

① `snapshot()`  $\rightarrow \Theta(n)$

The time complexity of this function is linear. The use of `vector::clear` function produces a linear time complexity due to its dependence on the size of the vector. The statement `buzzers = the_queue` also results in a linear time complexity. This is because the vector class overrides the '=' operator and performs an element-by-element copy, which is also dependent on the size.  
Hence,  $n + n = 2n \Rightarrow \Theta(n)$ .

② `length()`  $\rightarrow \Theta(1)$

Using the `vector::size()` results in a constant time complexity.  
Hence,  $1 \Rightarrow \Theta(1)$

③ `give_buzzer()`  $\rightarrow \Theta(1)$

In the if condition, vector member function `size()`, `back()`, and `pop_back()` were utilized. All these member functions results in a constant time complexity.

$$1(1+1) = 2 \Rightarrow \Theta(1)$$

The else statement only has one constant statement, therefore its time complexity is also  $\Theta(1)$ .

Lastly, the `vector::push_back()` function has an amortized constant time complexity ( $\Theta(1)$ ).

$$\text{Hence, } 1 + 1 + 1 = 3 \Rightarrow \Theta(1)$$

④ `seat()`  $\rightarrow \Theta(n)$

The if statement uses a `vector::size()` function with a constant time complexity  $\Theta(1)$ . However, in the else statement a `vector::erase()` function. This function erases a number of elements depending on the given range. Therefore, its time complexity will be linear.

$$\text{Hence, } 1 + n = n \Rightarrow \Theta(n)$$

⑤ `kickout()`  $\rightarrow \Theta(n)$

The `find()` function's time complexity is up to linear. Therefore, its worst-case scenario is  $O(n)$ . The if statement runs on a constant time complexity. However, the else statement is dominated by the  $\Theta(n)$  because of the use of `vector::erase()`. This function uses an iterator to move to the right to-be-deleted position.

$$\text{Hence, } n + 1 + n \times 1 = 2n \Rightarrow \Theta(n)$$

⑥ `take_bribe()`  $\rightarrow \Theta(n)$

This function has a similar runtime as `kickout()`. The only difference is that the else statement contains an `insert()` function with a time complexity of  $O(n)$ .

$$\text{Hence, } n + 1 + (n+n) \times 1 = 3n \Rightarrow \Theta(n)$$