

1.

$$A = LDL^T$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \underbrace{\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}}_{\begin{pmatrix} d_{11} & d_{11}l_{21} & d_{11}l_{31} \\ 0 & d_{22} & d_{22}l_{32} \\ 0 & 0 & d_{33} \end{pmatrix}} \begin{pmatrix} 1 & l_{21} & l_{31} \\ 0 & 1 & l_{32} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} d_{11} & d_{11}l_{21} & d_{11}l_{31} \\ 0 & d_{22} & d_{22}l_{32} \\ 0 & 0 & d_{33} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{11}l_{21} & d_{11}l_{31} \\ l_{21}d_{11} & d_{11}l_{21}^2 + d_{22} & d_{11}l_{21}l_{31} + d_{22}l_{32} \\ l_{31}d_{11} & d_{11}l_{31}l_{21} + l_{32}d_{22} & d_{11}l_{31}^2 + d_{22}l_{32}^2 + d_{33} \end{pmatrix}$$

$$d_{11} = a_{11}$$

$$d_{11}l_{21} = a_{12} \Rightarrow l_{21} = \frac{a_{12}}{d_{11}} = \frac{a_{12}}{a_{11}}$$

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$$d_{11}l_{31} = a_{31} \Rightarrow l_{31} = \frac{a_{31}}{d_{11}} = \frac{a_{31}}{a_{11}}$$

$$d_{11}l_{31} = a_{13} \Rightarrow l_{31} = \frac{a_{13}}{d_{11}} = \frac{a_{13}}{a_{11}}$$

Por ahora, vemos que se cumple: $d_{jj} = a_{jj}$, $j=1$ y $l_{ij} = \frac{a_{ij}}{d_{jj}}$ con $1 \leq i \leq 3$ $j=1$

$$a_{22} = d_{11}l_{21}^2 + d_{22} \Rightarrow d_{22} = a_{22} - d_{11}l_{21}^2$$

$$a_{32} = d_{11}l_{31}l_{21} + l_{32}d_{22} = a_{32} \Rightarrow l_{32} = \frac{a_{32} - d_{11}l_{31}l_{21}}{d_{22}}$$

Ahora, podemos extraer las sig. fórmulas:

$$d_{jj} = a_{jj} - d_{j-1,j-1} l_{j,j-1}^2 \quad \text{y} \quad l_{ij} = \frac{a_{ij} - d_{j-1,j-1} l_{i,j-1} l_{j,j-1}}{d_{jj}} \\ \text{para } 1 \leq j \leq 2 \quad \text{y} \quad \text{para } 1 \leq i \leq 3 \quad \text{y} \quad 1 \leq j \leq 2$$

$$a_{33} = d_{11} l_{31}^2 + d_{22} l_{32}^2 + d_{33} \Rightarrow d_{33} = a_{33} - d_{11} l_{31}^2 - d_{22} l_{32}^2$$

Vemos que:

$$a_{jj} = a_{jj} - d_{j-2,j-2} l_{jj-2}^2 - d_{j-1,j-1} l_{jj-1}^2 \quad 1 \leq j \leq 3$$

Así, llegamos a

$$a_{jj} = a_{jj} - \sum_{k=1}^{j-1} d_{kk} l_{jk}^2 \quad \left. \vphantom{\sum_{k=1}^{j-1}} \right\} \text{ fórmula para } D.$$

y con

$$l_{ij} = \frac{a_{ij}}{d_{jj}} \quad \text{para } j=1 \quad 2 \leq i \leq 3$$

$$l_{ij} = \frac{a_{ij} - d_{j-1,j-1} l_{ij-1} l_{jj-1}}{d_{jj}}$$

$$j=2, i=3$$

Podemos llegar a que se cumple lo siguiente:

$$l_{ij} = \frac{1}{d_{jj}} \left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{jk} d_{kk} \right), \quad 2 \leq i \leq 3, \quad 1 \leq j \leq 2$$

fórmula para L.