1. **Describe the following concepts in the context of logical reasoning as precisely and compact as possible:**

• **Enumeration**:

*Enumeration is going through all possible models of a given statement to check for truth values. This can done using a truth table where you arrange models so as to capture all possible combinations of the symbols .*

• **Validity**. **I.e. when is a sentence valid/invalid**:

*A valid sentence is one that is true for all the models(a structured world where the truth of false of a sentence can be evaluated)*

• **Satisfiability. I.e. when is a sentence satisfiable/ when is it unsatisfiable?**

*If at least one model proves to be true the statement is said to be satisfiable*

• **CNF and DNF. Give an example for each**

*CNF : Conjunctive Normal form ; A statement or a sentence that is expressed as conjunction(∧) of multiple clauses(multiple AND clauses)*
*eg:* $(A \vee B) \wedge (\neg B \vee C)$

*DNF: Disjunctive Normal Form; A logical sentence that is expressed as dis junction of multiple multiple clauses(multiple OR clauses)*
*eg:* $(A \wedge B) \vee (\neg B \wedge C)$

• **Resolution**:

*Resolution is a inference rule that can be used iteratively solve to prove the satisfiability of a statement. To perform this, first convert the statement to conjunctive normal form. Then if we want to show that the knowledge base entails the statement* $\alpha$ *(*$KB \models \alpha$*), we do this by iteratively finding pairs of* $\alpha$ *until we can prove that* $KB \models \neg\alpha$ *is unsatisfied.*

## 2. Consider the following Implication:

$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party]$

a) **Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable**

**Let T=Truth and F= False**

| Food (F) | Party (P) | Drinks (D) | $F \Rightarrow P$ | $D \Rightarrow P$ | $F \wedge D$ | (F $\Rightarrow P$) $\vee$ (D $\Rightarrow P$) | $(F \wedge D) \Rightarrow P$ | $[(\text{F} \Rightarrow P) \vee (D \Rightarrow P)] \Rightarrow [(\text{F} \wedge D) \Rightarrow P]$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | T |
| T | T | F | T | T | F | T | T | T |
| T | F | T | F | F | T | F | F | T |
| T | F | F | T | T | F | T | T | F |
| F | T | T | T | T | F | T | T | T |
| F | T | F | T | T | F | T | T | T |
| F | F | T | T | F | F | T | T | T |
| F | F | F | T | T | F | T | T | T |

Since for all models the sentence is true this is valid.

(b) **Convert the left-hand and right-hand sides of the main implication into CNF separately, showing each step, and explain how the results confirm your answer.**

$$[(\mathrm{F} \Rightarrow P) \vee (D \Rightarrow P)] \Rightarrow [(F \wedge D) \Rightarrow P]$$

**Reducing the implements sign using implication elimination**

$$[(\neg F \vee P) \vee (\neg D \vee P)] \Rightarrow [\neg(F \wedge D) \vee P]$$

**Using De morgen's law on right hand of implements sign**

$$[(\neg F \vee P) \vee (\neg D \vee P)] \Rightarrow [(\neg F \vee \neg D) \vee P]$$

**Impliment elimination**

$$\neg[(\neg F \vee P) \vee (\neg D \vee P)] \vee [(\neg F \vee \neg D) \vee P]$$

**Using De morgen's law**

$$\neg(\neg F \vee P) \wedge \neg(\neg D \vee P) \vee [(\neg F \vee \neg D) \vee P]$$

**Using De morgen's law and double negation elimination**

$$(\mathrm{F} \wedge \neg P) \wedge (D \wedge \neg P) \vee [(\neg F \vee \neg D) \vee P]$$

**Using associativity of** $\vee$

$$(\mathrm{F} \wedge \neg P) \wedge (D \wedge \neg P) \vee [(\neg F \vee (\neg D \vee P)]$$

**Elimination of duplicates P**

$$(\mathrm{F} \wedge D \wedge \neg P) \vee (\neg F \vee \neg D \vee P)$$

(c) **Prove your answer to (a) using resolution.**

| Food (F) | Party (P) | Drinks (D) | $\neg F$ | $\neg P$ | $\neg D$ | $F \wedge D \wedge \neg P$ | $\neg F \vee \neg D \vee P$ | (F $\wedge D \wedge \neg P)$ $\vee$ $(\neg F \vee \neg D \vee P)$ |
|---|---|---|---|---|---|---|---|---|
| F | F | F | T | T | T | F | T | T |
| F | F | T | T | T | F | F | T | T |
| F | T | F | T | F | T | F | T | T |
| F | T | T | T | F | F | F | T | T |
| T | F | F | F | T | T | F | T | T |
| T | F | T | F | T | F | T | F | T |
| T | T | F | F | F | T | F | T | T |
| T | T | T | F | F | F | F | T | T |

**3. Let A, B and C be propositional formulas such that A and B entails C** $(e.g A \wedge B \rightarrow C)$**. Then, is** $(A \rightarrow C) \vee (B \rightarrow C)$ **always true?**

$$A \vee B \Rightarrow C$$

Reducing the implements sign using implication elimination

$$\neg(A \vee B) \vee C$$

Expanding bracket using demorgens law

$$(\neg A \wedge \neg B) \vee C$$

Using distribuitivity of $\vee over \wedge$

$$(\neg A \vee C) \wedge (\neg B \vee C)$$

Using reverse of implication eliminiation

$$(A \Rightarrow C) \vee (B \Rightarrow C)$$

| A | B | C | $A \wedge B$ | $A \Rightarrow C$ | $B \Rightarrow C$ | $(A \wedge B) \Rightarrow C$ | $(A \Rightarrow C) \vee (B \Rightarrow C)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | F | F | T |
| T | F | T | F | F | T | T | T |
| T | F | F | F | F | T | T | T |
| F | T | T | F | T | T | T | T |
| F | T | F | F | T | F | T | T |
| F | F | T | F | T | T | T | T |
| F | F | F | F | T | T | T | T |

$(A \rightarrow C) \vee (B \rightarrow C)$ is always true(proved by Truth Table)

4. **Prove the following formulas:**

Let T=Truth, F=False

- $\neg P \wedge \neg Q \Longleftrightarrow \neg(P \vee Q)$

*Demorgen's law state that* $\neg P \wedge \neg Q \equiv \neg(P \vee Q)$ *therefore we can say that above equation holds true bidirectionally under Demorgen's law*

| P | Q | $\neg P$ | $\neg Q$ | $P \vee Q$ | $\neg P \wedge \neg Q$ | $\neg(P \vee Q)$ | $\neg P \wedge \neg Q \Longleftrightarrow \neg(P \vee Q)$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F | T |
| T | F | F | T | T | F | F | T |
| F | T | T | F | T | F | F | T |
| F | F | T | T | F | T | T | T |

- $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

*Demorgen's law state that* $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ *therefore we can say that above formula is true bidirectionally*

| P | Q | $\neg P$ | $\neg Q$ | $P \wedge Q$ | $\neg(P \wedge Q)$ | $\neg P \vee \neg Q$ | $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F | T |
| T | F | F | T | F | T | T | T |
| F | T | T | F | F | T | T | T |
| F | F | T | T | F | T | T | T |

- $P \vee (P \wedge Q) \Leftrightarrow P$

  Using distributivity of $\vee$ over $\wedge$

| P | Q | $P \wedge Q$ | $P \vee (P \wedge Q)$ | $P \vee (P \wedge Q) \Leftrightarrow P$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | F | F | T |
| F | F | F | F | T |

References :

Russell, Stuart and Norvig, Peter. Artificial Intelligence: A Modern Approach