bend right, "\varphi"'] & S_n \arrow[r, "\varphi_2"] & GL_n(\mathbb{R}) \end{tikzcd}$$

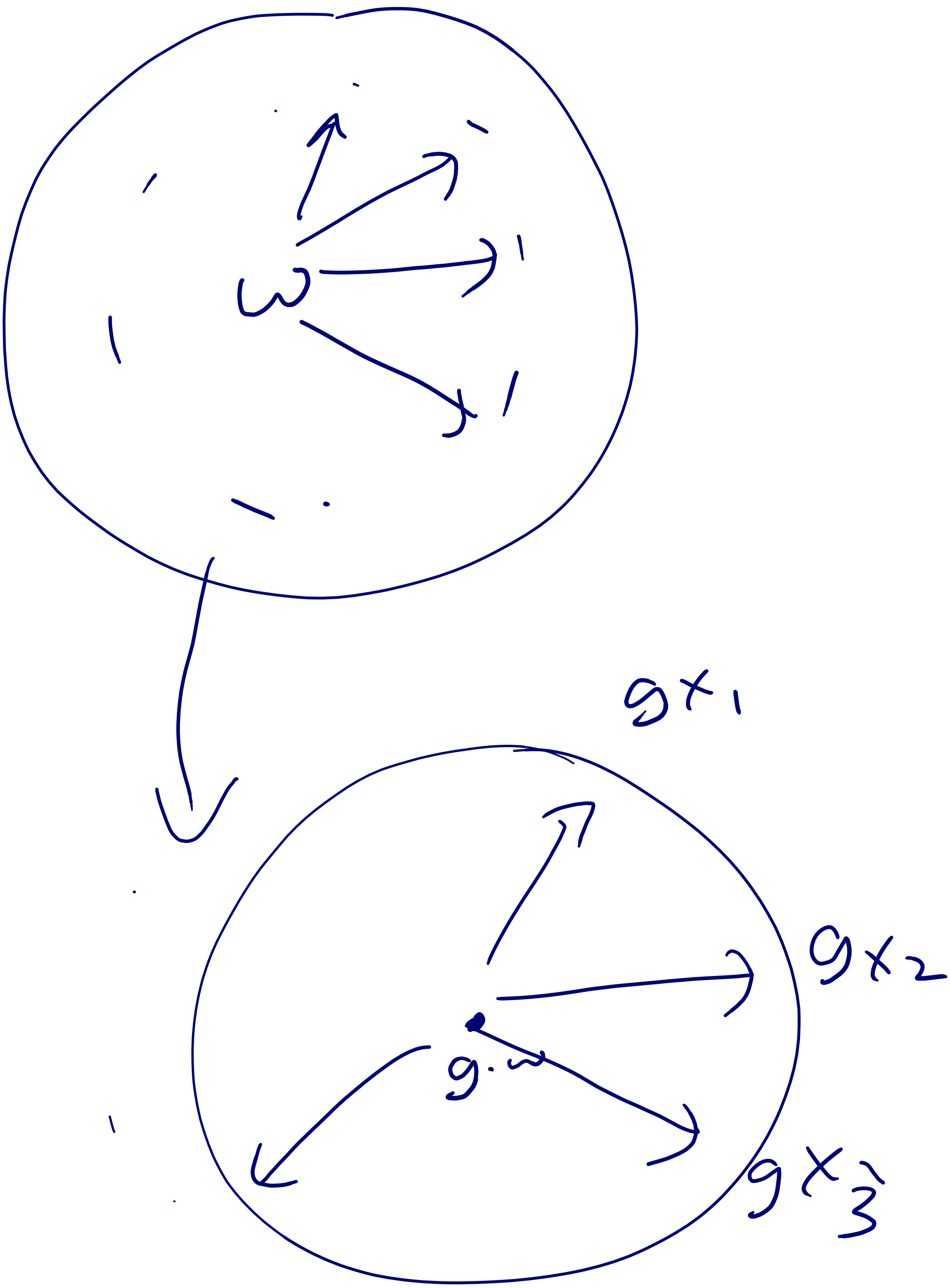
$G/_{\ker\varphi} \cong K \leq GL_n(\mathbb{R}).$

$\underline{\underline{|G|}}$

$\varphi_2 : S_n \longmapsto K$

$\sigma \longmapsto$

$$\hat{w} = \xrightarrow{\text{minimizes}} \sum_{i=1}^{n} d(w, x_i)^{2}$$



$g \cdot w = w$

$\Rightarrow g = e$

G is free $\Leftrightarrow G \overset{\oplus}{\curvearrowright} \Gamma(G, S)$