

## **2024 Senate and House of Representatives Candidates' Wikipedia Hyperlink Graphs Analysis**

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# Summary

This study investigates the network characteristics and community structures of the 2024 U.S. Senate and House candidates based on their Wikipedia pages, utilizing graph theory and network analysis techniques. By constructing and analyzing directed hyperlink graphs, we examined the connectivity and assortativity among candidates, focusing on attributes such as incumbency, party affiliation, state, and candidacy status. Our findings reveal distinct patterns of connectivity and reciprocity within the Senate and House candidate networks. In particular, we identify the presence of tightly-knit communities and significant variations in candidate connectivity across different states and political affiliations. Through visualizations including Sankey diagrams and adjacency matrix heatmaps, we provide a comprehensive overview of the underlying structures and attribute distributions within these political candidate networks. This analysis contributes to a deeper understanding of the digital presence and interconnectivity of political figures, offering insights into the dynamics of online political representation.

## 1. Data Collection

Candidates for the U.S. Senate and U.S. House of Representatives 2024 elections were scraped from their respective Ballotpedia pages. Ballotpedia ([https://ballotpedia.org/Main\\_Page](https://ballotpedia.org/Main_Page)) is a nonprofit and nonpartisan online political encyclopedia that covers elections and public policy in the United States. The desired candidates lists were located within tables organized by state and contained information such as candidate name, party affiliation, office and candidacy status. Python's "requests" and "BeautifulSoup" libraries were utilized to scrape the Ballotpedia webpages and extract the tables from the raw HTML content.

After identifying the required tables, additional preprocessing steps were necessary to clean the tables columns and append individual state's tables into two Python dataframes: one

for Senate candidates (dfs) and one for the House candidates (dfh). This process included extracting the “Incumbent” tag from within the candidate’s name column and introducing a new “inc incumbency” column. This involved identifying instances where candidates were tagged as “Incumbent” in their names, removing these tags, and creating a separate column to denote incumbency status.

A key aspect of this research involved collecting the Wikipedia page of each candidate. To achieve this, using the wikipediaapi Python module (<https://github.com/martin-majlis/Wikipedia-API>), a function was created to check if a Wikipedia page for a given name existed. However, finding a page did not imply that it correctly corresponded to the candidate, necessitating additional validation and processing steps. This initial automated approach helped identify potential Wikipedia pages associated with each candidate’s name.

To ensure the Wikipedia pages identified were correct, we performed the following additional processing: manual verification and cross-referencing with other sources to confirm the accuracy. Wikipedia itself had a page for both the 2024 Senate ([https://en.wikipedia.org/wiki/2024\\_United\\_States\\_Senate\\_elections](https://en.wikipedia.org/wiki/2024_United_States_Senate_elections)) and House of Representative ([https://en.wikipedia.org/wiki/2024\\_United\\_States\\_House\\_of\\_Representatives\\_elections](https://en.wikipedia.org/wiki/2024_United_States_House_of_Representatives_elections)) elections, which provided links to a candidate’s personal wikipedia page if it existed. The tables in these Wikipedia pages were not as thorough or updated as the Ballotpedia tables, but served as an initial automated method to cross-reference results found from searching Wikipedia for the candidates’ names.

For candidates who did not have Wikipedia pages identified through automated means, further efforts were made to manually search and verify their pages. This involved using

variations of their names and checking additional sources. This thorough process ensured that the dataset included the most accurate and comprehensive information possible.

Through this systematic approach, we ensured a comprehensive collection of candidate information, facilitating the construction of a robust network analysis based on the available Wikipedia pages for 2024 U.S. Senate and House of Representatives candidates. Next, we will discuss the creation of Senate and House hyperlink graphs using the validated Wikipedia pages for the collected data.

## 2. Creation of Hyperlink Graphs

The creation of hyperlink graphs is a crucial step in visualizing the relationships and connections among the candidates for the 2024 U.S. Senate and House of Representatives elections. By mapping out the hyperlinks between the Wikipedia pages of the candidates, we can analyze the interconnectedness and network structure of the political landscape.

### 2.1. Obtaining Edge Values

To begin, the validated Wikipedia pages for each candidate were used as the basis for constructing each hyperlink graph. Each Wikipedia page in both dataframes was inspected to extract hyperlinks pointing to other Wikipedia pages. These hyperlinks represent connections between different candidates. To obtain these hyperlinks, a for-loop utilizing the “wikipediaapi.Wikipedia” Python library was used to systematically traverse each candidate’s Wikipedia page and compile a list of hyperlinks. Within the loop, we retrieved, sorted, and filtered out self-referencing links shown on each candidate’s page. We stored the edge pairs in the hyperlink list when we found intersecting links with the validated Wikipedia pages. This approach allowed us to identify and capture all outgoing hyperlinks between candidates’ Wikipedia pages to be used as the edges in our network graph.

## 2.2. Building the Graphs

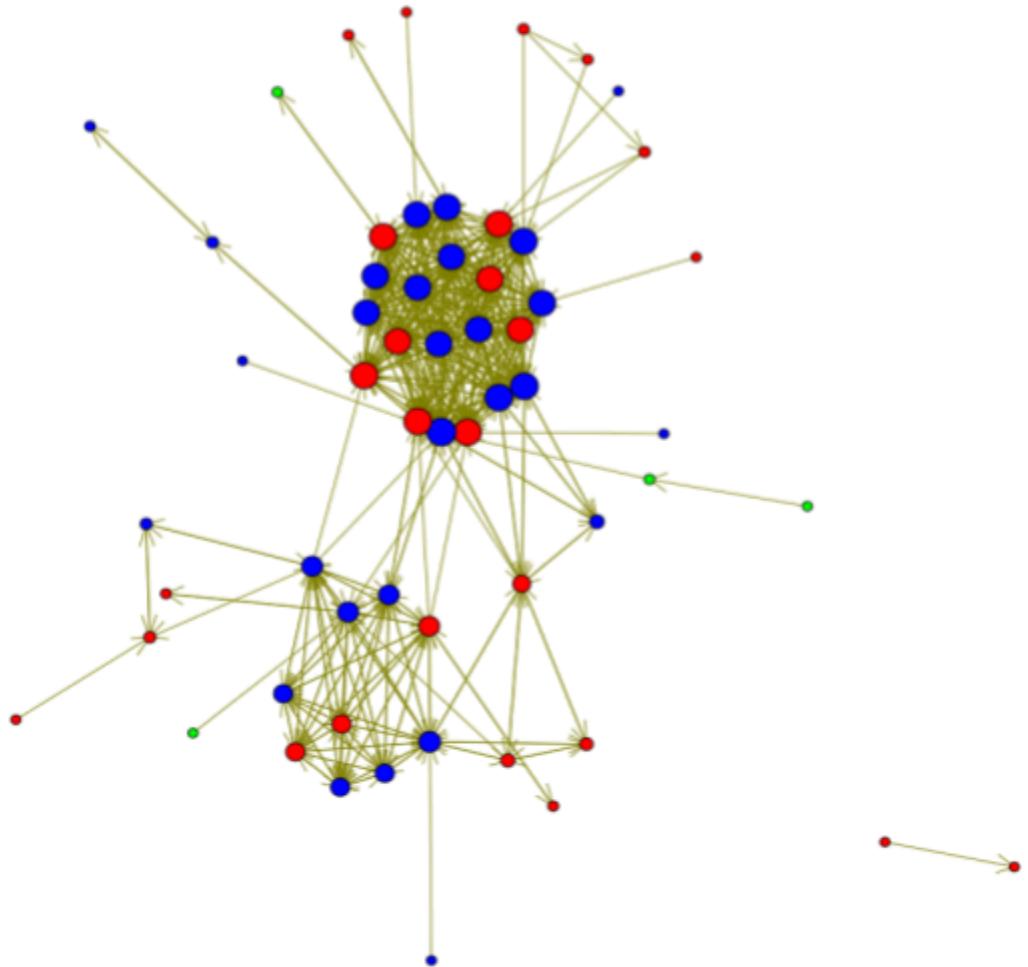
The two hyperlink graphs were constructed using the NetworkX library in Python (<https://networkx.org/>), which is designed for the creation, manipulation, and study of complex networks. Both the Senate candidate's graph ( $G_s$ ) and the House candidate's graph ( $G_h$ ) were initiated as a NetworkX DiGraph (directed graph). Each candidate was represented by his/her Wikipedia page as a node in their respective graph, and the hyperlinks between their Wikipedia pages (nodes) were represented as edges connecting these nodes.

- **Nodes:** Each candidate with a validated Wikipedia page was added as a node in the graph. Additional attributes such as party affiliation, office, incumbency status, and state were assigned to each node to provide context and facilitate further analysis.
- **Edges:** Hyperlinks extracted from the Wikipedia pages were used to create directed edges between nodes. These edges indicate a hyperlink from one candidate's Wikipedia page to another, capturing the nature of their interconnectedness.

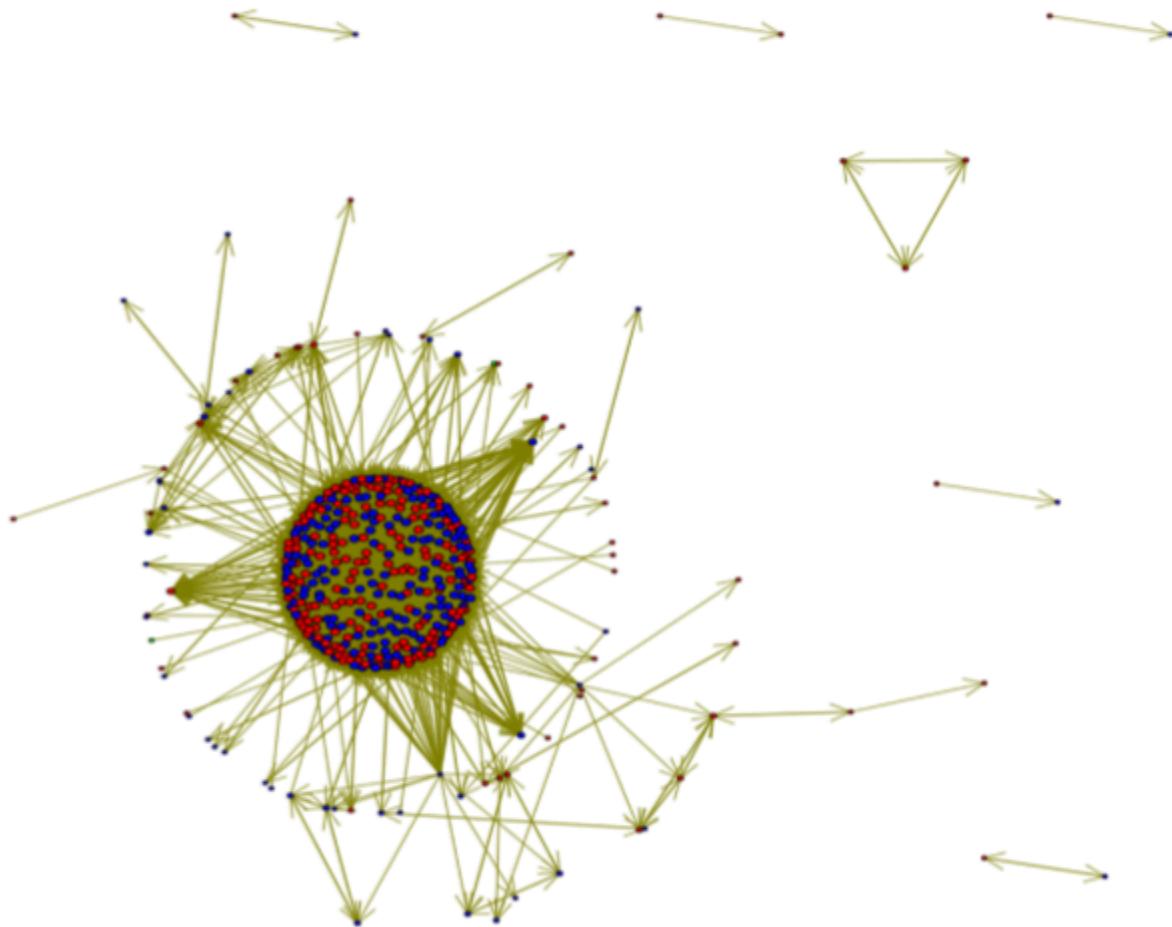
## 2.3. Visualizations

The constructed graphs were visualized using Python's "Holoviews" library (<https://www.holoviews.org/>). Figures 1 and 2 display the hyperlink graphs of the Senate and House candidates' Wikipedia pages, respectively. Different visual attributes were used to enhance the clarity and interpretability of the graphs. Nodes were sized by in-degree attribute and colored by party affiliation.

**Fig 1: Senate Candidates Hyperlink Graph of Wikipedia Pages**



**Fig 2: House Candidates Hyperlink Graph of Wikipedia Pages**



### 3. Connectivity of Hyperlink Graphs

In graph theory, connectivity is a fundamental concept that describes how nodes are connected to one another within a graph

([https://en.wikipedia.org/wiki/Connectivity\\_\(graph\\_theory\)](https://en.wikipedia.org/wiki/Connectivity_(graph_theory))). There are different types of connectivity depending on whether the graph is directed or undirected, each providing insights into the graph's properties and behavior. Before describing the connectedness of the Senate and House hyperlink graphs, we define these concepts.

### 3.1. Connectivity in Undirected Graphs

In an undirected graph, edges have no direction, meaning the connection between any two nodes is bidirectional.

- **Connected Component:** A connected component in an undirected graph is a subgraph in which any two nodes are connected to each other by paths, and which is connected to no additional nodes in the supergraph.
- **Giant Connected Component:** The giant connected component is the largest connected subgraph that typically contains a substantial proportion of the entire undirected graph's vertices and edges.

### 3.2. Connectivity in Directed Graphs

In directed graphs, edges have directions, indicating the relationship flows from one node to another. This gives rise to more nuanced concepts of connectivity: weak connectivity and strong connectivity.

- **Weakly Connected Component:** A weakly connected component is a subgraph where if the direction of edges is ignored, the subgraph is connected. This means that replacing all directed edges with undirected edges results in a connected component.
- **Giant Weakly Connected Component:** This is the largest weakly connected subgraph in the directed graph.
- **Strongly Connected Component:** A strongly connected component is a subgraph where for every pair of nodes  $(u, v)$  in the graph, there is a directed path from  $u$  to  $v$  and also another directed path from  $v$  to  $u$ .
- **Giant Strongly Connected Component:** This is the largest strongly connected subgraph in the directed graph.

### 3.3. Connectivity of Candidate Hyperlink Graphs

In the tables below, the enumeration column corresponds to the decreasing order of sizes of the corresponding connected components. We observe that the weakly connected component subgraphs contain more candidates compared to the strongly connected component subgraphs in both the Senate and the House, which is consistent with their definitions.

**Table 1: Table of Weakly Connected Component Subgraphs for Senate Candidates**

Enumeration	No. of candidates	No. vertices in subgraph	No. edges in subgraph
0	56	56	580
1	2	2	1

**Table 2: Table of Strongly Connected Component Subgraphs for Senate Candidates**

Enumeration	No. of candidates	No. vertices in subgraph	No. edges in subgraph
0	43	43	564

**Table 3: Table of Weakly Connected Component Subgraphs for House Candidates**

Enumeration	No. of candidates	No. nodes in subgraph	No. edges in subgraph
0	429	429	116708
1	3	3	6
2	2	2	1
3	2	2	1
4	2	2	1
5	2	2	2
6	2	2	2

**Table 4: Table of Strongly Connected Component Subgraphs for House Candidates**

Enumeration	No. of candidates	No. nodes in subgraph	No. edges in subgraph
0	386	386	116263
1	4	4	8
2	3	3	6
3	3	3	6
4	2	2	2
5	2	2	2
6	2	2	2
7	2	2	2

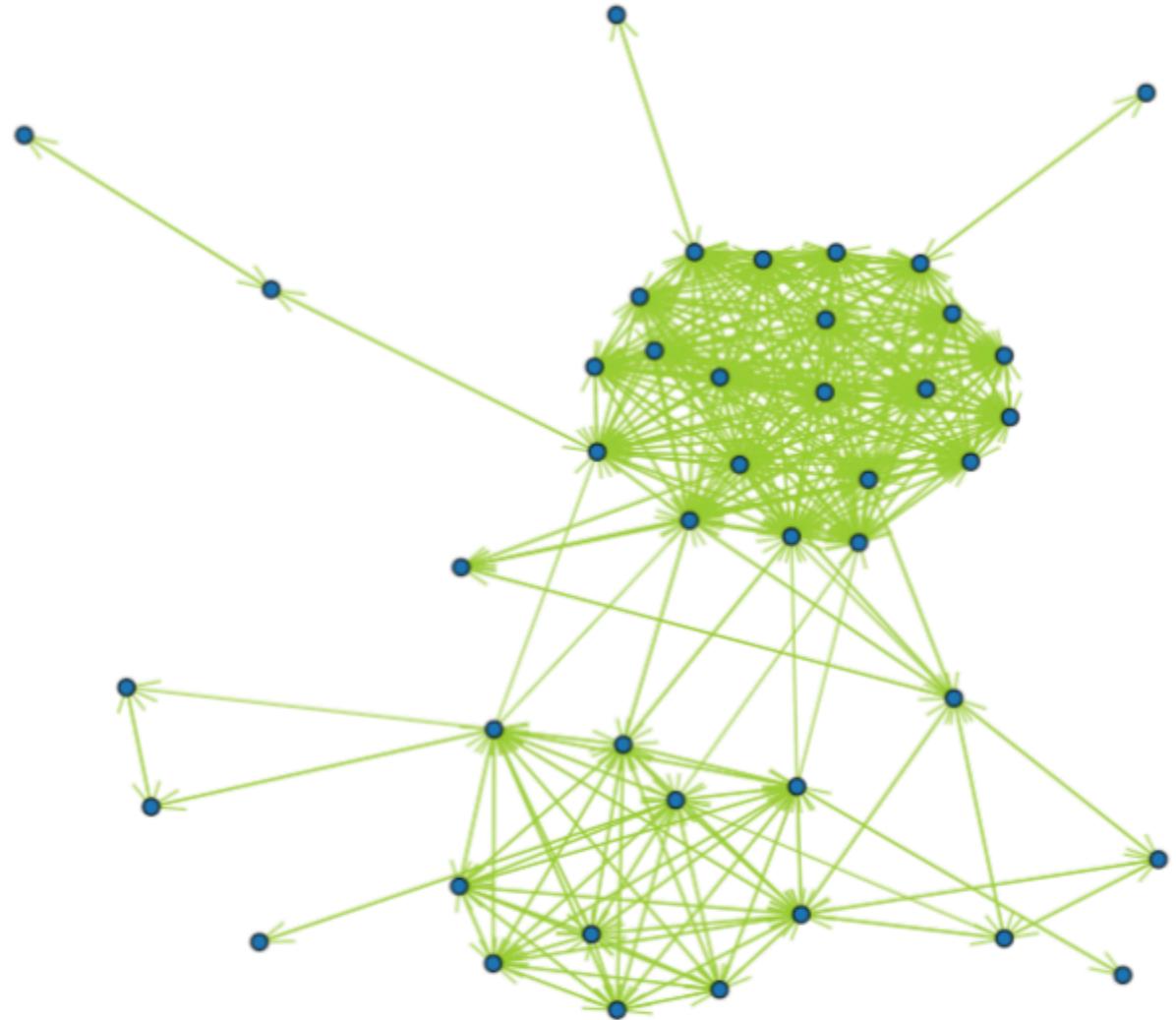
### 3.4. Plots of Candidate Hyperlink Graphs

**Fig 3: Giant Weakly Connected Component of the Senate Candidates**



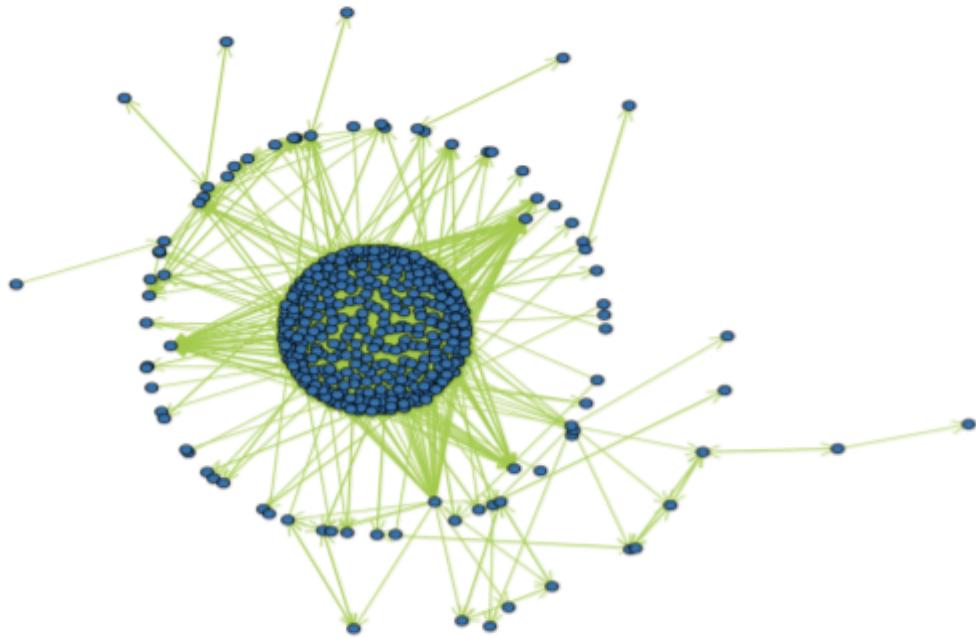
Giant weakly connected component of Senate Candidates graph has 56 nodes and 580 edges.

**Fig 4: Giant Strongly Connected Component of Senate Candidates**



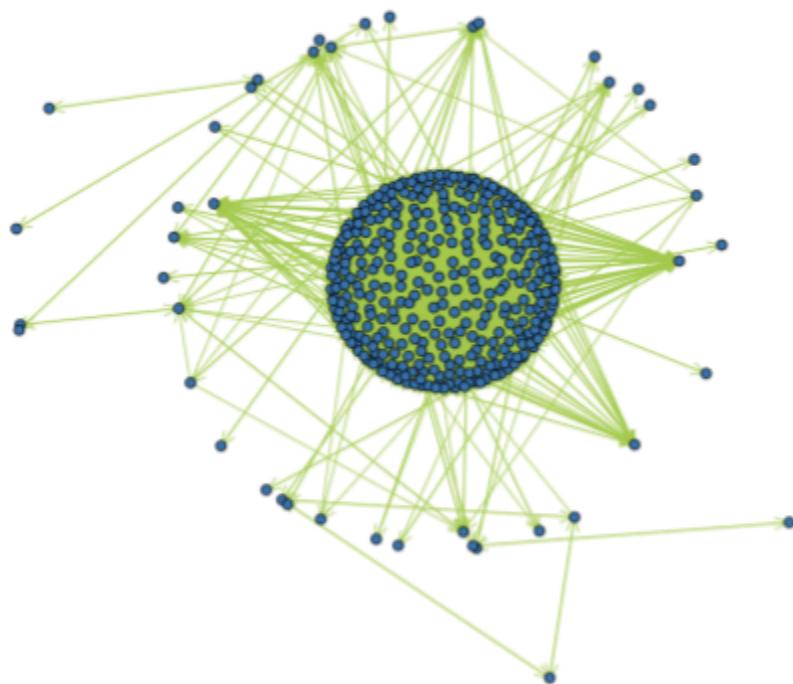
Giant strongly connected component of Senate Candidates graph has 43 nodes and 564 edges.

**Fig 5: Giant Weakly Connected Component of the House Candidates**



Giant weakly-connected component of the House candidates graph contains 429 nodes and 116,708 edges.

**Fig 6: Giant Strongly Connected Component of the House Candidates**



Giant strongly-connected component of the House candidates graph contains 386 nodes and 116,263 edges.

## 4. Reciprocated Subgraphs of Candidate Hyperlink Graphs

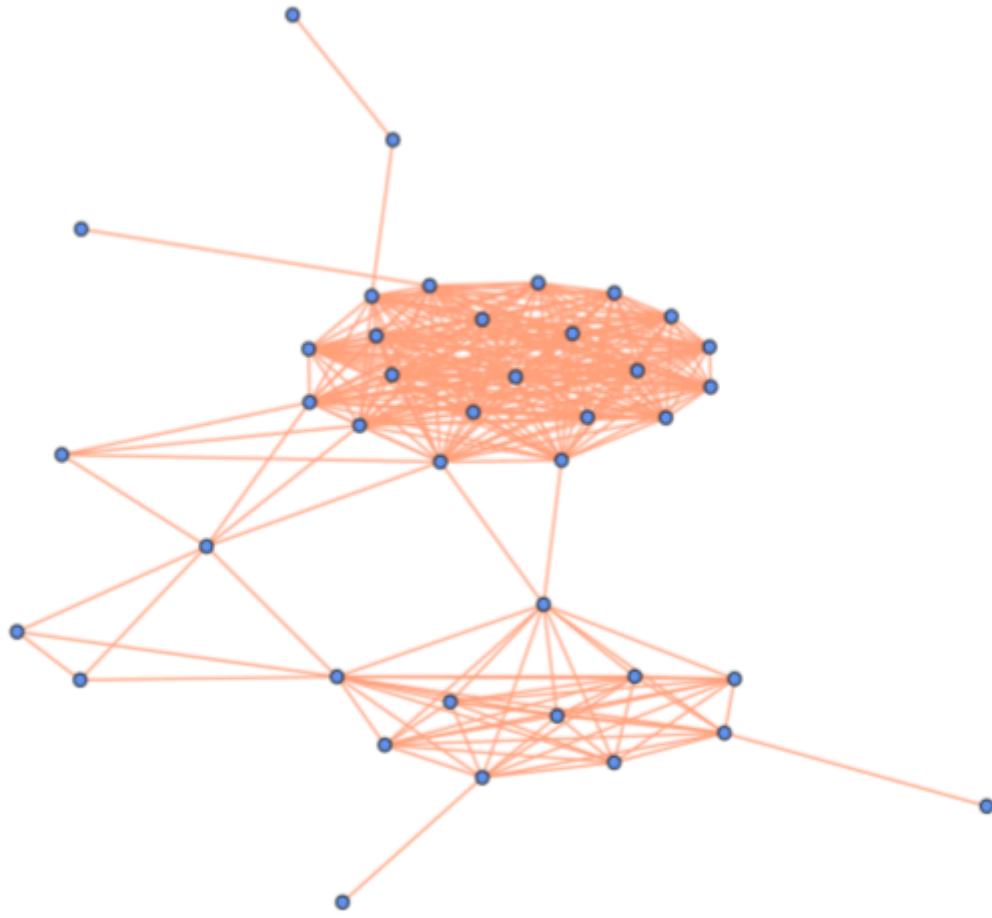
Evaluating reciprocity in directed graphs provides insight into the mutual connections between nodes. Concerning the case of candidate hyperlink graphs, to begin with, we created the reciprocated hyperlink subgraphs for both Senate (Gs) and House (Gh) candidates. Reciprocity in this context refers to pairs of candidates whose Wikipedia pages contain hyperlinks to each other. The reciprocated hyperlink subgraphs capture these mutual connections.

In a directed graph, a reciprocated edge is defined as one where, if there is a directed edge from node A to node B, there is also a directed edge from node B to node A (<https://www.smrfoundation.org/networks/overall-metrics-defined/>). A reciprocated graph is a subgraph consisting solely of reciprocated edges (conventionally identified as an undirected type of graph). To identify reciprocated edges for both the Gs and Gh, we iterated through all the edges in each graph and checked for a corresponding edge in the opposite direction. Using the NetworkX library, we created the reciprocated graphs, denoted as recGs and recGh, by selecting the subgraph produced from the identified reciprocated edges. To quantify reciprocity, we calculated the reciprocity coefficient, which is defined as the ratio of the number of reciprocated edges to the total number of edges in the graph ([https://en.wikipedia.org/wiki/Reciprocity\\_\(network\\_science\)](https://en.wikipedia.org/wiki/Reciprocity_(network_science))). This metric provides insight into the level of mutual pairwise connectivity within the network.

### 4.1. Reciprocity of Senate Candidates (recGs)

The reciprocity coefficient of Gs is 0.953, which represents a substantially high level of mutual pairwise connectivity among the Senate candidates' Wikipedia pages. The reciprocated graph of the US Senate candidates is composed of 43 nodes and 278 edges.

**Fig 7: Reciprocated Hyperlink Graph for Senate Candidates**

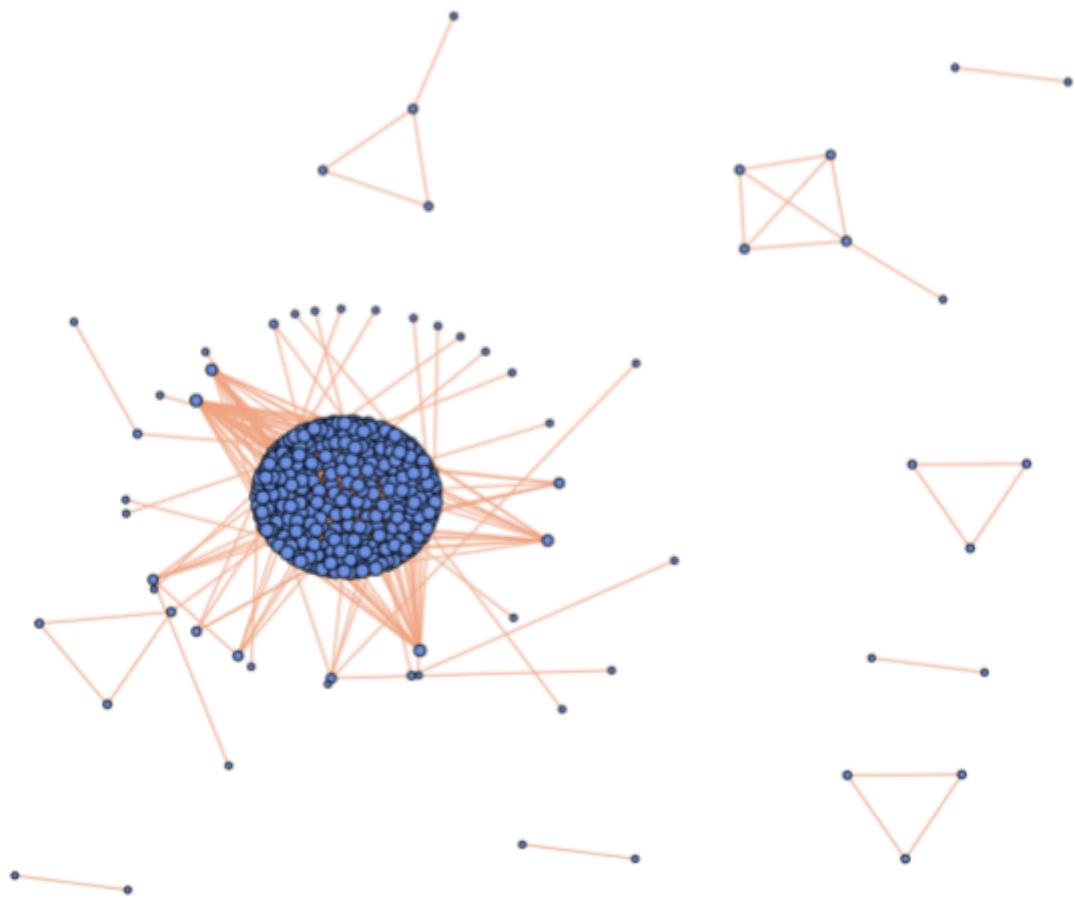


As shown in Figure 7, there is only 1 connected component in the Senate candidates reciprocated hyperlink graph.

#### 4.2. Reciprocity of House Candidates (recGh)

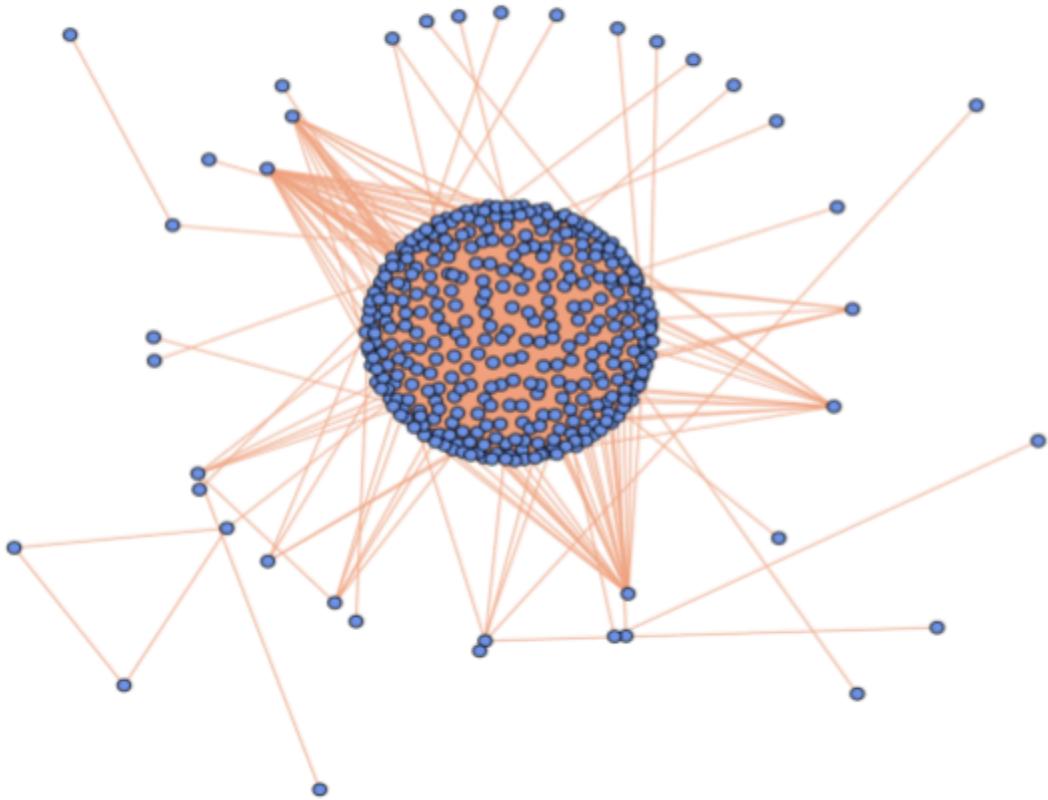
The reciprocity coefficient of Gh is 0.9903, indicating a significantly high level of mutual pairwise connectivity among the House candidates' Wikipedia pages. The reciprocated graph of the US House candidates contains 403 nodes and 57795 edges.

**Fig 8: Reciprocated Hyperlink Graph for House Candidates**



As seen in Figure 8, there are nine connected components in the House candidates reciprocated hyperlink graph. Figure 9 displays the giant connected component containing 380 nodes and 57,774 edges.

**Fig 9: Giant Connected Component of Reciprocated House Candidates Hyperlink Graph**



## 5. Attributes

In this section, we describe the process of assigning four key attributes to all candidates for both the Senate and House. These attributes are:

- Incumbency: This indicates whether the candidate currently holds the position or is a challenger.
- Party affiliation: This represents the candidate's political party, such as Democratic, Republican, Independent.
- State: This refers to the state from which the candidate is running for office.

- Candidacy status: This refers to the current status of the candidate's campaign, such as Candidacy Declared Primary, Candidacy Declared General, On the Ballot Primary, On the Ballot General, and On the Ballot Convention.

It is worth noting that we are not currently filtering out candidates who have withdrawn, been disqualified, or lost their status. Future work on this should include this preprocessing step.

From the Senate data frame (dfs) and the House data frame (dfh), we created four dictionaries that map each candidate to their respective attributes (incumbency, party, state, and status) and inherited these dictionaries to the nodes of the graphs Gs and Gh, respectively. By analyzing the subgraphs of Gs and Gh based on each attribute, we identified the number of nodes and edges of the largest weakly connected component for each unique attribute value. For example, we figured that there are 37 candidates having incumbency as "Challenger" and 21 candidates as "Incumbent" with 121 and 420 hyperlinks respectively. Among them, in the largest weakly connected components, there are 21 candidates as "Challenger" with 115 hyperlinks and 21 candidates as "Incumbent" with 420 hyperlinks.

Next, we assigned the attribute values to the nodes of the reciprocated graphs, recGs and recGh. By analyzing the subgraphs of recGs and recGh based on each attribute, we also derived the number of nodes and edges of the connected component for each unique attribute value. For instance, in the reciprocated graph, there are 19 "Challenger" candidates and 21 "Incumbent" candidates with 55 and 210 edges respectively. In the largest connected components, 16 "Challenger" candidates have 54 edges, and 21 "Incumbent" candidates have 210 edges. These attributes are used throughout the remainder of this paper to evaluate characteristics of the Senate and House graphs.

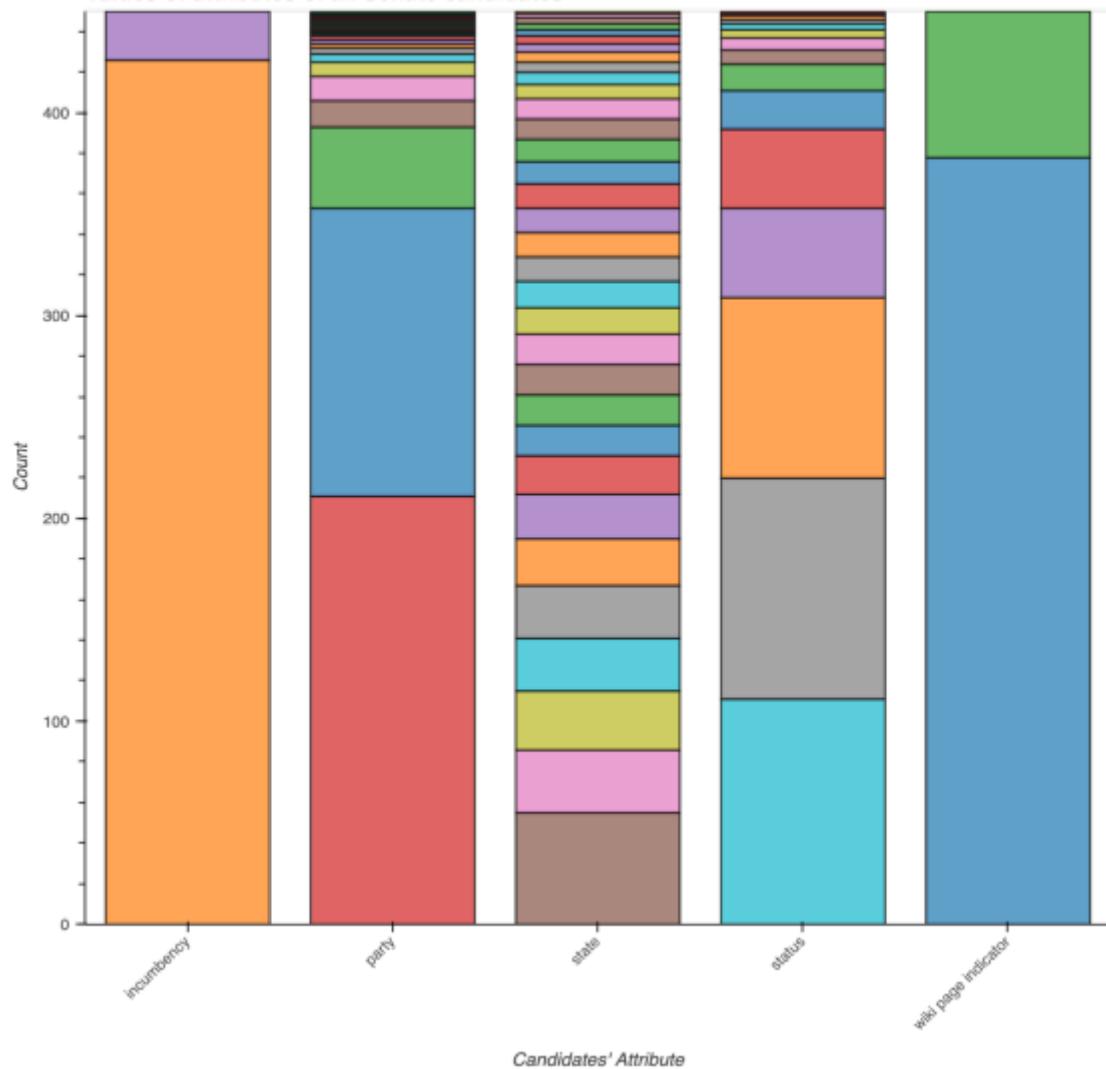
## 6. General Statistics

In this section, we provide an overview of general statistics of the Senate and House candidates graphs with stacked bar charts. The three graphs show the counts of candidates by different values in four attribute groups for overall candidates listed on Ballotpedia pages, candidates having Wikipedia pages, and candidates who are in our network graphs. The fourth graph displays the attribute value counts for the reciprocated candidates' graphs.

### 6.1. Senate Statistics

Figure 10 shows “Challenger” candidates (426) are significantly dominating the candidates who are running in the campaign. The number of candidates who do have a Wikipedia page is only 72 compared to 378. Table 5 displays the values represented in the graph.

**Fig 10: Values of Attributes of All Senate Candidates**



**Table 5: Attribute Counts of All Senate Candidates**

variable	value	count
inc incumbency	Challenger	426
inc incumbency	Incumbent	24
party	Republican	211
party	Democratic	142
party	Independent	40
party	Libertarian	13
party	Green	12

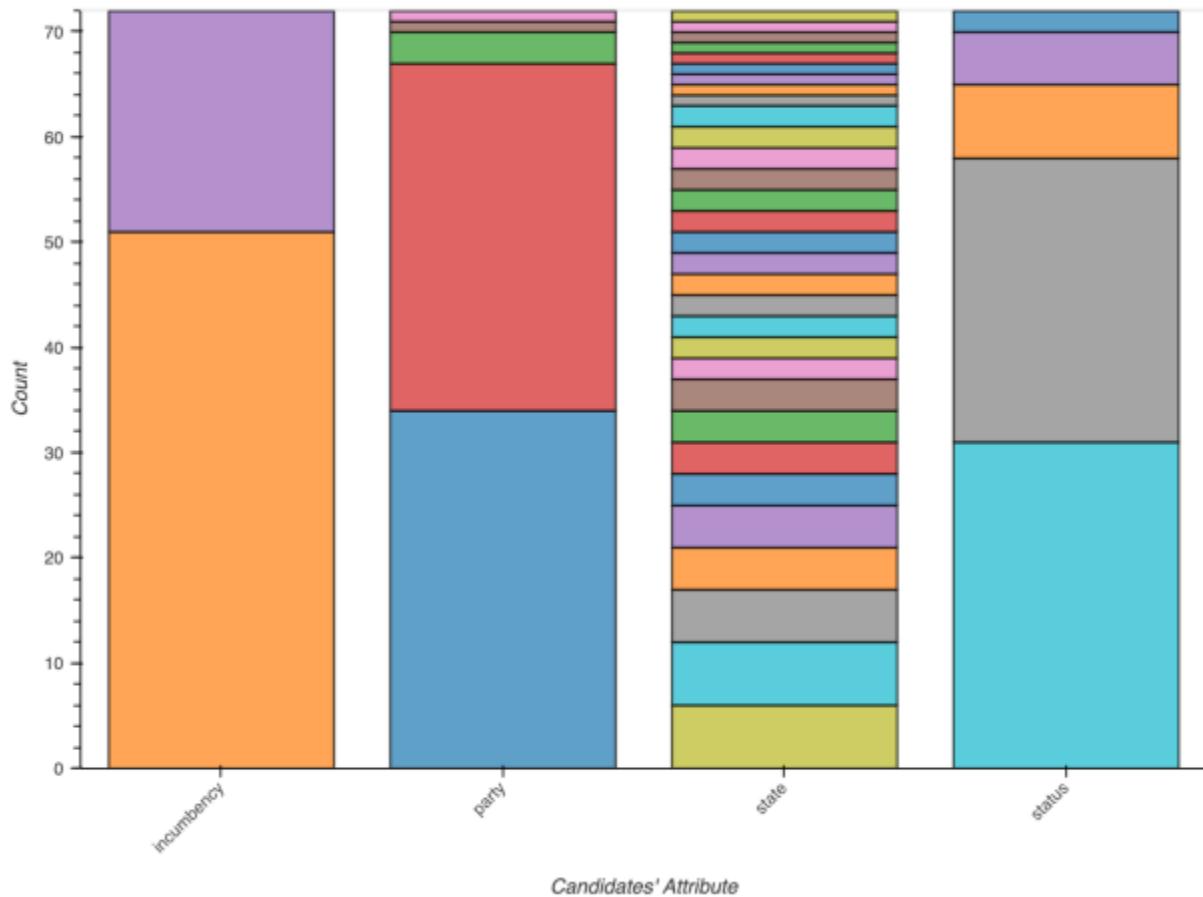
party	No party preference	7
party	No Political Party	4
party	No Party Affiliation	3
party	American Independent Party	2
party	Independent American Party of Utah	2
party	Legal Marijuana Now Party	2
party	American Independent Party of California	1
party	Constitution Party	1
party	Independence-Alliance Party of Minnesota	1
party	Independent American Party	1
party	Nonpartisan	1
party	One Earth Party	1
party	Progressive Party	1
party	Socialist Equality Party	1
party	Unenrolled	1
party	Vienmerisce Veittemeignzce USA	1
party	Wisdom People Party	1
party	Workers Party	1
state	CA	55
state	FL	31
state	MD	29
state	NV	26
state	TX	26
state	MI	23
state	NJ	22
state	UT	19
state	AZ	15

state	IN	15
state	NY	15
state	VA	15
state	TN	13
state	WV	13
state	MA	12
state	MO	12
state	MT	12
state	WI	12
state	PA	11
state	WA	11
state	MN	10
state	NM	10
state	NE	7
state	OH	6
state	DE	5
state	RI	5
state	CT	4
state	MS	4
state	HI	3
state	ME	3
state	ND	3
state	WY	2
state	VT	1
status	Candidacy Declared Primary	111
status	Withdrew Primary	109
status	On the Ballot Primary	89
status	Candidacy Declared General	44
status	Lost Primary	39

status	On the Ballot General	19
status	Withdrew General	13
status	On the Ballot Convention	7
status	Disqualified Primary	6
status	Lost (Write-in) Primary	4
status	Candidacy Declared (Write-in) General	3
status	On the Ballot Round 1	2
status	Withdrew (Write-in) Primary	2
status	Disqualified General	1
status	Withdrew Round 1	1
wiki page indicator	0	378
wiki page indicator	1	72

In Figure 11, out of the 72 candidates who have Wikipedia pages, 21 are Incumbent, and 51 are “Challenger”. Republican and Democratic candidates are nearly evenly split with a few independent candidates. Michigan and Nevada are the top two ranked states with more candidates. For status, “On the Ballot Primary” takes up the most group with 31 candidates, and “Candidacy Declared Primary” is the second group with 27 candidates. Table 6 displays the values represented in the graph.

**Fig 11: Values of Attributes of All Senate Candidates having Wiki Pages**



**Table 6: Attribute Counts of All Senate Candidates having Wiki Pages**

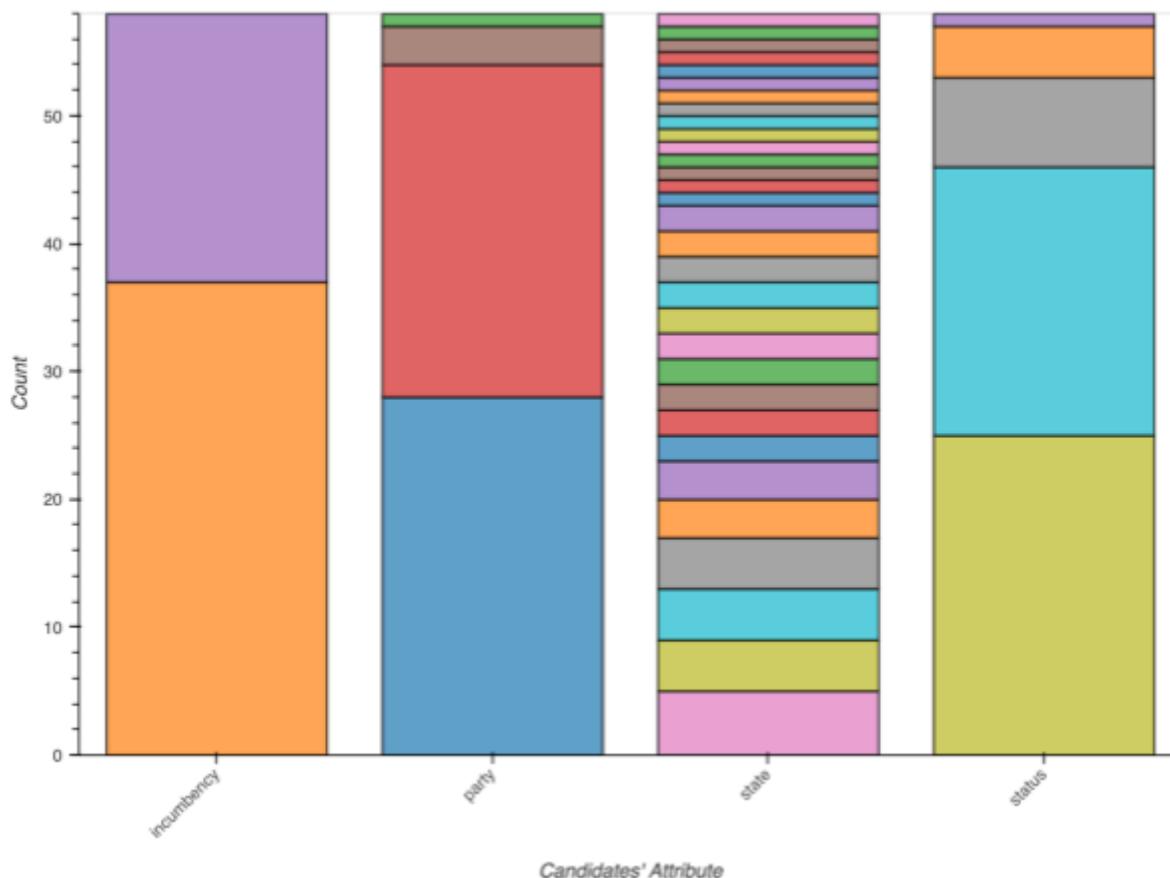
variable	value	count
incumbency	Challenger	51
incumbency	Incumbent	21
party	Democratic	34
party	Republican	33
party	Independent	3
party	Green	1
party	Nonpartisan	1
state	MI	6
state	NV	6
state	FL	5

state	MD	4
state	UT	4
state	AZ	3
state	MO	3
state	TN	3
state	WV	3
state	CA	2
state	DE	2
state	IN	2
state	MA	2
state	MN	2
state	MT	2
state	NE	2
state	NY	2
state	OH	2
state	PA	2
state	RI	2
state	TX	2
state	WI	2
state	CT	1
state	HI	1
state	MS	1
state	ND	1
state	NJ	1
state	NM	1
state	VA	1
state	WA	1
state	WY	1
status	On the Ballot Primary	31

status	Candidacy Declared Primary	27
status	On the Ballot General	7
status	Candidacy Declared General	5
status	On the Ballot Convention	2

As we looked at the number of candidates in the Figure 12, we found that the counts slightly changed in these attribute groups from the “having Wikipedia pages” graph, such as “Challenger” in incumbency, parties, and “On the Ballot Primary” status. Table 7 displays the values represented in the graph.

**Fig 12: Values of Attributes of All Senate Candidates in the Senate Graph**



**Table 7: Attribute Counts of All Senate Candidates in the Senate Graph**

variable	value	count
inc incumbency	Challenger	37
inc incumbency	Incumbent	21
party	Democratic	28
party	Republican	26
party	Independent	3
party	Green	1
state	MI	5
state	FL	4
state	MD	4
state	NV	4
state	TN	3
state	WV	3
state	AZ	2
state	CA	2
state	MA	2
state	MO	2
state	NE	2
state	OH	2
state	PA	2
state	TX	2
state	UT	2
state	WI	2
state	CT	1
state	DE	1
state	HI	1
state	IN	1
state	MN	1

state	MS	1
state	MT	1
state	ND	1
state	NJ	1
state	NM	1
state	NY	1
state	RI	1
state	VA	1
state	WA	1
state	WY	1
status	On the Ballot Primary	25
status	Candidacy Declared Primary	21
status	On the Ballot General	7
status	Candidacy Declared General	4
status	On the Ballot Convention	1

**Table 8: Top Candidates in Senate Graph by out-degree**

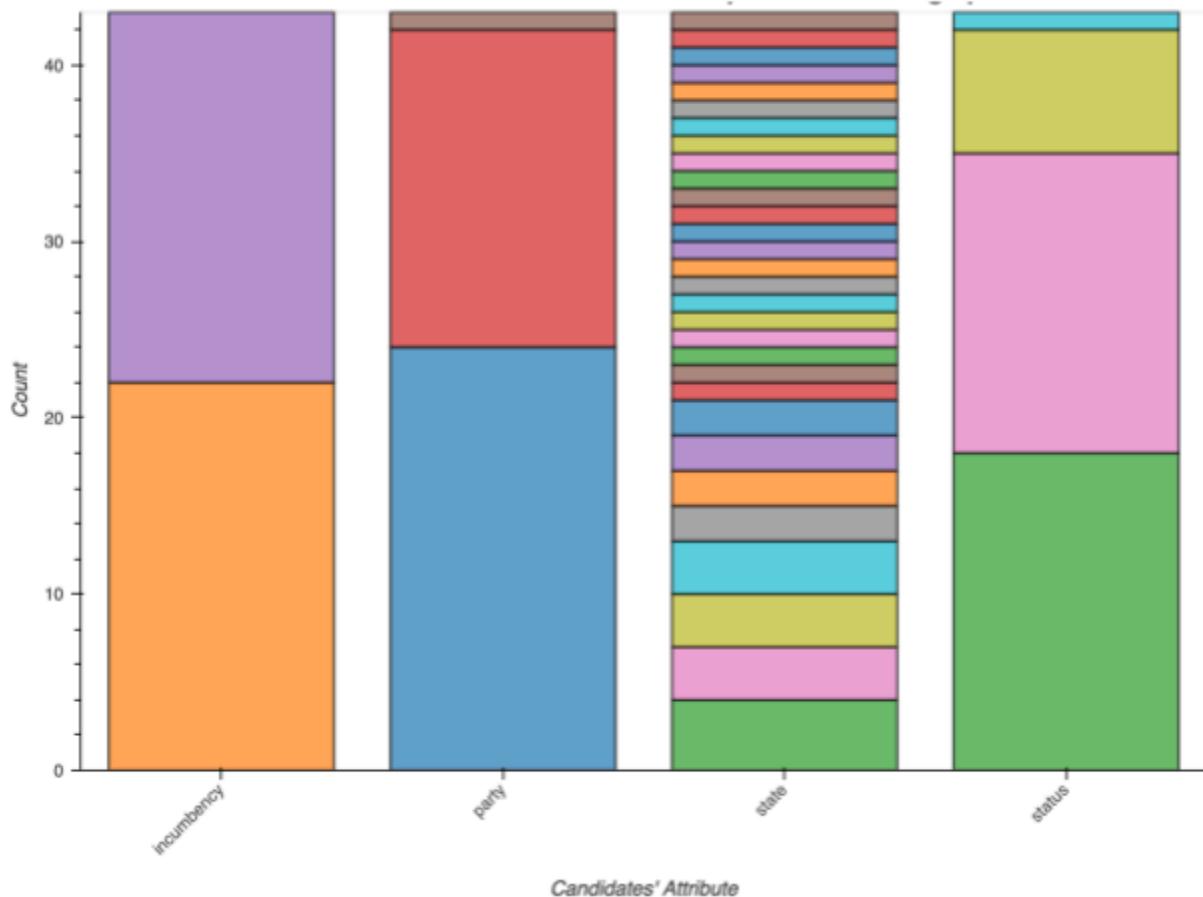
<b>candidate</b>	<b>incumbency</b>	<b>party</b>		<b>status</b>	<b>state</b>	<b>out_degree</b>
Elizabeth Warren	Incumbent	Democratic	Candidacy Declared Primary	MA		23
Amy Klobuchar	Incumbent	Democratic	Candidacy Declared Primary	MN		22
Kirsten Gillibrand	Incumbent	Democratic	Candidacy Declared Primary	NY		22
Ted Cruz	Incumbent	Republican	On the Ballot General	TX		21
Sherrod Brown	Incumbent	Democratic	On the Ballot General	OH		21
Rick Scott	Incumbent	Republican	Candidacy Declared Primary	FL		21
Deb Fischer	Incumbent	Republican	On the Ballot Primary	NE		21
Josh Hawley	Incumbent	Republican	Candidacy Declared Primary	MO		21
Bob Casey Jr.	Incumbent	Democratic	On the Ballot Primary	PA		20
Kevin Cramer	Incumbent	Republican	On the Ballot Primary	ND		20

**Table 9: Top Candidates in Senate Graph by in-degree**

<b>candidate</b>	<b>incumbency</b>	<b>party</b>		<b>status</b>	<b>state</b>	<b>in_degree</b>
Elizabeth Warren	Incumbent	Democratic	Candidacy Declared Primary	MA		26
Jacky Rosen	Incumbent	Democratic	On the Ballot Primary	NV		23
Ted Cruz	Incumbent	Republican	On the Ballot General	TX		23
Rick Scott	Incumbent	Republican	Candidacy Declared Primary	FL		22
Amy Klobuchar	Incumbent	Democratic	Candidacy Declared Primary	MN		22
Josh Hawley	Incumbent	Republican	Candidacy Declared Primary	MO		22
Marsha Blackburn	Incumbent	Republican	On the Ballot Primary	TN		22
Kirsten Gillibrand	Incumbent	Democratic	Candidacy Declared Primary	NY		22
Bob Casey Jr.	Incumbent	Democratic	On the Ballot Primary	PA		21
Deb Fischer	Incumbent	Republican	On the Ballot Primary	NE		21

The reciprocated Senate graph, Figure 13, is a smaller subset of the Senate graph, containing only bidirectional edges. For incumbency, the number of “Challenger” candidates decreased from 37 to 21. Similarly, the sizes of subgroup values for each other attribute reduce as the number of hyperlinks in the reciprocated graph diminishes. Table 10 displays the values represented in the graph.

**Fig 13: Values of Attributes of All Senate Candidates in the Reciprocated Senate Graph**



**Table 10: Attribute Counts of All Senate Candidates in the Reciprocated Senate Graph**

variable	value	count
inc incumbency	Challenger	22
inc incumbency	Incumbent	21
party	Democratic	24
party	Republican	18

party	Independent	1
state	MI	4
state	FL	3
state	MD	3
state	WV	3
state	CA	2
state	NE	2
state	OH	2
state	TX	2
state	AZ	1
state	DE	1
state	HI	1
state	IN	1
state	MA	1
state	MN	1
state	MO	1
state	MS	1
state	MT	1
state	ND	1
state	NJ	1
state	NM	1
state	NV	1
state	NY	1
state	PA	1
state	RI	1
state	TN	1
state	UT	1
state	VA	1
state	WA	1

state	WI	1
state	WY	1
status	Candidacy Declared Primary	18
status	On the Ballot Primary	17
status	On the Ballot General	7
status	Candidacy Declared General	1

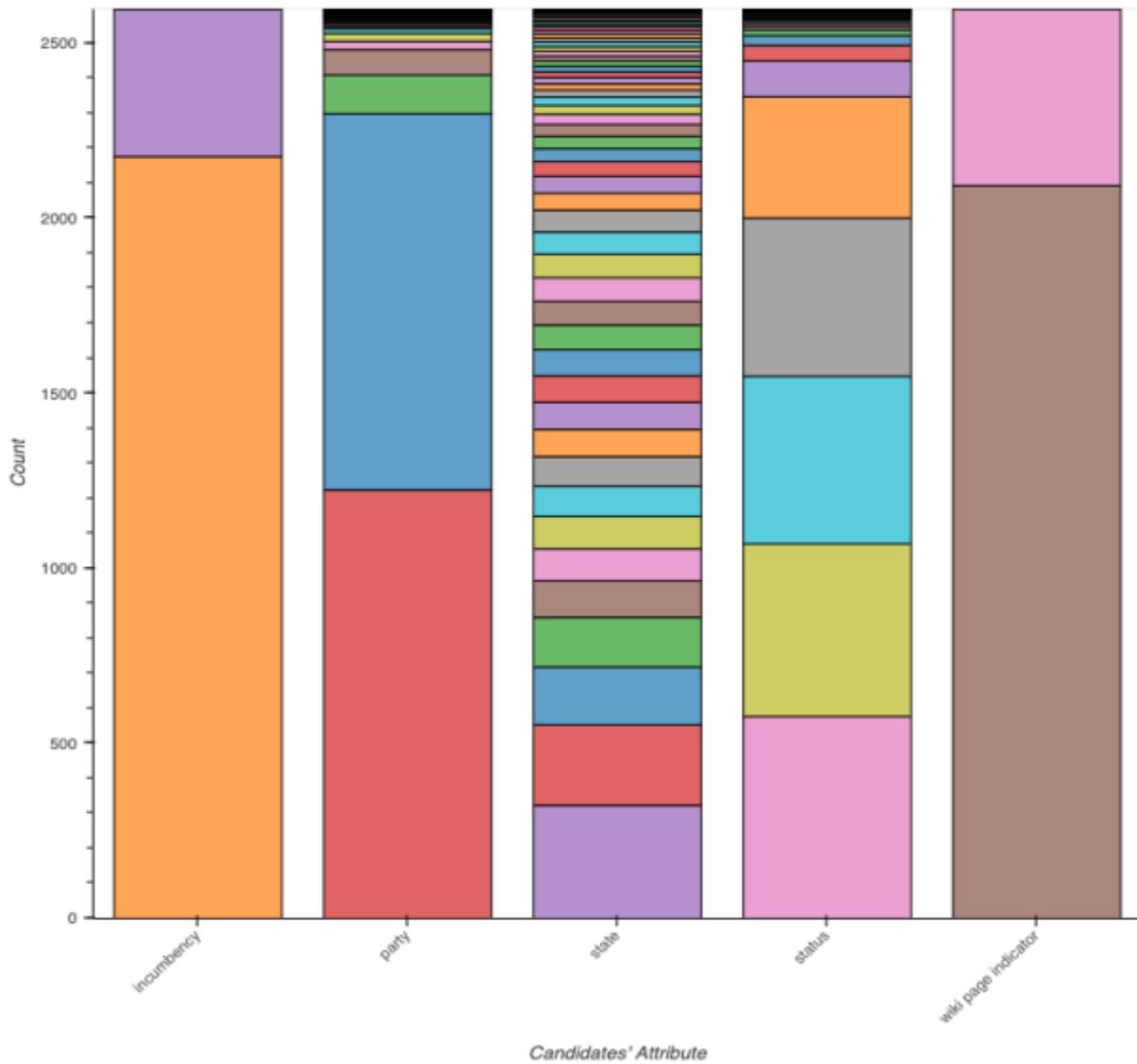
**Table 11: Top Candidate in Reciprocated Senate Graph by degree**

candidate	incumbency	party	status	state	degree
Elizabeth Warren	Incumbent	Democratic	Candidacy Declared Primary	MA	23
Amy Klobuchar	Incumbent	Democratic	Candidacy Declared Primary	MN	22
Kirsten Gillibrand	Incumbent	Democratic	Candidacy Declared Primary	NY	22
Ted Cruz	Incumbent	Republican	On the Ballot General	TX	21
Sherrod Brown	Incumbent	Democratic	On the Ballot General	OH	21
Deb Fischer	Incumbent	Republican	On the Ballot Primary	NE	21
Rick Scott	Incumbent	Republican	Candidacy Declared Primary	FL	21

## 6.2. House Statistics

Figure 14 displays the counts of the attributes for all the House candidates. The incumbency column shows that the majority of candidates (2,176) are of type “Challenger,” and 421 are of type “Incumbent.” The party attribute displays that Republican and Democratic are pretty evenly split and dominate the party attribute. Additionally, we see that a small proportion of the candidate’s have a Wikipedia page (504). Table 12 displays the values represented in the graph.

**Fig 14: Values of Attributes of All House Candidates**



**Table 12: Attribute Counts of All House Candidates**

variable	value	count
inc incumbency	Challenger	2176
inc incumbency	Incumbent	421
party	Republican	1224
party	Democratic	1075
party	Independent	110
party	Libertarian	73

party	No Party Affiliation	23
party	No party preference	23
party	Green	13
party	Constitution Party	6
party	Conservative Party	5
party	No Political Party	5
party	Independent American Party	4
party	Peace and Freedom Party	3
party	Unaffiliated	3
party	Working Families Party	3
party	Alliance Party	2
party	Unity Party	2
party	American Constitution Party	1
party	American Independent Party	1
party	American People's Freedom Party	1
party	Bipartisan Party	1
party	Calm Rational GOP Party	1
party	Colorado Forward Party	1
party	Congress Sucks Party	1
party	Connecticut Conservative Party	1
party	Constitutional Party	1
party	George Wallace Party	1
party	Independence Party	1
party	MAGA Democratic Party	1
party	MAGA Republican Party	1
party	People's Party	1
party	Progressive Party	1
party	Trump Republican Party	1
party	Unenrolled	1

party	Union Party	1
party	United Citizens Party	1
party	United Utah Party	1
party	Unity Party of Colorado	1
party	Veteran's Party	1
party	Wisdom People Party	1
state	CA	323
state	TX	230
state	FL	165
state	NY	142
state	MD	104
state	CO	92
state	IN	92
state	NC	86
state	OH	85
state	GA	78
state	AZ	77
state	VA	76
state	WA	75
state	IL	70
state	MI	68
state	NJ	67
state	PA	67
state	TN	64
state	MO	62
state	AL	49
state	SC	48
state	OR	42
state	NV	37

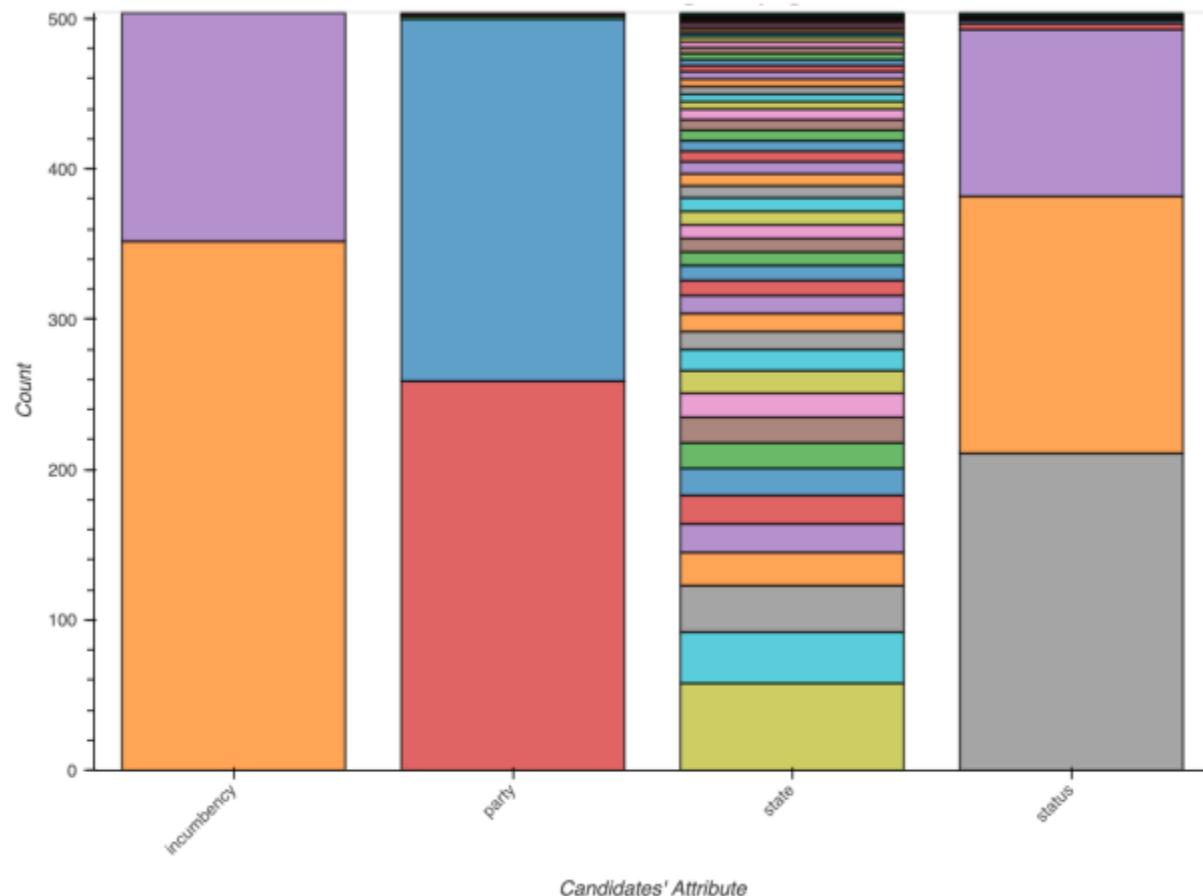
state	WI	35
state	MN	34
state	UT	29
state	KY	25
state	OK	24
state	MT	20
state	MA	18
state	NH	18
state	CT	16
state	KS	16
state	LA	15
state	IA	14
state	MS	14
state	AR	13
state	WV	13
state	ID	12
state	NE	12
state	ME	10
state	ND	10
state	DE	7
state	NM	7
state	DC	6
state	GU	6
state	SD	6
state	MP	4
state	AK	3
state	RI	3
state	AS	2
state	HI	2

state	VT	2
state	VI	1
state	WY	1
status	On the Ballot Primary	577
status	Withdrew Primary	493
status	On the Ballot General	478
status	Lost Primary	453
status	Candidacy Declared Primary	346
status	Candidacy Declared General	102
status	Disqualified Primary	45
status	Withdrew General	28
status	On the Ballot Primary Runoff	16
status	Candidacy Declared (Write-in) General	8
status	On the Ballot (Write-in) General	8
status	Lost (Write-in) Primary	6
status	On the Ballot Round 1	6
status	Disqualified General	4
status	Lost (unofficially withdrew) Primary	3
status	Lost Primary Runoff	3
status	On the Ballot (Write-in) Primary	3
status	Advanced Round 1	2
status	Candidacy Declared Round 1	2
status	Lost Convention	2
status	On the Ballot (unofficially withdrew) Primary	2
status	Withdrew (Write-in) General	2
status	Withdrew (unofficially withdrew) Primary	2
status	Candidacy Declared (Write-in) Primary	1

status	Lost (unofficially withdrew) Primary Runoff	1
status	On the Ballot (Write-in) Round 1	1
status	On the Ballot Convention	1
status	Withdrew Round 1	1
status	Won General	1
wiki page indicator	0	2093
wiki page indicator	1	504

In Figure 15, we look at the subset of candidates that have Wikipedia pages. The predominant value for the incumbency is ‘Incumbent’ with 352, compared to 152 of type Challenger. Republican and Democratic continue to remain almost evenly split for the party attribute. California has the largest number of candidates, followed by Texas and Florida. There are 10 values for status, however, “on the ballot general”, “on the ballot primary” and “candidacy declared primary” make up the majority. Table 13 displays the values represented in the graph.

**Fig 15: Values of Attributes of All House Candidates Having Wiki Pages**



**Table 13: Attribute Counts of All House Candidates having Wiki Pages**

variable	value	count
incumbency	Incumbent	352
incumbency	Challenger	152
party	Republican	259
party	Democratic	241
party	Independent	2
party	Alliance Party	1
party	Constitution Party	1
state	CA	58
state	TX	34
state	FL	31

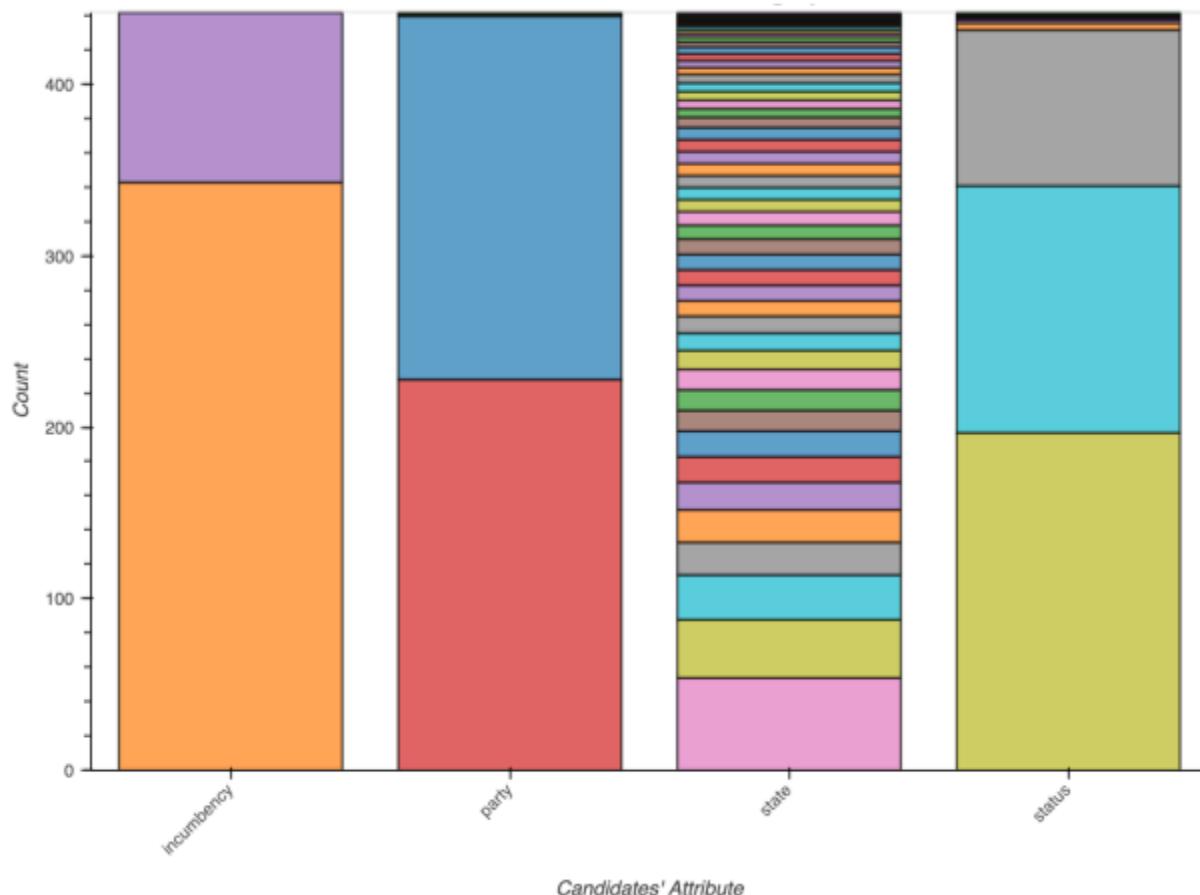
state	NY	22
state	AZ	19
state	PA	19
state	MI	18
state	GA	17
state	IL	17
state	OH	16
state	CO	15
state	WA	14
state	MN	12
state	MO	12
state	VA	12
state	OR	10
state	SC	10
state	IN	9
state	MA	9
state	MT	9
state	NC	9
state	TN	9
state	MD	8
state	NJ	8
state	WI	8
state	AL	7
state	CT	7
state	KY	7
state	LA	7
state	NV	7
state	IA	5
state	ND	5

state	NE	5
state	OK	5
state	UT	5
state	AR	4
state	KS	4
state	ME	4
state	MS	4
state	NH	4
state	ID	3
state	AK	2
state	GU	2
state	HI	2
state	NM	2
state	WV	2
state	DC	1
state	DE	1
state	RI	1
state	SD	1
state	VT	1
state	WY	1
status	On the Ballot General	211
status	On the Ballot Primary	171
status	Candidacy Declared Primary	111
status	On the Ballot Round 1	4
status	On the Ballot Primary Runoff	2
status	Candidacy Declared (Write-in) General	1
status	Candidacy Declared General	1
status	Lost Primary	1

status	Withdrew Primary	1
status	Won General	1

Figure 16, displays the subset of attributes that appears in the House candidates' graph (G<sub>h</sub>). The distribution of values for each attribute is similar to what we saw in Figure 15. One noticeable difference is the reduced number of "Challengers" in the incumbency attribute: 99 compared to 152. The number of "Incumbents" only reduced by 9. Table 14 displays the values represented in the graph.

**Fig 16: Values of Attributes of All House Candidates in the House Graph**



**Table 14: Attribute Counts of All House Candidates in the House Graph**

variable	value	count
inc incumbency	Incumbent	343
inc incumbency	Challenger	99
party	Republican	228
party	Democratic	212
party	Constitution Party	1
party	Independent	1
state	CA	54
state	TX	34
state	FL	26
state	NY	19
state	PA	19
state	IL	16
state	GA	15
state	OH	15
state	AZ	12
state	CO	12
state	MI	12
state	VA	11
state	OR	10
state	WA	10
state	MA	9
state	MN	9
state	MO	9
state	MT	9
state	TN	9
state	IN	8
state	SC	8

state	CT	7
state	KY	7
state	LA	7
state	MD	7
state	NC	7
state	NJ	7
state	WI	7
state	AL	6
state	IA	5
state	NE	5
state	NV	5
state	OK	5
state	UT	5
state	AR	4
state	KS	4
state	ME	4
state	MS	4
state	ID	3
state	ND	3
state	AK	2
state	HI	2
state	NM	2
state	DC	1
state	GU	1
state	NH	1
state	RI	1
state	SD	1
state	VT	1
state	WV	1

state	WY	1
status	On the Ballot General	197
status	On the Ballot Primary	144
status	Candidacy Declared Primary	91
status	On the Ballot Round 1	4
status	On the Ballot Primary Runoff	2
status	Candidacy Declared (Write-in) General	1
status	Lost Primary	1
status	Withdrew Primary	1
status	Won General	1

In Tables 15 and 16 we display the top number of candidates in the House candidates graph ( $G_h$ ) based on out-degree and in-degree, respectively. None of the candidates in the top out-degree are listed in the top in-degree list, and vice-versa.

**Table 15: Top Candidates in House Graph by out-degree**

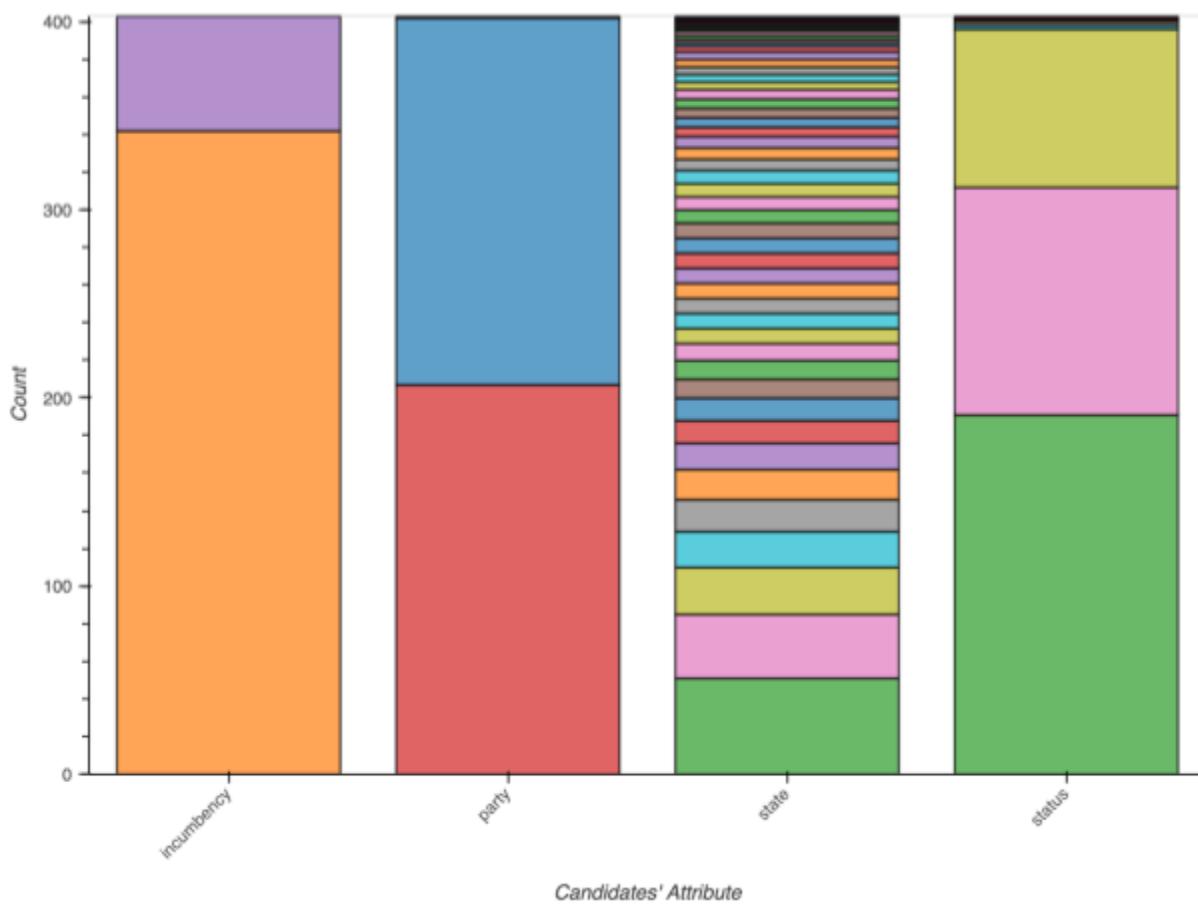
<b>candidate</b>	<b>incumbency</b>	<b>party</b>	<b>status</b>	<b>state</b>	<b>out_degree</b>
Young Kim	Incumbent	Republican	On the Ballot General	CA	345
Erin Houchin	Incumbent	Republican	On the Ballot General	IN	342
Victoria Spartz	Incumbent	Republican	On the Ballot General	IN	342
Jim Baird	Incumbent	Republican	On the Ballot General	IN	342
Judy Chu	Incumbent	Democratic	On the Ballot General	CA	342
Rudy Yakym	Incumbent	Republican	On the Ballot General	IN	342
Mary Peltola	Incumbent	Democratic	Candidacy Declared Primary	AK	342
Frank Mrvan	Incumbent	Democratic	On the Ballot General	IN	342
William Timmons	Incumbent	Republican	On the Ballot Primary	SC	342
Lori Chavez-DeRemer	Incumbent	Republican	On the Ballot Primary	OR	342
Paul Tonko	Incumbent	Democratic	Candidacy Declared Primary	NY	342
Steve Scalise	Incumbent	Republican	Candidacy Declared Primary	LA	342

**Table 16: Top Candidates in House Graph by in-degree**

<b>candidate</b>	<b>incumbency</b>	<b>party</b>	<b>status</b>	<b>state</b>	<b>in_degree</b>
Nancy Pelosi	Incumbent	Democratic	On the Ballot General	CA	349
Ryan K. Zinke	Incumbent	Republican	On the Ballot Primary	MT	346
Mike Johnson	Incumbent	Republican	Candidacy Declared Primary	LA	345
Derrick Van Orden	Incumbent	Republican	Candidacy Declared Primary	WI	344
Matt Gaetz	Incumbent	Republican	On the Ballot Primary	FL	344
Paul Gosar	Incumbent	Republican	On the Ballot Primary	AZ	344
Pat Ryan	Incumbent	Democratic	Candidacy Declared Primary	NY	344
Hakeem Jeffries	Incumbent	Democratic	Candidacy Declared Primary	NY	344
Bob Good	Incumbent	Republican	On the Ballot Primary	VA	344

Figure 17 displays the values of attributes from the reciprocated House graph. In the incumbency attribute, we see a fewer number of “Challengers” than we did in the full graph: 61 compared to 99. The number of “Incumbents” is only one fewer: 342 compared to 343. “Republicans” and “Democrats” still make up the majority of the party attribute, with a single candidate having value “Constitution Party.” The top 3 state attribute values remain California, Texas and Florida. The values of the status attribute remain similar to the full graph as well. Table 17 displays the values represented in the graph.

**Fig 17: Values of Attributes of All House Candidates in the Reciprocated House Graph**



**Table 17: Attribute Counts of All House Candidates in the Reciprocated House Graph**

variable	value	count
inc incumbency	Incumbent	342
inc incumbency	Challenger	61
party	Republican	207
party	Democratic	195
party	Constitution Party	1
state	CA	51
state	TX	34
state	FL	25
state	PA	19
state	NY	17
state	IL	16
state	OH	14
state	GA	12
state	MI	12
state	CO	10
state	OR	10
state	MA	9
state	AZ	8
state	IN	8
state	MN	8
state	MO	8
state	SC	8
state	TN	8
state	VA	8
state	WA	8

state	KY	7
state	LA	7
state	NC	7
state	NJ	7
state	AL	6
state	CT	6
state	MT	6
state	IA	5
state	MD	5
state	NE	5
state	OK	5
state	WI	5
state	AR	4
state	KS	4
state	MS	4
state	NV	4
state	UT	4
state	ID	3
state	AK	2
state	HI	2
state	ME	2
state	NM	2
state	DC	1
state	GU	1
state	NH	1
state	RI	1
state	SD	1
state	VT	1
state	WV	1

state	WY	1
status	On the Ballot General	191
status	On the Ballot Primary	121
status	Candidacy Declared Primary	84
status	On the Ballot Primary Runoff	2
status	On the Ballot Round 1	2
status	Lost Primary	1
status	Withdrew Primary	1
status	Won General	1

**Table 18: Top Candidate in Reciprocated House Graph by degree**

candidate	incumbency	party	status	state	degree
Jim Baird	Incumbent	Republican	On the Ballot General	IN	341
Paul Tonko	Incumbent	Democratic	Candidacy Declared Primary	NY	341
Victoria Spartz	Incumbent	Republican	On the Ballot General	IN	341
William Timmons	Incumbent	Republican	On the Ballot Primary	SC	341
Claudia Tenney	Incumbent	Republican	Candidacy Declared Primary	NY	341
Erin Houchin	Incumbent	Republican	On the Ballot General	IN	341
Nancy Pelosi	Incumbent	Democratic	On the Ballot General	CA	341
Judy Chu	Incumbent	Democratic	On the Ballot General	CA	341
Rudy Yakym	Incumbent	Republican	On the Ballot General	IN	341
Frank Mrvan	Incumbent	Democratic	On the Ballot General	IN	341
Lori Chavez-DeRemer	Incumbent	Republican	On the Ballot Primary	OR	341

## 7. Visualizations of Moderate Size Attributed Graphs

### 7.1. Senate Candidate Graphs

The four attributes were embedded in the graph as described in Section 5, therefore it is straightforward to create corresponding graphs based on any of these attributes. The Senate graph contains a total of 58 nodes, so we decided to include all of them in our graphs. Figure 18 displays the same as the reciprocated hyperlink graph we derived earlier, and without distinguishing by attributes.

**Fig 18: Senate Graph without Attributes**

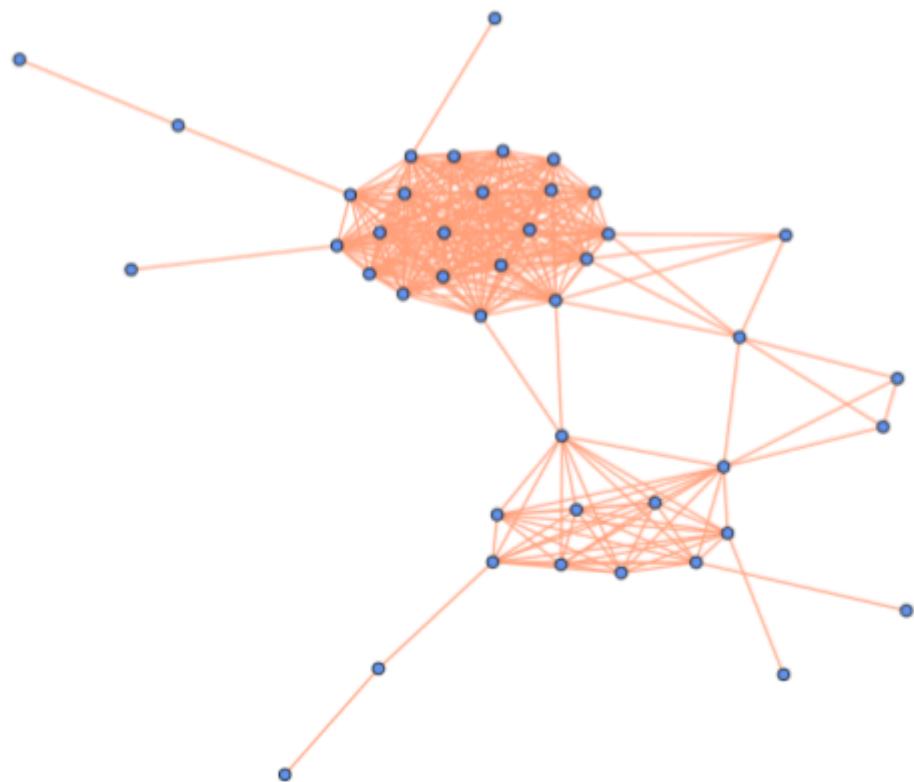
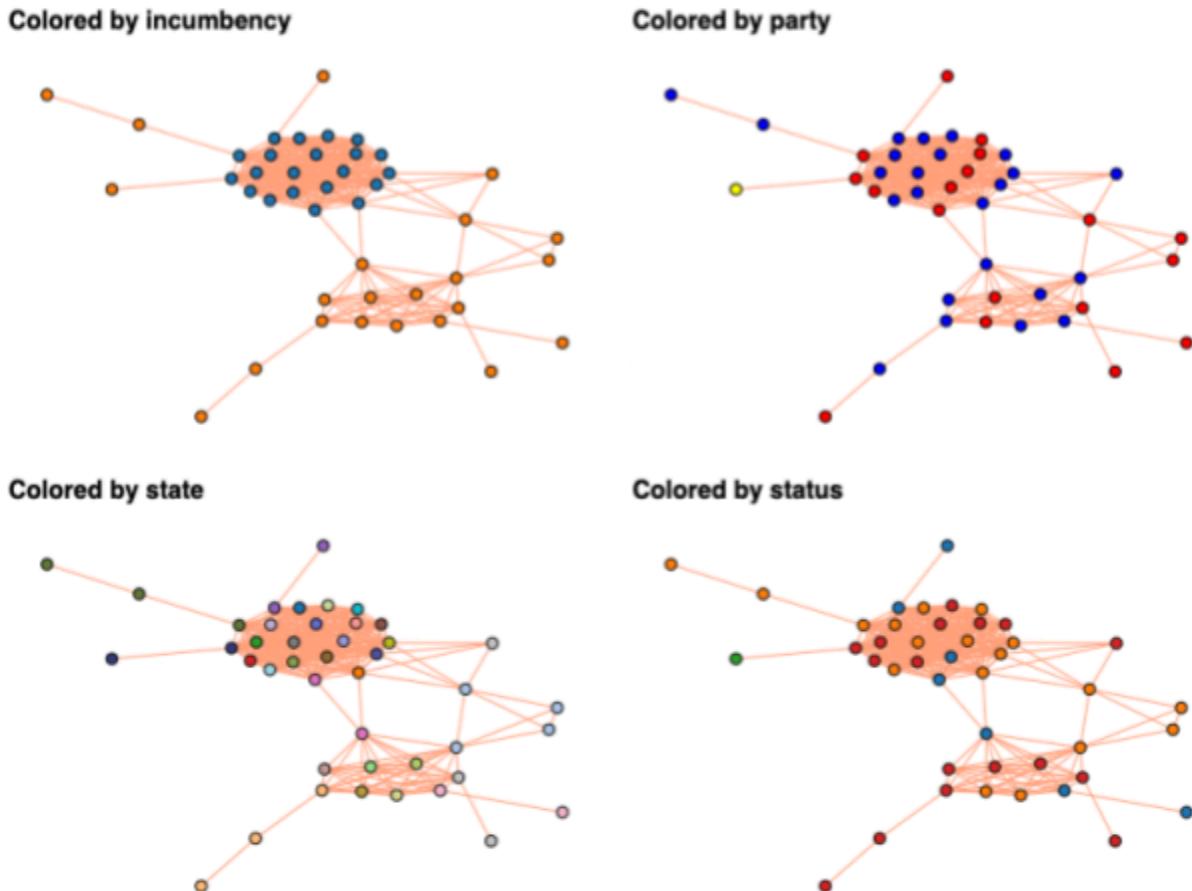


Figure 19 displays the reciprocated Senate graph with candidate nodes colored according to their respective attribute values. Each of the four graphs emphasizes a different attribute out of incumbency, party, state, and status. In the incumbency graph, 21 blue nodes

represent “Incumbent” candidates and 19 orange nodes represent “Challengers”. The party affiliation graph illustrates the distribution of political parties among the candidates: 24 nodes colored blue represent “Democratic” candidates, and 15 nodes colored red represent “Republican” candidates. Additionally, only 1 node is colored yellow, meaning “Independent” affiliation. The state graph includes 30 distinct states, with Michigan having the highest number of candidates, totaling 4. The last graph, the status-attributed graph has 4 groups, where 18 blue-colored nodes are “Candidacy Declared Primary”, 15 red-colored nodes are “On the Ballot Primary”, 6 orange-colored nodes are “On the Ballot General”, and only 1 green-colored nodes are “Candidacy Declared General”.

**Fig 19: 4 Senate Graphs with all Four Attributes**

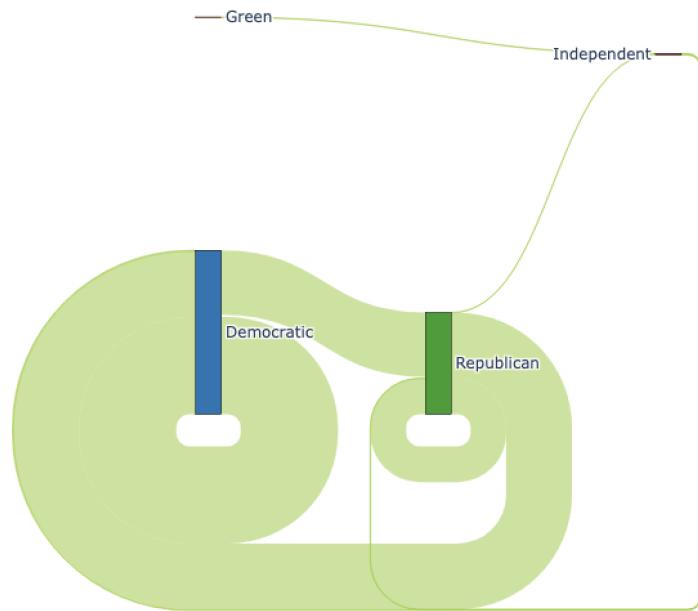


## 7.2. Senate Attributed Graphs as Sankey Diagrams

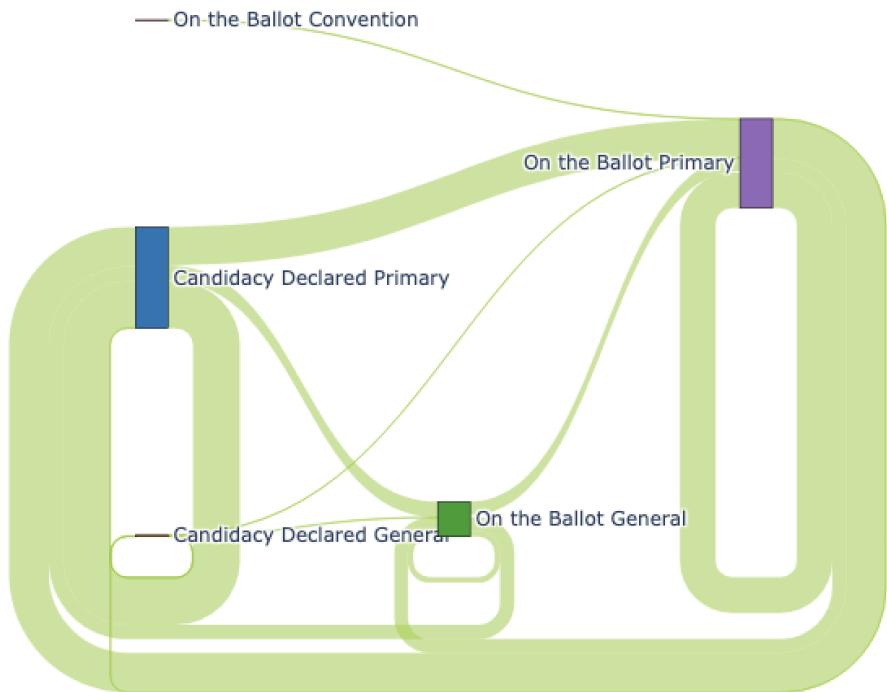
A Sankey diagram ([https://en.wikipedia.org/wiki/Sankey\\_diagram](https://en.wikipedia.org/wiki/Sankey_diagram)) is a type of flow diagram where the width of the arrows is proportional to the flow rate. The flows represent the directed edges (hyperlinks) between different attribute groups, where “source” is the starting point and “target” is the destination. In our interactive visualizations (Holoviews), we can find the number of hyperlinks by hovering over each flow, and we can also obtain the numbers of incoming and outgoing flows by hovering over each attribute type band.

In the following graphs, we display the incumbency, party affiliation and candidacy status attributes for the full Senate candidates hyperlink graph ( $G_s$ ) using Sankey diagrams.

**Fig 20-1: Sankey diagram for Senate party**



**Fig 20-2: Sankey diagram for Senate status**



**Fig 20-3: Sankey diagram for Senate incumbency**

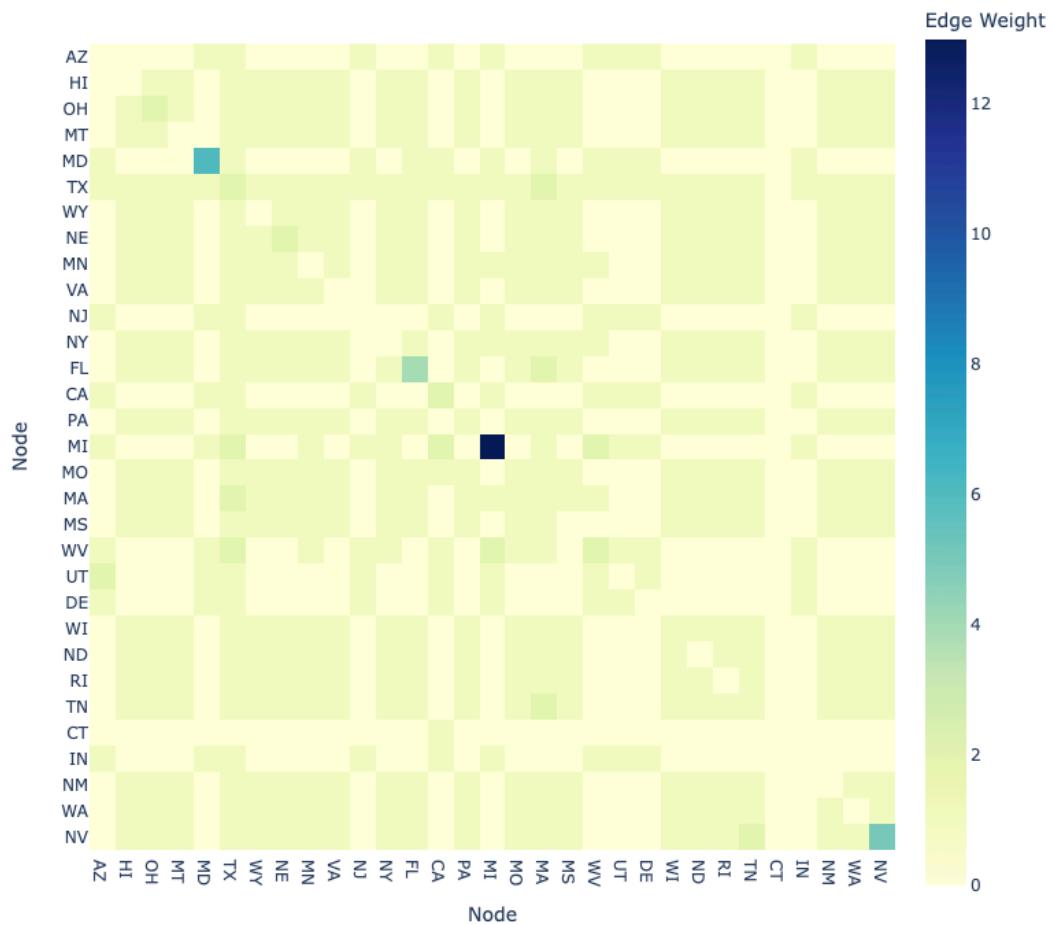


We are not including state-attributed Sankey diagrams due to the reason that the number of states in both Gs and Gh would make the graph unreadable, so we present the adjacency matrix heatmaps to visualize the relationships between candidate nodes from the state aspect.

### 7.3. State Attributed Heatmap of Senate Candidates Hyperlink Graph

Figure 21 displays the heatmap for the Senate Candidates' hyperlink graph. The edge weights are represented by the color intensity in each cell of the heatmap, meaning the weight of the edge between two states. The assortativity coefficient is a measure used to quantify the tendency of nodes in a network to connect to other nodes that are similar in some way. A higher intensity (darker color) indicates a stronger connection between two states, and a lower intensity (lighter color) indicates a weaker connection. The heatmap above allows us to obtain a clear and detailed visualization of the connectivity relationships between candidates in different states. For instance, Michigan (MI) and California (CA) exhibit relatively higher connections and assortativity. However, one limitation is that this heatmap cannot provide the direction of the assortativity between two states. This means that we cannot determine if the connections from MI to CA have higher assortativity or vice versa.

**Fig 21: State Attributed Senate Candidates Adjacency Matrix Heatmap**



## 7.4. House Candidate Graphs

In order to manageably visualize the attributes for the House candidates, we selected a subgraph of candidate's from the states of Colorado, Connecticut, Georgia, Illinois and Washington. The reciprocated subgraph of these states contains 52 nodes and 967 edges. Figure 22 illustrates the subgraph without attributes.

**Fig 22: Subgraph of the CO, CT, GA, IL, WA House Candidates**

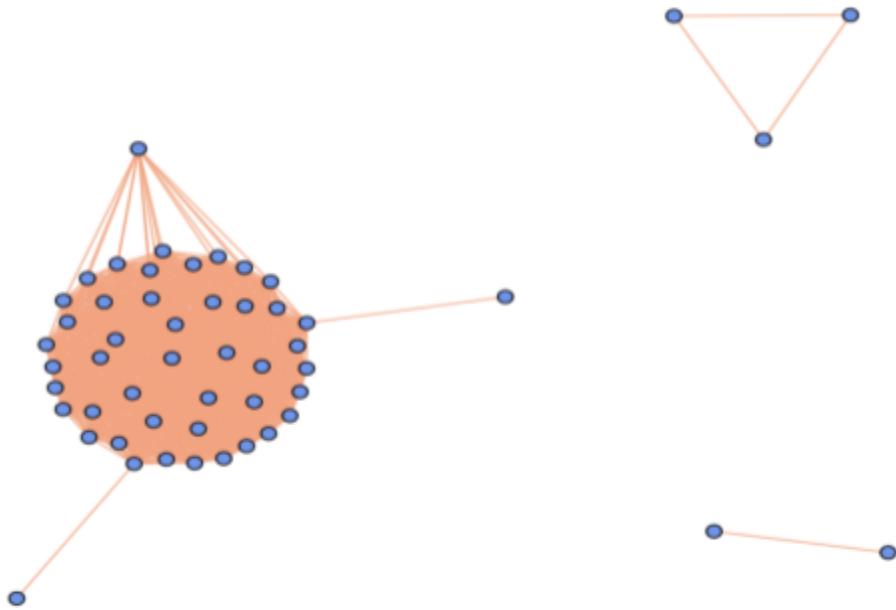
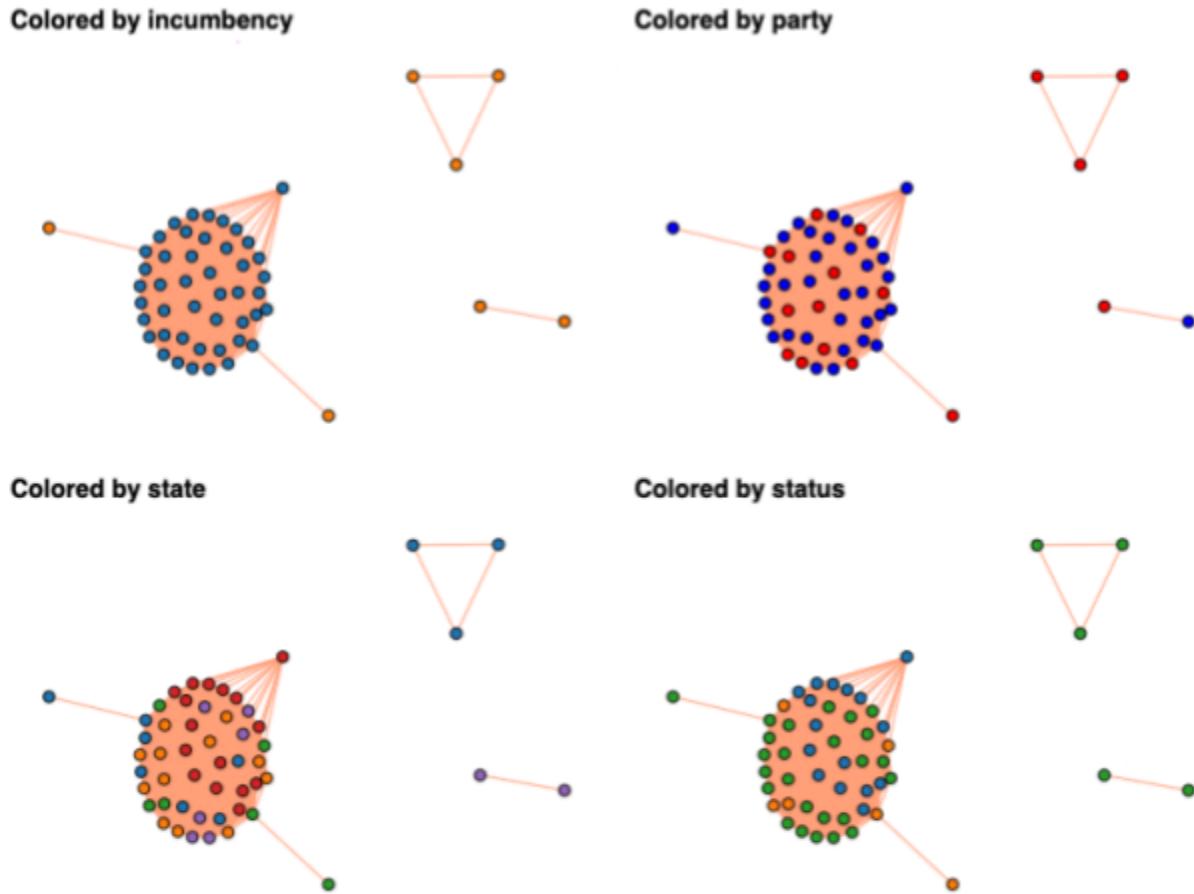


Figure 23 presents subgraphs where candidate nodes are colored based on their respective attribute values. Each of the four subgraphs highlights different attributes: incumbency, party affiliation, state, and candidacy status. In the incumbency graph, green nodes represent “Incumbents,” comprising the majority with 45 nodes and red nodes represent “Challengers,” accounting for 7 nodes. The party affiliation graph depicts “Democratic” as blue nodes, represented by 35 nodes and “Republican” as red nodes, represented by 17 nodes. The states graph colored Colorado as blue with 10 nodes, Connecticut as orange with 6 nodes, Georgia as green with 12 nodes, Illinois as red with 17 nodes and Washington as purple with 8 nodes. Lastly, the status attributed graph colored “candidacy declared primary” as blue with 6 nodes, “on the ballot general” as orange with 16 nodes and “on the ballot primary” as green with 30 nodes. Each subgraph visually emphasizes the distribution of candidates based on the

specific attribute, providing a clear and comparative view of how these attributes are represented within the network of candidates.

**Fig 23: Attribute Subgraphs of the CO, CT, GA, IL, WA House Candidates**



The four attribute subgraphs of the House candidates from Colorado, Connecticut, Georgia, Illinois and Washington.

## 7.5. House Attributed Graphs as Sankey Diagrams

In the following graphs, we display the incumbency, party affiliation and candidacy status attributes for the full House candidates hyperlink graph ( $G_h$ ) using Sankey diagrams.

Fig 24-1: Sankey Diagram of Incumbency for House Candidates

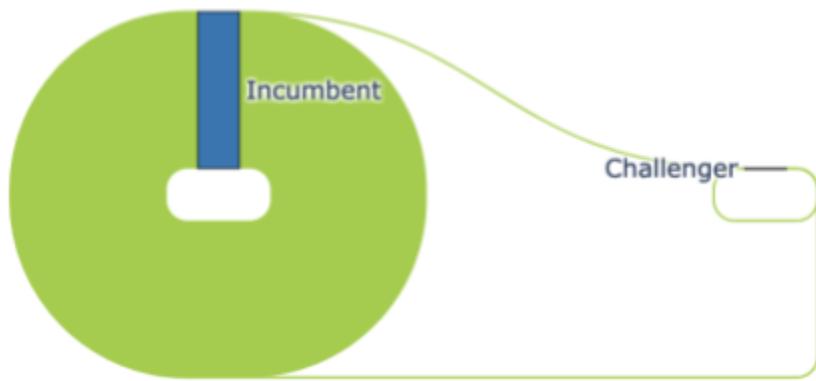
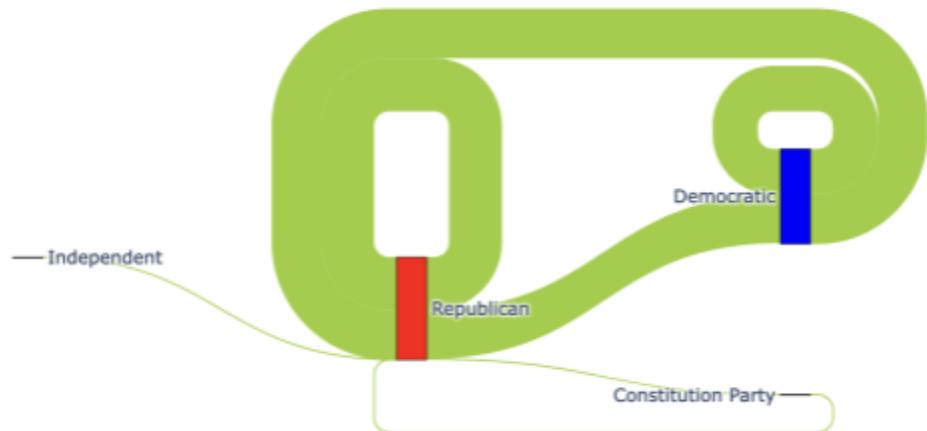
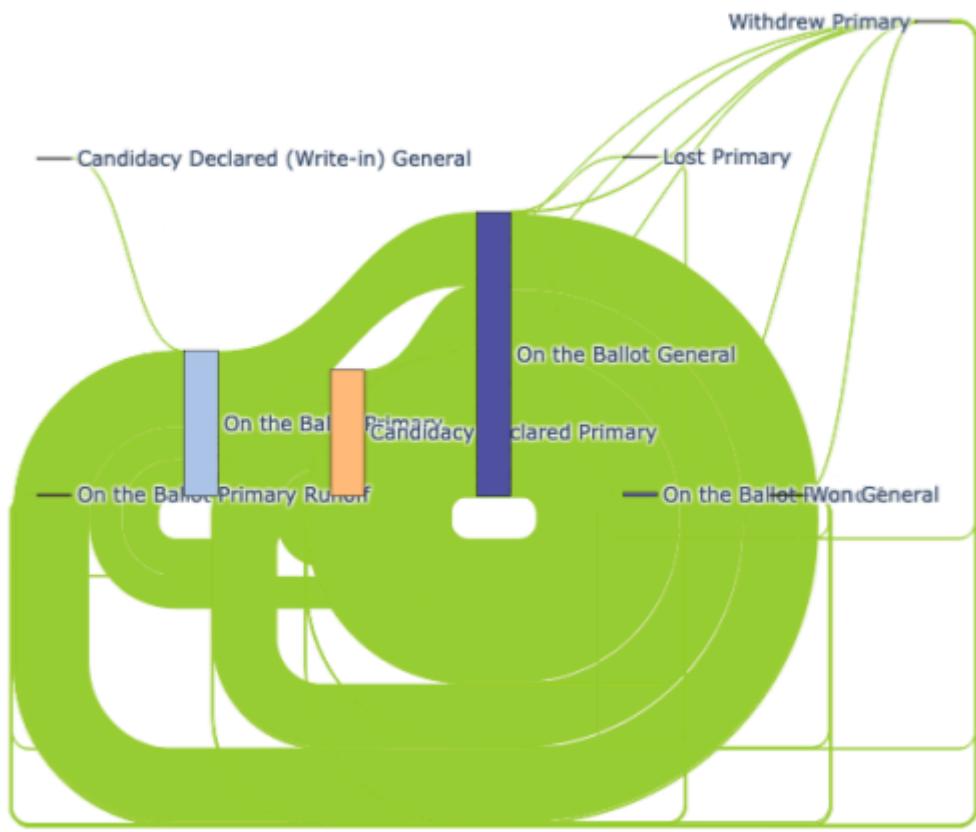


Fig 24-2: Sankey Diagram of Party for House Candidates



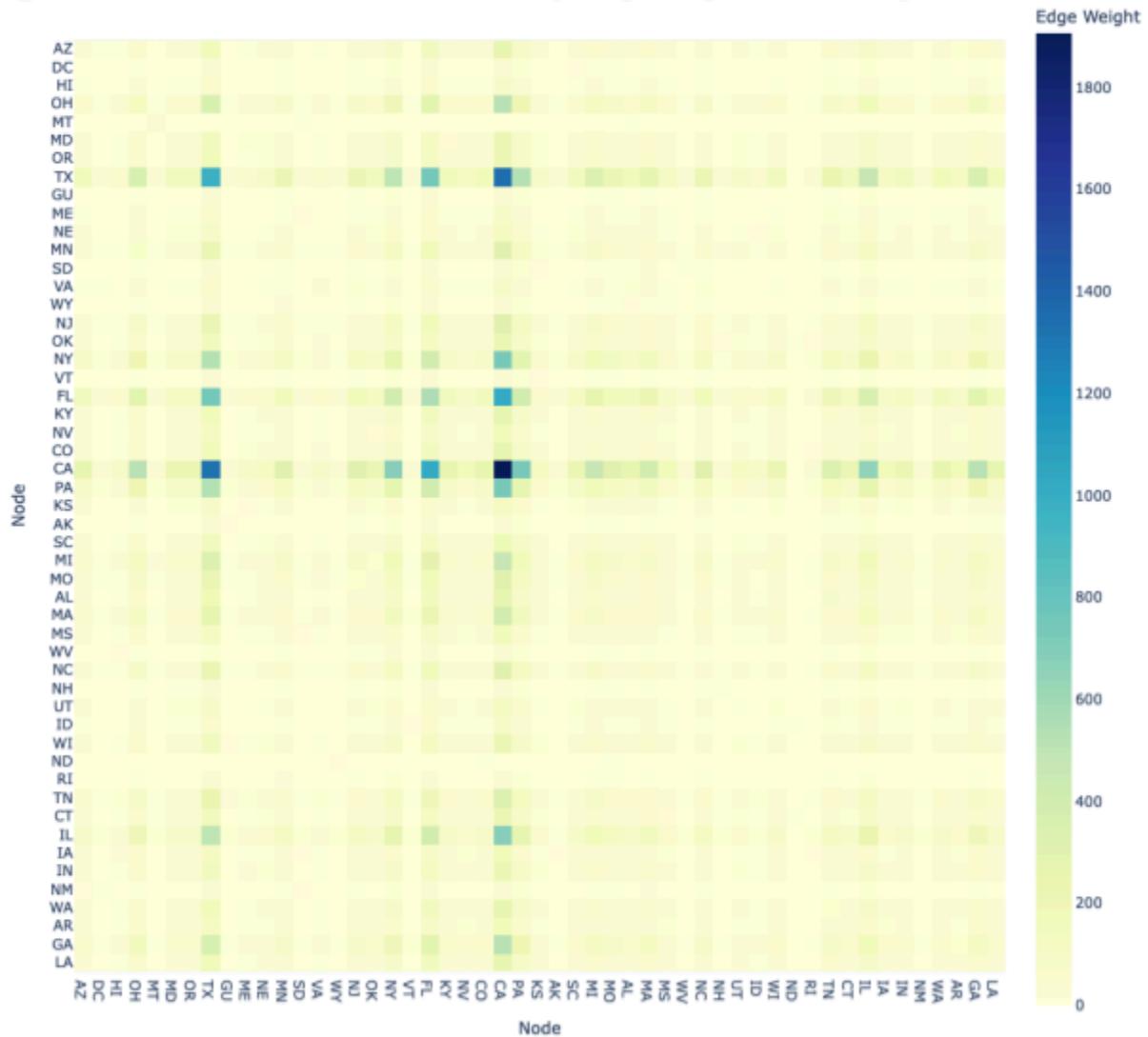
**Fig 24-3: Sankey Diagram of Status for House Candidates**



## 7.6. State Attributed Heatmap of House Candidates Hyperlink Graph

The heatmap below offers a detailed look at how House candidates are interconnected within the network, with the color intensity indicating the weight of the edge between the corresponding nodes. Nodes along the diagonal represent self-loops. For example, the darkest node shows California has a high connectivity to other candidate nodes also from California. There also appears to be many candidates from California and Texas connected to each other.

**Fig 25: State Attributed House Candidates Graph Adjacency Matrix Heatmap**



## 8. Nodal Centrality Indices

In this section we explore multiple centrality measures for nodes in both the Senate and House hyperlinks graph. We begin by defining the measure of centrality used in our analysis. Our calculations were performed using NetworkX's centrality functions.

- **Out-degree centrality:** In a directed graph, the degree centrality of a node is the number of nodes outgoing from the node divided by the total number of nodes in the

graph minus one

([https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.out\\_degree\\_centrality.html#networkx.algorithms.centrality.out\\_degree\\_centrality](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.out_degree_centrality.html#networkx.algorithms.centrality.out_degree_centrality)).

- **In-degree centrality:** In a directed graph, the degree centrality of a node is the number of nodes incoming to the node divided by the total number of nodes in the graph minus one

([https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.in\\_degree\\_centrality.html#networkx.algorithms.centrality.in\\_degree\\_centrality](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.in_degree_centrality.html#networkx.algorithms.centrality.in_degree_centrality)).

- **Degree centrality:** In an undirected graph, the degree centrality of a node is the number of nodes adjacent to the node divided by the total number of nodes in the graph minus one

([https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.degree\\_centrality.html](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.degree_centrality.html)).

- **Closeness centrality:** Closeness centrality quantifies how close a node is to all other nodes in the network by using the average shortest path distance to a node over all other reachable nodes (the exact mathematical definition is here

[https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.closeness\\_centrality.html#networkx.algorithms.centrality.closeness\\_centrality](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.closeness_centrality.html#networkx.algorithms.centrality.closeness_centrality)).

- **Betweenness centrality:** Betweenness centrality quantifies the importance of a node based on the number of times it acts as a bridge along the shortest path between two other nodes (the exact mathematical definition is here

<https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algo>

[rithms.centrality.betweenness\\_centrality.html#networkx.algorithms.centrality.betweenness\\_centrality](#).

- **Eigenvector centrality:** Eigenvector centrality iteratively computes the centrality of a node based on the centrality of its neighbors (the exact mathematical definition is here [https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.eigenvector\\_centrality](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.eigenvector_centrality.html#networkx.algorithms.centrality.eigenvector_centrality)).
- **HITS:** The HITS algorithm computes two numbers for a node- hubs and authorities. Authorities estimate the node value based on the incoming links. Hubs estimates the node value based on outgoing links  
([https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.link\\_analysis.hits\\_alg.hits](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.link_analysis.hits_alg.hits.html#networkx.algorithms.link_analysis.hits_alg.hits)).
- **Katz centrality:** Katz centrality measures the relative influence of a node within a network by considering the number of all paths leading to that node, while giving a diminishing weight to longer paths. It is defined using a parameter  $\alpha$  that scales the importance of the paths and a parameter  $\beta$  that adds a constant to account for each node's centrality (the exact mathematical definition is here [https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.katz\\_centrality](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.katz_centrality.html#networkx.algorithms.centrality.katz_centrality)).
- **PageRank centrality:** PageRank computes a ranking of the nodes in the graph based on the structure of the incoming links. It was originally designed as an algorithm to rank web pages  
([https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.link\\_analysis.page\\_rank\\_alg.page\\_rank](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.link_analysis.page_rank_alg.page_rank.html#networkx.algorithms.link_analysis.page_rank_alg.page_rank)).

- **Load centrality:** The load centrality of a node is the fraction of all shortest paths that pass through that node  
([https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.load\\_centrality.html#networkx.algorithms.centrality.load\\_centrality](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.load_centrality.html#networkx.algorithms.centrality.load_centrality)).
- **Communicability:** The communicability between pairs of nodes  $(u,v)$  in a graph is the sum of walks of different lengths starting at  $u$  and ending at  $v$   
([https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.communicability\\_alg.communicability.html#networkx.algorithms.communicability\\_alg.communicability](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.communicability_alg.communicability.html#networkx.algorithms.communicability_alg.communicability)).

## 8.1. Centrality Measures for Reciprocated (undirected) Senate Graph

**Table 19: Centrality Measures for Reciprocated Subgraph of Senate Graph**

node	degree	closeness	betweenness	eigenvector	HITS	Katz	PageRank	load	communicability
Colin Allred	0.261905	0.531646	0.358188	0.022944	0.004855	0.021787	0.028444	0.35707	0.470194
Elizabeth Warren	0.547619	0.626866	0.221332	0.221603	0.046895	0.221539	0.033383	0.218448	0.695074
Ted Cruz	0.5	0.591549	0.173868	0.218282	0.046192	0.218272	0.029273	0.173926	0.655624
Justin Amash	0.166667	0.482759	0.094774	0.035161	0.007441	0.034424	0.017956	0.095891	0.167885
Rick Scott	0.5	0.506024	0.092915	0.217709	0.046071	0.217721	0.032118	0.092915	0.577673
David Trone	0.238095	0.407767	0.092915	0.002064	0.000436	0.000812	0.031618	0.092915	0.137938
Elissa Slotkin	0.285714	0.424242	0.061556	0.003923	0.00083	0.002601	0.034377	0.06417	0.109371
Sherrod Brown	0.5	0.5	0.047619	0.217708	0.04607	0.217721	0.031036	0.047619	0.556273
Alexander Mooney	0.238095	0.403846	0.047619	0.002064	0.000436	0.000813	0.030796	0.047619	0.099174
Angela Alsobrooks	0.047619	0.295775	0.047619	0.000103	2.2e-05	-0.000661	0.014312	0.047619	0.048019
Deb Fischer	0.5	0.5	0.047619	0.217708	0.04607	0.217721	0.031036	0.047619	0.556273
Debbie Mucarsel-Powe	0.047619	0.344262	0.047619	0.010878	0.002302	0.010211	0.012138	0.047619	0.049317
Adam Schiff	0.238095	0.403846	0.047619	0.002064	0.000436	0.000813	0.030796	0.047619	0.099174
Amy Klobuchar	0.52381	0.525	0.038753	0.220513	0.046664	0.220501	0.031213	0.03886	0.58082
Kirsten Gillibrand	0.52381	0.525	0.038753	0.220513	0.046664	0.220501	0.031213	0.03886	0.58082
Don Blankenship	0.095238	0.411765	0.0	0.03478	0.00736	0.034182	0.009314	0.0	0.031078
Sheldon Whitehouse	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Mike Rogers	0.071429	0.371681	0.0	0.00205	0.000434	0.001247	0.011308	0.0	0.007111
Larry Hogan	0.02381	0.229508	0.0	5e-06	1e-06	-0.0007	0.009571	0.0	0.000185
Alan Grayson	0.02381	0.257669	0.0	0.000542	0.000115	-0.000156	0.008647	0.0	0.000122
Maria Cantwell	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Tim Kaine	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Peter Meijer	0.071429	0.371681	0.0	0.00205	0.000434	0.001247	0.011308	0.0	0.007111
Dan Osborn	0.02381	0.336	0.0	0.010851	0.002296	0.010219	0.004745	0.0	0.001775
Bernie Moreno	0.02381	0.336	0.0	0.010851	0.002296	0.010219	0.004745	0.0	0.001775
Ruben Gallego	0.214286	0.4	0.0	0.002059	0.000435	0.000843	0.025788	0.0	0.058759
Marsha Blackburn	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Josh Hawley	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Jon Tester	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Roger Wicker	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Bob Casey Jr.	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Jacky Rosen	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Mazie K. Hirono	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Martin Heinrich	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
John Barrasso	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Kevin Cramer	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Tammy Baldwin	0.47619	0.494118	0.0	0.217193	0.045961	0.217234	0.027107	0.0	0.534634
Steve Garvey	0.02381	0.289655	0.0	0.000103	2.2e-05	-0.000626	0.006106	0.0	0.000459
Jim Banks	0.214286	0.4	0.0	0.002059	0.000435	0.000843	0.025788	0.0	0.058759
John Curtis	0.214286	0.4	0.0	0.002059	0.000435	0.000843	0.025788	0.0	0.058759
Andrew Kim	0.214286	0.4	0.0	0.002059	0.000435	0.000843	0.025788	0.0	0.058759
Lisa Blunt Rochester	0.214286	0.4	0.0	0.002059	0.000435	0.000843	0.025788	0.0	0.058759
Jim Justice	0.02381	0.289655	0.0	0.000103	2.2e-05	-0.000626	0.006106	0.0	0.000459

## 8.2. Centrality Measures for Directed Senate Graph

**Table 20: Centrality Measures for Connected Subgraph of Senate Graph**

node	out_degree	in_degree	closeness	betweenness	eigenvector	HITS_hubs	HITS_auths	Katz	PageRank	load
Elizabeth Warren	0.418182	0.472727	0.597826	0.146002	0.221651	0.045116	0.046927	0.221477	0.040501	0.131152
Colin Allred	0.2	0.2	0.478261	0.109852	0.023966	0.005092	0.005415	0.022661	0.022476	0.088811
David Trone	0.236364	0.181818	0.376712	0.108876	0.003016	0.00531	0.000828	0.001609	0.026388	0.110723
Adam Schiff	0.181818	0.236364	0.474138	0.088159	0.013447	0.000676	0.00338	0.011998	0.028643	0.105017
Rick Scott	0.381818	0.4	0.561224	0.085753	0.217711	0.044307	0.045742	0.217694	0.033391	0.09064
Josh Hawley	0.381818	0.4	0.544554	0.08435	0.217195	0.04436	0.045737	0.217177	0.031483	0.105665
Ted Cruz	0.381818	0.418182	0.561224	0.073846	0.220002	0.044429	0.046365	0.219927	0.032416	0.060216
Justin Amash	0.145455	0.127273	0.44	0.059054	0.035182	0.009475	0.007376	0.034357	0.014221	0.059302
Alexander Mooney	0.218182	0.181818	0.371622	0.057095	0.003016	0.00527	0.00083	0.00161	0.023432	0.059289
Elissa Slotkin	0.218182	0.236364	0.387324	0.04438	0.004878	0.001174	0.001581	0.003357	0.027274	0.046263
Jacky Rosen	0.363636	0.418182	0.495495	0.035017	0.217052	0.044193	0.045817	0.217025	0.032578	0.035017
Debbie Mucarsel-Powell	0.036364	0.036364	0.366667	0.031987	0.010873	0.00222	0.002271	0.010125	0.009874	0.031987
Sherrod Brown	0.381818	0.381818	0.486726	0.031987	0.217566	0.04432	0.045469	0.217618	0.03091	0.031987
Debra Fischer	0.381818	0.381818	0.486726	0.031987	0.217566	0.04432	0.045469	0.217618	0.03091	0.031987
Angela Alsobrooks	0.036364	0.036364	0.280612	0.027609	0.000158	5.3e-05	0.000272	-0.000705	0.015787	0.027609
Amy Klobuchar	0.4	0.4	0.509259	0.023281	0.220371	0.04489	0.046175	0.220395	0.030855	0.023397
Kirsten Gillibrand	0.4	0.4	0.509259	0.023281	0.220371	0.04489	0.046175	0.220395	0.030855	0.023397
Marsha Blackburn	0.363636	0.4	0.491071	0.021044	0.217052	0.044199	0.045702	0.217063	0.031231	0.021044
Shiva Ayyadurai	0.018182	0.018182	0.018182	0.014478	0.0	0.002272	0.0	-0.000785	0.004955	0.014478
Larry Hogan	0.018182	0.054545	0.282051	0.014141	0.000158	1.3e-05	0.000275	-0.00074	0.013391	0.014141
Bob Casey Jr.	0.363636	0.381818	0.486726	0.014141	0.217052	0.04421	0.045475	0.2171	0.02983	0.014141
Tammy Baldwin	0.363636	0.381818	0.486726	0.014141	0.217052	0.04421	0.045475	0.2171	0.02983	0.014141
Mike Rogers	0.072727	0.054545	0.341615	0.000786	0.002101	0.000625	0.000568	0.001197	0.008359	0.000842
Sam Brown	0.036364	0.018182	0.018182	0.000168	0.0	0.00443	0.000114	-0.000785	0.003438	0.000168
Lucas Kunce	0.018182	0.0	0.0	0.0	0.0	0.002214	-0.0	-0.000748	0.002679	0.0
David McCormick	0.018182	0.0	0.0	0.0	0.0	0.002201	0.0	-0.000748	0.002679	0.0
Jim Justice	0.018182	0.018182	0.272277	0.0	0.000015	4e-05	0.000269	-0.000668	0.004338	0.0
Gloria Johnson	0.018182	0.0	0.0	0.0	0.0	0.002212	-0.0	-0.000748	0.002679	0.0
Marquita Bradshaw	0.018182	0.0	0.0	0.0	0.0	0.002272	-0.0	-0.000748	0.002679	0.0
Don Blankenship	0.072727	0.072727	0.395683	0.0	0.034751	0.007099	0.007382	0.034083	0.00807	0.0
Jim Merchant	0.018182	0.018182	0.018182	0.0	0.0	0.002218	0.000114	-0.000785	0.003438	0.0
Bernie Moreno	0.018182	0.018182	0.329341	0.0	0.010838	0.002201	0.002266	0.010133	0.00393	0.0
Robin Ficker	0.018182	0.0	0.0	0.0	0.0	1.3e-05	0.0	-0.000748	0.002679	0.0
Jeff Gunter	0.054545	0.0	0.0	0.0	0.0	0.002229	0.0	-0.000748	0.002679	0.0
Peter Meijer	0.054545	0.054545	0.341615	0.0	0.002101	0.000461	0.000576	0.001197	0.007898	0.0
Hill Harper	0.018182	0.0	0.0	0.0	0.0	7.7e-05	0.0	-0.000748	0.002679	0.0
Dan Osborn	0.018182	0.018182	0.329341	0.0	0.010838	0.002201	0.002266	0.010133	0.00393	0.0
Ruben Gallego	0.163636	0.163636	0.369128	0.0	0.003009	0.000787	0.001057	0.001642	0.019557	0.0
Randy Toler	0.018182	0.0	0.0	0.0	0.0	0.0	-0.0	-0.000748	0.002679	0.0
Alan Grayson	0.018182	0.018182	0.269608	0.0	0.000542	0.00011	0.000113	-0.000242	0.006875	0.0
Lisa Blunt Rochester	0.163636	0.163636	0.369128	0.0	0.003009	0.000787	0.001057	0.001642	0.019557	0.0
Andrew Kim	0.163636	0.163636	0.369128	0.0	0.003009	0.000787	0.001057	0.001642	0.019557	0.0
John Curtis	0.163636	0.163636	0.369128	0.0	0.003009	0.000787	0.001057	0.001642	0.019557	0.0
Jim Banks	0.163636	0.163636	0.369128	0.0	0.003009	0.000787	0.001057	0.001642	0.019557	0.0
Steve Garvey	0.018182	0.018182	0.323529	0.0	0.00067	0.000164	3.5e-05	-0.000148	0.005113	0.0
Robert Hyde	0.018182	0.0	0.0	0.0	0.0	0.000164	-0.0	-0.000748	0.002679	0.0
Kevin Cramer	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
John Barrasso	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
Martin Heinrich	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
Mazie K. Hirono	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
Roger Wicker	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
Jon Tester	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
Sheldon Whitehouse	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
Tim Kaine	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
Maria Cantwell	0.363636	0.363636	0.482456	0.0	0.217052	0.044215	0.045362	0.217136	0.027646	0.0
Eric Hovde	0.018182	0.0	0.0	0.0	0.0	0.002201	-0.0	-0.000748	0.002679	0.0

### 8.3. Centrality Measures for Reciprocated (undirected) House Graph (subset)

We consider the same subgraph of 5 states- Colorado, Connecticut, Georgia, Illinois and Washington- that we used in the section 6 visualizations. The reciprocated subgraph has 52 vertices and 967 edges. Table 21 displays the calculated centrality measures.

**Table 21: Centrality Measures for Reciprocated Subgraph of House Candidates**

node	degree	closeness	betweenness	eigenvector	HITS	Katz	PageRank	load
Lauren Boebert	0.862745	0.864379	0.035294	0.150212	0.022475	0.129893	0.022674	0.035294
Jahana Hayes	0.862745	0.864379	0.035294	0.150212	0.022475	0.129893	0.022674	0.035294
Lauren Underwood	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Eric Sorensen	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Robin Kelly	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Mary Miller	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Bill Foster	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Delia Ramirez	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Nikki Budzinski	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Sean Casten	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Darin LaHood	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Brad Schneider	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Jonathan Jackson	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Raja Krishnamoorthi	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Danny K. Davis	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Jan Schakowsky	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Mike Bost	0.862745	0.864379	0.001621	0.151327	0.022642	0.134383	0.020393	0.001621
Brittany Pettersen	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Jim Himes	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Rick Larsen	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Mike Collins	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Drew MacEwen	0.019608	0.019608	0.0	0.0	0.0	-0.163014	0.019231	0.0
Joe Neguse	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
George Logan	0.019608	0.446131	0.0	0.003484	0.000521	-0.148369	0.003323	0.0
Marilyn Strickland	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Barry Loudermilk	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Kim Schrier	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Michael Lynch (1)	0.039216	0.039216	0.0	0.0	-0.0	-0.172071	0.019231	0.0
Suzan DelBene	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Mike Quigley	0.294118	0.525192	0.0	0.052641	0.007876	-0.054077	0.008794	0.0
Diana DeGette	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Nikema Williams	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Joe Courtney	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Rich McCormick	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Marjorie Taylor Greene	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Pramila Jayapal	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Jason Crow	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Yadira Caraveo	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Sanford Bishop Jr.	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Emily Randall	0.019608	0.019608	0.0	0.0	-0.0	-0.163014	0.019231	0.0
David Scott	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Hank Johnson	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Dan Newhouse	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Gabe Evans	0.039216	0.039216	0.0	0.0	-0.0	-0.172071	0.019231	0.0
Lucy McBath	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Austin Scott	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Rick Allen	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Andrew Clyde	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Richard Holtorf	0.039216	0.039216	0.0	0.0	-0.0	-0.172071	0.019231	0.0
John Larson	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0
Adam Frisch	0.019608	0.446131	0.0	0.003484	0.000521	-0.148369	0.003323	0.0
Rosa L. DeLauro	0.843137	0.846739	0.0	0.150133	0.022463	0.136958	0.019895	0.0

Evaluating these centrality measures, we can see that most candidates are well connected to others in the reciprocated network. The high degree and closeness centrality

measure of candidate nodes indicate high accessibility between candidates' Wikipedia pages. Lauren Boebert and Jahana Hayes are two candidates that stand out due to their high values in betweenness and load centrality, indicating that they serve as connectors in the network.

#### 8.4. Centrality Measures for Directed House Graph (subset)

The weakly connected subgraph of the House hyperlink graph for Colorado, Connecticut, Georgia, Illinois and Washington contains 60 vertices and 1,969 edges. Table 22 displays the calculated centrality measures.

**Table 22: Centrality Measures for Connected Subgraph of House Candidates**

node	out_degree	in_degree	closeness	betweenness	eigenvector	HITS_hubs	HITS_auths	Katz	PageRank	load
Lauren Boebert	0.745763	0.779661	0.787228	0.054062	0.150546	0.022094	0.022552	0.112178	0.02612	0.054062
Jahana Hayes	0.745763	0.762712	0.760082	0.027762	0.150546	0.022095	0.02254	0.119202	0.022029	0.027762
Ron Hanks	0.016949	0.016949	0.016949	0.013735	0.0	0.000514	-0.0	-0.147149	0.00482	0.013735
Dan Newhouse	0.728814	0.762712	0.760082	0.013442	0.150546	0.022083	0.02254	0.118918	0.021664	0.013442
David Scott	0.728814	0.762712	0.760082	0.013442	0.150546	0.022083	0.02254	0.118918	0.021664	0.013442
Joe Courtney	0.728814	0.762712	0.760082	0.013442	0.150546	0.022083	0.02254	0.118918	0.021664	0.013442
Jonathan Jackson	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Delia Ramirez	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Raja Krishnamoorthi	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Mary Miller	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Eric Sorensen	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Darin LaHood	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Robin Kelly	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Bill Foster	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Nikki Budzinski	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Danny K. Davis	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Jan Schakowsky	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Mike Bost	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Lauren Underwood	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Sean Casten	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Brad Schneider	0.745763	0.745763	0.747199	0.000701	0.150546	0.022259	0.02254	0.125592	0.019501	0.000701
Barry Loudermilk	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Mike Quigley	0.745763	0.254237	0.46405	0.0	0.052095	0.022597	0.007716	-0.045948	0.008256	0.0
Joe Neguse	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Kim Schrier	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Michael Lynch (1)	0.033898	0.033898	0.033898	0.0	0.0	-0.0	-0.0	-0.155713	0.017368	0.0
Mike France	0.016949	0.0	0.0	0.0	0.0	0.000514	-0.0	-0.140142	0.002605	0.0
George Logan	0.016949	0.016949	0.408192	0.0	0.003475	0.000514	0.000511	-0.134182	0.003031	0.0
Suzan DelBene	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Brittany Pettersen	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Mike Collins	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Jim Himes	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Philip Singleton	0.016949	0.0	0.0	0.0	0.0	0.0	0.0	-0.140142	0.002605	0.0
Rick Larsen	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Marilyn Strickland	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Drew MacEwen	0.016949	0.016949	0.016949	0.0	0.0	-0.0	-0.0	-0.147518	0.017368	0.0
Nikema Williams	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Diana DeGette	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Rich McCormick	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Marjorie Taylor Greene	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Pramila Jayapal	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Jason Crow	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Yadira Caraveo	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Sanford Bishop Jr.	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Emily Randall	0.016949	0.016949	0.016949	0.0	0.0	-0.0	-0.0	-0.147518	0.017368	0.0
Hank Johnson	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Gabe Evans	0.033898	0.033898	0.033898	0.0	0.0	-0.0	-0.0	-0.155713	0.017368	0.0
Jerrod Sessler	0.016949	0.0	0.0	0.0	0.0	0.000514	-0.0	-0.140142	0.002605	0.0
Mandisha Thomas	0.0	0.016949	0.016949	0.0	0.0	-0.0	0.0	-0.147149	0.00482	0.0
Lucy McBath	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Mike Crane	0.016949	0.0	0.0	0.0	0.0	0.000514	-0.0	-0.140142	0.002605	0.0
Austin Scott	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Dave Williams	0.016949	0.0	0.0	0.0	0.0	-0.0	-0.0	-0.140142	0.002605	0.0
Rick Allen	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Andrew Clyde	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Joe Kent	0.0	0.0	0.0	0.0	0.0	-0.0	-0.0	-0.140142	0.002605	0.0
Richard Holtorf	0.033898	0.033898	0.033898	0.0	0.0	-0.0	-0.0	-0.155713	0.017368	0.0
John Larson	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0
Adam Frisch	0.016949	0.016949	0.415894	0.0	0.003475	0.000514	0.000511	-0.134533	0.00311	0.0
Rosa L. DeLauro	0.728814	0.745763	0.747199	0.0	0.150546	0.022083	0.022528	0.125592	0.019493	0.0

## 9. Attribute Assortativity Coefficients

Assortative mixing in a graph quantifies the tendency of nodes to connect with other ‘similar’ nodes over ‘dissimilar’ nodes (see Newman <https://arxiv.org/pdf/cond-mat/0209450>). The assortativity coefficient of a graph is a measure of the extent to which assortative mixing among nodes occurs in a graph. The range of the assortativity coefficient of a graph is between  $-1$  and  $1$ . Assortativity coefficients close to  $1$  indicate that there is a very high likelihood of two nodes sharing the same property to be adjacent (assortative graph). Assortativity coefficients close to  $-1$  indicate that there is very low likelihood of two nodes sharing the same property to be adjacent (disassortative graph). Graphs with assortativity coefficients close to zero are indeterminate in terms of their assortative mixing (neutral assortative graph).

Here we consider cases of assortative mixing corresponding to the attributes of our candidates’ graphs. In this section, we evaluate the assortativity for the Senate and House graphs based on the attributes we described in Section 5. The assortativity coefficients were calculated using NetworkX’s `attribute_assortativity_coefficient` function ([https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.assortativity.attribute\\_assortativity\\_coefficient.html#networkx.algorithms.assortativity.attribute\\_asortativity\\_coefficient](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.assortativity.attribute_assortativity_coefficient.html#networkx.algorithms.assortativity.attribute_asortativity_coefficient)).

### 9.1. Attribute Assortativity for Senate Candidate Graphs

The incumbency-attributed Senate graph, having 58 vertices and 581 edges, has a corresponding incumbency assortativity coefficient of  $0.823$ , which is strongly assortative. This indicates that candidates of the same type of incumbency tend to be linked together.

Nevertheless, the party-attributed Senate candidates graph has a negative party assortativity coefficient of -0.044, i.e. this graph is rather neutral assortative. In addition, the status-attributed Senate candidates graph is neutral assortative too, having a small status assortativity coefficient of 0.02. Finally, the state-attributed Senate candidates graph is also neutral assortative with a very low coefficient of 0.04. Therefore, although in terms of incumbency, the graph is assortative, we cannot conclude the same in terms of party affiliation, status, or state.

## 9.2. Attribute Assortativity for House Candidate Graphs

The incumbency-attributed graph, with an incumbency assortativity coefficient of 0.346, displays a moderate tendency for assortativity, i.e, House candidates tend to connect with others sharing the same incumbency status. This is visually supported by the Sankey diagram in Figure 24-1 from Section 7.5, which shows incumbents and challengers linking predominantly within their groups.

In contrast, the party-attributed graph exhibits a party assortativity coefficient of -0.00179, indicating that connections are nearly random with respect to party affiliation. This suggests that candidates are equally likely to connect with members of their own party as they are with members of other parties.

Similarly, the state-attributed graph has a state assortativity coefficient of 0.00014, implying almost no assortative mixing based on state. This means that there is no apparent tendency for candidates to preferentially connect with others from either the same state or different states.

The status-attributed graph shows a smallest status assortativity coefficient, at 0.00001, further reinforcing the notion of random connectivity. The negligible value suggests that candidate connections are uniformly distributed across various statuses.

Wondering about the found neutral assortativity of the party, state and status attributes, we looked into a further evaluation of candidate Wikipedia pages. Thus we found that at the bottom of most pages a table containing hyperlinks to all current Senators or House members was included (respectively). This inclusion of comprehensive link tables likely contributes to the random neutral mixing observed in the party, state, and status-attributed graphs, as it creates a baseline level of connectivity that is not influenced by these attributes.

## 10. Graph Partition Communities

In this section, we examine various community partition algorithms applied to both the Senate and House hyperlink graphs. We start by defining the community partition algorithms utilized in our analysis. The calculations were conducted using NetworkX functions.

- **Girvan-Newman algorithm:** Detects communities by progressively removing edges from the original network. The connected components of the remaining network are the communities (applied to any graph type  
[https://en.wikipedia.org/wiki/Girvan%E2%80%93Newman\\_algorithm](https://en.wikipedia.org/wiki/Girvan%E2%80%93Newman_algorithm)).
- **Louvain communities:** Two step method for detecting communities: first small communities are found by optimizing modularity ([https://en.wikipedia.org/wiki/Modularity\\_\(networks\)](https://en.wikipedia.org/wiki/Modularity_(networks))) locally on all nodes, then each small community is grouped into one node and the first step is repeated (applies to any graph type [https://en.wikipedia.org/wiki/Louvain\\_method](https://en.wikipedia.org/wiki/Louvain_method)).
- **Leiden communities:** A method for detecting communities in large networks that improves upon the Louvain algorithm, offering better accuracy and faster performance by refining community partitions more effectively (only for undirected graphs  
[https://en.wikipedia.org/wiki/Leiden\\_algorithm](https://en.wikipedia.org/wiki/Leiden_algorithm)).

- **Label propagation algorithm (LPA):** A community detection algorithm where each node adopts the label (community) that most of its neighbors belong to, iteratively updating labels until a consensus is reached (only for undirected graphs  
[https://en.wikipedia.org/wiki/Label\\_propagation\\_algorithm](https://en.wikipedia.org/wiki/Label_propagation_algorithm)).
- **Asynchronous label propagation algorithm:** A variation of the label propagation algorithm where nodes update their labels one at a time in a random order, which can help avoid oscillations and speed up convergence (applied to any graph  
[https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.community.label\\_propagation.asyn\\_lpa\\_communities.html#r40d53c2ae55e-1](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.community.label_propagation.asyn_lpa_communities.html#r40d53c2ae55e-1)).
- **Fluid communities:** A dynamic community detection algorithm that simulates the movement of fluid particles within the network, allowing communities to adapt and change fluidly based on the network's structure (only for connected simple undirected graphs  
[https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.community.asyn\\_fluid.asyn\\_fluidc.html](https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.community.asyn_fluid.asyn_fluidc.html)).

## 10.1. Community Partitions of Senate Candidates Graph

The Senate candidates' graph ( $G_s$ ) has 4 Girvan-Newman communities, 6 Louvain communities and 5 Asynchronous LPA communities. Table 23 presents a breakdown of the community partitions for the four attribute categories along with the Gini index to represent the sparsity of the attributed graph (see Goswami for the use of Gini index as a sparsity measure for network graph <https://www.sciencedirect.com/science/article/pii/S0020025518304158>). A Gini index greater than 0.4 is highlighted in the tables. Note that the counts for the attribute lists represent the alphabetically ordered attribute values. For example, the incumbency value

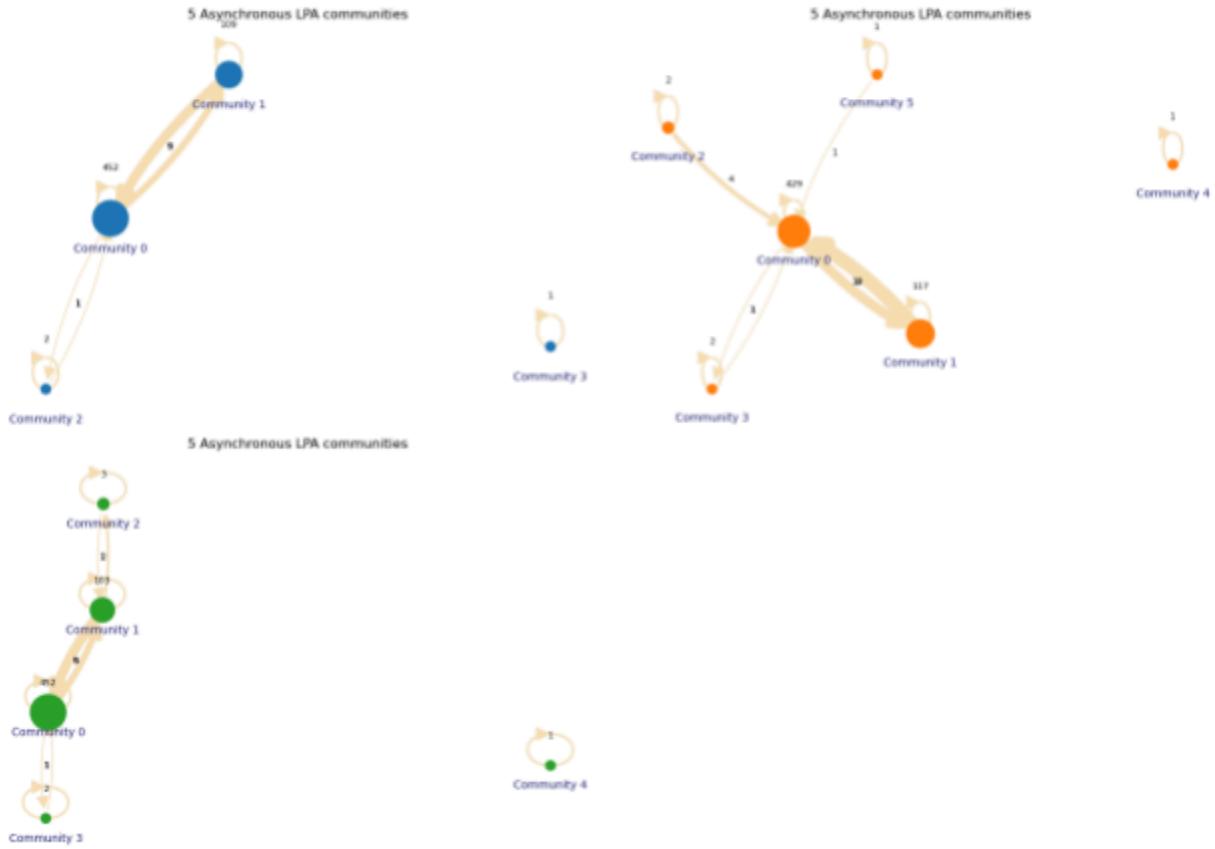
[14,21] represents 14 ‘challengers’ and 21 ‘incumbents.’ Figure 26 displays the three community image graphs.

**Table 23: Community Partitions by Attributes for the Senate Candidates’ Hyperlink Graph**

Community	inc incumbency	inc incumbency Gini index	party	party Gini index	state	state Gini index	status	status Gini index
Girvan-Newman community 0	[14, 21]	0.1	[17, 1, 2, 15]	0.435714	[0, 0, 0, 0, 2, 1, 0, 2, 0, 1, 1, 2, 1, 1, 1, 2, 0, 1, 4, 1, 2, 2, 1, 3, 1, 0, 1, 1, 2, 1, 1]	0.43318	[3, 13, 0, 4, 15]	0.457143
Girvan-Newman community 1	[19, 0]	0.5	[9, 0, 1, 9]	0.460526	[1, 2, 1, 1, 0, 0, 1, 0, 4, 4, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0]	0.75382	[1, 6, 0, 3, 9]	0.484211
Girvan-Newman community 2	[2, 0]	0.5	[2, 0, 0, 0]	0.75	[0, 0, 0, 0, 2, 0]	0.967742	[0, 2, 0, 0, 0]	0.8
Girvan-Newman community 3	[2, 0]	0.5	[0, 0, 0, 2]	0.75	[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]	0.935484	[0, 0, 1, 0, 1]	0.6
Louvain community 0	[7, 21]	0.25	[16, 0, 1, 11]	0.517857	[0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 2, 1, 1, 1, 2, 0, 1, 1, 1, 2, 2, 1, 3, 1, 0, 1, 1, 2, 0, 1]	0.449309	[1, 12, 0, 4, 11]	0.485714
Louvain community 1	[21, 0]	0.5	[10, 0, 1, 10]	0.464286	[1, 2, 1, 1, 0, 0, 1, 0, 4, 5, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 3, 0]	0.768049	[1, 7, 0, 3, 10]	0.495238
Louvain community 2	[3, 0]	0.5	[0, 0, 0, 3]	0.75	[0, 0]	0.967742	[0, 0, 0, 0, 3]	0.8
Louvain community 3	[2, 0]	0.5	[2, 0, 0, 0]	0.75	[0, 0, 0, 0, 2, 0]	0.967742	[0, 2, 0, 0, 0]	0.8
Louvain community 4	[2, 0]	0.5	[0, 0, 0, 2]	0.75	[1, 0]	0.935484	[0, 0, 1, 0, 1]	0.6
Louvain community 5	[2, 0]	0.5	[0, 1, 1, 0]	0.5	[0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.935484	[2, 0, 0, 0, 0]	0.8
Asynchronous label propagation	[14, 21]	0.1	[17, 1, 2, 15]	0.435714	[0, 0, 0, 0, 2, 1, 0, 2, 0, 1, 1, 2, 1, 1, 1, 2, 0, 1, 4, 1, 2, 2, 1, 3, 1, 0]	0.43318	[3, 13, 0, 4, 15]	0.457143

community 0					1, 1, 2, 1, 1]		
Asynchronous label propagation community 1	[16, 0]	0.5	[8, 0, 1, 7]	0.46875	[1, 2, 1, 1, 0, 0, 1, 0, 1, 4, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0]	0.733871	[1, 6, 0, 3, 6]
Asynchronous label propagation community 2	[3, 0]	0.5	[1, 0, 0, 2]	0.583333	[0, 0, 0, 0, 0, 0, 0, 3, 0]	0.967742	[0, 0, 0, 0, 3]
Asynchronous label propagation community 3	[2, 0]	0.5	[2, 0, 0, 0]	0.75	[0, 0, 0, 0, 2, 0]	0.967742	[0, 2, 0, 0, 0]

**Fig 26: Communities Image Graphs for the Senate Graph**



## 10.2. Community Partitions of Reciprocated Senate Candidates Graph

The reciprocated Senate candidates' graph (recGh) has 3 Girvan-Newman communities, 4 Louvain communities, 4 Leiden communities, 4 LPA communities, 5 Asynchronous LPA communities and 5 Fluid communities. Table 24 presents a breakdown of the community partitions for each of the four attributes along with their Gini index. Figure 27 displays the six community image graphs.

**Table 24: Community Partitions by Attributes for the Reciprocated Senate Candidates' Hyperlink Graph**

Community	incumbency	incumbency Gini index	party	party Gini index	state	state Gini index	status	status Gini index
Girvan-Newman community 0	[4, 21]	0.34	[14, 1, 10]	0.346667	[0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 2, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1]	0.289333	[1, 11, 4, 9]	0.35
Girvan-Newman community 1	[16, 0]	0.5	[8, 0, 8]	0.333333	[1, 2, 1, 0, 0, 1, 0, 3, 3, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0]	0.75	[0, 5, 3, 8]	0.40625
Girvan-Newman community 2	[2, 0]	0.5	[2, 0, 0]	0.666667	[0, 0, 0, 2, 0]	0.966667	[0, 2, 0, 0]	0.75
Louvain community 0	[2, 21]	0.413043	[13, 1, 9]	0.347826	[0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 2, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1]	0.355072	[1, 10, 4, 8]	0.336957
Louvain community 1	[14, 0]	0.5	[8, 0, 6]	0.380952	[1, 2, 1, 0, 0, 1, 0, 3, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0]	0.738095	[0, 3, 3, 8]	0.428571
Louvain community 2	[4, 0]	0.5	[1, 0, 3]	0.5	[0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0]	0.95	[0, 3, 0, 1]	0.625
Louvain community 3	[2, 0]	0.5	[2, 0, 0]	0.666667	[0, 0, 0, 2, 0]	0.966667	[0, 2, 0, 0]	0.75
Leiden community 0	[2, 21]	0.413043	[13, 1, 9]	0.347826	[0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 2, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1]	0.355072	[1, 10, 4, 8]	0.336957
Leiden community 1	[14, 0]	0.5	[8, 0, 6]	0.380952	[1, 2, 1, 0, 0, 1, 0, 3, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0]	0.738095	[0, 3, 3, 8]	0.428571
Leiden community 2	[4, 0]	0.5	[1, 0, 3]	0.5	[0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0]	0.95	[0, 3, 0, 1]	0.625
Leiden community 3	[2, 0]	0.5	[2, 0, 0]	0.666667	[0, 0, 0, 2, 0]	0.966667	[0, 2, 0, 0]	0.75

Label propagation community 0	[4, 21]	0.34	[14, 1, 10]	0.346667	[0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 2, 0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1]	0.289333	[1, 11, 4, 9]	0.35
Label propagation community 1	[14, 0]	0.5	[7, 0, 7]	0.333333	[1, 2, 1, 0, 0, 1, 0, 1, 3, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0]	0.738095	[0, 5, 3, 6]	0.357143
Label propagation community 2	[2, 0]	0.5	[2, 0, 0]	0.666667	[0, 0, 0, 2, 0]	0.966667	[0, 2, 0, 0]	0.75
Label propagation community 3	[2, 0]	0.5	[1, 0, 1]	0.333333	[0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.966667	[0, 0, 0, 2]	0.75
Asynchronous label propagation community 0	[4, 21]	0.34	[14, 1, 10]	0.346667	[0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 2, 0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1]	0.289333	[1, 11, 4, 9]	0.35
Asynchronous label propagation community 1	[12, 0]	0.5	[7, 0, 5]	0.388889	[1, 2, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0]	0.711111	[0, 3, 3, 6]	0.375
Asynchronous label propagation community 2	[2, 0]	0.5	[2, 0, 0]	0.666667	[0, 0, 0, 2, 0]	0.966667	[0, 2, 0, 0]	0.75
Asynchronous label propagation community 3	[2, 0]	0.5	[1, 0, 1]	0.333333	[0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.966667	[0, 0, 0, 2]	0.75
Asynchronous label propagation community 4	[2, 0]	0.5	[0, 0, 2]	0.666667	[0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.966667	[0, 2, 0, 0]	0.75
Fluid community 0	[3, 20]	0.369565	[14, 1, 8]	0.376812	[0, 0, 0, 3, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 2, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1]	0.413043	[1, 12, 2, 8]	0.423913
Fluid community 1	[11, 0]	0.5	[7, 0, 4]	0.424242	[1, 1, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0]	0.693939	[0, 3, 2, 6]	0.431818
Fluid community 2	[4, 0]	0.5	[1, 0, 3]	0.5	[0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0]	0.95	[0, 3, 0, 1]	0.625
Fluid community 3	[3, 0]	0.5	[1, 0, 2]	0.444444	[0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.944444	[0, 0, 1, 2]	0.583333
Fluid community 4	[1, 1]	0	[1, 0, 1]	0.333333	[0, 0]	0.966667	[0, 0, 2, 0]	0.75

**Fig 27: Communities Image Graphs for the Senate Reciprocated Graph**



### 10.3. Community Partitions of House Candidates Graph

The House candidates' graph ( $G_h$ ) has 9 Girvan-Newman communities, 24 Louvain communities and 18 Asynchronous LPA communities. Table 25 presents a breakdown of the community partitions for each of the four attributes along with their Gini index. Figure 28 displays the three community image graphs and Figure 29 shows an alternative view of the community image graphs that will also display the attribute breakdown for each community when viewed as an interactive (html) rendering.

**Table 25: Community Partitions by Attributes for the House Candidates' Hyperlink Graph**



					0, 0]			
Louvain community 8	[3, 1]	0.25	[0, 0, 0, 4]	0.75	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.980392	[0, 0, 0, 0, 4, 0, 0, 0, 0]	0.888889
Louvain community 9	[2, 2]	0	[0, 3, 0, 1]	0.625	[0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.960784	[0, 2, 0, 0, 2, 0, 0, 0, 0]	0.777778
Louvain community 10	[3, 1]	0.25	[0, 3, 0, 1]	0.625	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.980392	[0, 0, 0, 0, 4, 0, 0, 0, 0]	0.888889
Louvain community 11	[2, 1]	0.166667	[0, 1, 0, 2]	0.583333	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0]	0.980392	[0, 0, 0, 0, 0, 0, 3, 0, 0]	0.888889
Louvain community 12	[2, 1]	0.166667	[0, 3, 0, 0]	0.75	[0, 3, 0]	0.980392	[0, 3, 0, 0, 0, 0, 0, 0, 0]	0.888889
Louvain community 13	[3, 0]	0.5	[0, 0, 0, 3]	0.75	[0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.980392	[0, 0, 0, 0, 3, 0, 0, 0, 0]	0.888889
Louvain community 14	[1, 1]	0	[0, 2, 0, 0]	0.75	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0]	0.980392	[0, 2, 0, 0, 0, 0, 0, 0, 0]	0.888889
Louvain community 15	[2, 0]	0.5	[0, 1, 0, 1]	0.5	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.980392	[0, 0, 0, 2, 0, 0, 0, 0, 0]	0.888889

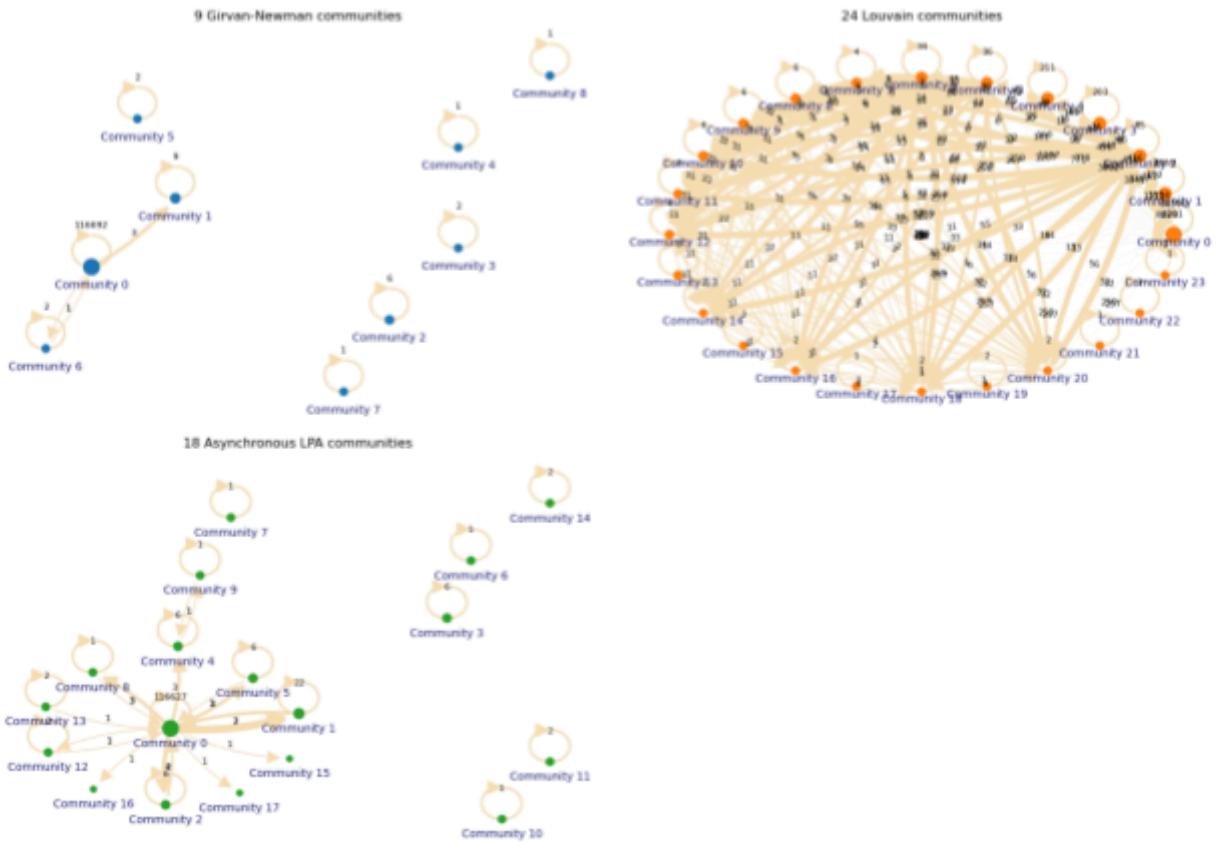




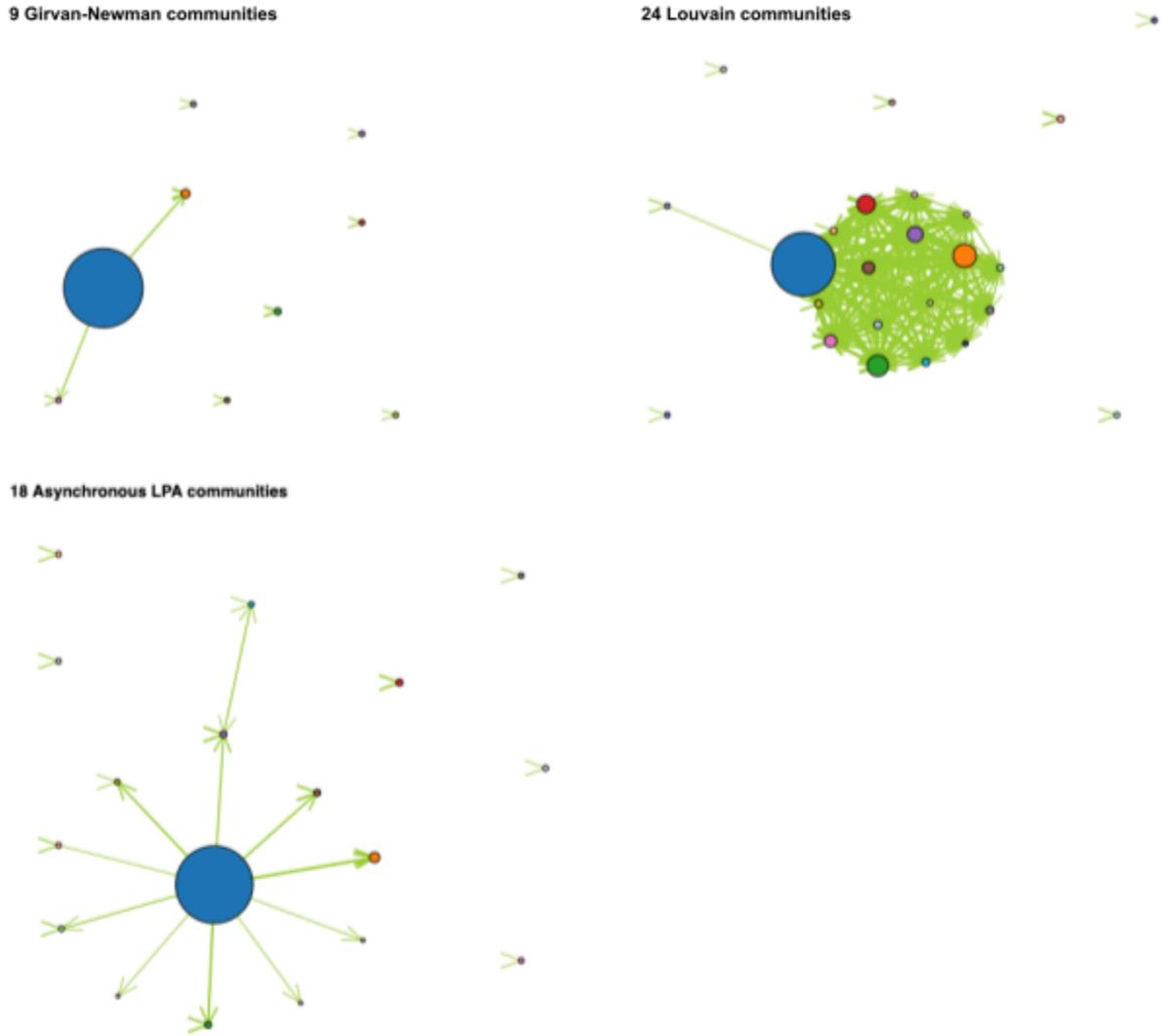


		0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]	
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**Fig 28: Communities Image Graphs for the House Candidates' Graph**



**Fig 29: Attributed House Candidates' Community Partitions**



#### 10.4. Community Partitions of Reciprocated House Candidates Graph

The reciprocated House candidates' graph (recGh) has 11 Girvan-Newman communities, 37 Louvain communities, 37 Leiden communities, 13 LPA communities and 13 Asynchronous LPA communities. Note that, unlike the reciprocated Senate graph, recGh is disconnected and thus Fluid communities cannot be calculated for this graph. Table 26 presents a breakdown of the community partitions for each of the four attributes along with their Gini index. Figures 30-31 display alternative views for the five community image graphs.

**Table 26: Community Partitions by Attributes for the Reciprocated House Candidates' Hyperlink Graph**



Louvain community 8	[5, 0]	0.5	[0, 4, 1]	0.533333	[0, 0, 0, 0, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 5, 0, 0, 0, 0, 0]	0.875
Louvain community 9	[3, 1]	0.25	[0, 0, 4]	0.666667	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 0, 4, 0, 0, 0, 0]	0.875
Louvain community 10	[3, 1]	0.25	[0, 4, 0]	0.666667	[0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.97	[0, 0, 1, 3, 0, 0, 0, 0]	0.8125
Louvain community 11	[4, 0]	0.5	[0, 0, 4]	0.666667	[0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.97	[0, 0, 0, 4, 0, 0, 0, 0]	0.875
Louvain community 12	[3, 1]	0.25	[0, 3, 1]	0.5	[0, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 0, 4, 0, 0, 0, 0]	0.875
Louvain community 13	[2, 1]	0.166667	[0, 2, 1]	0.444444	[0, 3, 0]	0.98	[0, 0, 3, 0, 0, 0, 0, 0]	0.875
Louvain community 14	[3, 0]	0.5	[0, 0, 3]	0.666667	[0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 0, 3, 0, 0, 0, 0]	0.875
Louvain community 15	[3, 0]	0.5	[0, 2, 1]	0.444444	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 0, 3, 0, 0, 0, 0]	0.875
Louvain community 16	[1, 1]	0	[0, 1, 1]	0.333333	[0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 0, 2, 0, 0, 0, 0]	0.875
Louvain community 17	[1, 1]	0	[0, 1, 1]	0.333333	[0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 2, 0, 0, 0, 0, 0]	0.875











Leiden community 31	[1, 1]	0	[0, 1, 1]	0.333333	[0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 0, 2, 0, 0, 0, 0]	0.875
Leiden community 32	[1, 1]	0	[0, 1, 1]	0.333333	[2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[2, 0, 0, 0, 0, 0, 0, 0]	0.875
Leiden community 33	[1, 1]	0	[1, 0, 1]	0.333333	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 0, 2, 0, 0, 0, 0]	0.875
Leiden community 34	[1, 1]	0	[0, 2, 0]	0.666667	[0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0]	0.96	[0, 0, 0, 2, 0, 0, 0, 0]	0.875
Leiden community 35	[2, 0]	0.5	[0, 0, 2]	0.666667	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 2, 0, 0, 0, 0, 0]	0.875
Leiden community 36	[1, 1]	0	[0, 1, 1]	0.333333	[0, 2, 0]	0.98	[0, 0, 0, 2, 0, 0, 0, 0]	0.875
Label propagation community 0	[29, 342]	0.421833	[1, 179, 191]	0.34142	[2, 6, 4, 8, 46, 7, 6, 1, 25, 12, 1, 2, 5, 3, 16, 8, 4, 7, 7, 9, 5, 2, 12, 8, 8, 4, 3, 7, 3, 1, 7, 2, 4, 17, 12, 5, 8, 17, 1, 6, 1, 8, 32, 3, 2, 1, 6, 5, 1, 1]	0.487278	[84, 1, 179, 102, 1, 2, 1, 1]	0.674191
Label propagation community 1	[5, 0]	0.5	[0, 4, 1]	0.533333	[0, 0, 0, 0, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 5, 0, 0, 0, 0, 0]	0.875
Label propagation community 2	[4, 0]	0.5	[0, 0, 4]	0.666667	[0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.97	[0, 0, 0, 4, 0, 0, 0, 0]	0.875
Label propagation community 3	[3, 0]	0.5	[0, 3, 0]	0.666667	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 0, 3, 0, 0, 0, 0]	0.875

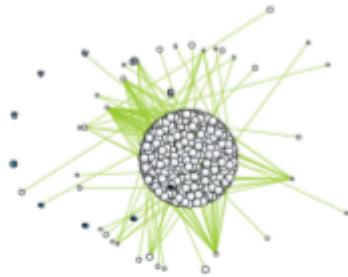




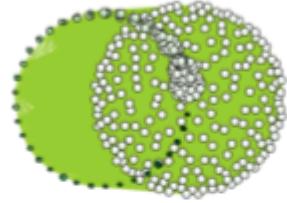
Asynchronous label propagation community 11	[2, 0]	0.5	[0, 1, 1]	0.333333	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0]	0.98	[0, 0, 1, 0, 1, 0, 0, 0]	0.75
Asynchronous label propagation community 12	[2, 0]	0.5	[0, 1, 1]	0.333333	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0]	0.98	[0, 0, 0, 2, 0, 0, 0, 0]	0.875

**Fig 30: Reciprocated House Candidates' Community Partitions**

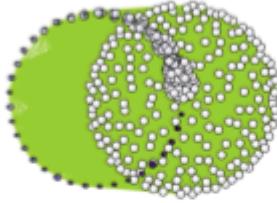
11 Girvan-Newman communities



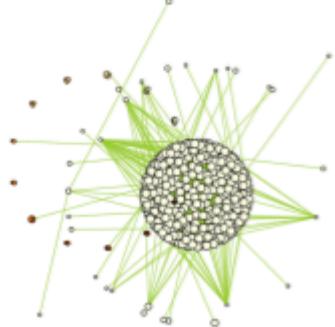
37 Louvain communities



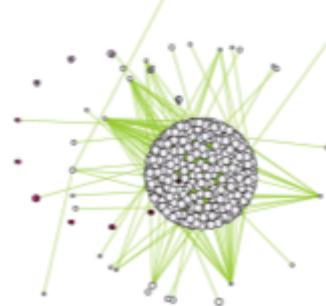
37 Leiden communities



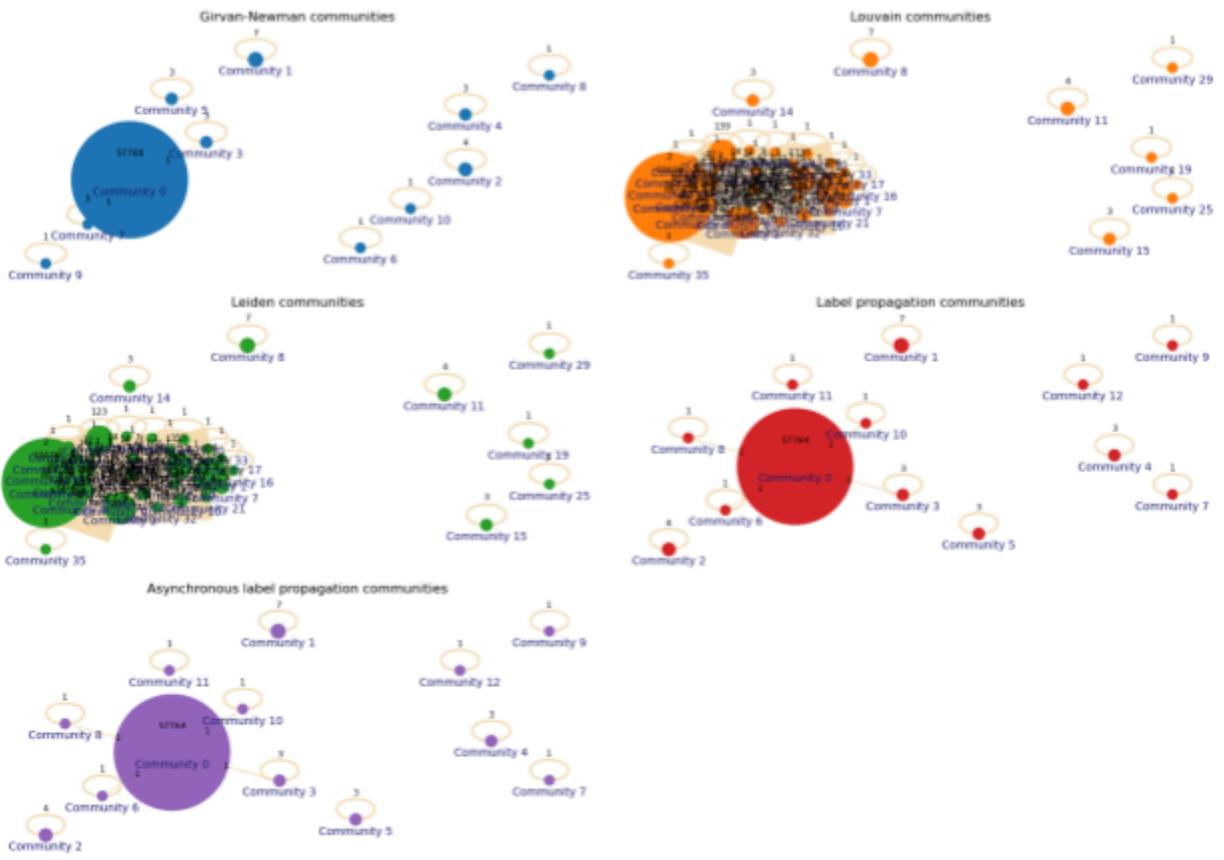
13 Label propagation communities



13 Asynchronous LPA communities



**Fig 31: Communities Image Graphs for the House Reciprocated Graph**



**Fig 32: Attributed Reciprocated House Candidates' Community Partitions**

