Priority Queues

Data Structures and Algorithms

Sep. 4th, 2014

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Priority Queue

- Priority queue
  - Add: insert a comparable key.
  - RemoveMax: find and remove the key with the highest priority.
- Application
  - Scheduling.
- Implementation
  - Lazy vs. Eager.
  - Heap.

Is the incoming order important for scheduling?

Priority Queue Abstract Class

```
public abstract class AbstractPriorityQueue<T extends Comparable<T>>
{
  abstract public void add(T key);
  abstract public T    removeMax();
  abstract public int  size();

  public boolean isEmpty()
  {
    return size() == 0;
  }

  protected void throwNoSuchElementException()
  {
    if (isEmpty())
      throw new NoSuchElementException("No key exists.");
  }
}
```

These are called?

defined?

visibility?

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Lazy vs. Eager Algorithms

- Lazy algorithm
  - Add: insert each key to the back of the list.
  - RemoveMax: Find the key with the highest priority and remove it from the list.
- Eager algorithm
  - Add: insert each key to the list in ascending order.
  - RemoveMax: remove the last key in the list.

	Add	RemoveMax
Lazy	$O(1)$	$O(n)$
Eager	$O(n)$	$O(1)$

$O(\log n)$

Eager Priority Queue

```
List<T> l_keys = new ArrayList<>();


public void add(T key)
{
  int index = Collections.binarySearch(l_keys, key);
  if (index < 0) index = -(index + 1);
  l_keys.add(index, key);
}
```

-(insertion point) - 1

Complexity?

```
public T removeMax()
{
  return l_keys.remove(l_keys.size()-1);
}
```

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Binary Heap

- What is a heap?
  - A tree where the key of each node is larger than or equal to the keys in its children.
  - The tree is guaranteed to be balanced.
  - What is a binary heap?

- Operations
  - Add: swim.
  - RemoveMax: sink.
  - Both operations can be done in O(log n).

Binary Heap Using List

Input = | 7 | 3 | 2 | 4 | 6 | 1 | 5 |

Binary Heap Using List

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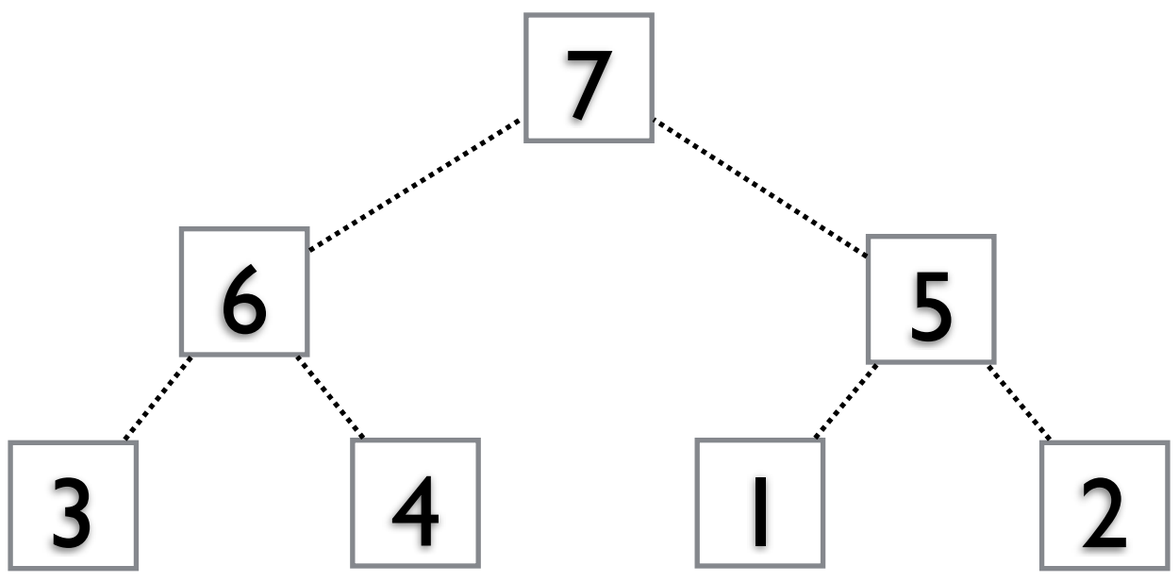
Index of the parent?

$k/2$

Index of the children?

$k*2$, k*2 + 1



list = | ∅ | 7 | 6 | 5 | 3 | 4 | 1 | 2 |

Why?

Binary Heap Using List

```
List<T> l_keys;
int n_size;


public BinaryHeap()
{
    l_keys = new ArrayList<>();
    l_keys.add(null);
    n_size = 0;
}
```

Add via Swim

Input = | 7 | 3 | 2 | 4 | 6 | 1 | 5 |

Add each key to the leftmost leaf.

Keep swapping the new node with its parent
until it satisfies the heap property.

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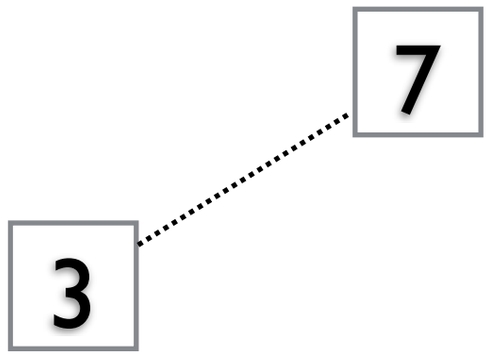

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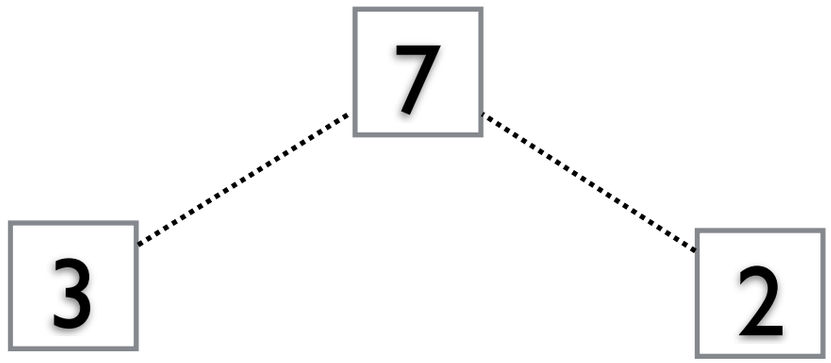

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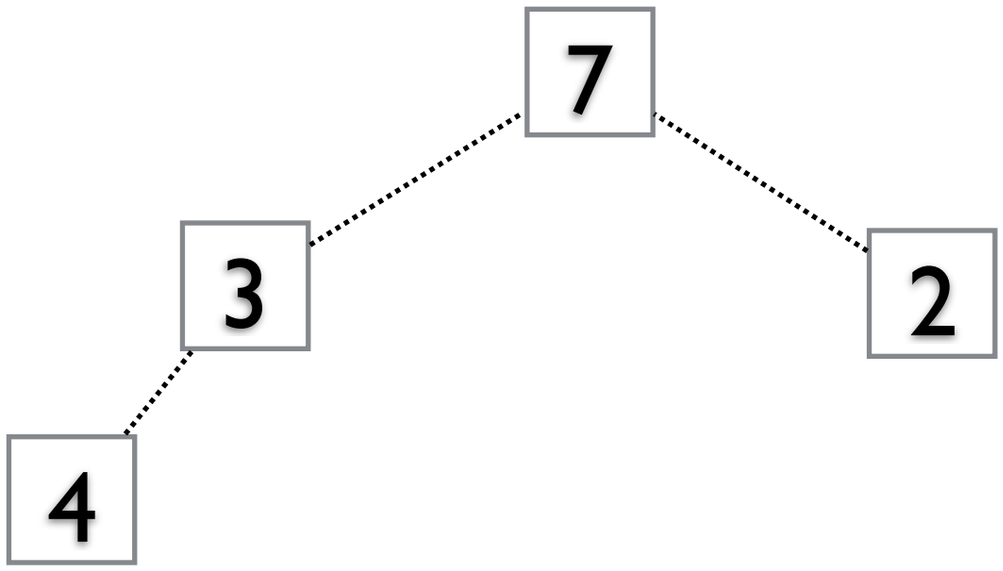

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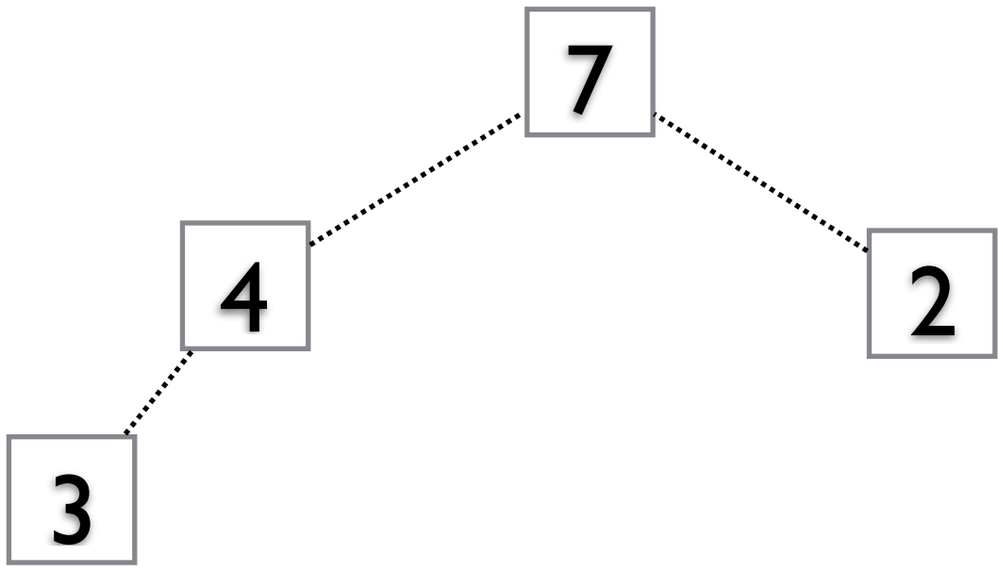

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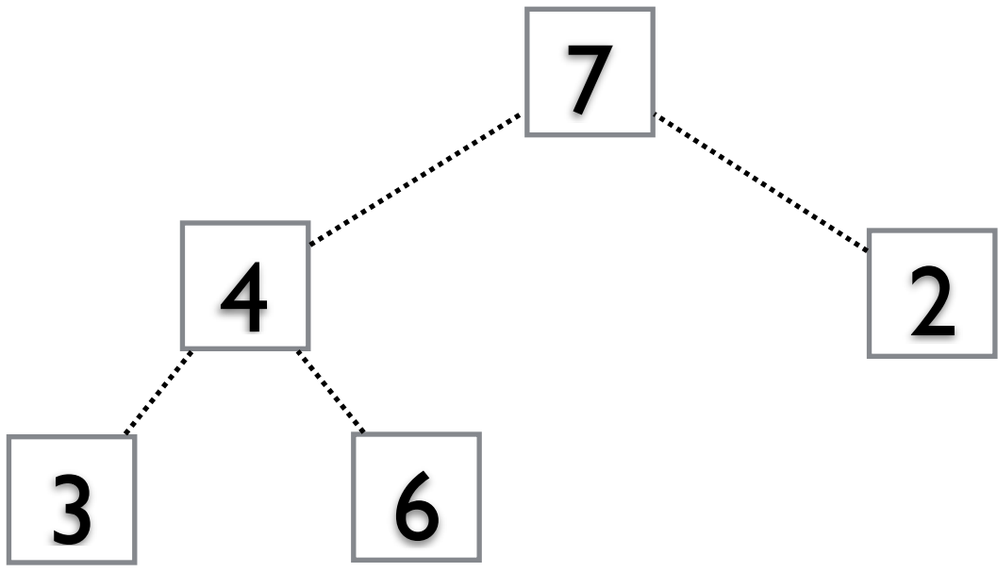

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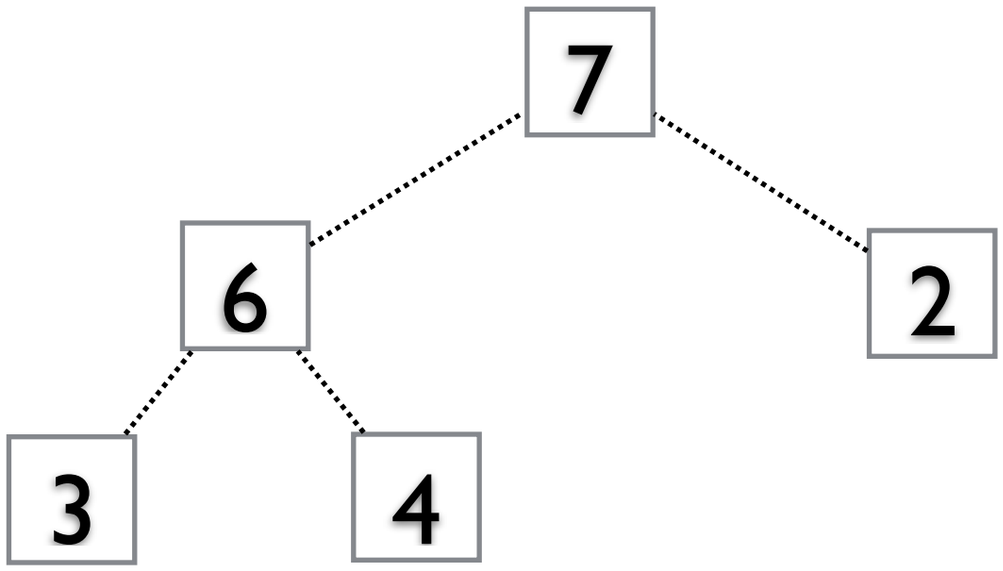

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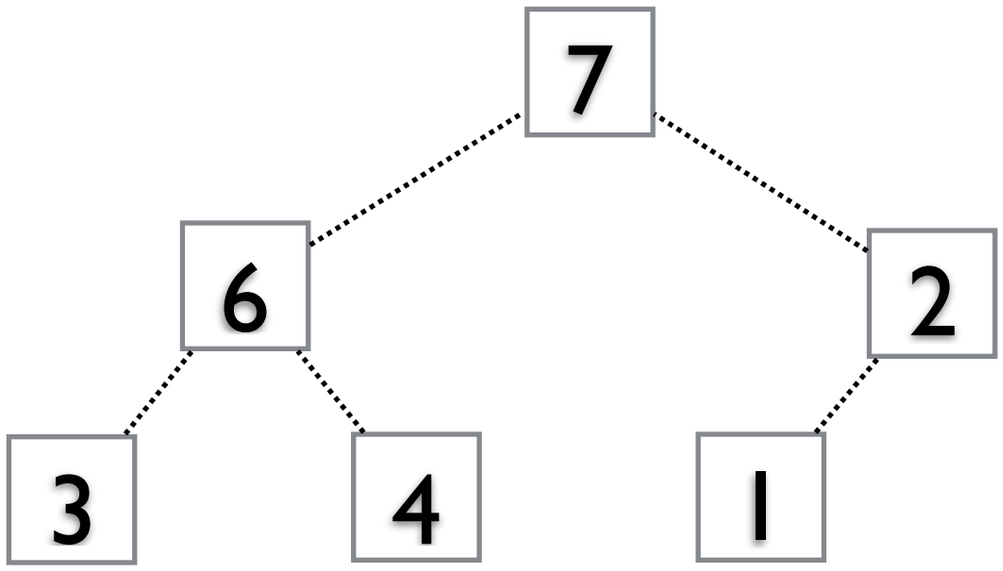

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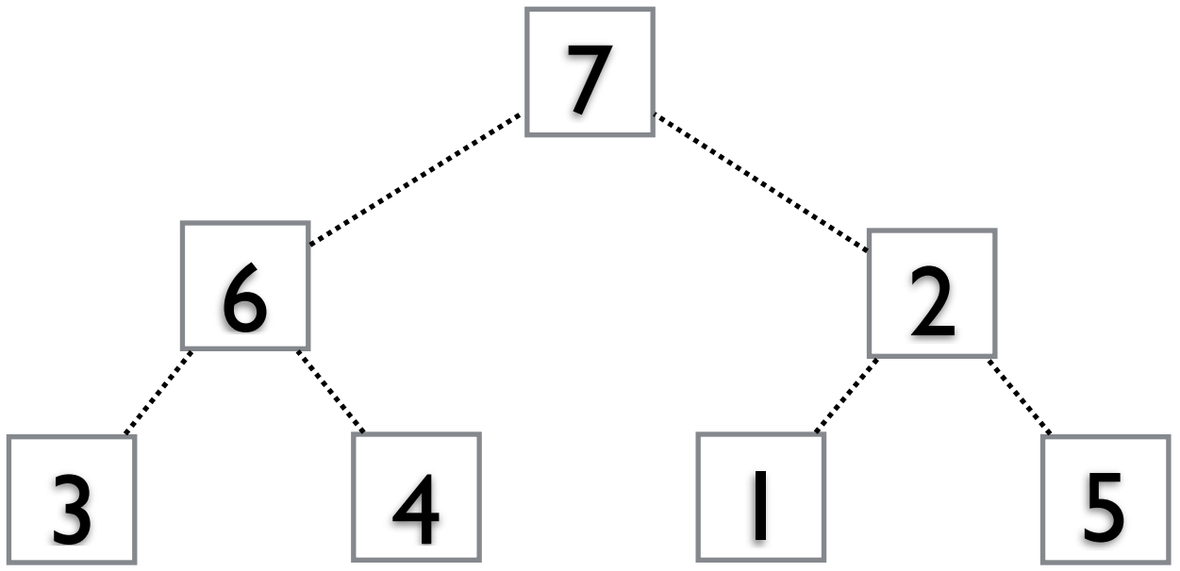

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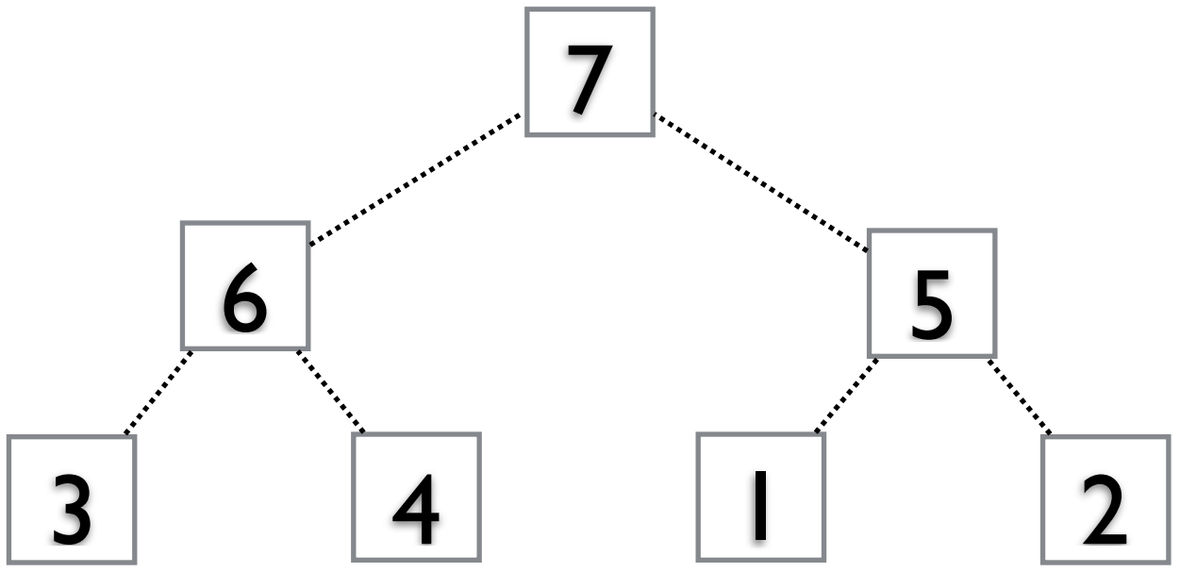

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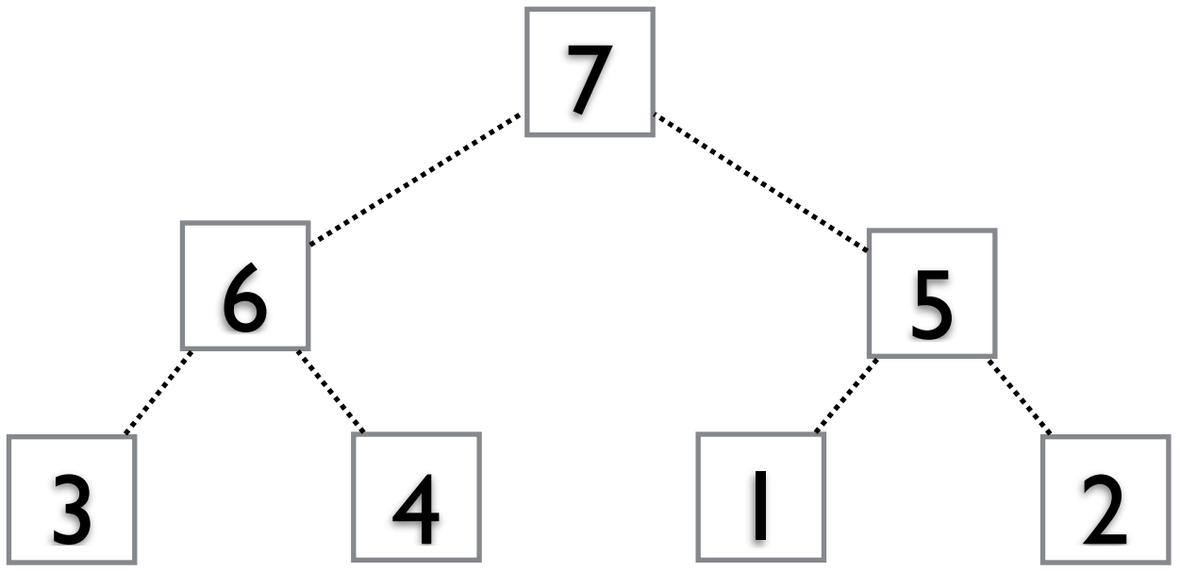

Complexity?

Add via Swim

```
public void add(T key)
{
  l_keys.add(key);
  swim(++n_size);
}
```

← Leftmost leaf

```
private void swim(int k)
{
  while (k > 1 && compareTo(k/2, k) < 0)
  {
      swap(k/2, k);
      k /= 2;
  }
}
```

↑ Parent

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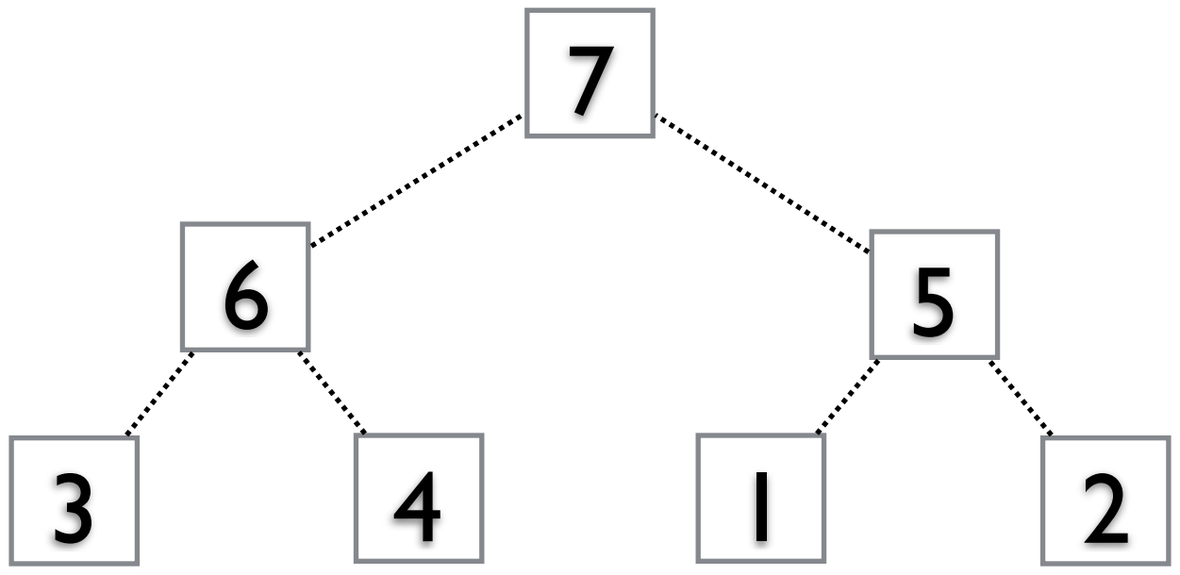
RemoveMax vis Sink

Replace the root with the rightmost leaf.

Keep swapping the root node with its child
until it satisfies the heap property.



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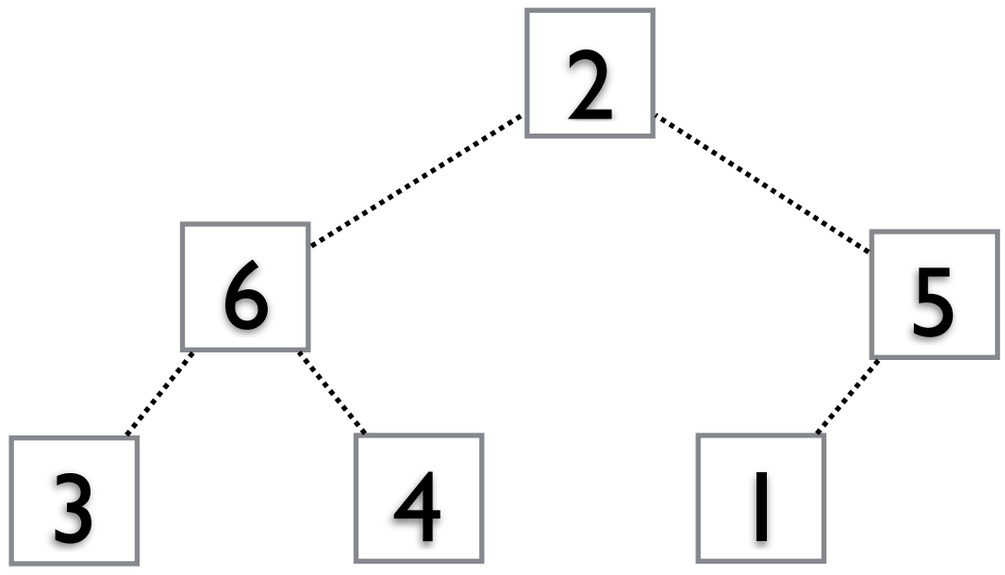

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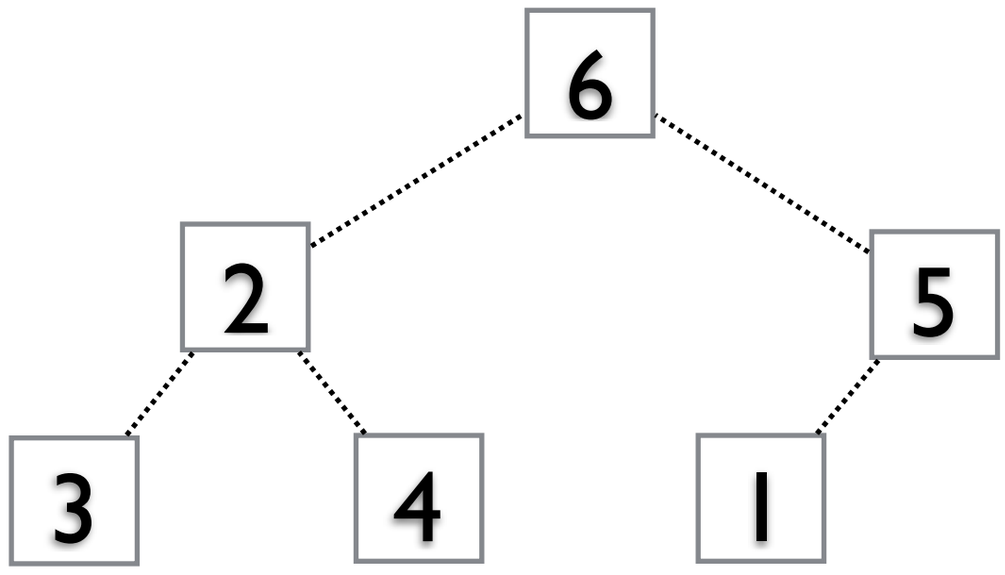

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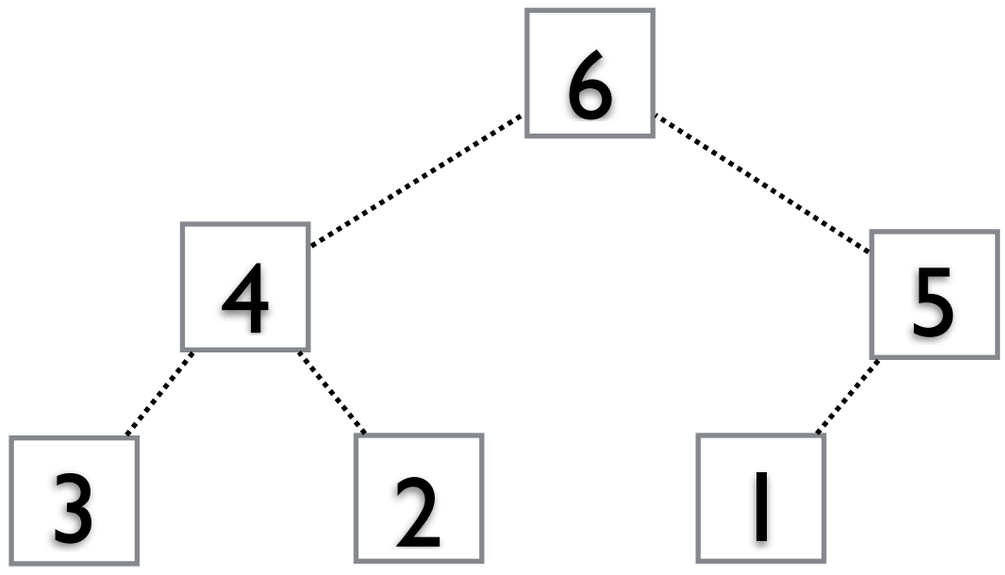

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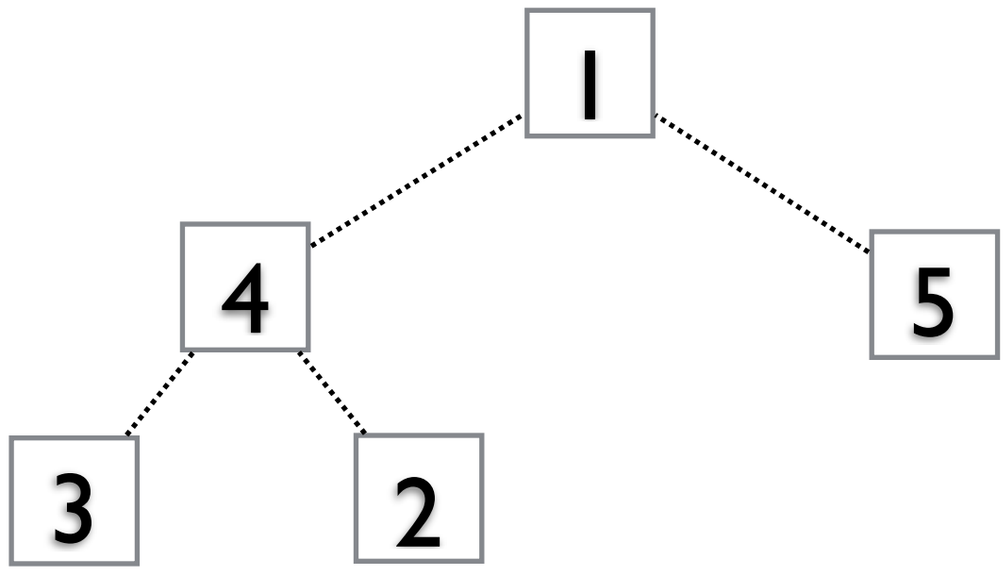

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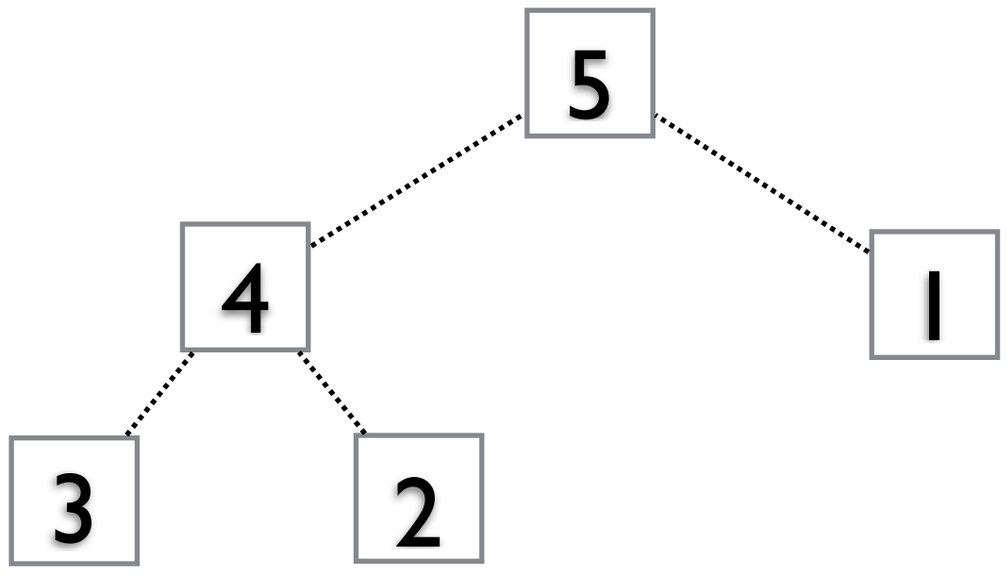

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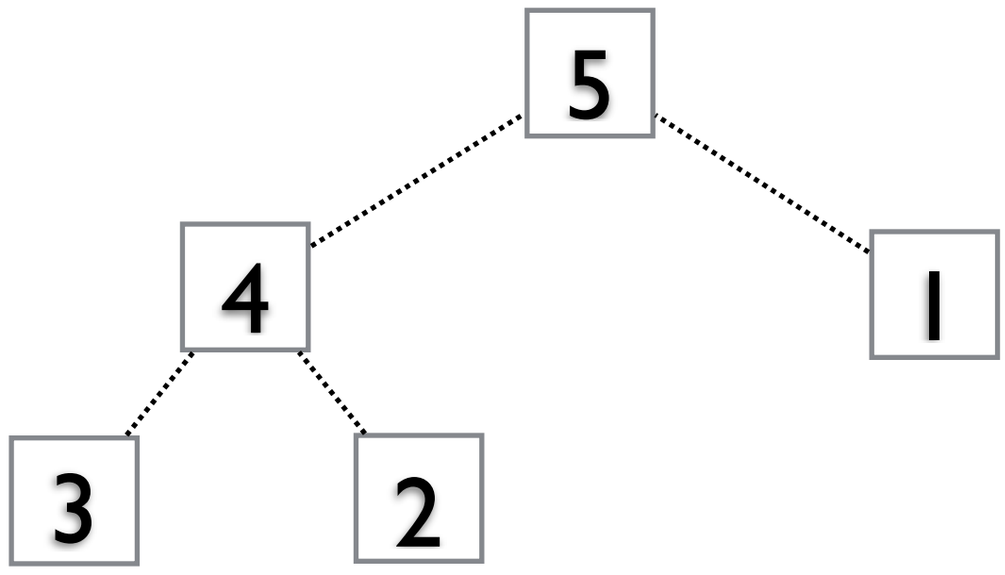

Complexity?

RemoveMax vis Sink

```
public T removeMax()
{
  throwNoSuchElementException();
  swap(1, n_size);
  T max = l_keys.remove(n_size--);
  sink(1);
  return max;
}
```

← Root

```
private void sink(int k)
{
  for (int i=k*2; i<=n_size; k=i,i=k*2)
  {
    if (i < n_size && compareTo(i, i+1) < 0) i++;
    if (compareTo(k, i) >= 0) break;
    swap(k, i);
  }
}
```

← Left child

↑ Compare between children

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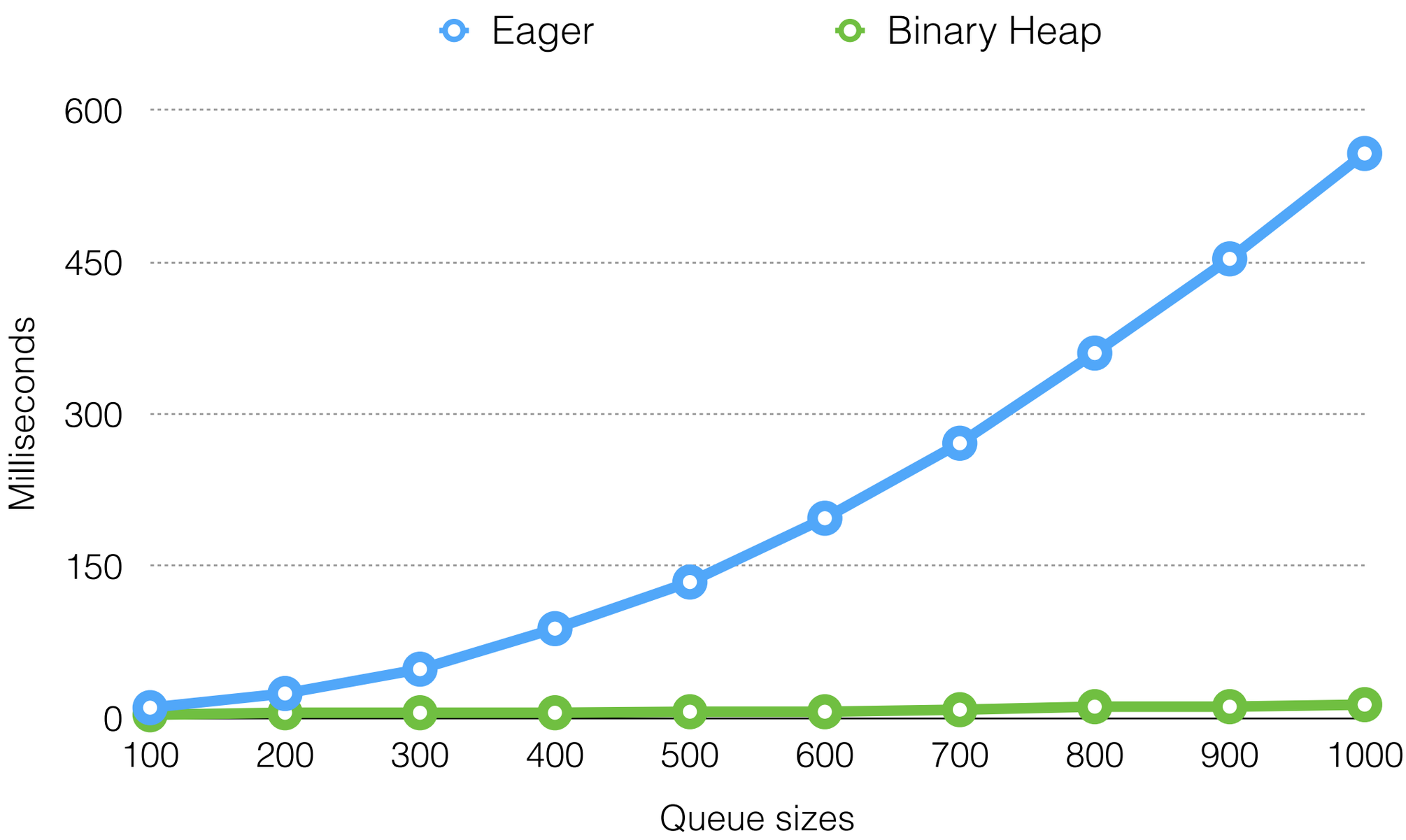
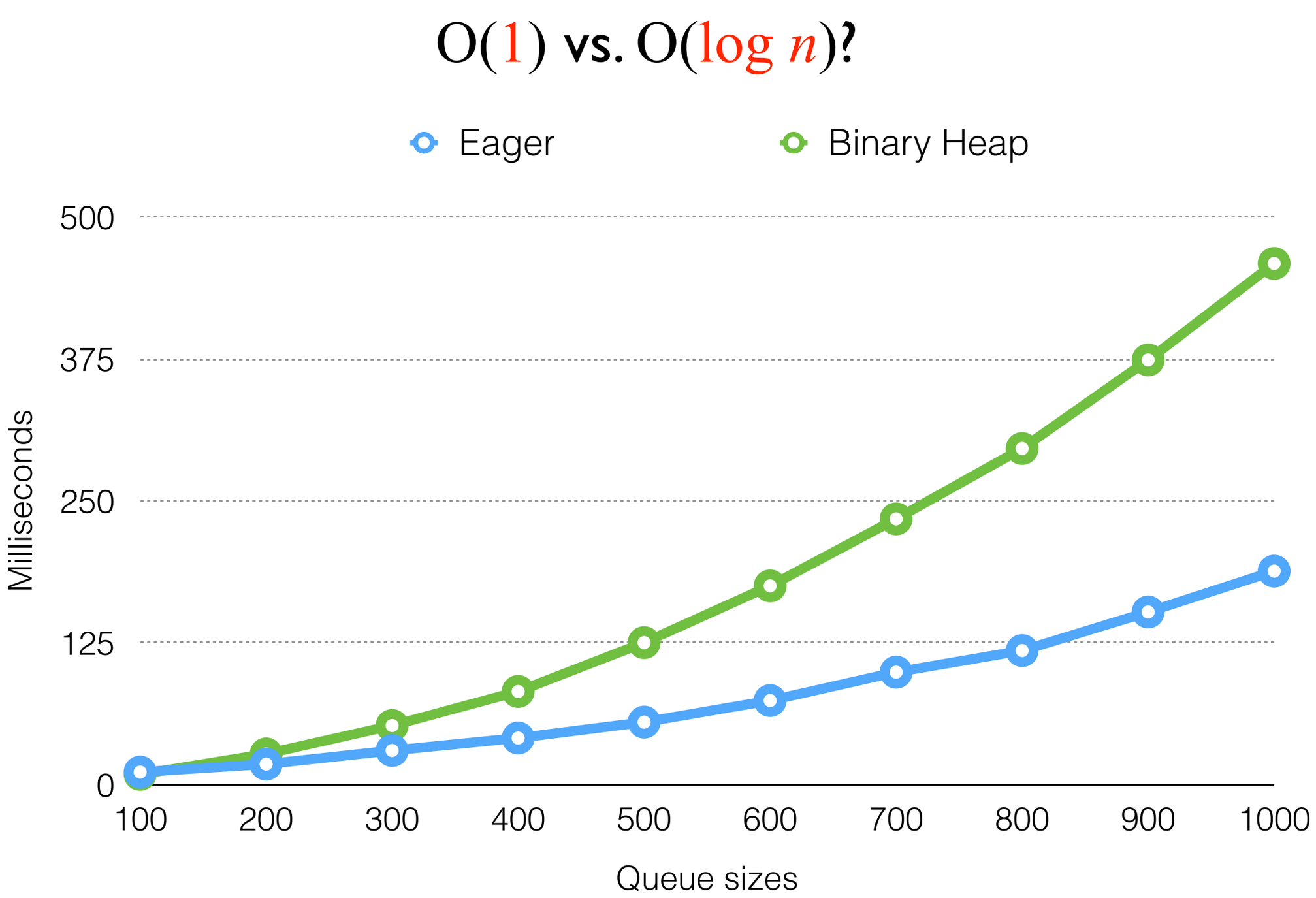
Speed Comparison - Add

Both O($\log n$)?



Speed Comparison - RemoveMax



Exercise - Extra Credit

- HeapSelect
  - Create a class called HeapSelect under the select package.
  - Use BinaryHeap for finding the k'th maximum key.
  - Compare the speeds between SmartSelect and HeapSelect.
  - https://github.com/jdchoi77/emory-courses/wiki/CS323:-Priority-Queues
  - Submit your answer by Sep. 11th (Thus).

- Next class reading
  - Sections 2.1, 2.2, 2. 4 - insertion sort, selection sort, merge sort.