Kalkulus 1 Latihan Soal 1.1

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1. Tentukan jenis bilangan real berikut ini termasuk bilangan bulat, bilangan rasional, atau bilangan irasional.

   (a) $-\frac{2}{7} = -0.\overline{285714}$ (Bilangan Rasional)

   (b) 0 (Bilangan Bulat)

   (c) $\frac{33}{11} = 3$ (Bilangan Bulat)

   (d) $0,75$ (Bilangan Rasional)

   (e) $\sqrt{\pi}$ (Bilangan Irasional)

   (f) $17,10200300...$ (Bilangan Irasional)

   (g) $-\sqrt{9} = -3$ (Bilangan Bulat)

   (h) $0,999...$ (Bilangan Rasional)

2. Gunakan cara serupa pada Contoh 1.1.2, jika perlu, untuk menyajikan bentuk desimal di bawah ini sebagai bentuk pecahan (pembagian bilangan bulat)

   (a) $0,12123123...$

   $\Leftrightarrow\ x = 0,12\overline{123}$

   $\Leftrightarrow\ 100x = 12,\overline{123}$

   $\Leftrightarrow\ 100.000x = 12.123,\overline{123}$

   $\Leftrightarrow\ 100.000x - 100x = 12.123,\overline{123} - 12,\overline{123}$

   $\Leftrightarrow\ 99.900x = 12.111$

   $\Leftrightarrow\ x = \frac{12.111}{99.900}$

   (b) $12,777...$

   $\Leftrightarrow\ x = 12,\overline{7}$

   $\Leftrightarrow\ 10x = 127,\overline{7}$

   $\Leftrightarrow\ 10x - x = 127,\overline{7} - 12,\overline{7}$

   $\Leftrightarrow\ 9x = 115$

   $\Leftrightarrow\ x = \frac{115}{9}$

   (c) $38,07818181...$

   $\Leftrightarrow\ x = 38,07\overline{81}$

$\Leftrightarrow\ 100x = 3.807,\overline{81}$

$\Leftrightarrow\ 10.000x = 380.781,\overline{81}$

$\Leftrightarrow\ 10.000x - 100x = 380.781,\overline{81} - 3.807,\overline{81}$

$\Leftrightarrow\ 9.900x = 376.974$

$\Leftrightarrow\ x = \frac{376974}{9.900}$

(d) $0,4296000...$

$\Leftrightarrow\ x = 0,4296\overline{0}$

$\Leftrightarrow\ 10.000x = 4.296,\overline{0}$

$\Leftrightarrow\ 100.000x = 42.960,\overline{0}$

$\Leftrightarrow\ 100.000x - 10.000x = 42.960 - 4.296$

$\Leftrightarrow\ 90.000x = 38.664$

$\Leftrightarrow\ x = \frac{38664}{90.000}$

(e) $0,41211211...$

$\Leftrightarrow\ x = 0,41\overline{211}$

$\Leftrightarrow\ 100x = 41,\overline{211}$

$\Leftrightarrow\ 10.000x = 41211,\overline{211}$

$\Leftrightarrow\ 10.000x - 100x = 41211,\overline{211} - 41,\overline{211}$

$\Leftrightarrow\ 9.900x = 41.170$

$\Leftrightarrow\ x = \frac{41.170}{9.900}$

(f) $0,123456712345671234567...$

$\Leftrightarrow\ x = 0,\overline{1234567}$

$\Leftrightarrow\ 10.000.000x = 1.234.567,\overline{1234567}$

$\Leftrightarrow\ 100.000.000.000.000x = 12.345.671.234.567,\overline{1234567}$

$\Leftrightarrow\ 100.000.000.000.000x - 10.000.000x =$
$12.345.671.234.567,\overline{1234567} - 1.234.567,\overline{1234567}$

$\Leftrightarrow\ 99.999.990.000.000x = 12.345.670.000.000$

$\Leftrightarrow\ x = \frac{12.345.670.000.000}{99.999.990.000.000}$

3. Jika $a \leq b$, maka relasi manakah yang bernilai benar?

   (a) $a - 3 \leq b - 3$ (benar)

   (b) $-a \leq -b$ (Salah)

   (c) $3 - a \leq 3 - b$ (Salah)

   (d) $7a \leq 7b$ (Benar)

   (e) $a^2 \leq ab$ (Salah)

   (f) $a^3 \leq a^2b$ (Benar)

4. Jika $a \geq b$, maka relasi berikut ini yang benar adalah...

   (a) $a^2b \geq ab^2$ (Benar)

(b) $a^3b^2 \leq a^2b^3$ (Salah)

(c) $a + 2b \leq 3b$ (Salah)

(d) $3a \geq a + 2b$ (Benar)

(e) $2b - 2a \geq b - a$ (Salah)

5. Jika $a \leq b$ dan $c \leq d$, maka relasi mana yang selalu benar?

   (a) $a + 2 \leq b + 2d$ (Tidak Selalu Benar)

   (b) $a - 2c \leq b - 2d$ (Tidak Selalu Benar)

   (c) $a + 2c \leq b + 2d$ (Selalu Benar)

6. Dari beberapa pernyataan berikut, manakah yang benar? jelaskan.

   (a) Jika $x = 2$, maka $x^2 = 4$
   Benar karena $2^2 = 4$

   (b) Jika $x^2 = 4$, maka $x = 2$
   Salah, Karena $\sqrt{4} = \pm 2$

   (c) Jika $x < 2$, maka $x^2 < 4$
   Salah, Karena $-3^2 = 9$

   (d) Jika $x > 2$, maka $x^2 > 4$
   Benar, karena $2^2 = 4, x > 2$

   (e) Jika $x^2 < 4$, maka $x < 2$
   Benar, karena $\sqrt{4} = 2, \sqrt{x^2} \geq 0$;

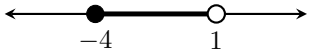
   (f) Jika $x^2 > 4$, maka $x > 2$
   Benar, karena $\sqrt{4} = 2$

   (g) Jika $-2 < x \leq 1$, maka $0 \leq x^2 < 4$
   benar

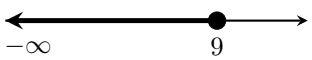
7. Buatlah sketsa pada garis bilangan yang menyajikan himpunan berikut ini.

   (a) $[-3, 2] \cup [1, 4]$

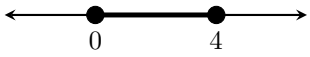
   (b) $[-4, 0) \cup (-4, 1)$



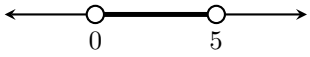
   (c) $(-\infty, 5) \cup (-5, 9]$



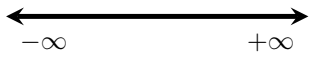
   (d) $[-2, 4] \cap [0, 5)$



(e) $(-\infty, 5) \cap (0, +\infty)$



(f) $[-\infty, 5] \cup [-5, +\infty]$



Untuk Soal 8, 9 dan 10, selesaikan pertidaksamaan yang diberikan dan buatlah sketsa penyelesaiannya pada garis bilangan.

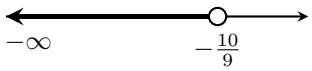
8. $2x - 1 > 11x + 9$

   $\Leftrightarrow\ 2x - 11x > 9 + 1$

   $\Leftrightarrow\ -9x > 10$

   $\Leftrightarrow\ x > -\frac{10}{9}$

   $\Leftrightarrow\ x \in (-\infty, -\frac{10}{9})$



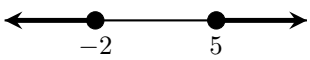
9. $x^2 - 3x - 10 \geq 0$

   $\Leftrightarrow\ (x + 2)(x - 5) \geq 0$

   $\Leftrightarrow\ x + 2 \leq 0 \vee x - 5 \geq 0$

   $\Leftrightarrow\ x \leq -2 \vee x \geq 5$

   $\Leftrightarrow\ x \in (-\infty, -2] \cup [5, +\infty)$



10. $\frac{\frac{1}{2}x-3}{4+x} > 1$

    $\Leftrightarrow\ \frac{\frac{1}{2}x-3}{4+x} > 1, x \neq -4$

    $\Leftrightarrow\ \frac{\frac{x-6}{2}}{4+x} > 1$

    $\Leftrightarrow\ \frac{x-6}{2(4+x)} > 1$

    $\Leftrightarrow\ \frac{x-6}{2(4+x)} - 1 > 0$

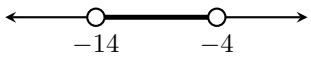
    $\Leftrightarrow\ \frac{x-6-2(4+x)}{2(4+x)} > 0$

    $\Leftrightarrow\ \frac{-x-14}{8+2x} > 0$

    $\Leftrightarrow\ -x - 14 < 0 \vee 8 + 2x < 0$

$\Leftrightarrow\ x > -14 \vee x < -4$

$\Leftrightarrow\ x \in (-14, -4), x \neq -4$



Untuk Soal 11 dan 12, tentukan semua nilai bilangan real x yang memenuhi pertidaksamaan yang diberikan.

11. $\sqrt{x^2 + x - 6} > 0$

    $\Leftrightarrow\ (x+3)(x-2) > 0$

    $\Leftrightarrow\ x + 3 < 0 \vee x - 2 > 0$

    $\Leftrightarrow\ x < -3 \vee x > -2$

    $\Leftrightarrow\ x \in (-\infty, -3) \cup (-2, +\infty)$

12. $\sqrt{\frac{x+2}{x-1}} \geq 0$

    $\Leftrightarrow\ \sqrt{\frac{x+2}{x-1}} \geq 0, x \neq 1$

    $\Leftrightarrow\ x + 2 \leq 0 \vee x - 1 > 0$

    $\Leftrightarrow\ x \in (-\infty, -2] \cup (1, +\infty), x \neq 1$

13. Suhu Fahrenheit dan Celcius dihubungkan oleh rumus $C = \frac{5}{9}(F - 32)$. Jika suhu dalam derajat Celcius antara interval $25 \leq C \leq 40$ pada hari tertentu, berapakah suhu tersebut dalam derajat Fahrenheit?

    $25 \leq \frac{5}{9}(F - 32) \leq 40$

    $\Leftrightarrow \begin{cases} \frac{5}{9}(F-32) & \geq & 25 \\ \frac{5}{9}(F-32) & \leq & 40 \end{cases}$

    $\Leftrightarrow \begin{cases} F & \geq & 77 \\ F & \leq & 104 \end{cases}$

    $\Leftrightarrow\ F \in [77, 104]$

    Jadi saat suhu dalam Derajat antara interval $25 \leq C \leq 40$, maka suhu tersebut dalam derajat Fahrenheit antara interval $77 \leq F \leq 104$

14. Harga komoditi A dan komoditi B mempunyai hubungan dalam bentuk $A = \frac{7}{3}(B - 49)$. Pada saat harga komoditi A bergerak pada interval $21 \leq A \leq 35$, bagaimanakah harga komoditi B?

    $21 \leq \frac{7}{3}(B - 49) \leq 35$

    $\Leftrightarrow \begin{cases} \frac{7}{3}(B-49) & \geq & 21 \\ \frac{7}{3}(B-49) & \leq & 35 \end{cases}$

    $\Leftrightarrow \begin{cases} B & \geq & 58 \\ B & \leq & 64 \end{cases}$

$\Leftrightarrow\ B \in [58, 64]$

Jadi saat harga komoditi A bergerak pada interval $21 \leq A \leq 35$, maka harga komodi B bergerak pada interval $58 \leq B \leq 64$

15. Selesaikan pertidaksamaan berikut:

(a) $x^3 + 2x^2 + x + 2 > 0$

$\Leftrightarrow\ x^2(x+2) + x + 2 > 0$

$\Leftrightarrow\ (x+2)(x^2+2) > 0$

$\Leftrightarrow\ (x+2)(x^2+2) > 0$

$\Leftrightarrow\ x + 2 > 0$

$\Leftrightarrow\ x > -2$

$\Leftrightarrow\ x \in (-2, +\infty)$

(b) $-x^3 + 4x^2 - 7x + 12 > 0$

$\Leftrightarrow\ -x^3 + 3x^2 + x^2 - 3x - 4x + 12 > 0$

$\Leftrightarrow\ -x^2(x-3) + x(x-3) - 4(x-3) > 0$

$\Leftrightarrow\ -(x-3)(x^2 - x + 4) > 0$

$\Leftrightarrow\ (x-3)(x^2 - x + 4) < 0$

$\Leftrightarrow\ x - 3 < 0$

$\Leftrightarrow\ x < 3$

$\Leftrightarrow\ x \in (-\infty, 3)$

(c) $-x^4 - x^3 - x^2 + x + 2 < 0$

$\Leftrightarrow\ -x^4 + x^3 - 2x^3 + 2x^2 - 3x^2 + 3x - 2x + 2 < 0$

$\Leftrightarrow\ -x^3(x-1) - 2x^2(x-1) - 3x(x-1) - 2(x-1) < 0$

$\Leftrightarrow\ -(x-1)(x^3 + 2x^2 + 3x + 2) < 0$

$\Leftrightarrow\ -(x-1)(x^3 + x^2 + x^2 + 2x + x + 2) < 0$

$\Leftrightarrow\ -(x-1)(x^2(x+1) + x(x+1) + 2(x+1)) < 0$

$\Leftrightarrow\ -(x-1)(x+1)(x^2 + x + 2) < 0$

$\Leftrightarrow\ (x-1)(x+1)(x^2 + x + 2) > 0$

$\Leftrightarrow\ x \in (-\infty, -1) \cup (1, +\infty)$

(d) $x^4 - x^2 < 0$

$\Leftrightarrow\ x^2(x^2 - 1) < 0$

$\Leftrightarrow\ x \in (-1, 0) \cup (0, 1)$

(e) $2 \leq x^2 - x < 6$

$\Leftrightarrow\ x^2 - x \geq 2, x^2 - x < 6$

$\Leftrightarrow\ x \in (-\infty, -1] \cup [2, +\infty), x \in (-2, 3)$

$\Leftrightarrow\ x \in (-2, -1] \cup [2, 3)$

(f) $\frac{x^2+2x+3}{6x^2-x-2} \geq 0$

$\Leftrightarrow\ \begin{cases} x^2 + 2x + 3 & \geq & 0 \\ 6x^2 - x - 2 & > & 0 \end{cases}$

$\Leftrightarrow\ x \in (-\frac{1}{2}, \frac{2}{3}) \cup (\frac{2}{3}, +\infty)$

(g) $\frac{x^3+x^2-2x-2}{6x^2+7x+2} \leq 0$

$$\Leftrightarrow \begin{cases} x^3+x^2-2x-2 & \leq & 0 \\ 6x^2+7x+2 & > & 0 \\ x^3+x^2-2x-2 & \geq & 0 \\ 6x^2+7x+2 & < & 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & (-\infty, -\sqrt{2}] \cup [-1, \sqrt{2}] \\ x & \in & (-\infty, -\frac{2}{3}) \cup (-\frac{1}{2}, +\infty) \\ x & \in & (-\sqrt{2}, -1) \cup (\sqrt{2}, +\infty) \\ x & \in & (-\frac{2}{3}, -\frac{1}{2}) \end{cases}$$

$\Leftrightarrow\ x \in (-\infty, -\sqrt{2}] \cup [-1, -\frac{2}{3}) \cup (-\frac{1}{2}, \sqrt{2}]$

(h) $\frac{x^3-2x^2}{10x^2-29x+10} \geq 0$

$$\Leftrightarrow \begin{cases} x^3-2x^2 & \geq & 0 \\ 10x^2-29x+10 & > & 0 \\ x^3-2x^2 & \leq & 0 \\ 10x^2-29x+10 & < & 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & [2, +\infty) \cup 0 \\ x & \in & (-\infty, \frac{2}{5}) \cup (\frac{5}{2}, +\infty) \\ x & \in & (-\infty, 2] \\ x & \in & (\frac{2}{5}, \frac{5}{2}) \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & (\frac{5}{2}, +\infty) \cup \{0\} \\ x & \in & (\frac{2}{5}, 2] \end{cases}$$

$\Leftrightarrow\ x \in (\frac{2}{5}, 2] \cup (\frac{5}{2}, +\infty) \cup \{0\}$

(i) $\frac{2}{x} \geq x+1$

$\Leftrightarrow\ \frac{2}{x} \geq x+1, x \neq 0$

$\Leftrightarrow\ \frac{2}{x} - x - 1 \geq 0$

$\Leftrightarrow\ \frac{2}{x} \geq 0$

$$\Leftrightarrow \begin{cases} 2-x^2-x & \geq & 0 \\ x & > & 0 \\ 2-x^2-x & \leq & 0 \\ x & < & 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & [-2, 1] \\ x & > & 0 \\ x & \in & (-\infty, -2] \cup [1, +\infty) \\ x & < & 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & (0, 1] \\ x & \in & (-\infty, -2] \end{cases}$$

$\Leftrightarrow\ x \in (-\infty, -2] \cup (0, 1]$

(j) $\frac{x+1}{2-x} \geq \frac{x}{x+3}$

$\Leftrightarrow\ \frac{x+1}{2-x} \geq \frac{x}{x+3}, x \neq 2, x \neq -3$

$\Leftrightarrow \frac{x+1}{2-x} - \frac{x}{x+3} \geq 0$

$\Leftrightarrow \frac{(x+3)(x+1)-x(2-x)}{(2-x)(x+3)} \geq 0$

$\Leftrightarrow \frac{x^2+x+3x+3-2x+x^2}{(2-x)(x+3)} \geq 0$

$\Leftrightarrow \frac{2x^2+2x+3}{(2-x)(x+3)} \geq 0$

$$\Leftrightarrow \begin{cases} 2x^2+2x+3 & \geq & 0 \\ (2-x)(x+3) & > & 0 \end{cases}$$

$\Leftrightarrow x \in (-3, 2)$

(k) $2x+1 \leq x^3 \leq 2x+4$

$$\Leftrightarrow \begin{cases} x^3 & \geq & 2x+1 \\ x^3 & \leq & 2x+4 \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & [-1, \frac{1-\sqrt{5}}{2}] \cup [\frac{1+\sqrt{5}}{2}, +\infty) \\ x & \in & (-\infty, 2] \end{cases}$$

$\Leftrightarrow x \in [-1, \frac{1-\sqrt{5}}{2}] \cup [\frac{1+\sqrt{5}}{2}, 2]$

(l) $2 \leq \frac{x^2+1}{x} < x+3$

$\Leftrightarrow 2 \leq \frac{x^2+1}{x} < x+3, x \neq 0$

$$\Leftrightarrow \begin{cases} \frac{x^2+1}{x} & \geq & 2 \\ \frac{x^2+1}{x} & < & x+3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & (0, +\infty) \\ x & \in & (-\infty, 0) \cup (\frac{1}{3}, +\infty) \end{cases}$$

$\Leftrightarrow x \in (\frac{1}{3}, +\infty), x \neq 0$

(m) $\frac{x-2}{x^2} \leq \frac{x+1}{x+3}$

$\Leftrightarrow \frac{x-2}{x^2} \leq \frac{x+1}{x+3}, x \neq 0, x \neq -3$

$\Leftrightarrow \frac{x-2}{x^2} - \frac{x+1}{x+3} \leq 0$

$\Leftrightarrow \frac{(x+3)(x+2)-x^2(x+1)}{x^2(x+3)} \leq 0$

$\Leftrightarrow \frac{x^2-2x+3x-6-x^3-x^2}{x^2(x+3)} \leq 0$

$\Leftrightarrow \frac{x-6-3^3}{x^2(x+3)} \leq 0$

$$\Leftrightarrow \begin{cases} x-6-x^3 & \leq & 0 \\ x^2(x+3) & > & 0 \\ x-6-x^3 & \geq & 0 \\ x^2(x+3) & < & 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & [-2, +\infty) \\ x & \in & (-3, 0) \cup (0, +\infty) \\ x & \in & (-\infty, -2] \\ x & \in & (-\infty, -3) \end{cases}$$

$$\Leftrightarrow \begin{cases} x & \in & [-2, 0) \cup (0, +\infty) \\ x & \in & (-\infty, -3) \end{cases}$$

$\Leftrightarrow\ x \in (-\infty, -3) \cup [-2, 0) \cup (0, +\infty)$

16. Tentukan penyelesaian dari:

(a) $\frac{5x-8}{2x-6} \leq 2$

$\Leftrightarrow\ \frac{5x-8}{2x-6} - 2 \leq 0$

$\Leftrightarrow\ \frac{5x-8-(4x-12)}{2x-6} \leq 0$

$\Leftrightarrow\ \frac{5x-8-4x+12}{2x-6} \leq 0$

$\Leftrightarrow\ \frac{x+4}{2x-6} \leq 0$

$\Leftrightarrow\ x + 4 = 0 \vee 2x - 6 \neq 0$

$\Leftrightarrow\ x = -4 \vee x \neq 3$

(b) $\frac{1}{x-5} \geq 3$

$\Leftrightarrow\ \frac{1}{x-5} - 3 \geq 0$

$\Leftrightarrow\ \frac{1-3(x-5)}{x-5} \geq 0$

$\Leftrightarrow\ \frac{1-3x+15}{x-5} \geq 0$

$\Leftrightarrow\ \frac{-3x+16}{x-5} \geq 0$

$\Leftrightarrow\ -3x + 16 = 0 \vee x - 5 \neq 0$

$\Leftrightarrow\ x = \frac{16}{3} \vee x \neq 5$

(c) $\frac{3}{2-x} \geq \frac{4}{2x-3}$

$\Leftrightarrow\ \frac{3}{2-x} \geq \frac{4}{2x-3}, x \neq 2, x \neq \frac{3}{2}$

$\Leftrightarrow\ \frac{3}{2-x} - \frac{4}{2x-3} \geq 0$

$\Leftrightarrow\ \frac{3(2x-3)-4(2-x)}{(2-x)(2x-3)} \geq 0$

$\Leftrightarrow\ \frac{6xe-9-8-4x}{(2-x)(2x-3)} \geq 0$

$\Leftrightarrow\ \frac{10x-17}{(2-x)(2x-3)} \geq 0$

$$\Leftrightarrow\ \begin{cases} 10x - 17 & \geq & 0 \\ (2-x)(2x-3) & > & 0 \\ 10x - 17 & \leq & 0 \\ (2-x)(2x-3) & < & 0 \end{cases}$$

$$\Leftrightarrow\ \begin{cases} x & \geq & \frac{17}{10}, x \in (\frac{3}{2}, 2) \\ x & \leq & \frac{17}{10}, x \in (-\infty, \frac{3}{2}) \cup (2, +\infty) \end{cases}$$

$$\Leftrightarrow\ \begin{cases} x & \in & [\frac{17}{10}, 2) \\ x & \in & (-\infty, \frac{3}{2}) \end{cases}$$

$\Leftrightarrow\ x \in (-\infty, \frac{3}{2}) \cup [\frac{17}{10}, 2), x \neq 2, x \neq \frac{3}{2}$