ProjectStartUp1 LeastSquares-Classifier

December 2, 2020

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[]:
[29]: from scipy.io import loadmat
     import numpy as np
     #LASSO Solver - provided code in earlier assignment
     def ista_solve_hot( A, d, la_array ):
         # ista solve hot: Iterative soft-thresholding for multiple values of
         # lambda with hot start for each case - the converged value for the previous
         # value of lambda is used as an initial condition for the current lambda.
         # this function solves the minimization problem
         # Minimize |Ax-d| 2^2 + lambda*|x| 1 (Lasso regression)
         # using iterative soft-thresholding.
         \max iter = 10**4
         tol = 10**(-3)
         tau = 1/np.linalg.norm(A,2)**2
         n = A.shape[1]
         w = np.zeros((n,1))
         num_lam = len(la_array)
         X = np.zeros((n, num_lam))
         for i, each_lambda in enumerate(la_array):
             for j in range(max_iter):
                 z = w - tau*(A.T@(A@w-d))
                 w \text{ old} = w
                 w = np.sign(z) * np.clip(np.abs(z)-tau*each_lambda/2, 0, np.inf)
                 X[:, i:i+1] = w
                 if np.linalg.norm(w - w_old) < tol:</pre>
                     break
         return X
     #SETUP
     X = loadmat("RawData.mat")['X']
     y = loadmat("RawData.mat")['y']
     Xones = np.ones((len(X),1))
     #Single threshold for classification
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thresh = 1500
for i in range(len(X)):
          if(y[i] <= thresh):</pre>
                     y[i] = -1
          if(y[i] > thresh):
                     y[i] = 1
print(np.mean(y))
#Optional: Eliminate high values
#for i in range(110):
           X = np.delete(X, np.argmax(y), 0)
          y = np.delete(y, np.argmax(y), 0)
          X = np.delete(X, np.argmin(y), 0)
           y = np.delete(y, np.arqmin(y), 0)
TwoNormCol = np.zeros((len(X.T),1))
#Remove keyword columns (poorly treated data)
X = np.delete(X, 17, 1)
#Normalize columns to 2-norm
for i in range(len(X.T)):
          TwoNormCol[i] = np.sqrt(X[i,:]@X[i,:])
          X[i,:] = X[i,:]/TwoNormCol[i]
#print(TwoNormCol)
#print(X[0,:])
#print(y)
#Form subsets (indices, first group is full X)
Xsubs = np.
  \rightarrowarray([[0,49],[0,5],[5,7],[7,9],[9,11],[11,17],[17,20],[20,26],[28,33],[33,37],[37,45],[45,
#Create and run over 11 sets of 3604 entries from X and y,
#using 1 as a primary test set (when needed for lambda)
setArr = np.array([[0,int(len(X)/11)],[int(len(X)/11),int(2*len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],[int(len(X)/11)],
 \rightarrow11)],[int(2*len(X)/11),int(3*len(X)/11)],\
                                                   [int(3*len(X)/11), int(4*len(X)/11)], [int(4*len(X)/11)]
  \rightarrow11), int(5*len(X)/11)],\
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[int(5*len(X)/11), int(6*len(X)/11)], [int(6*len(X)/11)]
 \hookrightarrow11),int(7*len(X)/11)],\
                    [int(7*len(X)/11), int(8*len(X)/11)], [int(8*len(X)/11)]
\rightarrow11), int(9*len(X)/11)],\
                    [int(9*len(X)/11), int(10*len(X)/11)], [int(10*len(X)/11)]
\rightarrow11), int(len(X))]])
#print(setArr)
#Set up lambda
lambdaCount = 20
lambdaTest = 5*np.logspace(-6, 3, lambdaCount)
#Error tally storage
errorOrig = np.zeros((12,1))
#errorOrigOnes = np.zeros((11,1))
errorLASSO = np.zeros((12,1))
#errorRidge = np.zeros((11,1))
################################
#Loops over (1) all X subsets, (2 and 3) over all 11 sets for testing (1320_{\sqcup}
\rightarrow loops total)
for Xset in range(len(Xsubs)):
    for i in range(11):
        for j in range(11):
             if i != j:
                 print("X subset = ", Xset, ", i = ",i, ", j = ",j)
                 testTally = 0
                 #Set up Training and testing sets
                 for k in range(11):
                     if i == k:
                          XTest1 = X[setArr[i,0]:setArr[i,1],Xsubs[Xset,0]:
→Xsubs[Xset,1]]
                          #print(XTest1)
                          yTest1 = y[setArr[i,0]:setArr[i,1]]
                          #print(yTest1)
                     if j == k:
                          XTest2 = X[setArr[j,0]:setArr[j,1],Xsubs[Xset,0]:
→Xsubs[Xset,1]]
                          #print(XTest2)
                          yTest2 = y[setArr[j,0]:setArr[j,1]]
                          #print(yTest2)
                     if k != j and k != i:
                          if testTally == 0:
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XTrain = X[setArr[k,0]:setArr[k,1],Xsubs[Xset,0]:
yTrain = y[setArr[k,0]:setArr[k,1]]
                           testTally = 1
                       else:
                           XTrain = np.concatenate((XTrain, X[setArr[k,0]:
\rightarrowsetArr[k,1], Xsubs[Xset,0]: Xsubs[Xset,1]]), \
                                                  axis=0)
                           yTrain = np.concatenate((yTrain, y[setArr[k,0]:
\rightarrowsetArr[k,1]]), axis=0)
               #print(len(XTrain))
               #print(len(XTrain.T))
               #Add a column of ones for comparison
               Xones = np.ones((len(XTrain),1))
               XTrainwithOne = np.concatenate((XTrain, Xones), axis=1)
               Xones2 = np.ones((len(XTest2),1))
               XTest2withOne = np.concatenate((XTest2, Xones2), axis=1)
               #Additional polynomial tests
               #UPDATE 11/13: SOME RESULT IN SINGULAR MATRICES, NOT TESTING
\hookrightarrow FURTHER
               \#Xpoly2 = np.concatenate((XTrain, XTrain*XTrain, Xones), axis=1)
               →XTrain*XTrain*XTrain, Xones), axis=1)
               #Training 1 (for sets that need it)
               #Lowest error is based on lowest average differences (abs val)
\rightarrow between y and Xw
               WLASSO = ista_solve_hot(XTrain,yTrain,lambdaTest)
               # Storage for current iteration ridge regression
               \#Wrid = np.zeros((len(X.T), lambdaCount))
               #UPDATE 11/15: Has been too much for the kernel to handle_
→ (using optimized setup per activity 17)
               #Not continuing with Ridge Regression
               # Ridge regression
               #for index in range(lambdaCount):
                   #WRID[:, index] = np.linalq.inv(XTrain.
\hookrightarrow T@XTrain+lambdaTest[index]*
                                                 #np.
→ identity(len(XTrain)))@XTrain.T@yTrain[:,0]
               #check with Test Set 1
               y_lasso1 = XTest1@WLASSO
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#y_rid1 = XTest1@WRID
              errorTallyLasso = np.zeros((lambdaCount,1))
              #errorTallyRid = np.zeros((lambdaCount,1))
              for lam in range(lambdaCount):
                  for index in range(len(XTest1)):
                      if (yTest1[index] > 0 and y_lasso1[index,lam] < 0) or__
→(yTest1[index] < 0 and y_lasso1[index,lam] > 0 or y_lasso1[index,lam] == 0):
                          errorTallyLasso[lam] = errorTallyLasso[lam] + 1
                  #if (max(y_lasso1[:,lam]) < 1e-6):
                      print(max(y_lasso1[:,lam]))
                  #print(errorTallyLasso[lam])
              #print(errorTallyLasso)
              #print(errorTallyRid)
              OptimalLambdaLasso = np.argmin(errorTallyLasso)
              \#print("Optimal\ lambda:\ ",lambdaTest[OptimalLambdaLasso],"\ with
\rightarrowmin ",min(errorTallyLasso)/(len(X)/11))
              #print(np.argmin(errorTallyLasso))
              #OptimalLambdaRid = np.arqmin(errorTallyRid)
              #print(np.argmin(errorTallyRid))
              #Running on Testing 2
              wOrig = np.linalg.inv(XTrain.T@XTrain)@XTrain.T@yTrain
              yTestOrig = XTest2@wOrig
              #print(yTestOriq)
              yTestLASS0 = XTest2@WLASS0[:,OptimalLambdaLasso]
              errorOrigTemp = 0
              errorLASSOTemp = 0
              for index in range(len(XTest2)):
                  if (yTest1[index] > 0 and yTestOrig[index] < 0) or__</pre>
errorOrigTemp = errorOrigTemp + 1
                  if (yTest1[index] > 0 and yTestLASS0[index] < 0) or__
errorLASSOTemp = errorLASSOTemp + 1
              errorOrigTemp = errorOrigTemp/len(XTest2)
              errorLASSOTemp = errorLASSOTemp/len(XTest2)
              print("Mean error from standard least squares: ",errorOrigTemp)
              print("Mean error from LASSO: ",errorLASSOTemp)
              errorOrig[Xset] = errorOrig[Xset] + errorOrigTemp
              errorLASSO[Xset] = errorLASSO[Xset] + errorLASSOTemp
              #UPDATE 11/13: SOME ONES X RESULT IN SINGULAR MATRIX, NOT
→ TESTING FURTHER
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#wOrigOnes = np.linalg.inv(XTrainwithOne.
 \hookrightarrow T@XTrainwithOne)@XTrainwithOne.T@yTrain
                 #yTestOrigOne = XTest2withOne@wOrigOnes
                 #print(yTestOrigOne)
GlobalMeanOrigError = np.zeros((11,1))
GlobalMeanLASSOError = np.zeros((11,1))
for index in range(11):
    GlobalMeanOrigError[index] = errorOrig[index]/110
    GlobalMeanLASSOError[index] = errorLASSO[index]/110
    print("Mean Error for pure least squares, X subset ",(index+1)," =__
 →",GlobalMeanOrigError[index])
    print("Mean Error for LASSO, X subset ",(index+1)," =_
 →",GlobalMeanLASSOError[index])
-0.07986076077086066
X \text{ subset} = 0, i = 0, j = 1
Mean error from standard least squares: 0.49278579356270813
Mean error from LASSO: 0.49250832408435075
X \text{ subset} = 0, i = 0, j = 2
Mean error from standard least squares: 0.5016648168701443
Mean error from LASSO: 0.49778024417314093
X \text{ subset} = 0, i = 0, j = 3
Mean error from standard least squares: 0.5047169811320755
Mean error from LASSO: 0.4972253052164262
X \text{ subset} = 0, i = 0, j = 4
Mean error from standard least squares: 0.4855715871254162
Mean error from LASSO: 0.5002774694783574
X \text{ subset} = 0, i = 0, j = 5
Mean error from standard least squares: 0.4997225305216426
Mean error from LASSO: 0.4894561598224195
X \text{ subset} = 0, i = 0, j = 6
Mean error from standard least squares: 0.5049944506104328
Mean error from LASSO: 0.4911209766925638
X \text{ subset} = 0, i = 0, j = 7
Mean error from standard least squares: 0.5074916759156493
Mean error from LASSO: 0.4936182019977802
X \text{ subset} = 0, i = 0, j = 8
Mean error from standard least squares: 0.5102663706992231
Mean error from LASSO: 0.49528301886792453
X \text{ subset} = 0, i = 0, j = 9
Mean error from standard least squares: 0.5097114317425083
Mean error from LASSO: 0.4975027746947836
X \text{ subset} = 0, i = 0, j = 10
Mean error from standard least squares: 0.5011098779134295
Mean error from LASSO: 0.5049944506104328
X subset = 0 , i = 1 , j = 0
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Mean error from standard least squares: 0.5066592674805771
Mean error from LASSO: 0.4811320754716981
X \text{ subset} = 0, i = 1, j = 2
Mean error from standard least squares: 0.5083240843507214
Mean error from LASSO: 0.48668146503884574
X subset = 0 , i = 1 , j = 3
Mean error from standard least squares: 0.5102663706992231
Mean error from LASSO: 0.4653163152053274
X subset = 0 , i = 1 , j = 4
Mean error from standard least squares: 0.48335183129855713
Mean error from LASSO: 0.462819089900111
X \text{ subset} = 0, i = 1, j = 5
Mean error from standard least squares: 0.4975027746947836
Mean error from LASSO: 0.4675360710321865
X subset = 0 , i = 1 , j = 6
Mean error from standard least squares: 0.5038845726970034
Mean error from LASSO: 0.4653163152053274
X \text{ subset} = 0, i = 1, j = 7
Mean error from standard least squares: 0.4986126526082131
Mean error from LASSO: 0.46170921198668147
X \text{ subset} = 0, i = 1, j = 8
Mean error from standard least squares: 0.5055493895671476
Mean error from LASSO: 0.4794672586015538
X \text{ subset} = 0, i = 1, j = 9
Mean error from standard least squares: 0.52330743618202
Mean error from LASSO: 0.4572697003329634
X \text{ subset} = 0 , i = 1 , j = 10
Mean error from standard least squares: 0.5147058823529411
Mean error from LASSO: 0.4542175360710322
X subset = 0 , i = 2 , j = 0
Mean error from standard least squares: 0.4936182019977802
Mean error from LASSO: 0.5396781354051055
X \text{ subset} = 0, i = 2, j = 1
Mean error from standard least squares: 0.46642619311875694
Mean error from LASSO: 0.5502219755826859
X subset = 0 , i = 2 , j = 3
Mean error from standard least squares: 0.4936182019977802
Mean error from LASSO: 0.5749167591564928
X \text{ subset} = 0, i = 2, j = 4
Mean error from standard least squares: 0.49306326304106546
Mean error from LASSO: 0.5735294117647058
X subset = 0 , i = 2 , j = 5
Mean error from standard least squares: 0.5094339622641509
Mean error from LASSO: 0.5738068812430632
X subset = 0 , i = 2 , j = 6
Mean error from standard least squares: 0.4769700332963374
Mean error from LASSO: 0.574361820199778
X \text{ subset} = 0, i = 2, j = 7
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Mean error from standard least squares: 0.48612652608213097
Mean error from LASSO: 0.5768590455049944
X \text{ subset} = 0, i = 2, j = 8
Mean error from standard least squares: 0.47835738068812433
Mean error from LASSO: 0.5721420643729189
X subset = 0 , i = 2 , j = 9
Mean error from standard least squares: 0.4983351831298557
Mean error from LASSO: 0.5635405105438401
X \text{ subset} = 0 , i = 2 , j = 10
Mean error from standard least squares: 0.4528301886792453
Mean error from LASSO: 0.5541065482796892
X \text{ subset} = 0, i = 3, j = 0
Mean error from standard least squares: 0.4900110987791343
Mean error from LASSO: 0.5294117647058824
X \text{ subset} = 0 , i = 3 , j = 1
Mean error from standard least squares: 0.4830743618201998
Mean error from LASSO: 0.5482796892341842
X \text{ subset} = 0, i = 3, j = 2
Mean error from standard least squares: 0.5041620421753608
Mean error from LASSO: 0.531354051054384
X \text{ subset} = 0, i = 3, j = 4
Mean error from standard least squares: 0.5074916759156493
Mean error from LASSO: 0.560488346281909
X \text{ subset} = 0, i = 3, j = 5
Mean error from standard least squares: 0.4916759156492786
Mean error from LASSO: 0.5665926748057714
X \text{ subset} = 0 , i = 3 , j = 6
Mean error from standard least squares: 0.4936182019977802
Mean error from LASSO: 0.5518867924528302
X subset = 0 , i = 3 , j = 7
Mean error from standard least squares: 0.4913984461709212
Mean error from LASSO: 0.5738068812430632
X \text{ subset} = 0, i = 3, j = 8
Mean error from standard least squares: 0.5177580466148723
Mean error from LASSO: 0.5552164261931187
X \text{ subset} = 0 , i = 3 , j = 9
Mean error from standard least squares: 0.5047169811320755
Mean error from LASSO: 0.5693673695893452
X \text{ subset} = 0 , i = 3 , j = 10
Mean error from standard least squares: 0.4855715871254162
Mean error from LASSO: 0.5499445061043285
X subset = 0 , i = 4 , j = 0
Mean error from standard least squares: 0.4955604883462819
Mean error from LASSO: 0.5094339622641509
X \text{ subset} = 0, i = 4, j = 1
Mean error from standard least squares: 0.5072142064372919
Mean error from LASSO: 0.5149833518312985
X \text{ subset} = 0, i = 4, j = 2
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Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.5241398446170921
X \text{ subset} = 0, i = 4, j = 3
Mean error from standard least squares: 0.5044395116537181
Mean error from LASSO: 0.5310765815760267
X subset = 0 , i = 4 , j = 5
Mean error from standard least squares: 0.4961154273029967
Mean error from LASSO: 0.5288568257491676
X subset = 0 , i = 4 , j = 6
Mean error from standard least squares: 0.5005549389567148
Mean error from LASSO: 0.5283018867924528
X \text{ subset} = 0, i = 4, j = 7
Mean error from standard least squares: 0.49528301886792453
Mean error from LASSO: 0.529134295227525
X subset = 0 , i = 4 , j = 8
Mean error from standard least squares: 0.4700332963374029
Mean error from LASSO: 0.5380133185349611
X \text{ subset} = 0, i = 4, j = 9
Mean error from standard least squares: 0.5130410654827969
Mean error from LASSO: 0.5274694783573807
X \text{ subset} = 0 , i = 4 , j = 10
Mean error from standard least squares: 0.49223085460599336
Mean error from LASSO: 0.5310765815760267
X \text{ subset} = 0, i = 5, j = 0
Mean error from standard least squares: 0.5019422863485017
Mean error from LASSO: 0.49250832408435075
X \text{ subset} = 0, i = 5, j = 1
Mean error from standard least squares: 0.5024972253052165
Mean error from LASSO: 0.48640399556048836
X \text{ subset} = 0 , i = 5 , j = 2
Mean error from standard least squares: 0.49528301886792453
Mean error from LASSO: 0.5002774694783574
X \text{ subset} = 0, i = 5, j = 3
Mean error from standard least squares: 0.5038845726970034
Mean error from LASSO: 0.49667036625971145
X \text{ subset} = 0, i = 5, j = 4
Mean error from standard least squares: 0.4975027746947836
Mean error from LASSO: 0.49084350721420644
X \text{ subset} = 0, i = 5, j = 6
Mean error from standard least squares: 0.49250832408435075
Mean error from LASSO: 0.49223085460599336
X \text{ subset} = 0, i = 5, j = 7
Mean error from standard least squares: 0.5102663706992231
Mean error from LASSO: 0.503607103218646
X subset = 0 , i = 5 , j = 8
Mean error from standard least squares: 0.5155382907880133
Mean error from LASSO: 0.49694783573806883
X \text{ subset} = 0, i = 5, j = 9
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Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.49472807991120976
X \text{ subset} = 0 , i = 5 , j = 10
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.4955604883462819
X subset = 0 , i = 6 , j = 0
Mean error from standard least squares: 0.51165371809101
Mean error from LASSO: 0.49916759156492785
X \text{ subset} = 0, i = 6, j = 1
Mean error from standard least squares: 0.5102663706992231
Mean error from LASSO: 0.4980577136514983
X \text{ subset} = 0, i = 6, j = 2
Mean error from standard least squares: 0.5102663706992231
Mean error from LASSO: 0.4958379578246393
X subset = 0 , i = 6 , j = 3
Mean error from standard least squares: 0.4958379578246393
Mean error from LASSO: 0.5088790233074362
X \text{ subset} = 0, i = 6, j = 4
Mean error from standard least squares: 0.5022197558268591
Mean error from LASSO: 0.5052719200887902
X \text{ subset} = 0, i = 6, j = 5
Mean error from standard least squares: 0.4733629300776915
Mean error from LASSO: 0.49889012208657046
X \text{ subset} = 0, i = 6, j = 7
Mean error from standard least squares: 0.5027746947835738
Mean error from LASSO: 0.49916759156492785
X \text{ subset} = 0, i = 6, j = 8
Mean error from standard least squares: 0.4869589345172031
Mean error from LASSO: 0.5130410654827969
X \text{ subset} = 0, i = 6, j = 9
Mean error from standard least squares: 0.5002774694783574
Mean error from LASSO: 0.4913984461709212
X \text{ subset} = 0, i = 6, j = 10
Mean error from standard least squares: 0.5069367369589345
Mean error from LASSO: 0.5083240843507214
X subset = 0 , i = 7 , j = 0
Mean error from standard least squares: 0.4972253052164262
Mean error from LASSO: 0.5185904550499445
X \text{ subset} = 0, i = 7, j = 1
Mean error from standard least squares: 0.4900110987791343
Mean error from LASSO: 0.517480577136515
X \text{ subset} = 0, i = 7, j = 2
Mean error from standard least squares: 0.5041620421753608
Mean error from LASSO: 0.5269145394006659
X subset = 0 , i = 7 , j = 3
Mean error from standard least squares: 0.4814095449500555
Mean error from LASSO: 0.5321864594894562
X \text{ subset} = 0, i = 7, j = 4
```

```
Mean error from standard least squares: 0.49694783573806883
Mean error from LASSO: 0.5302441731409545
X \text{ subset} = 0, i = 7, j = 5
Mean error from standard least squares: 0.5002774694783574
Mean error from LASSO: 0.5382907880133185
X subset = 0 , i = 7 , j = 6
Mean error from standard least squares: 0.5033296337402886
Mean error from LASSO: 0.5238623751387348
X \text{ subset} = 0, i = 7, j = 8
Mean error from standard least squares: 0.4869589345172031
Mean error from LASSO: 0.5258046614872364
X \text{ subset} = 0, i = 7, j = 9
Mean error from standard least squares: 0.49223085460599336
Mean error from LASSO: 0.5152608213096559
X subset = 0 , i = 7 , j = 10
Mean error from standard least squares: 0.4741953385127636
Mean error from LASSO: 0.5305216426193119
X \text{ subset} = 0, i = 8, j = 0
Mean error from standard least squares: 0.5049944506104328
Mean error from LASSO: 0.5055493895671476
X \text{ subset} = 0, i = 8, j = 1
Mean error from standard least squares: 0.5022197558268591
Mean error from LASSO: 0.5283018867924528
X \text{ subset} = 0, i = 8, j = 2
Mean error from standard least squares: 0.5105438401775805
Mean error from LASSO: 0.5188679245283019
X \text{ subset} = 0, i = 8, j = 3
Mean error from standard least squares: 0.4986126526082131
Mean error from LASSO: 0.5319089900110988
X \text{ subset} = 0, i = 8, j = 4
Mean error from standard least squares: 0.46587125416204217
Mean error from LASSO: 0.5366259711431742
X \text{ subset} = 0, i = 8, j = 5
Mean error from standard least squares: 0.4797447280799112
Mean error from LASSO: 0.5341287458379578
X \text{ subset} = 0 , i = 8 , j = 6
Mean error from standard least squares: 0.4911209766925638
Mean error from LASSO: 0.5391231964483907
X \text{ subset} = 0, i = 8, j = 7
Mean error from standard least squares: 0.49694783573806883
Mean error from LASSO: 0.5471698113207547
X \text{ subset} = 0, i = 8, j = 9
Mean error from standard least squares: 0.47807991120976695
Mean error from LASSO: 0.5427302996670367
X subset = 0 , i = 8 , j = 10
Mean error from standard least squares: 0.4894561598224195
Mean error from LASSO: 0.5374583795782464
X \text{ subset} = 0, i = 9, j = 0
```

```
Mean error from standard least squares: 0.4986126526082131
Mean error from LASSO: 0.5366259711431742
X \text{ subset} = 0, i = 9, j = 1
Mean error from standard least squares: 0.48057713651498335
Mean error from LASSO: 0.5360710321864595
X subset = 0 , i = 9 , j = 2
Mean error from standard least squares: 0.5219200887902331
Mean error from LASSO: 0.554661487236404
X \text{ subset} = 0, i = 9, j = 3
Mean error from standard least squares: 0.5013873473917869
Mean error from LASSO: 0.5629855715871254
X \text{ subset} = 0, i = 9, j = 4
Mean error from standard least squares: 0.508046614872364
Mean error from LASSO: 0.5701997780244173
X \text{ subset} = 0 , i = 9 , j = 5
Mean error from standard least squares: 0.47225305216426194
Mean error from LASSO: 0.5785238623751388
X \text{ subset} = 0, i = 9, j = 6
Mean error from standard least squares: 0.46365149833518315
Mean error from LASSO: 0.5724195338512763
X \text{ subset} = 0, i = 9, j = 7
Mean error from standard least squares: 0.4714206437291898
Mean error from LASSO: 0.5688124306326304
X \text{ subset} = 0 , i = 9 , j = 8
Mean error from standard least squares: 0.48612652608213097
Mean error from LASSO: 0.5615982241953386
X \text{ subset} = 0 , i = 9 , j = 10
Mean error from standard least squares: 0.46170921198668147
Mean error from LASSO: 0.5610432852386238
X subset = 0 , i = 10 , j = 0
Mean error from standard least squares: 0.5019422863485017
Mean error from LASSO: 0.5418978912319645
X \text{ subset} = 0, i = 10, j = 1
Mean error from standard least squares: 0.48917869034406214
Mean error from LASSO: 0.532741398446171
X subset = 0 , i = 10 , j = 2
Mean error from standard least squares: 0.5005549389567148
Mean error from LASSO: 0.5507769145394007
X \text{ subset} = 0 , i = 10 , j = 3
Mean error from standard least squares: 0.5063817980022197
Mean error from LASSO: 0.566315205327414
X \text{ subset} = 0, i = 10, j = 4
Mean error from standard least squares: 0.4775249722530522
Mean error from LASSO: 0.556881243063263
X subset = 0 , i = 10 , j = 5
Mean error from standard least squares: 0.5044395116537181
Mean error from LASSO: 0.5696448390677026
X \text{ subset} = 0, i = 10, j = 6
```

```
Mean error from standard least squares: 0.5069367369589345
Mean error from LASSO: 0.5718645948945617
X \text{ subset} = 0, i = 10, j = 7
Mean error from standard least squares: 0.4572697003329634
Mean error from LASSO: 0.574361820199778
X subset = 0 , i = 10 , j = 8
Mean error from standard least squares: 0.47780244173140957
Mean error from LASSO: 0.5665926748057714
X \text{ subset} = 0 , i = 10 , j = 9
Mean error from standard least squares: 0.46476137624861263
Mean error from LASSO: 0.5549389567147613
X \text{ subset} = 1, i = 0, j = 1
Mean error from standard least squares: 0.5052719200887902
Mean error from LASSO: 0.5038845726970034
X subset = 1 , i = 0 , j = 2
Mean error from standard least squares: 0.5124861265260822
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 1, i = 0, j = 3
Mean error from standard least squares: 0.5030521642619312
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 1, i = 0, j = 4
Mean error from standard least squares: 0.5024972253052165
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 1, i = 0, j = 5
Mean error from standard least squares: 0.49889012208657046
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 1, i = 0, j = 6
Mean error from standard least squares: 0.505826859045505
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 1, i = 0, j = 7
Mean error from standard least squares: 0.5055493895671476
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 1, i = 0, j = 8
Mean error from standard least squares: 0.5047169811320755
Mean error from LASSO: 0.5038845726970034
X subset = 1 , i = 0 , j = 9
Mean error from standard least squares: 0.5002774694783574
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 1 , i = 0 , j = 10
Mean error from standard least squares: 0.5055493895671476
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 1, i = 1, j = 0
Mean error from standard least squares: 0.5441176470588235
Mean error from LASSO: 0.5463374028856826
X subset = 1 , i = 1 , j = 2
Mean error from standard least squares: 0.5374583795782464
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 1 , i = 1 , j = 3
```

```
Mean error from standard least squares: 0.5341287458379578
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 1 , i = 1 , j = 4
Mean error from standard least squares: 0.5413429522752498
Mean error from LASSO: 0.5463374028856826
X subset = 1 , i = 1 , j = 5
Mean error from standard least squares: 0.5352386237513873
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 1, i = 1, j = 6
Mean error from standard least squares: 0.5382907880133185
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 1, i = 1, j = 7
Mean error from standard least squares: 0.5316315205327414
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 1 , i = 1 , j = 8
Mean error from standard least squares: 0.5418978912319645
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 1 , i = 1 , j = 9
Mean error from standard least squares: 0.5474472807991121
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 1 , i = 1 , j = 10
Mean error from standard least squares: 0.5388457269700333
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 1, i = 2, j = 0
Mean error from standard least squares: 0.4167591564927858
Mean error from LASSO: 0.40288568257491675
X subset = 1 , i = 2 , j = 1
Mean error from standard least squares: 0.413984461709212
Mean error from LASSO: 0.40288568257491675
X subset = 1 , i = 2 , j = 3
Mean error from standard least squares: 0.42314095449500555
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 1, i = 2, j = 4
Mean error from standard least squares: 0.41315205327413984
Mean error from LASSO: 0.40288568257491675
X subset = 1 , i = 2 , j = 5
Mean error from standard least squares: 0.42480577136514985
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 1 , i = 2 , j = 6
Mean error from standard least squares: 0.41731409544950054
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 1, i = 2, j = 7
Mean error from standard least squares: 0.42702552719200887
Mean error from LASSO: 0.40288568257491675
X subset = 1 , i = 2 , j = 8
Mean error from standard least squares: 0.42563817980022195
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 1 , i = 2 , j = 9
```

```
Mean error from standard least squares: 0.416204217536071
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 1 , i = 2 , j = 10
Mean error from standard least squares: 0.427857935627081
Mean error from LASSO: 0.40288568257491675
X subset = 1 , i = 3 , j = 0
Mean error from standard least squares: 0.42730299667036625
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1, i = 3, j = 1
Mean error from standard least squares: 0.42314095449500555
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1, i = 3, j = 2
Mean error from standard least squares: 0.42147613762486125
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1 , i = 3 , j = 4
Mean error from standard least squares: 0.4322974472807991
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1, i = 3, j = 5
Mean error from standard least squares: 0.44034406215316313
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1 , i = 3 , j = 6
Mean error from standard least squares: 0.43118756936736957
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1, i = 3, j = 7
Mean error from standard least squares: 0.4367369589345172
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1 , i = 3 , j = 8
Mean error from standard least squares: 0.4422863485016648
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1, i = 3, j = 9
Mean error from standard least squares: 0.42591564927857933
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1 , i = 3 , j = 10
Mean error from standard least squares: 0.4325749167591565
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 1, i = 4, j = 0
Mean error from standard least squares: 0.470865704772475
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1 , i = 4 , j = 1
Mean error from standard least squares: 0.47031076581576026
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1, i = 4, j = 2
Mean error from standard least squares: 0.46892341842397334
Mean error from LASSO: 0.46448390677025525
X subset = 1 , i = 4 , j = 3
Mean error from standard least squares: 0.4608768035516093
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1 , i = 4 , j = 5
```

```
Mean error from standard least squares: 0.4827968923418424
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1 , i = 4 , j = 6
Mean error from standard least squares: 0.46420643729189787
Mean error from LASSO: 0.46448390677025525
X subset = 1 , i = 4 , j = 7
Mean error from standard least squares: 0.4711431742508324
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1, i = 4, j = 8
Mean error from standard least squares: 0.47807991120976695
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1, i = 4, j = 9
Mean error from standard least squares: 0.47613762486126526
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1 , i = 4 , j = 10
Mean error from standard least squares: 0.470865704772475
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1, i = 5, j = 0
Mean error from standard least squares: 0.5033296337402886
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1, i = 5, j = 1
Mean error from standard least squares: 0.5066592674805771
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1, i = 5, j = 2
Mean error from standard least squares: 0.5049944506104328
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1, i = 5, j = 3
Mean error from standard least squares: 0.5005549389567148
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1, i = 5, j = 4
Mean error from standard least squares: 0.5063817980022197
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1, i = 5, j = 6
Mean error from standard least squares: 0.5097114317425083
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1 , i = 5 , j = 7
Mean error from standard least squares: 0.5024972253052165
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1, i = 5, j = 8
Mean error from standard least squares: 0.5083240843507214
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1, i = 5, j = 9
Mean error from standard least squares: 0.5119311875693674
Mean error from LASSO: 0.5061043285238623
X subset = 1 , i = 5 , j = 10
Mean error from standard least squares: 0.5124861265260822
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 1 , i = 6 , j = 0
```

```
Mean error from standard least squares: 0.5019422863485017
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1 , i = 6 , j = 1
Mean error from standard least squares: 0.5002774694783574
Mean error from LASSO: 0.4983351831298557
X subset = 1 , i = 6 , j = 2
Mean error from standard least squares: 0.4986126526082131
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1, i = 6, j = 3
Mean error from standard least squares: 0.5005549389567148
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1, i = 6, j = 4
Mean error from standard least squares: 0.5038845726970034
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1 , i = 6 , j = 5
Mean error from standard least squares: 0.49916759156492785
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1, i = 6, j = 7
Mean error from standard least squares: 0.49916759156492785
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1, i = 6, j = 8
Mean error from standard least squares: 0.49223085460599336
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1 , i = 6 , j = 9
Mean error from standard least squares: 0.4997225305216426
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1 , i = 6 , j = 10
Mean error from standard least squares: 0.48612652608213097
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 1 , i = 7 , j = 0
Mean error from standard least squares: 0.46337402885682577
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1, i = 7, j = 1
Mean error from standard least squares: 0.4700332963374029
Mean error from LASSO: 0.46448390677025525
X subset = 1 , i = 7 , j = 2
Mean error from standard least squares: 0.4653163152053274
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1 , i = 7 , j = 3
Mean error from standard least squares: 0.47058823529411764
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1, i = 7, j = 4
Mean error from standard least squares: 0.46892341842397334
Mean error from LASSO: 0.46448390677025525
X subset = 1 , i = 7 , j = 5
Mean error from standard least squares: 0.47835738068812433
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1 , i = 7 , j = 6
```

```
Mean error from standard least squares: 0.46892341842397334
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1 , i = 7 , j = 8
Mean error from standard least squares: 0.46392896781354054
Mean error from LASSO: 0.46448390677025525
X subset = 1 , i = 7 , j = 9
Mean error from standard least squares: 0.4728079911209767
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1 , i = 7 , j = 10
Mean error from standard least squares: 0.48029966703662597
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 1, i = 8, j = 0
Mean error from standard least squares: 0.4667036625971143
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1 , i = 8 , j = 1
Mean error from standard least squares: 0.45754716981132076
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1, i = 8, j = 2
Mean error from standard least squares: 0.47447280799112096
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1, i = 8, j = 3
Mean error from standard least squares: 0.46170921198668147
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1, i = 8, j = 4
Mean error from standard least squares: 0.4589345172031077
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1 , i = 8 , j = 5
Mean error from standard least squares: 0.46059933407325193
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1, i = 8, j = 6
Mean error from standard least squares: 0.4655937846836848
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1, i = 8, j = 7
Mean error from standard least squares: 0.46476137624861263
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1 , i = 8 , j = 9
Mean error from standard least squares: 0.46614872364039955
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1 , i = 8 , j = 10
Mean error from standard least squares: 0.46781354051054386
Mean error from LASSO: 0.4514428412874584
X \text{ subset} = 1, i = 9, j = 0
Mean error from standard least squares: 0.4209211986681465
Mean error from LASSO: 0.40427302996670367
X subset = 1 , i = 9 , j = 1
Mean error from standard least squares: 0.4109322974472808
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 9 , j = 2
```

```
Mean error from standard least squares: 0.43174250832408434
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 9 , j = 3
Mean error from standard least squares: 0.42924528301886794
Mean error from LASSO: 0.40427302996670367
X subset = 1 , i = 9 , j = 4
Mean error from standard least squares: 0.4320199778024417
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1, i = 9, j = 5
Mean error from standard least squares: 0.4364594894561598
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1, i = 9, j = 6
Mean error from standard least squares: 0.4261931187569367
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1, i = 9, j = 7
Mean error from standard least squares: 0.4284128745837958
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1, i = 9, j = 8
Mean error from standard least squares: 0.4284128745837958
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 9 , j = 10
Mean error from standard least squares: 0.422031076581576
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 10 , j = 0
Mean error from standard least squares: 0.4137069922308546
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 10 , j = 1
Mean error from standard least squares: 0.42064372918978915
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1, i = 10, j = 2
Mean error from standard least squares: 0.42758046614872364
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 10 , j = 3
Mean error from standard least squares: 0.4320199778024417
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 10 , j = 4
Mean error from standard least squares: 0.4253607103218646
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 10 , j = 5
Mean error from standard least squares: 0.4281354051054384
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1, i = 10, j = 6
Mean error from standard least squares: 0.42924528301886794
Mean error from LASSO: 0.40427302996670367
X subset = 1 , i = 10 , j = 7
Mean error from standard least squares: 0.43479467258601556
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 10 , j = 8
```

```
Mean error from standard least squares: 0.42758046614872364
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 1 , i = 10 , j = 9
Mean error from standard least squares: 0.43507214206437295
Mean error from LASSO: 0.40427302996670367
X subset = 2 , i = 0 , j = 1
Mean error from standard least squares: 0.5022197558268591
Mean error from LASSO: 0.5027746947835738
X \text{ subset} = 2, i = 0, j = 2
Mean error from standard least squares: 0.49667036625971145
Mean error from LASSO: 0.49056603773584906
X \text{ subset} = 2, i = 0, j = 3
Mean error from standard least squares: 0.49278579356270813
Mean error from LASSO: 0.491953385127636
X \text{ subset} = 2 \text{ , i = 0 , j = 4}
Mean error from standard least squares: 0.48862375138734737
Mean error from LASSO: 0.5047169811320755
X \text{ subset} = 2, i = 0, j = 5
Mean error from standard least squares: 0.5019422863485017
Mean error from LASSO: 0.5002774694783574
X \text{ subset} = 2, i = 0, j = 6
Mean error from standard least squares: 0.4753052164261931
Mean error from LASSO: 0.4753052164261931
X \text{ subset} = 2, i = 0, j = 7
Mean error from standard least squares: 0.5038845726970034
Mean error from LASSO: 0.5061043285238623
X \text{ subset} = 2, i = 0, j = 8
Mean error from standard least squares: 0.48196448390677027
Mean error from LASSO: 0.4916759156492786
X \text{ subset} = 2 \text{ , } i = 0 \text{ , } j = 9
Mean error from standard least squares: 0.45449500554938954
Mean error from LASSO: 0.44200887902330743
X \text{ subset} = 2, i = 0, j = 10
Mean error from standard least squares: 0.45810210876803553
Mean error from LASSO: 0.4522752497225305
X \text{ subset} = 2, i = 1, j = 0
Mean error from standard least squares: 0.5230299667036626
Mean error from LASSO: 0.5266370699223085
X \text{ subset} = 2, i = 1, j = 2
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.47475027746947834
X \text{ subset} = 2, i = 1, j = 3
Mean error from standard least squares: 0.5027746947835738
Mean error from LASSO: 0.4550499445061043
X subset = 2 , i = 1 , j = 4
Mean error from standard least squares: 0.5055493895671476
Mean error from LASSO: 0.5077691453940066
X \text{ subset} = 2 , i = 1 , j = 5
```

```
Mean error from standard least squares: 0.5113762486126526
Mean error from LASSO: 0.5202552719200888
X \text{ subset} = 2, i = 1, j = 6
Mean error from standard least squares: 0.5110987791342952
Mean error from LASSO: 0.5144284128745837
X subset = 2 , i = 1 , j = 7
Mean error from standard least squares: 0.5199778024417314
Mean error from LASSO: 0.5208102108768036
X \text{ subset} = 2, i = 1, j = 8
Mean error from standard least squares: 0.4739178690344062
Mean error from LASSO: 0.47835738068812433
X \text{ subset} = 2, i = 1, j = 9
Mean error from standard least squares: 0.4669811320754717
Mean error from LASSO: 0.46503884572697
X \text{ subset} = 2 , i = 1 , j = 10
Mean error from standard least squares: 0.46614872364039955
Mean error from LASSO: 0.4223085460599334
X \text{ subset} = 2, i = 2, j = 0
Mean error from standard least squares: 0.4830743618201998
Mean error from LASSO: 0.4839067702552719
X \text{ subset} = 2, i = 2, j = 1
Mean error from standard least squares: 0.4672586015538291
Mean error from LASSO: 0.4683684794672586
X \text{ subset} = 2, i = 2, j = 3
Mean error from standard least squares: 0.4800221975582686
Mean error from LASSO: 0.49306326304106546
X \text{ subset} = 2, i = 2, j = 4
Mean error from standard least squares: 0.4836293007769145
Mean error from LASSO: 0.4841842397336293
X \text{ subset} = 2, i = 2, j = 5
Mean error from standard least squares: 0.46059933407325193
Mean error from LASSO: 0.46059933407325193
X \text{ subset} = 2, i = 2, j = 6
Mean error from standard least squares: 0.46809100998890124
Mean error from LASSO: 0.46781354051054386
X \text{ subset} = 2, i = 2, j = 7
Mean error from standard least squares: 0.47891231964483905
Mean error from LASSO: 0.48085460599334073
X \text{ subset} = 2, i = 2, j = 8
Mean error from standard least squares: 0.44339622641509435
Mean error from LASSO: 0.4422863485016648
X \text{ subset} = 2, i = 2, j = 9
Mean error from standard least squares: 0.46198668146503885
Mean error from LASSO: 0.46809100998890124
X subset = 2 , i = 2 , j = 10
Mean error from standard least squares: 0.4542175360710322
Mean error from LASSO: 0.4572697003329634
X \text{ subset} = 2 \text{ , } i = 3 \text{ , } j = 0
```

```
Mean error from standard least squares: 0.47641509433962265
Mean error from LASSO: 0.49667036625971145
X \text{ subset} = 2 \text{ , } i = 3 \text{ , } j = 1
Mean error from standard least squares: 0.4675360710321865
Mean error from LASSO: 0.46226415094339623
X subset = 2 , i = 3 , j = 2
Mean error from standard least squares: 0.49278579356270813
Mean error from LASSO: 0.5019422863485017
X \text{ subset} = 2, i = 3, j = 4
Mean error from standard least squares: 0.494173140954495
Mean error from LASSO: 0.5044395116537181
X \text{ subset} = 2, i = 3, j = 5
Mean error from standard least squares: 0.4614317425083241
Mean error from LASSO: 0.4614317425083241
X \text{ subset} = 2, i = 3, j = 6
Mean error from standard least squares: 0.48917869034406214
Mean error from LASSO: 0.48890122086570476
X \text{ subset} = 2, i = 3, j = 7
Mean error from standard least squares: 0.47780244173140957
Mean error from LASSO: 0.49889012208657046
X \text{ subset} = 2, i = 3, j = 8
Mean error from standard least squares: 0.48085460599334073
Mean error from LASSO: 0.48668146503884574
X \text{ subset} = 2, i = 3, j = 9
Mean error from standard least squares: 0.4556048834628191
Mean error from LASSO: 0.46337402885682577
X \text{ subset} = 2 , i = 3 , j = 10
Mean error from standard least squares: 0.4528301886792453
Mean error from LASSO: 0.46809100998890124
X \text{ subset} = 2, i = 4, j = 0
Mean error from standard least squares: 0.5016648168701443
Mean error from LASSO: 0.4958379578246393
X \text{ subset} = 2, i = 4, j = 1
Mean error from standard least squares: 0.47918978912319643
Mean error from LASSO: 0.4583795782463929
X \text{ subset} = 2, i = 4, j = 2
Mean error from standard least squares: 0.47807991120976695
Mean error from LASSO: 0.47807991120976695
X \text{ subset} = 2, i = 4, j = 3
Mean error from standard least squares: 0.4936182019977802
Mean error from LASSO: 0.49528301886792453
X \text{ subset} = 2, i = 4, j = 5
Mean error from standard least squares: 0.4800221975582686
Mean error from LASSO: 0.48057713651498335
X subset = 2 , i = 4 , j = 6
Mean error from standard least squares: 0.47891231964483905
Mean error from LASSO: 0.47863485016648166
X \text{ subset} = 2 \text{ , } i = 4 \text{ , } j = 7
```

```
Mean error from standard least squares: 0.4858490566037736
Mean error from LASSO: 0.4894561598224195
X \text{ subset} = 2, i = 4, j = 8
Mean error from standard least squares: 0.4716981132075472
Mean error from LASSO: 0.470865704772475
X subset = 2 , i = 4 , j = 9
Mean error from standard least squares: 0.4589345172031077
Mean error from LASSO: 0.45810210876803553
X \text{ subset} = 2 , i = 4 , j = 10
Mean error from standard least squares: 0.4447835738068812
Mean error from LASSO: 0.45394006659267483
X \text{ subset} = 2, i = 5, j = 0
Mean error from standard least squares: 0.5022197558268591
Mean error from LASSO: 0.4972253052164262
X \text{ subset} = 2, i = 5, j = 1
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.5019422863485017
X \text{ subset} = 2, i = 5, j = 2
Mean error from standard least squares: 0.5069367369589345
Mean error from LASSO: 0.5091564927857936
X \text{ subset} = 2, i = 5, j = 3
Mean error from standard least squares: 0.5044395116537181
Mean error from LASSO: 0.5044395116537181
X \text{ subset} = 2, i = 5, j = 4
Mean error from standard least squares: 0.4961154273029967
Mean error from LASSO: 0.5008324084350722
X \text{ subset} = 2, i = 5, j = 6
Mean error from standard least squares: 0.48335183129855713
Mean error from LASSO: 0.4858490566037736
X \text{ subset} = 2, i = 5, j = 7
Mean error from standard least squares: 0.48029966703662597
Mean error from LASSO: 0.4841842397336293
X \text{ subset} = 2, i = 5, j = 8
Mean error from standard least squares: 0.4586570477247503
Mean error from LASSO: 0.46392896781354054
X \text{ subset} = 2, i = 5, j = 9
Mean error from standard least squares: 0.46476137624861263
Mean error from LASSO: 0.4614317425083241
X \text{ subset} = 2 , i = 5 , j = 10
Mean error from standard least squares: 0.44367369589345174
Mean error from LASSO: 0.4442286348501665
X \text{ subset} = 2, i = 6, j = 0
Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.49250832408435075
X subset = 2 , i = 6 , j = 1
Mean error from standard least squares: 0.49694783573806883
Mean error from LASSO: 0.49667036625971145
X \text{ subset} = 2, i = 6, j = 2
```

```
Mean error from standard least squares: 0.48501664816870144
Mean error from LASSO: 0.4858490566037736
X \text{ subset} = 2, i = 6, j = 3
Mean error from standard least squares: 0.49334073251942284
Mean error from LASSO: 0.49306326304106546
X subset = 2 , i = 6 , j = 4
Mean error from standard least squares: 0.4975027746947836
Mean error from LASSO: 0.4980577136514983
X \text{ subset} = 2, i = 6, j = 5
Mean error from standard least squares: 0.4894561598224195
Mean error from LASSO: 0.48473917869034405
X \text{ subset} = 2, i = 6, j = 7
Mean error from standard least squares: 0.49223085460599336
Mean error from LASSO: 0.49250832408435075
X \text{ subset} = 2 , i = 6 , j = 8
Mean error from standard least squares: 0.4611542730299667
Mean error from LASSO: 0.456992230854606
X \text{ subset} = 2, i = 6, j = 9
Mean error from standard least squares: 0.45172031076581576
Mean error from LASSO: 0.4508879023307436
X \text{ subset} = 2 , i = 6 , j = 10
Mean error from standard least squares: 0.43729189789123196
Mean error from LASSO: 0.43590455049944504
X \text{ subset} = 2, i = 7, j = 0
Mean error from standard least squares: 0.48335183129855713
Mean error from LASSO: 0.4872364039955605
X \text{ subset} = 2 \text{ , i} = 7 \text{ , j} = 1
Mean error from standard least squares: 0.47225305216426194
Mean error from LASSO: 0.47253052164261933
X \text{ subset} = 2, i = 7, j = 2
Mean error from standard least squares: 0.4841842397336293
Mean error from LASSO: 0.4852941176470588
X \text{ subset} = 2, i = 7, j = 3
Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.49639289678135406
X subset = 2 , i = 7 , j = 4
Mean error from standard least squares: 0.4900110987791343
Mean error from LASSO: 0.4916759156492786
X \text{ subset} = 2, i = 7, j = 5
Mean error from standard least squares: 0.4594894561598224
Mean error from LASSO: 0.459211986681465
X \text{ subset} = 2, i = 7, j = 6
Mean error from standard least squares: 0.47669256381798003
Mean error from LASSO: 0.47613762486126526
X subset = 2 , i = 7 , j = 8
Mean error from standard least squares: 0.4594894561598224
Mean error from LASSO: 0.4586570477247503
X \text{ subset} = 2 , i = 7 , j = 9
```

```
Mean error from standard least squares: 0.46059933407325193
Mean error from LASSO: 0.4594894561598224
X \text{ subset} = 2 , i = 7 , j = 10
Mean error from standard least squares: 0.451165371809101
Mean error from LASSO: 0.45172031076581576
X subset = 2 , i = 8 , j = 0
Mean error from standard least squares: 0.4980577136514983
Mean error from LASSO: 0.4980577136514983
X \text{ subset} = 2, i = 8, j = 1
Mean error from standard least squares: 0.4728079911209767
Mean error from LASSO: 0.4697558268590455
X \text{ subset} = 2, i = 8, j = 2
Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.4955604883462819
X \text{ subset} = 2, i = 8, j = 3
Mean error from standard least squares: 0.48029966703662597
Mean error from LASSO: 0.48501664816870144
X \text{ subset} = 2, i = 8, j = 4
Mean error from standard least squares: 0.49250832408435075
Mean error from LASSO: 0.49250832408435075
X \text{ subset} = 2, i = 8, j = 5
Mean error from standard least squares: 0.4700332963374029
Mean error from LASSO: 0.4672586015538291
X \text{ subset} = 2, i = 8, j = 6
Mean error from standard least squares: 0.46587125416204217
Mean error from LASSO: 0.46226415094339623
X \text{ subset} = 2, i = 8, j = 7
Mean error from standard least squares: 0.4716981132075472
Mean error from LASSO: 0.46809100998890124
X \text{ subset} = 2, i = 8, j = 9
Mean error from standard least squares: 0.44311875693673697
Mean error from LASSO: 0.44173140954495005
X \text{ subset} = 2 , i = 8 , j = 10
Mean error from standard least squares: 0.43729189789123196
Mean error from LASSO: 0.4522752497225305
X \text{ subset} = 2, i = 9, j = 0
Mean error from standard least squares: 0.46032186459489455
Mean error from LASSO: 0.46365149833518315
X \text{ subset} = 2, i = 9, j = 1
Mean error from standard least squares: 0.4653163152053274
Mean error from LASSO: 0.4655937846836848
X \text{ subset} = 2, i = 9, j = 2
Mean error from standard least squares: 0.49472807991120976
Mean error from LASSO: 0.4980577136514983
X subset = 2 , i = 9 , j = 3
Mean error from standard least squares: 0.4894561598224195
Mean error from LASSO: 0.5055493895671476
X \text{ subset} = 2 \text{ , } i = 9 \text{ , } j = 4
```

```
Mean error from standard least squares: 0.4830743618201998
Mean error from LASSO: 0.48446170921198667
X \text{ subset} = 2 \text{ , } i = 9 \text{ , } j = 5
Mean error from standard least squares: 0.448945615982242
Mean error from LASSO: 0.4470033296337403
X subset = 2 , i = 9 , j = 6
Mean error from standard least squares: 0.48446170921198667
Mean error from LASSO: 0.4836293007769145
X \text{ subset} = 2, i = 9, j = 7
Mean error from standard least squares: 0.4811320754716981
Mean error from LASSO: 0.4811320754716981
X \text{ subset} = 2, i = 9, j = 8
Mean error from standard least squares: 0.4481132075471698
Mean error from LASSO: 0.4506104328523862
X \text{ subset} = 2 \text{ , i} = 9 \text{ , j} = 10
Mean error from standard least squares: 0.45394006659267483
Mean error from LASSO: 0.4583795782463929
X \text{ subset} = 2, i = 10, j = 0
Mean error from standard least squares: 0.48473917869034405
Mean error from LASSO: 0.4872364039955605
X subset = 2 , i = 10 , j = 1
Mean error from standard least squares: 0.47613762486126526
Mean error from LASSO: 0.468645948945616
X \text{ subset} = 2 , i = 10 , j = 2
Mean error from standard least squares: 0.4916759156492786
Mean error from LASSO: 0.5019422863485017
X \text{ subset} = 2 , i = 10 , j = 3
Mean error from standard least squares: 0.49306326304106546
Mean error from LASSO: 0.5008324084350722
X \text{ subset} = 2, i = 10, j = 4
Mean error from standard least squares: 0.49472807991120976
Mean error from LASSO: 0.5011098779134295
X \text{ subset} = 2, i = 10, j = 5
Mean error from standard least squares: 0.46032186459489455
Mean error from LASSO: 0.46004439511653716
X \text{ subset} = 2 \text{ , i} = 10 \text{ , j} = 6
Mean error from standard least squares: 0.4694783573806881
Mean error from LASSO: 0.4711431742508324
X \text{ subset} = 2 , i = 10 , j = 7
Mean error from standard least squares: 0.4772475027746948
Mean error from LASSO: 0.4775249722530522
X \text{ subset} = 2, i = 10, j = 8
Mean error from standard least squares: 0.4586570477247503
Mean error from LASSO: 0.4733629300776915
X subset = 2 , i = 10 , j = 9
Mean error from standard least squares: 0.4522752497225305
Mean error from LASSO: 0.45338512763596006
X \text{ subset} = 3, i = 0, j = 1
```

```
Mean error from standard least squares: 0.4442286348501665
Mean error from LASSO: 0.4442286348501665
X \text{ subset} = 3, i = 0, j = 2
Mean error from standard least squares: 0.4264705882352941
Mean error from LASSO: 0.42591564927857933
X subset = 3 , i = 0 , j = 3
Mean error from standard least squares: 0.462819089900111
Mean error from LASSO: 0.462819089900111
X \text{ subset} = 3, i = 0, j = 4
Mean error from standard least squares: 0.4655937846836848
Mean error from LASSO: 0.4653163152053274
X \text{ subset} = 3, i = 0, j = 5
Mean error from standard least squares: 0.4714206437291898
Mean error from LASSO: 0.4730854605993341
X subset = 3 , i = 0 , j = 6
Mean error from standard least squares: 0.4900110987791343
Mean error from LASSO: 0.48196448390677027
X \text{ subset} = 3, i = 0, j = 7
Mean error from standard least squares: 0.4900110987791343
Mean error from LASSO: 0.4897336293007769
X \text{ subset} = 3 , i = 0 , j = 8
Mean error from standard least squares: 0.49334073251942284
Mean error from LASSO: 0.4936182019977802
X \text{ subset} = 3, i = 0, j = 9
Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.4961154273029967
X \text{ subset} = 3 , i = 0 , j = 10
Mean error from standard least squares: 0.4830743618201998
Mean error from LASSO: 0.4841842397336293
X \text{ subset} = 3 , i = 1 , j = 0
Mean error from standard least squares: 0.4264705882352941
Mean error from LASSO: 0.42286348501664817
X \text{ subset} = 3, i = 1, j = 2
Mean error from standard least squares: 0.42591564927857933
Mean error from LASSO: 0.42147613762486125
X \text{ subset} = 3, i = 1, j = 3
Mean error from standard least squares: 0.42591564927857933
Mean error from LASSO: 0.42452830188679247
X \text{ subset} = 3, i = 1, j = 4
Mean error from standard least squares: 0.46198668146503885
Mean error from LASSO: 0.44977802441731407
X \text{ subset} = 3, i = 1, j = 5
Mean error from standard least squares: 0.4525527192008879
Mean error from LASSO: 0.4525527192008879
X \text{ subset} = 3 , i = 1 , j = 6
Mean error from standard least squares: 0.4775249722530522
Mean error from LASSO: 0.4753052164261931
X \text{ subset} = 3 , i = 1 , j = 7
```

```
Mean error from standard least squares: 0.4877913429522753
Mean error from LASSO: 0.4714206437291898
X \text{ subset} = 3, i = 1, j = 8
Mean error from standard least squares: 0.48501664816870144
Mean error from LASSO: 0.47058823529411764
X \text{ subset} = 3 , i = 1 , j = 9
Mean error from standard least squares: 0.4730854605993341
Mean error from LASSO: 0.4553274139844617
X \text{ subset} = 3 , i = 1 , j = 10
Mean error from standard least squares: 0.462819089900111
Mean error from LASSO: 0.45394006659267483
X \text{ subset} = 3, i = 2, j = 0
Mean error from standard least squares: 0.49278579356270813
Mean error from LASSO: 0.49250832408435075
X subset = 3 , i = 2 , j = 1
Mean error from standard least squares: 0.4880688124306326
Mean error from LASSO: 0.4880688124306326
X \text{ subset} = 3, i = 2, j = 3
Mean error from standard least squares: 0.5152608213096559
Mean error from LASSO: 0.5158157602663707
X \text{ subset} = 3, i = 2, j = 4
Mean error from standard least squares: 0.4911209766925638
Mean error from LASSO: 0.4911209766925638
X \text{ subset} = 3, i = 2, j = 5
Mean error from standard least squares: 0.5344062153163152
Mean error from LASSO: 0.5377358490566038
X \text{ subset} = 3, i = 2, j = 6
Mean error from standard least squares: 0.5427302996670367
Mean error from LASSO: 0.5418978912319645
X \text{ subset} = 3, i = 2, j = 7
Mean error from standard least squares: 0.5513318534961155
Mean error from LASSO: 0.5513318534961155
X \text{ subset} = 3, i = 2, j = 8
Mean error from standard least squares: 0.5183129855715871
Mean error from LASSO: 0.5177580466148723
X \text{ subset} = 3, i = 2, j = 9
Mean error from standard least squares: 0.548834628190899
Mean error from LASSO: 0.5499445061043285
X \text{ subset} = 3 , i = 2 , j = 10
Mean error from standard least squares: 0.5394006659267481
Mean error from LASSO: 0.5388457269700333
X \text{ subset} = 3, i = 3, j = 0
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.4975027746947836
X subset = 3 , i = 3 , j = 1
Mean error from standard least squares: 0.48501664816870144
Mean error from LASSO: 0.48501664816870144
X \text{ subset} = 3 , i = 3 , j = 2
```

```
Mean error from standard least squares: 0.48029966703662597
Mean error from LASSO: 0.4841842397336293
X \text{ subset} = 3, i = 3, j = 4
Mean error from standard least squares: 0.5266370699223085
Mean error from LASSO: 0.5283018867924528
X \text{ subset} = 3 , i = 3 , j = 5
Mean error from standard least squares: 0.5241398446170921
Mean error from LASSO: 0.5241398446170921
X \text{ subset} = 3, i = 3, j = 6
Mean error from standard least squares: 0.5416204217536071
Mean error from LASSO: 0.5421753607103219
X \text{ subset} = 3, i = 3, j = 7
Mean error from standard least squares: 0.5441176470588235
Mean error from LASSO: 0.553274139844617
X \text{ subset} = 3, i = 3, j = 8
Mean error from standard least squares: 0.5449500554938956
Mean error from LASSO: 0.5471698113207547
X \text{ subset} = 3, i = 3, j = 9
Mean error from standard least squares: 0.5554938956714761
Mean error from LASSO: 0.5577136514983352
X \text{ subset} = 3, i = 3, j = 10
Mean error from standard least squares: 0.5377358490566038
Mean error from LASSO: 0.5374583795782464
X \text{ subset} = 3, i = 4, j = 0
Mean error from standard least squares: 0.46420643729189787
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 3, i = 4, j = 1
Mean error from standard least squares: 0.44950055493895674
Mean error from LASSO: 0.44922308546059936
X \text{ subset} = 3, i = 4, j = 2
Mean error from standard least squares: 0.468645948945616
Mean error from LASSO: 0.4683684794672586
X \text{ subset} = 3, i = 4, j = 3
Mean error from standard least squares: 0.4753052164261931
Mean error from LASSO: 0.4797447280799112
X \text{ subset} = 3, i = 4, j = 5
Mean error from standard least squares: 0.5055493895671476
Mean error from LASSO: 0.5055493895671476
X \text{ subset} = 3, i = 4, j = 6
Mean error from standard least squares: 0.5033296337402886
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 3, i = 4, j = 7
Mean error from standard least squares: 0.5086015538290788
Mean error from LASSO: 0.5086015538290788
X \text{ subset} = 3 , i = 4 , j = 8
Mean error from standard least squares: 0.519700332963374
Mean error from LASSO: 0.5188679245283019
X \text{ subset} = 3 , i = 4 , j = 9
```

```
Mean error from standard least squares: 0.5141509433962265
Mean error from LASSO: 0.5138734739178691
X \text{ subset} = 3 , i = 4 , j = 10
Mean error from standard least squares: 0.5152608213096559
Mean error from LASSO: 0.5185904550499445
X subset = 3 , i = 5 , j = 0
Mean error from standard least squares: 0.43590455049944504
Mean error from LASSO: 0.4353496115427303
X \text{ subset} = 3, i = 5, j = 1
Mean error from standard least squares: 0.44783573806881244
Mean error from LASSO: 0.4483906770255272
X \text{ subset} = 3, i = 5, j = 2
Mean error from standard least squares: 0.44755826859045506
Mean error from LASSO: 0.44728079911209767
X \text{ subset} = 3, i = 5, j = 3
Mean error from standard least squares: 0.46032186459489455
Mean error from LASSO: 0.46059933407325193
X \text{ subset} = 3, i = 5, j = 4
Mean error from standard least squares: 0.4630965593784684
Mean error from LASSO: 0.46420643729189787
X \text{ subset} = 3, i = 5, j = 6
Mean error from standard least squares: 0.4855715871254162
Mean error from LASSO: 0.48446170921198667
X \text{ subset} = 3, i = 5, j = 7
Mean error from standard least squares: 0.49084350721420644
Mean error from LASSO: 0.4877913429522753
X \text{ subset} = 3, i = 5, j = 8
Mean error from standard least squares: 0.4938956714761376
Mean error from LASSO: 0.49334073251942284
X \text{ subset} = 3, i = 5, j = 9
Mean error from standard least squares: 0.5102663706992231
Mean error from LASSO: 0.5102663706992231
X \text{ subset} = 3, i = 5, j = 10
Mean error from standard least squares: 0.4972253052164262
Mean error from LASSO: 0.49694783573806883
X \text{ subset} = 3 , i = 6 , j = 0
Mean error from standard least squares: 0.4547724750277469
Mean error from LASSO: 0.4553274139844617
X \text{ subset} = 3, i = 6, j = 1
Mean error from standard least squares: 0.4550499445061043
Mean error from LASSO: 0.4550499445061043
X \text{ subset} = 3, i = 6, j = 2
Mean error from standard least squares: 0.43756936736958935
Mean error from LASSO: 0.43729189789123196
X subset = 3 , i = 6 , j = 3
Mean error from standard least squares: 0.45394006659267483
Mean error from LASSO: 0.45394006659267483
X \text{ subset} = 3, i = 6, j = 4
```

```
Mean error from standard least squares: 0.47058823529411764
Mean error from LASSO: 0.470865704772475
X \text{ subset} = 3 , i = 6 , j = 5
Mean error from standard least squares: 0.4667036625971143
Mean error from LASSO: 0.46642619311875694
X subset = 3 , i = 6 , j = 7
Mean error from standard least squares: 0.5016648168701443
Mean error from LASSO: 0.5013873473917869
X \text{ subset} = 3, i = 6, j = 8
Mean error from standard least squares: 0.5047169811320755
Mean error from LASSO: 0.5044395116537181
X \text{ subset} = 3, i = 6, j = 9
Mean error from standard least squares: 0.5013873473917869
Mean error from LASSO: 0.5011098779134295
X \text{ subset} = 3 , i = 6 , j = 10
Mean error from standard least squares: 0.4897336293007769
Mean error from LASSO: 0.49223085460599336
X \text{ subset} = 3, i = 7, j = 0
Mean error from standard least squares: 0.451165371809101
Mean error from LASSO: 0.45172031076581576
X \text{ subset} = 3, i = 7, j = 1
Mean error from standard least squares: 0.4630965593784684
Mean error from LASSO: 0.4630965593784684
X \text{ subset} = 3, i = 7, j = 2
Mean error from standard least squares: 0.4542175360710322
Mean error from LASSO: 0.4542175360710322
X \text{ subset} = 3, i = 7, j = 3
Mean error from standard least squares: 0.48057713651498335
Mean error from LASSO: 0.48057713651498335
X \text{ subset} = 3, i = 7, j = 4
Mean error from standard least squares: 0.48057713651498335
Mean error from LASSO: 0.48057713651498335
X \text{ subset} = 3, i = 7, j = 5
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.49778024417314093
X subset = 3 , i = 7 , j = 6
Mean error from standard least squares: 0.5011098779134295
Mean error from LASSO: 0.5011098779134295
X \text{ subset} = 3, i = 7, j = 8
Mean error from standard least squares: 0.4986126526082131
Mean error from LASSO: 0.4986126526082131
X \text{ subset} = 3, i = 7, j = 9
Mean error from standard least squares: 0.5141509433962265
Mean error from LASSO: 0.5141509433962265
X subset = 3 , i = 7 , j = 10
Mean error from standard least squares: 0.5260821309655938
Mean error from LASSO: 0.5271920088790233
X \text{ subset} = 3 , i = 8 , j = 0
```

```
Mean error from standard least squares: 0.4714206437291898
Mean error from LASSO: 0.4714206437291898
X \text{ subset} = 3 , i = 8 , j = 1
Mean error from standard least squares: 0.462819089900111
Mean error from LASSO: 0.462819089900111
X subset = 3 , i = 8 , j = 2
Mean error from standard least squares: 0.4816870144284129
Mean error from LASSO: 0.4816870144284129
X \text{ subset} = 3, i = 8, j = 3
Mean error from standard least squares: 0.47891231964483905
Mean error from LASSO: 0.47891231964483905
X \text{ subset} = 3, i = 8, j = 4
Mean error from standard least squares: 0.47669256381798003
Mean error from LASSO: 0.47669256381798003
X \text{ subset} = 3 , i = 8 , j = 5
Mean error from standard least squares: 0.49944506104328523
Mean error from LASSO: 0.5002774694783574
X \text{ subset} = 3, i = 8, j = 6
Mean error from standard least squares: 0.5102663706992231
Mean error from LASSO: 0.5177580466148723
X \text{ subset} = 3, i = 8, j = 7
Mean error from standard least squares: 0.5158157602663707
Mean error from LASSO: 0.516093229744728
X \text{ subset} = 3, i = 8, j = 9
Mean error from standard least squares: 0.5133185349611543
Mean error from LASSO: 0.5124861265260822
X \text{ subset} = 3 , i = 8 , j = 10
Mean error from standard least squares: 0.5258046614872364
Mean error from LASSO: 0.5271920088790233
X \text{ subset} = 3, i = 9, j = 0
Mean error from standard least squares: 0.48251942286348504
Mean error from LASSO: 0.4827968923418424
X \text{ subset} = 3, i = 9, j = 1
Mean error from standard least squares: 0.49334073251942284
Mean error from LASSO: 0.49334073251942284
X \text{ subset} = 3, i = 9, j = 2
Mean error from standard least squares: 0.4955604883462819
Mean error from LASSO: 0.4958379578246393
X \text{ subset} = 3, i = 9, j = 3
Mean error from standard least squares: 0.5208102108768036
Mean error from LASSO: 0.521087680355161
X \text{ subset} = 3, i = 9, j = 4
Mean error from standard least squares: 0.5280244173140954
Mean error from LASSO: 0.5277469478357381
X \text{ subset} = 3 , i = 9 , j = 5
Mean error from standard least squares: 0.5341287458379578
Mean error from LASSO: 0.5341287458379578
X \text{ subset} = 3 , i = 9 , j = 6
```

```
Mean error from standard least squares: 0.5410654827968924
Mean error from LASSO: 0.5413429522752498
X \text{ subset} = 3, i = 9, j = 7
Mean error from standard least squares: 0.554661487236404
Mean error from LASSO: 0.5554938956714761
X subset = 3 , i = 9 , j = 8
Mean error from standard least squares: 0.5571587125416204
Mean error from LASSO: 0.5574361820199778
X \text{ subset} = 3 , i = 9 , j = 10
Mean error from standard least squares: 0.5477247502774695
Mean error from LASSO: 0.5521642619311876
X \text{ subset} = 3, i = 10, j = 0
Mean error from standard least squares: 0.503607103218646
Mean error from LASSO: 0.503607103218646
X subset = 3 , i = 10 , j = 1
Mean error from standard least squares: 0.4938956714761376
Mean error from LASSO: 0.4938956714761376
X \text{ subset} = 3, i = 10, j = 2
Mean error from standard least squares: 0.49306326304106546
Mean error from LASSO: 0.49278579356270813
X \text{ subset} = 3 , i = 10 , j = 3
Mean error from standard least squares: 0.5138734739178691
Mean error from LASSO: 0.5149833518312985
X \text{ subset} = 3 , i = 10 , j = 4
Mean error from standard least squares: 0.5244173140954494
Mean error from LASSO: 0.5244173140954494
X \text{ subset} = 3 , i = 10 , j = 5
Mean error from standard least squares: 0.5363485016648168
Mean error from LASSO: 0.5385682574916759
X \text{ subset} = 3 , i = 10 , j = 6
Mean error from standard least squares: 0.5610432852386238
Mean error from LASSO: 0.5593784683684795
X \text{ subset} = 3, i = 10, j = 7
Mean error from standard least squares: 0.5701997780244173
Mean error from LASSO: 0.5701997780244173
X \text{ subset} = 3 , i = 10 , j = 8
Mean error from standard least squares: 0.5571587125416204
Mean error from LASSO: 0.556881243063263
X \text{ subset} = 3 , i = 10 , j = 9
Mean error from standard least squares: 0.5510543840177581
Mean error from LASSO: 0.5507769145394007
X \text{ subset} = 4, i = 0, j = 1
Mean error from standard least squares: 0.5033296337402886
Mean error from LASSO: 0.5038845726970034
X subset = 4 , i = 0 , j = 2
Mean error from standard least squares: 0.5044395116537181
Mean error from LASSO: 0.5041620421753608
X \text{ subset} = 4, i = 0, j = 3
```

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Mean error from standard least squares: 0.5052719200887902
Mean error from LASSO: 0.5049944506104328
X \text{ subset} = 4, i = 0, j = 4
Mean error from standard least squares: 0.5019422863485017
Mean error from LASSO: 0.5044395116537181
X subset = 4 , i = 0 , j = 5
Mean error from standard least squares: 0.5016648168701443
Mean error from LASSO: 0.5022197558268591
X \text{ subset} = 4, i = 0, j = 6
Mean error from standard least squares: 0.5072142064372919
Mean error from LASSO: 0.5072142064372919
X \text{ subset} = 4, i = 0, j = 7
Mean error from standard least squares: 0.5061043285238623
Mean error from LASSO: 0.5047169811320755
X \text{ subset} = 4 \text{ , i} = 0 \text{ , j} = 8
Mean error from standard least squares: 0.5072142064372919
Mean error from LASSO: 0.5030521642619312
X \text{ subset} = 4, i = 0, j = 9
Mean error from standard least squares: 0.5091564927857936
Mean error from LASSO: 0.5047169811320755
X \text{ subset} = 4 , i = 0 , j = 10
Mean error from standard least squares: 0.5024972253052165
Mean error from LASSO: 0.49944506104328523
X \text{ subset} = 4 , i = 1 , j = 0
Mean error from standard least squares: 0.5430077691453941
Mean error from LASSO: 0.5443951165371809
X \text{ subset} = 4 , i = 1 , j = 2
Mean error from standard least squares: 0.5430077691453941
Mean error from LASSO: 0.5460599334073252
X \text{ subset} = 4, i = 1, j = 3
Mean error from standard least squares: 0.5338512763596004
Mean error from LASSO: 0.5480022197558269
X \text{ subset} = 4, i = 1, j = 4
Mean error from standard least squares: 0.5421753607103219
Mean error from LASSO: 0.5463374028856826
X \text{ subset} = 4 , i = 1 , j = 5
Mean error from standard least squares: 0.5480022197558269
Mean error from LASSO: 0.5474472807991121
X \text{ subset} = 4 , i = 1 , j = 6
Mean error from standard least squares: 0.5421753607103219
Mean error from LASSO: 0.5418978912319645
X \text{ subset} = 4, i = 1, j = 7
Mean error from standard least squares: 0.5443951165371809
Mean error from LASSO: 0.5449500554938956
X \text{ subset} = 4 , i = 1 , j = 8
Mean error from standard least squares: 0.5360710321864595
Mean error from LASSO: 0.5332963374028857
X \text{ subset} = 4 , i = 1 , j = 9
```

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Mean error from standard least squares: 0.5441176470588235
Mean error from LASSO: 0.5455049944506104
X \text{ subset} = 4 , i = 1 , j = 10
Mean error from standard least squares: 0.5471698113207547
Mean error from LASSO: 0.5496670366259712
X subset = 4 , i = 2 , j = 0
Mean error from standard least squares: 0.43174250832408434
Mean error from LASSO: 0.40316315205327413
X \text{ subset} = 4, i = 2, j = 1
Mean error from standard least squares: 0.4239733629300777
Mean error from LASSO: 0.4051054384017758
X \text{ subset} = 4, i = 2, j = 3
Mean error from standard least squares: 0.45782463928967815
Mean error from LASSO: 0.4056603773584906
X \text{ subset} = 4, i = 2, j = 4
Mean error from standard least squares: 0.4553274139844617
Mean error from LASSO: 0.4034406215316315
X \text{ subset} = 4, i = 2, j = 5
Mean error from standard least squares: 0.42730299667036625
Mean error from LASSO: 0.40843507214206437
X \text{ subset} = 4, i = 2, j = 6
Mean error from standard least squares: 0.41897891231964485
Mean error from LASSO: 0.4056603773584906
X \text{ subset} = 4, i = 2, j = 7
Mean error from standard least squares: 0.4225860155382908
Mean error from LASSO: 0.40593784683684797
X \text{ subset} = 4 , i = 2 , j = 8
Mean error from standard least squares: 0.4370144284128746
Mean error from LASSO: 0.4153718091009989
X \text{ subset} = 4 , i = 2 , j = 9
Mean error from standard least squares: 0.43923418423973365
Mean error from LASSO: 0.42147613762486125
X \text{ subset} = 4, i = 2, j = 10
Mean error from standard least squares: 0.45033296337402884
Mean error from LASSO: 0.4306326304106548
X subset = 4 , i = 3 , j = 0
Mean error from standard least squares: 0.4467258601553829
Mean error from LASSO: 0.4167591564927858
X \text{ subset} = 4, i = 3, j = 1
Mean error from standard least squares: 0.4242508324084351
Mean error from LASSO: 0.4109322974472808
X \text{ subset} = 4, i = 3, j = 2
Mean error from standard least squares: 0.45810210876803553
Mean error from LASSO: 0.41703662597114316
X subset = 4 , i = 3 , j = 4
Mean error from standard least squares: 0.44783573806881244
Mean error from LASSO: 0.413984461709212
X \text{ subset} = 4 , i = 3 , j = 5
```

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Mean error from standard least squares: 0.44006659267480575
Mean error from LASSO: 0.41703662597114316
X \text{ subset} = 4 , i = 3 , j = 6
Mean error from standard least squares: 0.42758046614872364
Mean error from LASSO: 0.4159267480577136
X \text{ subset} = 4 , i = 3 , j = 7
Mean error from standard least squares: 0.43784683684794673
Mean error from LASSO: 0.4167591564927858
X \text{ subset} = 4, i = 3, j = 8
Mean error from standard least squares: 0.4397891231964484
Mean error from LASSO: 0.42563817980022195
X \text{ subset} = 4, i = 3, j = 9
Mean error from standard least squares: 0.44755826859045506
Mean error from LASSO: 0.422031076581576
X \text{ subset} = 4 , i = 3 , j = 10
Mean error from standard least squares: 0.45199778024417314
Mean error from LASSO: 0.4370144284128746
X \text{ subset} = 4, i = 4, j = 0
Mean error from standard least squares: 0.47447280799112096
Mean error from LASSO: 0.46420643729189787
X \text{ subset} = 4, i = 4, j = 1
Mean error from standard least squares: 0.4694783573806881
Mean error from LASSO: 0.4655937846836848
X subset = 4, i = 4, j = 2
Mean error from standard least squares: 0.46614872364039955
Mean error from LASSO: 0.46476137624861263
X \text{ subset} = 4 , i = 4 , j = 3
Mean error from standard least squares: 0.47891231964483905
Mean error from LASSO: 0.4655937846836848
X \text{ subset} = 4 , i = 4 , j = 5
Mean error from standard least squares: 0.4736403995560488
Mean error from LASSO: 0.46614872364039955
X \text{ subset} = 4, i = 4, j = 6
Mean error from standard least squares: 0.4675360710321865
Mean error from LASSO: 0.46781354051054386
X \text{ subset} = 4, i = 4, j = 7
Mean error from standard least squares: 0.46365149833518315
Mean error from LASSO: 0.46420643729189787
X \text{ subset} = 4 , i = 4 , j = 8
Mean error from standard least squares: 0.46032186459489455
Mean error from LASSO: 0.4608768035516093
X \text{ subset} = 4, i = 4, j = 9
Mean error from standard least squares: 0.4869589345172031
Mean error from LASSO: 0.4736403995560488
X subset = 4 , i = 4 , j = 10
Mean error from standard least squares: 0.47863485016648166
Mean error from LASSO: 0.4733629300776915
X \text{ subset} = 4, i = 5, j = 0
```

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Mean error from standard least squares: 0.5033296337402886
Mean error from LASSO: 0.505826859045505
X \text{ subset} = 4, i = 5, j = 1
Mean error from standard least squares: 0.5038845726970034
Mean error from LASSO: 0.5049944506104328
X subset = 4 , i = 5 , j = 2
Mean error from standard least squares: 0.5013873473917869
Mean error from LASSO: 0.5024972253052165
X \text{ subset} = 4, i = 5, j = 3
Mean error from standard least squares: 0.4972253052164262
Mean error from LASSO: 0.5072142064372919
X \text{ subset} = 4, i = 5, j = 4
Mean error from standard least squares: 0.5113762486126526
Mean error from LASSO: 0.5066592674805771
X subset = 4 , i = 5 , j = 6
Mean error from standard least squares: 0.5074916759156493
Mean error from LASSO: 0.5072142064372919
X \text{ subset} = 4, i = 5, j = 7
Mean error from standard least squares: 0.505826859045505
Mean error from LASSO: 0.503607103218646
X \text{ subset} = 4 , i = 5 , j = 8
Mean error from standard least squares: 0.503607103218646
Mean error from LASSO: 0.5063817980022197
X \text{ subset} = 4, i = 5, j = 9
Mean error from standard least squares: 0.5027746947835738
Mean error from LASSO: 0.5019422863485017
X \text{ subset} = 4 , i = 5 , j = 10
Mean error from standard least squares: 0.5094339622641509
Mean error from LASSO: 0.5055493895671476
X \text{ subset} = 4, i = 6, j = 0
Mean error from standard least squares: 0.49889012208657046
Mean error from LASSO: 0.4980577136514983
X \text{ subset} = 4, i = 6, j = 1
Mean error from standard least squares: 0.49889012208657046
Mean error from LASSO: 0.5
X \text{ subset} = 4, i = 6, j = 2
Mean error from standard least squares: 0.5030521642619312
Mean error from LASSO: 0.5030521642619312
X \text{ subset} = 4 , i = 6 , j = 3
Mean error from standard least squares: 0.49889012208657046
Mean error from LASSO: 0.4972253052164262
X \text{ subset} = 4, i = 6, j = 4
Mean error from standard least squares: 0.5069367369589345
Mean error from LASSO: 0.5005549389567148
X \text{ subset} = 4 , i = 6 , j = 5
Mean error from standard least squares: 0.5002774694783574
Mean error from LASSO: 0.5011098779134295
X \text{ subset} = 4, i = 6, j = 7
```

```
Mean error from standard least squares: 0.503607103218646
Mean error from LASSO: 0.5024972253052165
X \text{ subset} = 4 , i = 6 , j = 8
Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.49472807991120976
X \text{ subset} = 4 , i = 6 , j = 9
Mean error from standard least squares: 0.5044395116537181
Mean error from LASSO: 0.5052719200887902
X \text{ subset} = 4 , i = 6 , j = 10
Mean error from standard least squares: 0.4880688124306326
Mean error from LASSO: 0.4900110987791343
X \text{ subset} = 4, i = 7, j = 0
Mean error from standard least squares: 0.46642619311875694
Mean error from LASSO: 0.46420643729189787
X \text{ subset} = 4, i = 7, j = 1
Mean error from standard least squares: 0.46892341842397334
Mean error from LASSO: 0.4683684794672586
X \text{ subset} = 4, i = 7, j = 2
Mean error from standard least squares: 0.47863485016648166
Mean error from LASSO: 0.46365149833518315
X \text{ subset} = 4 , i = 7 , j = 3
Mean error from standard least squares: 0.4653163152053274
Mean error from LASSO: 0.462819089900111
X \text{ subset} = 4, i = 7, j = 4
Mean error from standard least squares: 0.4683684794672586
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 4, i = 7, j = 5
Mean error from standard least squares: 0.46365149833518315
Mean error from LASSO: 0.4655937846836848
X \text{ subset} = 4, i = 7, j = 6
Mean error from standard least squares: 0.4669811320754717
Mean error from LASSO: 0.4672586015538291
X \text{ subset} = 4, i = 7, j = 8
Mean error from standard least squares: 0.4675360710321865
Mean error from LASSO: 0.4692008879023307
X subset = 4 , i = 7 , j = 9
Mean error from standard least squares: 0.470865704772475
Mean error from LASSO: 0.4730854605993341
X \text{ subset} = 4 , i = 7 , j = 10
Mean error from standard least squares: 0.4589345172031077
Mean error from LASSO: 0.4589345172031077
X \text{ subset} = 4, i = 8, j = 0
Mean error from standard least squares: 0.45394006659267483
Mean error from LASSO: 0.4528301886792453
X \text{ subset} = 4 , i = 8 , j = 1
Mean error from standard least squares: 0.4328523862375139
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 4 , i = 10 , j = 5
```

```
Mean error from standard least squares: 0.42036625971143177
Mean error from LASSO: 0.41037735849056606
X \text{ subset} = 4 , i = 10 , j = 6
Mean error from standard least squares: 0.4092674805771365
Mean error from LASSO: 0.408157602663707
X subset = 4 , i = 10 , j = 7
Mean error from standard least squares: 0.40871254162042175
Mean error from LASSO: 0.40732519422863483
X \text{ subset} = 4 , i = 10 , j = 8
Mean error from standard least squares: 0.4225860155382908
Mean error from LASSO: 0.41453940066592676
X \text{ subset} = 4, i = 10, j = 9
Mean error from standard least squares: 0.44145394006659266
Mean error from LASSO: 0.42286348501664817
X \text{ subset} = 5, i = 0, j = 1
Mean error from standard least squares: 0.42563817980022195
Mean error from LASSO: 0.42563817980022195
X \text{ subset} = 5, i = 0, j = 2
Mean error from standard least squares: 0.413984461709212
Mean error from LASSO: 0.413984461709212
X \text{ subset} = 5, i = 0, j = 3
Mean error from standard least squares: 0.41897891231964485
Mean error from LASSO: 0.41897891231964485
X \text{ subset} = 5, i = 0, j = 4
Mean error from standard least squares: 0.3995560488346282
Mean error from LASSO: 0.3995560488346282
X \text{ subset} = 5, i = 0, j = 5
Mean error from standard least squares: 0.4181465038845727
Mean error from LASSO: 0.4181465038845727
X \text{ subset} = 5, i = 0, j = 6
Mean error from standard least squares: 0.44006659267480575
Mean error from LASSO: 0.44006659267480575
X \text{ subset} = 5, i = 0, j = 7
Mean error from standard least squares: 0.4309100998890122
Mean error from LASSO: 0.4309100998890122
X \text{ subset} = 5, i = 0, j = 8
Mean error from standard least squares: 0.4439511653718091
Mean error from LASSO: 0.4439511653718091
X \text{ subset} = 5, i = 0, j = 9
Mean error from standard least squares: 0.44034406215316313
Mean error from LASSO: 0.44034406215316313
X \text{ subset} = 5, i = 0, j = 10
Mean error from standard least squares: 0.41731409544950054
Mean error from LASSO: 0.41731409544950054
X subset = 5 , i = 1 , j = 0
Mean error from standard least squares: 0.419811320754717
Mean error from LASSO: 0.419811320754717
X \text{ subset} = 5, i = 1, j = 2
```

```
Mean error from standard least squares: 0.427857935627081
Mean error from LASSO: 0.427857935627081
X \text{ subset} = 5, i = 1, j = 3
Mean error from standard least squares: 0.4281354051054384
Mean error from LASSO: 0.4281354051054384
X subset = 5 , i = 1 , j = 4
Mean error from standard least squares: 0.416204217536071
Mean error from LASSO: 0.416204217536071
X \text{ subset} = 5, i = 1, j = 5
Mean error from standard least squares: 0.44922308546059936
Mean error from LASSO: 0.44922308546059936
X \text{ subset} = 5, i = 1, j = 6
Mean error from standard least squares: 0.46503884572697
Mean error from LASSO: 0.46503884572697
X \text{ subset} = 5, i = 1, j = 7
Mean error from standard least squares: 0.43590455049944504
Mean error from LASSO: 0.43590455049944504
X \text{ subset} = 5, i = 1, j = 8
Mean error from standard least squares: 0.44339622641509435
Mean error from LASSO: 0.44339622641509435
X \text{ subset} = 5, i = 1, j = 9
Mean error from standard least squares: 0.44755826859045506
Mean error from LASSO: 0.44755826859045506
X \text{ subset} = 5, i = 1, j = 10
Mean error from standard least squares: 0.4320199778024417
Mean error from LASSO: 0.4320199778024417
X \text{ subset} = 5, i = 2, j = 0
Mean error from standard least squares: 0.41759156492785793
Mean error from LASSO: 0.41759156492785793
X \text{ subset} = 5, i = 2, j = 1
Mean error from standard least squares: 0.4034406215316315
Mean error from LASSO: 0.4034406215316315
X \text{ subset} = 5, i = 2, j = 3
Mean error from standard least squares: 0.39705882352941174
Mean error from LASSO: 0.39705882352941174
X \text{ subset} = 5, i = 2, j = 4
Mean error from standard least squares: 0.4076026637069922
Mean error from LASSO: 0.4076026637069922
X \text{ subset} = 5, i = 2, j = 5
Mean error from standard least squares: 0.40288568257491675
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 5, i = 2, j = 6
Mean error from standard least squares: 0.3978912319644839
Mean error from LASSO: 0.3978912319644839
X subset = 5 , i = 2 , j = 7
Mean error from standard least squares: 0.4092674805771365
Mean error from LASSO: 0.4092674805771365
X \text{ subset} = 5 , i = 2 , j = 8
```

```
Mean error from standard least squares: 0.4039955604883463
Mean error from LASSO: 0.4039955604883463
X \text{ subset} = 5, i = 2, j = 9
Mean error from standard least squares: 0.3806881243063263
Mean error from LASSO: 0.3806881243063263
X subset = 5 , i = 2 , j = 10
Mean error from standard least squares: 0.376803551609323
Mean error from LASSO: 0.376803551609323
X \text{ subset} = 5, i = 3, j = 0
Mean error from standard least squares: 0.40677025527192007
Mean error from LASSO: 0.40677025527192007
X \text{ subset} = 5, i = 3, j = 1
Mean error from standard least squares: 0.41481687014428414
Mean error from LASSO: 0.41481687014428414
X \text{ subset} = 5, i = 3, j = 2
Mean error from standard least squares: 0.4095449500554939
Mean error from LASSO: 0.4095449500554939
X \text{ subset} = 5, i = 3, j = 4
Mean error from standard least squares: 0.39900110987791343
Mean error from LASSO: 0.39900110987791343
X \text{ subset} = 5, i = 3, j = 5
Mean error from standard least squares: 0.40288568257491675
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 5, i = 3, j = 6
Mean error from standard least squares: 0.41759156492785793
Mean error from LASSO: 0.41759156492785793
X \text{ subset} = 5, i = 3, j = 7
Mean error from standard least squares: 0.4120421753607103
Mean error from LASSO: 0.4120421753607103
X \text{ subset} = 5, i = 3, j = 8
Mean error from standard least squares: 0.3953940066592675
Mean error from LASSO: 0.3953940066592675
X \text{ subset} = 5, i = 3, j = 9
Mean error from standard least squares: 0.379023307436182
Mean error from LASSO: 0.379023307436182
X \text{ subset} = 5, i = 3, j = 10
Mean error from standard least squares: 0.3776359600443951
Mean error from LASSO: 0.3776359600443951
X \text{ subset} = 5, i = 4, j = 0
Mean error from standard least squares: 0.41703662597114316
Mean error from LASSO: 0.41703662597114316
X \text{ subset} = 5, i = 4, j = 1
Mean error from standard least squares: 0.4298002219755827
Mean error from LASSO: 0.4298002219755827
X \text{ subset} = 5, i = 4, j = 2
Mean error from standard least squares: 0.42452830188679247
Mean error from LASSO: 0.42452830188679247
X \text{ subset} = 5, i = 4, j = 3
```

```
Mean error from standard least squares: 0.4225860155382908
Mean error from LASSO: 0.4225860155382908
X \text{ subset} = 5, i = 4, j = 5
Mean error from standard least squares: 0.42286348501664817
Mean error from LASSO: 0.42286348501664817
X \text{ subset} = 5, i = 4, j = 6
Mean error from standard least squares: 0.43479467258601556
Mean error from LASSO: 0.43479467258601556
X \text{ subset} = 5, i = 4, j = 7
Mean error from standard least squares: 0.4100998890122087
Mean error from LASSO: 0.4100998890122087
X \text{ subset} = 5, i = 4, j = 8
Mean error from standard least squares: 0.4184239733629301
Mean error from LASSO: 0.4184239733629301
X \text{ subset} = 5, i = 4, j = 9
Mean error from standard least squares: 0.41065482796892344
Mean error from LASSO: 0.41065482796892344
X \text{ subset} = 5, i = 4, j = 10
Mean error from standard least squares: 0.4142619311875694
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 5, i = 5, j = 0
Mean error from standard least squares: 0.40149833518312983
Mean error from LASSO: 0.40149833518312983
X \text{ subset} = 5, i = 5, j = 1
Mean error from standard least squares: 0.4200887902330744
Mean error from LASSO: 0.4200887902330744
X \text{ subset} = 5, i = 5, j = 2
Mean error from standard least squares: 0.4120421753607103
Mean error from LASSO: 0.4120421753607103
X \text{ subset} = 5, i = 5, j = 3
Mean error from standard least squares: 0.4178690344062153
Mean error from LASSO: 0.4178690344062153
X \text{ subset} = 5, i = 5, j = 4
Mean error from standard least squares: 0.4153718091009989
Mean error from LASSO: 0.4153718091009989
X \text{ subset} = 5, i = 5, j = 6
Mean error from standard least squares: 0.4284128745837958
Mean error from LASSO: 0.4284128745837958
X \text{ subset} = 5, i = 5, j = 7
Mean error from standard least squares: 0.44145394006659266
Mean error from LASSO: 0.44145394006659266
X \text{ subset} = 5, i = 5, j = 8
Mean error from standard least squares: 0.4467258601553829
Mean error from LASSO: 0.4467258601553829
X \text{ subset} = 5, i = 5, j = 9
Mean error from standard least squares: 0.4156492785793563
Mean error from LASSO: 0.4156492785793563
X \text{ subset} = 5 , i = 5 , j = 10
```

```
Mean error from standard least squares: 0.41453940066592676
Mean error from LASSO: 0.41453940066592676
X \text{ subset} = 5, i = 6, j = 0
Mean error from standard least squares: 0.40677025527192007
Mean error from LASSO: 0.40677025527192007
X subset = 5 , i = 6 , j = 1
Mean error from standard least squares: 0.427857935627081
Mean error from LASSO: 0.427857935627081
X \text{ subset} = 5, i = 6, j = 2
Mean error from standard least squares: 0.4134295227524972
Mean error from LASSO: 0.4134295227524972
X \text{ subset} = 5, i = 6, j = 3
Mean error from standard least squares: 0.4264705882352941
Mean error from LASSO: 0.4264705882352941
X \text{ subset} = 5, i = 6, j = 4
Mean error from standard least squares: 0.4209211986681465
Mean error from LASSO: 0.4209211986681465
X \text{ subset} = 5, i = 6, j = 5
Mean error from standard least squares: 0.41731409544950054
Mean error from LASSO: 0.41731409544950054
X \text{ subset} = 5, i = 6, j = 7
Mean error from standard least squares: 0.4298002219755827
Mean error from LASSO: 0.4298002219755827
X \text{ subset} = 5, i = 6, j = 8
Mean error from standard least squares: 0.4306326304106548
Mean error from LASSO: 0.4306326304106548
X \text{ subset} = 5, i = 6, j = 9
Mean error from standard least squares: 0.40871254162042175
Mean error from LASSO: 0.40871254162042175
X \text{ subset} = 5 , i = 6 , j = 10
Mean error from standard least squares: 0.416204217536071
Mean error from LASSO: 0.416204217536071
X \text{ subset} = 5, i = 7, j = 0
Mean error from standard least squares: 0.4109322974472808
Mean error from LASSO: 0.4109322974472808
X \text{ subset} = 5 , i = 7 , j = 1
Mean error from standard least squares: 0.42563817980022195
Mean error from LASSO: 0.42563817980022195
X \text{ subset} = 5, i = 7, j = 2
Mean error from standard least squares: 0.4239733629300777
Mean error from LASSO: 0.4239733629300777
X \text{ subset} = 5, i = 7, j = 3
Mean error from standard least squares: 0.41065482796892344
Mean error from LASSO: 0.41065482796892344
X subset = 5 , i = 7 , j = 4
Mean error from standard least squares: 0.39622641509433965
Mean error from LASSO: 0.39622641509433965
X \text{ subset} = 5, i = 7, j = 5
```

```
Mean error from standard least squares: 0.42064372918978915
Mean error from LASSO: 0.42064372918978915
X \text{ subset} = 5, i = 7, j = 6
Mean error from standard least squares: 0.41453940066592676
Mean error from LASSO: 0.41453940066592676
X subset = 5 , i = 7 , j = 8
Mean error from standard least squares: 0.42924528301886794
Mean error from LASSO: 0.42924528301886794
X \text{ subset} = 5, i = 7, j = 9
Mean error from standard least squares: 0.3998335183129856
Mean error from LASSO: 0.3998335183129856
X \text{ subset} = 5, i = 7, j = 10
Mean error from standard least squares: 0.4020532741398446
Mean error from LASSO: 0.4020532741398446
X \text{ subset} = 5, i = 8, j = 0
Mean error from standard least squares: 0.4092674805771365
Mean error from LASSO: 0.4092674805771365
X \text{ subset} = 5, i = 8, j = 1
Mean error from standard least squares: 0.41759156492785793
Mean error from LASSO: 0.41759156492785793
X \text{ subset} = 5, i = 8, j = 2
Mean error from standard least squares: 0.4164816870144284
Mean error from LASSO: 0.4164816870144284
X \text{ subset} = 5, i = 8, j = 3
Mean error from standard least squares: 0.4153718091009989
Mean error from LASSO: 0.4153718091009989
X \text{ subset} = 5, i = 8, j = 4
Mean error from standard least squares: 0.3973362930077691
Mean error from LASSO: 0.3973362930077691
X \text{ subset} = 5, i = 8, j = 5
Mean error from standard least squares: 0.4159267480577136
Mean error from LASSO: 0.4159267480577136
X \text{ subset} = 5, i = 8, j = 6
Mean error from standard least squares: 0.41731409544950054
Mean error from LASSO: 0.41731409544950054
X \text{ subset} = 5, i = 8, j = 7
Mean error from standard least squares: 0.4125971143174251
Mean error from LASSO: 0.4125971143174251
X \text{ subset} = 5, i = 8, j = 9
Mean error from standard least squares: 0.4020532741398446
Mean error from LASSO: 0.4020532741398446
X \text{ subset} = 5, i = 8, j = 10
Mean error from standard least squares: 0.3937291897891232
Mean error from LASSO: 0.3937291897891232
X subset = 5 , i = 9 , j = 0
Mean error from standard least squares: 0.40871254162042175
Mean error from LASSO: 0.40871254162042175
X \text{ subset} = 5, i = 9, j = 1
```

```
Mean error from standard least squares: 0.41148723640399554
Mean error from LASSO: 0.41148723640399554
X \text{ subset} = 5, i = 9, j = 2
Mean error from standard least squares: 0.40704772475027745
Mean error from LASSO: 0.40704772475027745
X \text{ subset} = 5 , i = 9 , j = 3
Mean error from standard least squares: 0.3992785793562708
Mean error from LASSO: 0.3992785793562708
X \text{ subset} = 5, i = 9, j = 4
Mean error from standard least squares: 0.40260821309655936
Mean error from LASSO: 0.40260821309655936
X \text{ subset} = 5, i = 9, j = 5
Mean error from standard least squares: 0.3981687014428413
Mean error from LASSO: 0.3981687014428413
X \text{ subset} = 5, i = 9, j = 6
Mean error from standard least squares: 0.3992785793562708
Mean error from LASSO: 0.3992785793562708
X \text{ subset} = 5, i = 9, j = 7
Mean error from standard least squares: 0.3909544950055494
Mean error from LASSO: 0.3909544950055494
X \text{ subset} = 5 , i = 9 , j = 8
Mean error from standard least squares: 0.3873473917869034
Mean error from LASSO: 0.3873473917869034
X \text{ subset} = 5, i = 9, j = 10
Mean error from standard least squares: 0.3745837957824639
Mean error from LASSO: 0.3745837957824639
X \text{ subset} = 5 , i = 10 , j = 0
Mean error from standard least squares: 0.40621531631520535
Mean error from LASSO: 0.40621531631520535
X \text{ subset} = 5, i = 10, j = 1
Mean error from standard least squares: 0.41287458379578246
Mean error from LASSO: 0.41287458379578246
X \text{ subset} = 5, i = 10, j = 2
Mean error from standard least squares: 0.41315205327413984
Mean error from LASSO: 0.41315205327413984
X \text{ subset} = 5 , i = 10 , j = 3
Mean error from standard least squares: 0.4181465038845727
Mean error from LASSO: 0.4181465038845727
X \text{ subset} = 5, i = 10, j = 4
Mean error from standard least squares: 0.38984461709211987
Mean error from LASSO: 0.38984461709211987
X \text{ subset} = 5, i = 10, j = 5
Mean error from standard least squares: 0.39872364039955605
Mean error from LASSO: 0.39872364039955605
X subset = 5 , i = 10 , j = 6
Mean error from standard least squares: 0.4064927857935627
Mean error from LASSO: 0.4064927857935627
X \text{ subset} = 5, i = 10, j = 7
```

```
Mean error from standard least squares: 0.390677025527192
Mean error from LASSO: 0.390677025527192
X \text{ subset} = 5 , i = 10 , j = 8
Mean error from standard least squares: 0.3978912319644839
Mean error from LASSO: 0.3978912319644839
X subset = 5 , i = 10 , j = 9
Mean error from standard least squares: 0.3706992230854606
Mean error from LASSO: 0.3706992230854606
X \text{ subset} = 6, i = 0, j = 1
Mean error from standard least squares: 0.35405105438401774
Mean error from LASSO: 0.35405105438401774
X \text{ subset} = 6, i = 0, j = 2
Mean error from standard least squares: 0.3867924528301887
Mean error from LASSO: 0.3867924528301887
X \text{ subset} = 6, i = 0, j = 3
Mean error from standard least squares: 0.41287458379578246
Mean error from LASSO: 0.41287458379578246
X \text{ subset} = 6, i = 0, j = 4
Mean error from standard least squares: 0.4209211986681465
Mean error from LASSO: 0.4209211986681465
X \text{ subset} = 6, i = 0, j = 5
Mean error from standard least squares: 0.43312985571587126
Mean error from LASSO: 0.43312985571587126
X \text{ subset} = 6, i = 0, j = 6
Mean error from standard least squares: 0.4364594894561598
Mean error from LASSO: 0.4364594894561598
X \text{ subset} = 6, i = 0, j = 7
Mean error from standard least squares: 0.4408990011098779
Mean error from LASSO: 0.4408990011098779
X \text{ subset} = 6, i = 0, j = 8
Mean error from standard least squares: 0.422031076581576
Mean error from LASSO: 0.42175360710321863
X \text{ subset} = 6, i = 0, j = 9
Mean error from standard least squares: 0.4109322974472808
Mean error from LASSO: 0.4109322974472808
X \text{ subset} = 6 , i = 0 , j = 10
Mean error from standard least squares: 0.40621531631520535
Mean error from LASSO: 0.40621531631520535
X \text{ subset} = 6, i = 1, j = 0
Mean error from standard least squares: 0.3088235294117647
Mean error from LASSO: 0.3088235294117647
X \text{ subset} = 6, i = 1, j = 2
Mean error from standard least squares: 0.35987791342952274
Mean error from LASSO: 0.35987791342952274
X subset = 6 , i = 1 , j = 3
Mean error from standard least squares: 0.37569367369589346
Mean error from LASSO: 0.37569367369589346
X \text{ subset} = 6 , i = 1 , j = 4
```

```
Mean error from standard least squares: 0.38041065482796893
Mean error from LASSO: 0.38041065482796893
X \text{ subset} = 6 , i = 1 , j = 5
Mean error from standard least squares: 0.4009433962264151
Mean error from LASSO: 0.4009433962264151
X \text{ subset} = 6 , i = 1 , j = 6
Mean error from standard least squares: 0.402330743618202
Mean error from LASSO: 0.402330743618202
X \text{ subset} = 6, i = 1, j = 7
Mean error from standard least squares: 0.4037180910099889
Mean error from LASSO: 0.4037180910099889
X \text{ subset} = 6, i = 1, j = 8
Mean error from standard least squares: 0.39261931187569366
Mean error from LASSO: 0.39261931187569366
X \text{ subset} = 6 , i = 1 , j = 9
Mean error from standard least squares: 0.36958934517203107
Mean error from LASSO: 0.36958934517203107
X \text{ subset} = 6, i = 1, j = 10
Mean error from standard least squares: 0.35960044395116536
Mean error from LASSO: 0.35960044395116536
X \text{ subset} = 6, i = 2, j = 0
Mean error from standard least squares: 0.4098224195338513
Mean error from LASSO: 0.4098224195338513
X \text{ subset} = 6, i = 2, j = 1
Mean error from standard least squares: 0.43340732519422864
Mean error from LASSO: 0.43340732519422864
X \text{ subset} = 6, i = 2, j = 3
Mean error from standard least squares: 0.5022197558268591
Mean error from LASSO: 0.5022197558268591
X \text{ subset} = 6, i = 2, j = 4
Mean error from standard least squares: 0.508046614872364
Mean error from LASSO: 0.508046614872364
X \text{ subset} = 6, i = 2, j = 5
Mean error from standard least squares: 0.5258046614872364
Mean error from LASSO: 0.5258046614872364
X subset = 6 , i = 2 , j = 6
Mean error from standard least squares: 0.5285793562708102
Mean error from LASSO: 0.5285793562708102
X \text{ subset} = 6, i = 2, j = 7
Mean error from standard least squares: 0.5330188679245284
Mean error from LASSO: 0.5330188679245284
X \text{ subset} = 6, i = 2, j = 8
Mean error from standard least squares: 0.5105438401775805
Mean error from LASSO: 0.5108213096559379
X subset = 6 , i = 2 , j = 9
Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.4944506104328524
X \text{ subset} = 6 , i = 2 , j = 10
```

```
Mean error from standard least squares: 0.4811320754716981
Mean error from LASSO: 0.4811320754716981
X \text{ subset} = 6 , i = 3 , j = 0
Mean error from standard least squares: 0.39900110987791343
Mean error from LASSO: 0.39900110987791343
X \text{ subset} = 6 , i = 3 , j = 1
Mean error from standard least squares: 0.4267480577136515
Mean error from LASSO: 0.4267480577136515
X \text{ subset} = 6, i = 3, j = 2
Mean error from standard least squares: 0.4481132075471698
Mean error from LASSO: 0.4481132075471698
X \text{ subset} = 6, i = 3, j = 4
Mean error from standard least squares: 0.4958379578246393
Mean error from LASSO: 0.4958379578246393
X \text{ subset} = 6 , i = 3 , j = 5
Mean error from standard least squares: 0.5166481687014428
Mean error from LASSO: 0.5166481687014428
X \text{ subset} = 6, i = 3, j = 6
Mean error from standard least squares: 0.51165371809101
Mean error from LASSO: 0.51165371809101
X \text{ subset} = 6, i = 3, j = 7
Mean error from standard least squares: 0.5258046614872364
Mean error from LASSO: 0.5258046614872364
X \text{ subset} = 6, i = 3, j = 8
Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.49667036625971145
X \text{ subset} = 6, i = 3, j = 9
Mean error from standard least squares: 0.4916759156492786
Mean error from LASSO: 0.4916759156492786
X \text{ subset} = 6 , i = 3 , j = 10
Mean error from standard least squares: 0.4733629300776915
Mean error from LASSO: 0.4733629300776915
X \text{ subset} = 6, i = 4, j = 0
Mean error from standard least squares: 0.36403995560488345
Mean error from LASSO: 0.36403995560488345
X \text{ subset} = 6, i = 4, j = 1
Mean error from standard least squares: 0.384850166481687
Mean error from LASSO: 0.384850166481687
X \text{ subset} = 6, i = 4, j = 2
Mean error from standard least squares: 0.4195338512763596
Mean error from LASSO: 0.4195338512763596
X \text{ subset} = 6, i = 4, j = 3
Mean error from standard least squares: 0.44950055493895674
Mean error from LASSO: 0.44950055493895674
X subset = 6 , i = 4 , j = 5
Mean error from standard least squares: 0.47253052164261933
Mean error from LASSO: 0.47253052164261933
X \text{ subset} = 6 , i = 4 , j = 6
```

```
Mean error from standard least squares: 0.47475027746947834
Mean error from LASSO: 0.47475027746947834
X \text{ subset} = 6, i = 4, j = 7
Mean error from standard least squares: 0.47835738068812433
Mean error from LASSO: 0.47835738068812433
X subset = 6 , i = 4 , j = 8
Mean error from standard least squares: 0.4625416204217536
Mean error from LASSO: 0.462819089900111
X \text{ subset} = 6, i = 4, j = 9
Mean error from standard least squares: 0.44561598224195337
Mean error from LASSO: 0.44561598224195337
X \text{ subset} = 6 , i = 4 , j = 10
Mean error from standard least squares: 0.43895671476137627
Mean error from LASSO: 0.43895671476137627
X \text{ subset} = 6 , i = 5 , j = 0
Mean error from standard least squares: 0.3346281908990011
Mean error from LASSO: 0.3346281908990011
X \text{ subset} = 6, i = 5, j = 1
Mean error from standard least squares: 0.34988901220865704
Mean error from LASSO: 0.34988901220865704
X \text{ subset} = 6, i = 5, j = 2
Mean error from standard least squares: 0.3867924528301887
Mean error from LASSO: 0.3867924528301887
X \text{ subset} = 6, i = 5, j = 3
Mean error from standard least squares: 0.41148723640399554
Mean error from LASSO: 0.41148723640399554
X \text{ subset} = 6, i = 5, j = 4
Mean error from standard least squares: 0.41509433962264153
Mean error from LASSO: 0.41509433962264153
X \text{ subset} = 6 , i = 5 , j = 6
Mean error from standard least squares: 0.43590455049944504
Mean error from LASSO: 0.43590455049944504
X \text{ subset} = 6, i = 5, j = 7
Mean error from standard least squares: 0.4447835738068812
Mean error from LASSO: 0.4447835738068812
X \text{ subset} = 6, i = 5, j = 8
Mean error from standard least squares: 0.42147613762486125
Mean error from LASSO: 0.42147613762486125
X \text{ subset} = 6, i = 5, j = 9
Mean error from standard least squares: 0.40843507214206437
Mean error from LASSO: 0.40843507214206437
X \text{ subset} = 6, i = 5, j = 10
Mean error from standard least squares: 0.40038845726970035
Mean error from LASSO: 0.40038845726970035
X subset = 6 , i = 6 , j = 0
Mean error from standard least squares: 0.341842397336293
Mean error from LASSO: 0.341842397336293
X \text{ subset} = 6, i = 6, j = 1
```

```
Mean error from standard least squares: 0.35960044395116536
Mean error from LASSO: 0.35960044395116536
X \text{ subset} = 6 , i = 6 , j = 2
Mean error from standard least squares: 0.38845726970033295
Mean error from LASSO: 0.38845726970033295
X \text{ subset} = 6 , i = 6 , j = 3
Mean error from standard least squares: 0.42147613762486125
Mean error from LASSO: 0.42147613762486125
X \text{ subset} = 6, i = 6, j = 4
Mean error from standard least squares: 0.42591564927857933
Mean error from LASSO: 0.42591564927857933
X \text{ subset} = 6, i = 6, j = 5
Mean error from standard least squares: 0.4406215316315205
Mean error from LASSO: 0.4406215316315205
X \text{ subset} = 6, i = 6, j = 7
Mean error from standard least squares: 0.4464483906770255
Mean error from LASSO: 0.4464483906770255
X \text{ subset} = 6, i = 6, j = 8
Mean error from standard least squares: 0.43340732519422864
Mean error from LASSO: 0.43340732519422864
X \text{ subset} = 6 , i = 6 , j = 9
Mean error from standard least squares: 0.41065482796892344
Mean error from LASSO: 0.41065482796892344
X \text{ subset} = 6 , i = 6 , j = 10
Mean error from standard least squares: 0.41065482796892344
Mean error from LASSO: 0.41065482796892344
X \text{ subset} = 6, i = 7, j = 0
Mean error from standard least squares: 0.36847946725860153
Mean error from LASSO: 0.36847946725860153
X \text{ subset} = 6, i = 7, j = 1
Mean error from standard least squares: 0.38623751387347394
Mean error from LASSO: 0.38623751387347394
X \text{ subset} = 6, i = 7, j = 2
Mean error from standard least squares: 0.4209211986681465
Mean error from LASSO: 0.4209211986681465
X subset = 6 , i = 7 , j = 3
Mean error from standard least squares: 0.45005549389567145
Mean error from LASSO: 0.45005549389567145
X \text{ subset} = 6, i = 7, j = 4
Mean error from standard least squares: 0.4553274139844617
Mean error from LASSO: 0.4553274139844617
X \text{ subset} = 6, i = 7, j = 5
Mean error from standard least squares: 0.4772475027746948
Mean error from LASSO: 0.4772475027746948
X subset = 6 , i = 7 , j = 6
Mean error from standard least squares: 0.47253052164261933
Mean error from LASSO: 0.47253052164261933
X \text{ subset} = 6 , i = 7 , j = 8
```

```
Mean error from standard least squares: 0.4567147613762486
Mean error from LASSO: 0.4567147613762486
X \text{ subset} = 6 , i = 7 , j = 9
Mean error from standard least squares: 0.43951165371809103
Mean error from LASSO: 0.43951165371809103
X subset = 6 , i = 7 , j = 10
Mean error from standard least squares: 0.4386792452830189
Mean error from LASSO: 0.4386792452830189
X \text{ subset} = 6, i = 8, j = 0
Mean error from standard least squares: 0.36847946725860153
Mean error from LASSO: 0.36847946725860153
X \text{ subset} = 6, i = 8, j = 1
Mean error from standard least squares: 0.3981687014428413
Mean error from LASSO: 0.3981687014428413
X \text{ subset} = 6 , i = 8 , j = 2
Mean error from standard least squares: 0.42341842397336293
Mean error from LASSO: 0.42341842397336293
X \text{ subset} = 6, i = 8, j = 3
Mean error from standard least squares: 0.45643729189789123
Mean error from LASSO: 0.45643729189789123
X \text{ subset} = 6 , i = 8 , j = 4
Mean error from standard least squares: 0.46503884572697
Mean error from LASSO: 0.46503884572697
X \text{ subset} = 6, i = 8, j = 5
Mean error from standard least squares: 0.4816870144284129
Mean error from LASSO: 0.4816870144284129
X \text{ subset} = 6, i = 8, j = 6
Mean error from standard least squares: 0.48668146503884574
Mean error from LASSO: 0.48668146503884574
X \text{ subset} = 6, i = 8, j = 7
Mean error from standard least squares: 0.4938956714761376
Mean error from LASSO: 0.4938956714761376
X \text{ subset} = 6, i = 8, j = 9
Mean error from standard least squares: 0.4597669256381798
Mean error from LASSO: 0.4597669256381798
X \text{ subset} = 6 , i = 8 , j = 10
Mean error from standard least squares: 0.4486681465038846
Mean error from LASSO: 0.4486681465038846
X \text{ subset} = 6, i = 9, j = 0
Mean error from standard least squares: 0.4076026637069922
Mean error from LASSO: 0.4076026637069922
X \text{ subset} = 6, i = 9, j = 1
Mean error from standard least squares: 0.42563817980022195
Mean error from LASSO: 0.42563817980022195
X subset = 6 , i = 9 , j = 2
Mean error from standard least squares: 0.46503884572697
Mean error from LASSO: 0.46503884572697
X \text{ subset} = 6 , i = 9 , j = 3
```

```
Mean error from standard least squares: 0.4955604883462819
Mean error from LASSO: 0.4955604883462819
X \text{ subset} = 6, i = 9, j = 4
Mean error from standard least squares: 0.505826859045505
Mean error from LASSO: 0.505826859045505
X \text{ subset} = 6 , i = 9 , j = 5
Mean error from standard least squares: 0.5274694783573807
Mean error from LASSO: 0.5274694783573807
X \text{ subset} = 6, i = 9, j = 6
Mean error from standard least squares: 0.5269145394006659
Mean error from LASSO: 0.5269145394006659
X \text{ subset} = 6, i = 9, j = 7
Mean error from standard least squares: 0.5283018867924528
Mean error from LASSO: 0.5283018867924528
X \text{ subset} = 6 , i = 9 , j = 8
Mean error from standard least squares: 0.5049944506104328
Mean error from LASSO: 0.5047169811320755
X \text{ subset} = 6 , i = 9 , j = 10
Mean error from standard least squares: 0.4841842397336293
Mean error from LASSO: 0.4841842397336293
X \text{ subset} = 6 , i = 10 , j = 0
Mean error from standard least squares: 0.4100998890122087
Mean error from LASSO: 0.4100998890122087
X \text{ subset} = 6, i = 10, j = 1
Mean error from standard least squares: 0.4239733629300777
Mean error from LASSO: 0.4239733629300777
X \text{ subset} = 6 , i = 10 , j = 2
Mean error from standard least squares: 0.4630965593784684
Mean error from LASSO: 0.4630965593784684
X \text{ subset} = 6 , i = 10 , j = 3
Mean error from standard least squares: 0.4972253052164262
Mean error from LASSO: 0.4972253052164262
X \text{ subset} = 6, i = 10, j = 4
Mean error from standard least squares: 0.49916759156492785
Mean error from LASSO: 0.49916759156492785
X \text{ subset} = 6 , i = 10 , j = 5
Mean error from standard least squares: 0.5230299667036626
Mean error from LASSO: 0.5230299667036626
X \text{ subset} = 6 , i = 10 , j = 6
Mean error from standard least squares: 0.5266370699223085
Mean error from LASSO: 0.5266370699223085
X \text{ subset} = 6, i = 10, j = 7
Mean error from standard least squares: 0.5310765815760267
Mean error from LASSO: 0.5310765815760267
X subset = 6 , i = 10 , j = 8
Mean error from standard least squares: 0.5072142064372919
Mean error from LASSO: 0.5072142064372919
X \text{ subset} = 6 , i = 10 , j = 9
```

```
Mean error from standard least squares: 0.4894561598224195
Mean error from LASSO: 0.4894561598224195
X \text{ subset} = 7, i = 0, j = 1
Mean error from standard least squares: 0.4597669256381798
Mean error from LASSO: 0.4597669256381798
X subset = 7 , i = 0 , j = 2
Mean error from standard least squares: 0.4769700332963374
Mean error from LASSO: 0.4769700332963374
X \text{ subset} = 7, i = 0, j = 3
Mean error from standard least squares: 0.4694783573806881
Mean error from LASSO: 0.4694783573806881
X \text{ subset} = 7, i = 0, j = 4
Mean error from standard least squares: 0.44755826859045506
Mean error from LASSO: 0.44755826859045506
X \text{ subset} = 7, i = 0, j = 5
Mean error from standard least squares: 0.462819089900111
Mean error from LASSO: 0.462819089900111
X \text{ subset} = 7, i = 0, j = 6
Mean error from standard least squares: 0.4741953385127636
Mean error from LASSO: 0.4741953385127636
X \text{ subset} = 7, i = 0, j = 7
Mean error from standard least squares: 0.4741953385127636
Mean error from LASSO: 0.4741953385127636
X \text{ subset} = 7, i = 0, j = 8
Mean error from standard least squares: 0.4816870144284129
Mean error from LASSO: 0.4816870144284129
X \text{ subset} = 7, i = 0, j = 9
Mean error from standard least squares: 0.4692008879023307
Mean error from LASSO: 0.4692008879023307
X \text{ subset} = 7 , i = 0 , j = 10
Mean error from standard least squares: 0.47253052164261933
Mean error from LASSO: 0.47253052164261933
X \text{ subset} = 7, i = 1, j = 0
Mean error from standard least squares: 0.49472807991120976
Mean error from LASSO: 0.49472807991120976
X subset = 7 , i = 1 , j = 2
Mean error from standard least squares: 0.5083240843507214
Mean error from LASSO: 0.5083240843507214
X \text{ subset} = 7, i = 1, j = 3
Mean error from standard least squares: 0.505826859045505
Mean error from LASSO: 0.505826859045505
X \text{ subset} = 7, i = 1, j = 4
Mean error from standard least squares: 0.4897336293007769
Mean error from LASSO: 0.4897336293007769
X subset = 7 , i = 1 , j = 5
Mean error from standard least squares: 0.47613762486126526
Mean error from LASSO: 0.47613762486126526
X \text{ subset} = 7, i = 1, j = 6
```

```
Mean error from standard least squares: 0.5122086570477248
Mean error from LASSO: 0.5122086570477248
X \text{ subset} = 7, i = 1, j = 7
Mean error from standard least squares: 0.5047169811320755
Mean error from LASSO: 0.5047169811320755
X subset = 7 , i = 1 , j = 8
Mean error from standard least squares: 0.5083240843507214
Mean error from LASSO: 0.5083240843507214
X \text{ subset} = 7, i = 1, j = 9
Mean error from standard least squares: 0.5005549389567148
Mean error from LASSO: 0.5005549389567148
X \text{ subset} = 7, i = 1, j = 10
Mean error from standard least squares: 0.4997225305216426
Mean error from LASSO: 0.4997225305216426
X \text{ subset} = 7, i = 2, j = 0
Mean error from standard least squares: 0.38623751387347394
Mean error from LASSO: 0.38623751387347394
X \text{ subset} = 7, i = 2, j = 1
Mean error from standard least squares: 0.3948390677025527
Mean error from LASSO: 0.3948390677025527
X \text{ subset} = 7, i = 2, j = 3
Mean error from standard least squares: 0.39261931187569366
Mean error from LASSO: 0.39261931187569366
X \text{ subset} = 7, i = 2, j = 4
Mean error from standard least squares: 0.3992785793562708
Mean error from LASSO: 0.3992785793562708
X subset = 7 , i = 2 , j = 5
Mean error from standard least squares: 0.37846836847946724
Mean error from LASSO: 0.37846836847946724
X \text{ subset} = 7, i = 2, j = 6
Mean error from standard least squares: 0.3940066592674806
Mean error from LASSO: 0.3940066592674806
X \text{ subset} = 7, i = 2, j = 7
Mean error from standard least squares: 0.3831853496115427
Mean error from LASSO: 0.3831853496115427
X subset = 7 , i = 2 , j = 8
Mean error from standard least squares: 0.39456159822419534
Mean error from LASSO: 0.39456159822419534
X \text{ subset} = 7, i = 2, j = 9
Mean error from standard least squares: 0.3865149833518313
Mean error from LASSO: 0.3865149833518313
X \text{ subset} = 7, i = 2, j = 10
Mean error from standard least squares: 0.38235294117647056
Mean error from LASSO: 0.38235294117647056
X subset = 7 , i = 3 , j = 0
Mean error from standard least squares: 0.3934517203107658
Mean error from LASSO: 0.3934517203107658
X \text{ subset} = 7, i = 3, j = 1
```

```
Mean error from standard least squares: 0.39900110987791343
Mean error from LASSO: 0.39900110987791343
X \text{ subset} = 7, i = 3, j = 2
Mean error from standard least squares: 0.39622641509433965
Mean error from LASSO: 0.39622641509433965
X subset = 7 , i = 3 , j = 4
Mean error from standard least squares: 0.3934517203107658
Mean error from LASSO: 0.3934517203107658
X \text{ subset} = 7, i = 3, j = 5
Mean error from standard least squares: 0.3973362930077691
Mean error from LASSO: 0.3973362930077691
X \text{ subset} = 7, i = 3, j = 6
Mean error from standard least squares: 0.3940066592674806
Mean error from LASSO: 0.3940066592674806
X \text{ subset} = 7, i = 3, j = 7
Mean error from standard least squares: 0.4017758046614872
Mean error from LASSO: 0.4017758046614872
X \text{ subset} = 7, i = 3, j = 8
Mean error from standard least squares: 0.3951165371809101
Mean error from LASSO: 0.3951165371809101
X \text{ subset} = 7, i = 3, j = 9
Mean error from standard least squares: 0.39456159822419534
Mean error from LASSO: 0.39456159822419534
X \text{ subset} = 7, i = 3, j = 10
Mean error from standard least squares: 0.40427302996670367
Mean error from LASSO: 0.40427302996670367
X \text{ subset} = 7, i = 4, j = 0
Mean error from standard least squares: 0.42036625971143177
Mean error from LASSO: 0.42036625971143177
X \text{ subset} = 7, i = 4, j = 1
Mean error from standard least squares: 0.4295227524972253
Mean error from LASSO: 0.4295227524972253
X \text{ subset} = 7, i = 4, j = 2
Mean error from standard least squares: 0.45005549389567145
Mean error from LASSO: 0.45005549389567145
X \text{ subset} = 7, i = 4, j = 3
Mean error from standard least squares: 0.44783573806881244
Mean error from LASSO: 0.44783573806881244
X \text{ subset} = 7, i = 4, j = 5
Mean error from standard least squares: 0.4353496115427303
Mean error from LASSO: 0.4353496115427303
X \text{ subset} = 7, i = 4, j = 6
Mean error from standard least squares: 0.44034406215316313
Mean error from LASSO: 0.44034406215316313
X subset = 7 , i = 4 , j = 7
Mean error from standard least squares: 0.44589345172031075
Mean error from LASSO: 0.44589345172031075
X \text{ subset} = 7, i = 4, j = 8
```

```
Mean error from standard least squares: 0.4408990011098779
Mean error from LASSO: 0.4408990011098779
X \text{ subset} = 7, i = 4, j = 9
Mean error from standard least squares: 0.4422863485016648
Mean error from LASSO: 0.4422863485016648
X subset = 7 , i = 4 , j = 10
Mean error from standard least squares: 0.4309100998890122
Mean error from LASSO: 0.4309100998890122
X \text{ subset} = 7, i = 5, j = 0
Mean error from standard least squares: 0.4714206437291898
Mean error from LASSO: 0.4714206437291898
X \text{ subset} = 7, i = 5, j = 1
Mean error from standard least squares: 0.46642619311875694
Mean error from LASSO: 0.46642619311875694
X \text{ subset} = 7, i = 5, j = 2
Mean error from standard least squares: 0.4852941176470588
Mean error from LASSO: 0.4852941176470588
X \text{ subset} = 7, i = 5, j = 3
Mean error from standard least squares: 0.47225305216426194
Mean error from LASSO: 0.47225305216426194
X \text{ subset} = 7, i = 5, j = 4
Mean error from standard least squares: 0.47641509433962265
Mean error from LASSO: 0.47641509433962265
X \text{ subset} = 7, i = 5, j = 6
Mean error from standard least squares: 0.47669256381798003
Mean error from LASSO: 0.47669256381798003
X \text{ subset} = 7, i = 5, j = 7
Mean error from standard least squares: 0.4669811320754717
Mean error from LASSO: 0.4669811320754717
X \text{ subset} = 7, i = 5, j = 8
Mean error from standard least squares: 0.47197558268590456
Mean error from LASSO: 0.47197558268590456
X \text{ subset} = 7, i = 5, j = 9
Mean error from standard least squares: 0.459211986681465
Mean error from LASSO: 0.459211986681465
X subset = 7 , i = 5 , j = 10
Mean error from standard least squares: 0.48335183129855713
Mean error from LASSO: 0.48335183129855713
X \text{ subset} = 7, i = 6, j = 0
Mean error from standard least squares: 0.4716981132075472
Mean error from LASSO: 0.4716981132075472
X \text{ subset} = 7, i = 6, j = 1
Mean error from standard least squares: 0.46781354051054386
Mean error from LASSO: 0.46781354051054386
X subset = 7 , i = 6 , j = 2
Mean error from standard least squares: 0.47475027746947834
Mean error from LASSO: 0.47475027746947834
X \text{ subset} = 7, i = 6, j = 3
```

```
Mean error from standard least squares: 0.4655937846836848
Mean error from LASSO: 0.4655937846836848
X \text{ subset} = 7, i = 6, j = 4
Mean error from standard least squares: 0.47031076581576026
Mean error from LASSO: 0.47031076581576026
X subset = 7 , i = 6 , j = 5
Mean error from standard least squares: 0.44977802441731407
Mean error from LASSO: 0.44977802441731407
X \text{ subset} = 7, i = 6, j = 7
Mean error from standard least squares: 0.4614317425083241
Mean error from LASSO: 0.4614317425083241
X \text{ subset} = 7, i = 6, j = 8
Mean error from standard least squares: 0.470865704772475
Mean error from LASSO: 0.470865704772475
X \text{ subset} = 7, i = 6, j = 9
Mean error from standard least squares: 0.4611542730299667
Mean error from LASSO: 0.4611542730299667
X \text{ subset} = 7, i = 6, j = 10
Mean error from standard least squares: 0.45643729189789123
Mean error from LASSO: 0.45643729189789123
X \text{ subset} = 7, i = 7, j = 0
Mean error from standard least squares: 0.44006659267480575
Mean error from LASSO: 0.44006659267480575
X \text{ subset} = 7, i = 7, j = 1
Mean error from standard least squares: 0.4397891231964484
Mean error from LASSO: 0.4397891231964484
X \text{ subset} = 7 \text{ , i} = 7 \text{ , j} = 2
Mean error from standard least squares: 0.4447835738068812
Mean error from LASSO: 0.4447835738068812
X \text{ subset} = 7, i = 7, j = 3
Mean error from standard least squares: 0.43729189789123196
Mean error from LASSO: 0.43729189789123196
X \text{ subset} = 7, i = 7, j = 4
Mean error from standard least squares: 0.4353496115427303
Mean error from LASSO: 0.4353496115427303
X subset = 7 , i = 7 , j = 5
Mean error from standard least squares: 0.422031076581576
Mean error from LASSO: 0.422031076581576
X \text{ subset} = 7, i = 7, j = 6
Mean error from standard least squares: 0.4447835738068812
Mean error from LASSO: 0.4447835738068812
X \text{ subset} = 7, i = 7, j = 8
Mean error from standard least squares: 0.4486681465038846
Mean error from LASSO: 0.4486681465038846
X subset = 7 , i = 7 , j = 9
Mean error from standard least squares: 0.44617092119866814
Mean error from LASSO: 0.44617092119866814
X \text{ subset} = 7 , i = 7 , j = 10
```

```
Mean error from standard least squares: 0.4309100998890122
Mean error from LASSO: 0.4309100998890122
X \text{ subset} = 7, i = 8, j = 0
Mean error from standard least squares: 0.4261931187569367
Mean error from LASSO: 0.4261931187569367
X subset = 7 , i = 8 , j = 1
Mean error from standard least squares: 0.4281354051054384
Mean error from LASSO: 0.4281354051054384
X \text{ subset} = 7, i = 8, j = 2
Mean error from standard least squares: 0.4353496115427303
Mean error from LASSO: 0.4353496115427303
X \text{ subset} = 7, i = 8, j = 3
Mean error from standard least squares: 0.43146503884572696
Mean error from LASSO: 0.43146503884572696
X \text{ subset} = 7, i = 8, j = 4
Mean error from standard least squares: 0.42314095449500555
Mean error from LASSO: 0.42314095449500555
X \text{ subset} = 7, i = 8, j = 5
Mean error from standard least squares: 0.40316315205327413
Mean error from LASSO: 0.40316315205327413
X \text{ subset} = 7, i = 8, j = 6
Mean error from standard least squares: 0.4300776914539401
Mean error from LASSO: 0.4300776914539401
X \text{ subset} = 7, i = 8, j = 7
Mean error from standard least squares: 0.4236958934517203
Mean error from LASSO: 0.4236958934517203
X \text{ subset} = 7, i = 8, j = 9
Mean error from standard least squares: 0.4253607103218646
Mean error from LASSO: 0.4253607103218646
X \text{ subset} = 7 , i = 8 , j = 10
Mean error from standard least squares: 0.4295227524972253
Mean error from LASSO: 0.4295227524972253
X \text{ subset} = 7, i = 9, j = 0
Mean error from standard least squares: 0.38013318534961155
Mean error from LASSO: 0.38013318534961155
X \text{ subset} = 7, i = 9, j = 1
Mean error from standard least squares: 0.3895671476137625
Mean error from LASSO: 0.3895671476137625
X \text{ subset} = 7, i = 9, j = 2
Mean error from standard least squares: 0.390677025527192
Mean error from LASSO: 0.390677025527192
X \text{ subset} = 7, i = 9, j = 3
Mean error from standard least squares: 0.3867924528301887
Mean error from LASSO: 0.3867924528301887
X subset = 7 , i = 9 , j = 4
Mean error from standard least squares: 0.38290788013318533
Mean error from LASSO: 0.38290788013318533
X \text{ subset} = 7, i = 9, j = 5
```

```
Mean error from standard least squares: 0.384850166481687
Mean error from LASSO: 0.384850166481687
X \text{ subset} = 7, i = 9, j = 6
Mean error from standard least squares: 0.38623751387347394
Mean error from LASSO: 0.38623751387347394
X subset = 7 , i = 9 , j = 7
Mean error from standard least squares: 0.3915094339622642
Mean error from LASSO: 0.3915094339622642
X \text{ subset} = 7, i = 9, j = 8
Mean error from standard least squares: 0.39012208657047726
Mean error from LASSO: 0.39012208657047726
X \text{ subset} = 7, i = 9, j = 10
Mean error from standard least squares: 0.39428412874583796
Mean error from LASSO: 0.39428412874583796
X subset = 7 , i = 10 , j = 0
Mean error from standard least squares: 0.3892896781354051
Mean error from LASSO: 0.3892896781354051
X \text{ subset} = 7, i = 10, j = 1
Mean error from standard least squares: 0.3765260821309656
Mean error from LASSO: 0.3765260821309656
X \text{ subset} = 7 , i = 10 , j = 2
Mean error from standard least squares: 0.39456159822419534
Mean error from LASSO: 0.39456159822419534
X \text{ subset} = 7, i = 10, j = 3
Mean error from standard least squares: 0.39844617092119866
Mean error from LASSO: 0.39844617092119866
X \text{ subset} = 7, i = 10, j = 4
Mean error from standard least squares: 0.38457269700332963
Mean error from LASSO: 0.38457269700332963
X \text{ subset} = 7, i = 10, j = 5
Mean error from standard least squares: 0.3770810210876804
Mean error from LASSO: 0.3770810210876804
X \text{ subset} = 7, i = 10, j = 6
Mean error from standard least squares: 0.38623751387347394
Mean error from LASSO: 0.38623751387347394
X \text{ subset} = 7 , i = 10 , j = 7
Mean error from standard least squares: 0.37597114317425084
Mean error from LASSO: 0.37597114317425084
X \text{ subset} = 7, i = 10, j = 8
Mean error from standard least squares: 0.3779134295227525
Mean error from LASSO: 0.3779134295227525
X \text{ subset} = 7, i = 10, j = 9
Mean error from standard least squares: 0.39705882352941174
Mean error from LASSO: 0.39705882352941174
X subset = 8 , i = 0 , j = 1
Mean error from standard least squares: 0.49944506104328523
Mean error from LASSO: 0.49889012208657046
X \text{ subset} = 8 \text{ , i} = 0 \text{ , j} = 2
```

```
Mean error from standard least squares: 0.49223085460599336
Mean error from LASSO: 0.49223085460599336
X \text{ subset} = 8 , i = 0 , j = 3
Mean error from standard least squares: 0.5049944506104328
Mean error from LASSO: 0.5041620421753608
X subset = 8 , i = 0 , j = 4
Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.49639289678135406
X \text{ subset} = 8, i = 0, j = 5
Mean error from standard least squares: 0.4980577136514983
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 8 , i = 0 , j = 6
Mean error from standard least squares: 0.4958379578246393
Mean error from LASSO: 0.4958379578246393
X \text{ subset} = 8, i = 0, j = 7
Mean error from standard least squares: 0.4894561598224195
Mean error from LASSO: 0.4894561598224195
X \text{ subset} = 8, i = 0, j = 8
Mean error from standard least squares: 0.5011098779134295
Mean error from LASSO: 0.5019422863485017
X \text{ subset} = 8, i = 0, j = 9
Mean error from standard least squares: 0.5180355160932297
Mean error from LASSO: 0.5185904550499445
X \text{ subset} = 8 , i = 0 , j = 10
Mean error from standard least squares: 0.5013873473917869
Mean error from LASSO: 0.5013873473917869
X \text{ subset} = 8 , i = 1 , j = 0
Mean error from standard least squares: 0.4916759156492786
Mean error from LASSO: 0.4916759156492786
X \text{ subset} = 8, i = 1, j = 2
Mean error from standard least squares: 0.5124861265260822
Mean error from LASSO: 0.5124861265260822
X \text{ subset} = 8, i = 1, j = 3
Mean error from standard least squares: 0.48501664816870144
Mean error from LASSO: 0.4852941176470588
X \text{ subset} = 8 , i = 1 , j = 4
Mean error from standard least squares: 0.48335183129855713
Mean error from LASSO: 0.48335183129855713
X \text{ subset} = 8, i = 1, j = 5
Mean error from standard least squares: 0.49944506104328523
Mean error from LASSO: 0.4997225305216426
X \text{ subset} = 8, i = 1, j = 6
Mean error from standard least squares: 0.5288568257491676
Mean error from LASSO: 0.5277469478357381
X \text{ subset} = 8 , i = 1 , j = 7
Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.49500554938956715
X \text{ subset} = 8 , i = 1 , j = 8
```

```
Mean error from standard least squares: 0.5083240843507214
Mean error from LASSO: 0.5083240843507214
X \text{ subset} = 8 , i = 1 , j = 9
Mean error from standard least squares: 0.5235849056603774
Mean error from LASSO: 0.5235849056603774
X subset = 8 , i = 1 , j = 10
Mean error from standard least squares: 0.5005549389567148
Mean error from LASSO: 0.5008324084350722
X subset = 8, i = 2, j = 0
Mean error from standard least squares: 0.531354051054384
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8, i = 2, j = 1
Mean error from standard least squares: 0.48668146503884574
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8 , i = 2 , j = 3
Mean error from standard least squares: 0.5110987791342952
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8, i = 2, j = 4
Mean error from standard least squares: 0.5219200887902331
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8, i = 2, j = 5
Mean error from standard least squares: 0.4877913429522753
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8, i = 2, j = 6
Mean error from standard least squares: 0.48473917869034405
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8 , i = 2 , j = 7
Mean error from standard least squares: 0.4955604883462819
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8 , i = 2 , j = 8
Mean error from standard least squares: 0.48917869034406214
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8, i = 2, j = 9
Mean error from standard least squares: 0.4880688124306326
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8 , i = 2 , j = 10
Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.40288568257491675
X \text{ subset} = 8 , i = 3 , j = 0
Mean error from standard least squares: 0.5061043285238623
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8, i = 3, j = 1
Mean error from standard least squares: 0.5135960044395117
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8 , i = 3 , j = 2
Mean error from standard least squares: 0.5066592674805771
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8 , i = 3 , j = 4
```

```
Mean error from standard least squares: 0.5002774694783574
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8 , i = 3 , j = 5
Mean error from standard least squares: 0.5105438401775805
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8 , i = 3 , j = 6
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8, i = 3, j = 7
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8, i = 3, j = 8
Mean error from standard least squares: 0.5019422863485017
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8 , i = 3 , j = 9
Mean error from standard least squares: 0.49694783573806883
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8 , i = 3 , j = 10
Mean error from standard least squares: 0.4980577136514983
Mean error from LASSO: 0.4142619311875694
X \text{ subset} = 8 , i = 4 , j = 0
Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.4955604883462819
X \text{ subset} = 8, i = 4, j = 1
Mean error from standard least squares: 0.491953385127636
Mean error from LASSO: 0.49223085460599336
X \text{ subset} = 8, i = 4, j = 2
Mean error from standard least squares: 0.5088790233074362
Mean error from LASSO: 0.5088790233074362
X \text{ subset} = 8, i = 4, j = 3
Mean error from standard least squares: 0.5094339622641509
Mean error from LASSO: 0.5094339622641509
X \text{ subset} = 8, i = 4, j = 5
Mean error from standard least squares: 0.5044395116537181
Mean error from LASSO: 0.5044395116537181
X \text{ subset} = 8 , i = 4 , j = 6
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.49778024417314093
X \text{ subset} = 8, i = 4, j = 7
Mean error from standard least squares: 0.49528301886792453
Mean error from LASSO: 0.494173140954495
X \text{ subset} = 8, i = 4, j = 8
Mean error from standard least squares: 0.4836293007769145
Mean error from LASSO: 0.47863485016648166
X \text{ subset} = 8 , i = 4 , j = 9
Mean error from standard least squares: 0.49694783573806883
Mean error from LASSO: 0.49667036625971145
X \text{ subset} = 8 , i = 4 , j = 10
```

```
Mean error from standard least squares: 0.5177580466148723
Mean error from LASSO: 0.517480577136515
X \text{ subset} = 8 , i = 5 , j = 0
Mean error from standard least squares: 0.4986126526082131
Mean error from LASSO: 0.5002774694783574
X subset = 8 , i = 5 , j = 1
Mean error from standard least squares: 0.4830743618201998
Mean error from LASSO: 0.4830743618201998
X \text{ subset} = 8, i = 5, j = 2
Mean error from standard least squares: 0.4986126526082131
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 8, i = 5, j = 3
Mean error from standard least squares: 0.5069367369589345
Mean error from LASSO: 0.5094339622641509
X \text{ subset} = 8 , i = 5 , j = 4
Mean error from standard least squares: 0.4938956714761376
Mean error from LASSO: 0.49250832408435075
X \text{ subset} = 8, i = 5, j = 6
Mean error from standard least squares: 0.4958379578246393
Mean error from LASSO: 0.4958379578246393
X \text{ subset} = 8, i = 5, j = 7
Mean error from standard least squares: 0.5108213096559379
Mean error from LASSO: 0.5108213096559379
X \text{ subset} = 8, i = 5, j = 8
Mean error from standard least squares: 0.4961154273029967
Mean error from LASSO: 0.4958379578246393
X \text{ subset} = 8, i = 5, j = 9
Mean error from standard least squares: 0.48612652608213097
Mean error from LASSO: 0.48612652608213097
X \text{ subset} = 8 , i = 5 , j = 10
Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.49639289678135406
X \text{ subset} = 8, i = 6, j = 0
Mean error from standard least squares: 0.4955604883462819
Mean error from LASSO: 0.4955604883462819
X \text{ subset} = 8, i = 6, j = 1
Mean error from standard least squares: 0.4938956714761376
Mean error from LASSO: 0.494173140954495
X \text{ subset} = 8, i = 6, j = 2
Mean error from standard least squares: 0.5052719200887902
Mean error from LASSO: 0.5052719200887902
X \text{ subset} = 8, i = 6, j = 3
Mean error from standard least squares: 0.5063817980022197
Mean error from LASSO: 0.5077691453940066
X subset = 8 , i = 6 , j = 4
Mean error from standard least squares: 0.4972253052164262
Mean error from LASSO: 0.5
X \text{ subset} = 8 , i = 6 , j = 5
```

```
Mean error from standard least squares: 0.4958379578246393
Mean error from LASSO: 0.4958379578246393
X \text{ subset} = 8 , i = 6 , j = 7
Mean error from standard least squares: 0.4897336293007769
Mean error from LASSO: 0.48917869034406214
X \text{ subset} = 8 , i = 6 , j = 8
Mean error from standard least squares: 0.5110987791342952
Mean error from LASSO: 0.51165371809101
X \text{ subset} = 8, i = 6, j = 9
Mean error from standard least squares: 0.48668146503884574
Mean error from LASSO: 0.4894561598224195
X \text{ subset} = 8 , i = 6 , j = 10
Mean error from standard least squares: 0.5110987791342952
Mean error from LASSO: 0.5122086570477248
X \text{ subset} = 8 \text{ , i} = 7 \text{ , j} = 0
Mean error from standard least squares: 0.4938956714761376
Mean error from LASSO: 0.4938956714761376
X \text{ subset} = 8, i = 7, j = 1
Mean error from standard least squares: 0.49944506104328523
Mean error from LASSO: 0.49916759156492785
X \text{ subset} = 8, i = 7, j = 2
Mean error from standard least squares: 0.5030521642619312
Mean error from LASSO: 0.5030521642619312
X \text{ subset} = 8, i = 7, j = 3
Mean error from standard least squares: 0.5113762486126526
Mean error from LASSO: 0.5105438401775805
X \text{ subset} = 8 \text{ , i} = 7 \text{ , j} = 4
Mean error from standard least squares: 0.4911209766925638
Mean error from LASSO: 0.491953385127636
X \text{ subset} = 8, i = 7, j = 5
Mean error from standard least squares: 0.5049944506104328
Mean error from LASSO: 0.5041620421753608
X \text{ subset} = 8, i = 7, j = 6
Mean error from standard least squares: 0.48917869034406214
Mean error from LASSO: 0.48917869034406214
X subset = 8 , i = 7 , j = 8
Mean error from standard least squares: 0.4972253052164262
Mean error from LASSO: 0.4972253052164262
X \text{ subset} = 8, i = 7, j = 9
Mean error from standard least squares: 0.48917869034406214
Mean error from LASSO: 0.48917869034406214
X \text{ subset} = 8 , i = 7 , j = 10
Mean error from standard least squares: 0.4936182019977802
Mean error from LASSO: 0.49334073251942284
X \text{ subset} = 8 , i = 8 , j = 0
Mean error from standard least squares: 0.5127635960044395
Mean error from LASSO: 0.5130410654827969
X \text{ subset} = 8 , i = 8 , j = 1
```

```
Mean error from standard least squares: 0.5013873473917869
Mean error from LASSO: 0.5013873473917869
X \text{ subset} = 8 , i = 8 , j = 2
Mean error from standard least squares: 0.505826859045505
Mean error from LASSO: 0.505826859045505
X \text{ subset} = 8 , i = 8 , j = 3
Mean error from standard least squares: 0.52330743618202
Mean error from LASSO: 0.5235849056603774
X \text{ subset} = 8, i = 8, j = 4
Mean error from standard least squares: 0.49472807991120976
Mean error from LASSO: 0.49500554938956715
X \text{ subset} = 8, i = 8, j = 5
Mean error from standard least squares: 0.5069367369589345
Mean error from LASSO: 0.5069367369589345
X \text{ subset} = 8 , i = 8 , j = 6
Mean error from standard least squares: 0.49916759156492785
Mean error from LASSO: 0.49916759156492785
X \text{ subset} = 8, i = 8, j = 7
Mean error from standard least squares: 0.4875138734739179
Mean error from LASSO: 0.4877913429522753
X \text{ subset} = 8, i = 8, j = 9
Mean error from standard least squares: 0.4872364039955605
Mean error from LASSO: 0.4875138734739179
X \text{ subset} = 8 , i = 8 , j = 10
Mean error from standard least squares: 0.49334073251942284
Mean error from LASSO: 0.49334073251942284
X \text{ subset} = 8 , i = 9 , j = 0
Mean error from standard least squares: 0.508046614872364
Mean error from LASSO: 0.5083240843507214
X \text{ subset} = 8 , i = 9 , j = 1
Mean error from standard least squares: 0.5141509433962265
Mean error from LASSO: 0.51165371809101
X \text{ subset} = 8, i = 9, j = 2
Mean error from standard least squares: 0.5172031076581576
Mean error from LASSO: 0.5172031076581576
X \text{ subset} = 8, i = 9, j = 3
Mean error from standard least squares: 0.51165371809101
Mean error from LASSO: 0.5038845726970034
X \text{ subset} = 8, i = 9, j = 4
Mean error from standard least squares: 0.4972253052164262
Mean error from LASSO: 0.49639289678135406
X \text{ subset} = 8, i = 9, j = 5
Mean error from standard least squares: 0.48890122086570476
Mean error from LASSO: 0.4900110987791343
X subset = 8 , i = 9 , j = 6
Mean error from standard least squares: 0.4858490566037736
Mean error from LASSO: 0.48196448390677027
X \text{ subset} = 8 , i = 9 , j = 7
```

```
Mean error from standard least squares: 0.4755826859045505
Mean error from LASSO: 0.4758601553829079
X \text{ subset} = 8 , i = 9 , j = 8
Mean error from standard least squares: 0.48890122086570476
Mean error from LASSO: 0.4836293007769145
X \text{ subset} = 8 , i = 9 , j = 10
Mean error from standard least squares: 0.48446170921198667
Mean error from LASSO: 0.4669811320754717
X \text{ subset} = 8 , i = 10 , j = 0
Mean error from standard least squares: 0.5094339622641509
Mean error from LASSO: 0.5088790233074362
X \text{ subset} = 8 , i = 10 , j = 1
Mean error from standard least squares: 0.51165371809101
Mean error from LASSO: 0.4938956714761376
X \text{ subset} = 8 , i = 10 , j = 2
Mean error from standard least squares: 0.5205327413984462
Mean error from LASSO: 0.508046614872364
X \text{ subset} = 8 , i = 10 , j = 3
Mean error from standard least squares: 0.525527192008879
Mean error from LASSO: 0.5188679245283019
X \text{ subset} = 8 , i = 10 , j = 4
Mean error from standard least squares: 0.5219200887902331
Mean error from LASSO: 0.5052719200887902
X \text{ subset} = 8 , i = 10 , j = 5
Mean error from standard least squares: 0.4983351831298557
Mean error from LASSO: 0.45754716981132076
X \text{ subset} = 8 , i = 10 , j = 6
Mean error from standard least squares: 0.4852941176470588
Mean error from LASSO: 0.47807991120976695
X \text{ subset} = 8, i = 10, j = 7
Mean error from standard least squares: 0.48640399556048836
Mean error from LASSO: 0.4542175360710322
X \text{ subset} = 8 , i = 10 , j = 8
Mean error from standard least squares: 0.48224195338512765
Mean error from LASSO: 0.462819089900111
X \text{ subset} = 8 , i = 10 , j = 9
Mean error from standard least squares: 0.4772475027746948
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 9, i = 0, j = 1
Mean error from standard least squares: 0.49778024417314093
Mean error from LASSO: 0.5002774694783574
X \text{ subset} = 9, i = 0, j = 2
Mean error from standard least squares: 0.4841842397336293
Mean error from LASSO: 0.5
X subset = 9 , i = 0 , j = 3
Mean error from standard least squares: 0.5055493895671476
Mean error from LASSO: 0.5011098779134295
X \text{ subset} = 9 , i = 0 , j = 4
```

```
Mean error from standard least squares: 0.4983351831298557
Mean error from LASSO: 0.5019422863485017
X \text{ subset} = 9 , i = 0 , j = 5
Mean error from standard least squares: 0.49250832408435075
Mean error from LASSO: 0.494173140954495
X subset = 9 , i = 0 , j = 6
Mean error from standard least squares: 0.4997225305216426
Mean error from LASSO: 0.4958379578246393
X \text{ subset} = 9, i = 0, j = 7
Mean error from standard least squares: 0.4961154273029967
Mean error from LASSO: 0.49694783573806883
X \text{ subset} = 9 , i = 0 , j = 8
Mean error from standard least squares: 0.48057713651498335
Mean error from LASSO: 0.4772475027746948
X \text{ subset} = 9 , i = 0 , j = 9
Mean error from standard least squares: 0.45810210876803553
Mean error from LASSO: 0.4597669256381798
X \text{ subset} = 9, i = 0, j = 10
Mean error from standard least squares: 0.4481132075471698
Mean error from LASSO: 0.451165371809101
X \text{ subset} = 9 , i = 1 , j = 0
Mean error from standard least squares: 0.5305216426193119
Mean error from LASSO: 0.5344062153163152
X \text{ subset} = 9, i = 1, j = 2
Mean error from standard least squares: 0.5319089900110988
Mean error from LASSO: 0.5432852386237513
X \text{ subset} = 9 , i = 1 , j = 3
Mean error from standard least squares: 0.5366259711431742
Mean error from LASSO: 0.5302441731409545
X \text{ subset} = 9 , i = 1 , j = 4
Mean error from standard least squares: 0.5366259711431742
Mean error from LASSO: 0.5405105438401776
X \text{ subset} = 9, i = 1, j = 5
Mean error from standard least squares: 0.5374583795782464
Mean error from LASSO: 0.5385682574916759
X \text{ subset} = 9 , i = 1 , j = 6
Mean error from standard least squares: 0.529134295227525
Mean error from LASSO: 0.5324639289678136
X \text{ subset} = 9, i = 1, j = 7
Mean error from standard least squares: 0.5396781354051055
Mean error from LASSO: 0.5369034406215316
X \text{ subset} = 9, i = 1, j = 8
Mean error from standard least squares: 0.5110987791342952
Mean error from LASSO: 0.5119311875693674
X subset = 9 , i = 1 , j = 9
Mean error from standard least squares: 0.49306326304106546
Mean error from LASSO: 0.49778024417314093
X \text{ subset} = 9 , i = 1 , j = 10
```

```
Mean error from standard least squares: 0.49334073251942284
Mean error from LASSO: 0.4916759156492786
X \text{ subset} = 9 \text{ , i} = 2 \text{ , j} = 0
Mean error from standard least squares: 0.44006659267480575
Mean error from LASSO: 0.39900110987791343
X subset = 9 , i = 2 , j = 1
Mean error from standard least squares: 0.42591564927857933
Mean error from LASSO: 0.40038845726970035
X \text{ subset} = 9, i = 2, j = 3
Mean error from standard least squares: 0.46587125416204217
Mean error from LASSO: 0.4009433962264151
X \text{ subset} = 9, i = 2, j = 4
Mean error from standard least squares: 0.43951165371809103
Mean error from LASSO: 0.4009433962264151
X \text{ subset} = 9 , i = 2 , j = 5
Mean error from standard least squares: 0.42508324084350724
Mean error from LASSO: 0.39678135405105436
X \text{ subset} = 9, i = 2, j = 6
Mean error from standard least squares: 0.41315205327413984
Mean error from LASSO: 0.39456159822419534
X \text{ subset} = 9, i = 2, j = 7
Mean error from standard least squares: 0.4223085460599334
Mean error from LASSO: 0.3973362930077691
X \text{ subset} = 9, i = 2, j = 8
Mean error from standard least squares: 0.40732519422863483
Mean error from LASSO: 0.3831853496115427
X \text{ subset} = 9 , i = 2 , j = 9
Mean error from standard least squares: 0.40260821309655936
Mean error from LASSO: 0.36764705882352944
X \text{ subset} = 9 , i = 2 , j = 10
Mean error from standard least squares: 0.4159267480577136
Mean error from LASSO: 0.3662597114317425
X \text{ subset} = 9, i = 3, j = 0
Mean error from standard least squares: 0.44200887902330743
Mean error from LASSO: 0.41148723640399554
X \text{ subset} = 9, i = 3, j = 1
Mean error from standard least squares: 0.43590455049944504
Mean error from LASSO: 0.40899001109877914
X \text{ subset} = 9, i = 3, j = 2
Mean error from standard least squares: 0.48834628190899
Mean error from LASSO: 0.41148723640399554
X \text{ subset} = 9, i = 3, j = 4
Mean error from standard least squares: 0.4397891231964484
Mean error from LASSO: 0.4117647058823529
X \text{ subset} = 9 , i = 3 , j = 5
Mean error from standard least squares: 0.4328523862375139
Mean error from LASSO: 0.40677025527192007
X \text{ subset} = 9 , i = 3 , j = 6
```

```
Mean error from standard least squares: 0.4261931187569367
Mean error from LASSO: 0.4053829078801332
X \text{ subset} = 9, i = 3, j = 7
Mean error from standard least squares: 0.4361820199778024
Mean error from LASSO: 0.40843507214206437
X subset = 9 , i = 3 , j = 8
Mean error from standard least squares: 0.419811320754717
Mean error from LASSO: 0.3940066592674806
X \text{ subset} = 9 , i = 3 , j = 9
Mean error from standard least squares: 0.4034406215316315
Mean error from LASSO: 0.3737513873473918
X \text{ subset} = 9 , i = 3 , j = 10
Mean error from standard least squares: 0.3981687014428413
Mean error from LASSO: 0.3748612652608213
X \text{ subset} = 9 , i = 4 , j = 0
Mean error from standard least squares: 0.4714206437291898
Mean error from LASSO: 0.4597669256381798
X \text{ subset} = 9, i = 4, j = 1
Mean error from standard least squares: 0.4653163152053274
Mean error from LASSO: 0.4614317425083241
X \text{ subset} = 9, i = 4, j = 2
Mean error from standard least squares: 0.46892341842397334
Mean error from LASSO: 0.46032186459489455
X \text{ subset} = 9, i = 4, j = 3
Mean error from standard least squares: 0.4894561598224195
Mean error from LASSO: 0.46170921198668147
X \text{ subset} = 9, i = 4, j = 5
Mean error from standard least squares: 0.4653163152053274
Mean error from LASSO: 0.45643729189789123
X \text{ subset} = 9 , i = 4 , j = 6
Mean error from standard least squares: 0.4630965593784684
Mean error from LASSO: 0.45643729189789123
X \text{ subset} = 9, i = 4, j = 7
Mean error from standard least squares: 0.4583795782463929
Mean error from LASSO: 0.4572697003329634
X \text{ subset} = 9, i = 4, j = 8
Mean error from standard least squares: 0.4447835738068812
Mean error from LASSO: 0.4367369589345172
X \text{ subset} = 9, i = 4, j = 9
Mean error from standard least squares: 0.43174250832408434
Mean error from LASSO: 0.42452830188679247
X \text{ subset} = 9, i = 4, j = 10
Mean error from standard least squares: 0.43312985571587126
Mean error from LASSO: 0.4184239733629301
X \text{ subset} = 9, i = 5, j = 0
Mean error from standard least squares: 0.48890122086570476
Mean error from LASSO: 0.5019422863485017
X \text{ subset} = 9, i = 5, j = 1
```

```
Mean error from standard least squares: 0.5019422863485017
Mean error from LASSO: 0.5022197558268591
X \text{ subset} = 9, i = 5, j = 2
Mean error from standard least squares: 0.491953385127636
Mean error from LASSO: 0.5008324084350722
X \text{ subset} = 9 , i = 5 , j = 3
Mean error from standard least squares: 0.49916759156492785
Mean error from LASSO: 0.49916759156492785
X \text{ subset} = 9, i = 5, j = 4
Mean error from standard least squares: 0.5066592674805771
Mean error from LASSO: 0.5044395116537181
X \text{ subset} = 9, i = 5, j = 6
Mean error from standard least squares: 0.4897336293007769
Mean error from LASSO: 0.491953385127636
X \text{ subset} = 9 , i = 5 , j = 7
Mean error from standard least squares: 0.5030521642619312
Mean error from LASSO: 0.49778024417314093
X \text{ subset} = 9, i = 5, j = 8
Mean error from standard least squares: 0.4816870144284129
Mean error from LASSO: 0.4800221975582686
X \text{ subset} = 9 , i = 5 , j = 9
Mean error from standard least squares: 0.45588235294117646
Mean error from LASSO: 0.4594894561598224
X \text{ subset} = 9 , i = 5 , j = 10
Mean error from standard least squares: 0.4508879023307436
Mean error from LASSO: 0.4506104328523862
X \text{ subset} = 9, i = 6, j = 0
Mean error from standard least squares: 0.49500554938956715
Mean error from LASSO: 0.49250832408435075
X \text{ subset} = 9, i = 6, j = 1
Mean error from standard least squares: 0.49056603773584906
Mean error from LASSO: 0.4938956714761376
X \text{ subset} = 9, i = 6, j = 2
Mean error from standard least squares: 0.4980577136514983
Mean error from LASSO: 0.48890122086570476
X \text{ subset} = 9, i = 6, j = 3
Mean error from standard least squares: 0.49306326304106546
Mean error from LASSO: 0.49778024417314093
X \text{ subset} = 9, i = 6, j = 4
Mean error from standard least squares: 0.4916759156492786
Mean error from LASSO: 0.49250832408435075
X \text{ subset} = 9, i = 6, j = 5
Mean error from standard least squares: 0.49278579356270813
Mean error from LASSO: 0.4872364039955605
X \text{ subset} = 9, i = 6, j = 7
Mean error from standard least squares: 0.4872364039955605
Mean error from LASSO: 0.4894561598224195
X \text{ subset} = 9 , i = 6 , j = 8
```

```
Mean error from standard least squares: 0.4692008879023307
Mean error from LASSO: 0.46809100998890124
X \text{ subset} = 9 , i = 6 , j = 9
Mean error from standard least squares: 0.45782463928967815
Mean error from LASSO: 0.45810210876803553
X \text{ subset} = 9 , i = 6 , j = 10
Mean error from standard least squares: 0.45588235294117646
Mean error from LASSO: 0.45338512763596006
X \text{ subset} = 9, i = 7, j = 0
Mean error from standard least squares: 0.46892341842397334
Mean error from LASSO: 0.46032186459489455
X \text{ subset} = 9, i = 7, j = 1
Mean error from standard least squares: 0.4614317425083241
Mean error from LASSO: 0.462819089900111
X \text{ subset} = 9 \text{ , i} = 7 \text{ , j} = 2
Mean error from standard least squares: 0.4692008879023307
Mean error from LASSO: 0.4614317425083241
X \text{ subset} = 9, i = 7, j = 3
Mean error from standard least squares: 0.47475027746947834
Mean error from LASSO: 0.46032186459489455
X \text{ subset} = 9 , i = 7 , j = 4
Mean error from standard least squares: 0.4683684794672586
Mean error from LASSO: 0.46226415094339623
X \text{ subset} = 9, i = 7, j = 5
Mean error from standard least squares: 0.4594894561598224
Mean error from LASSO: 0.45615982241953384
X \text{ subset} = 9, i = 7, j = 6
Mean error from standard least squares: 0.4567147613762486
Mean error from LASSO: 0.45615982241953384
X \text{ subset} = 9, i = 7, j = 8
Mean error from standard least squares: 0.43951165371809103
Mean error from LASSO: 0.4406215316315205
X \text{ subset} = 9, i = 7, j = 9
Mean error from standard least squares: 0.4303551609322974
Mean error from LASSO: 0.4242508324084351
X subset = 9 , i = 7 , j = 10
Mean error from standard least squares: 0.42036625971143177
Mean error from LASSO: 0.4156492785793563
X \text{ subset} = 9 , i = 8 , j = 0
Mean error from standard least squares: 0.46004439511653716
Mean error from LASSO: 0.4481132075471698
X \text{ subset} = 9, i = 8, j = 1
Mean error from standard least squares: 0.4464483906770255
Mean error from LASSO: 0.4483906770255272
X \text{ subset} = 9 , i = 8 , j = 2
Mean error from standard least squares: 0.4547724750277469
Mean error from LASSO: 0.44728079911209767
X \text{ subset} = 9 , i = 8 , j = 3
```

```
Mean error from standard least squares: 0.4800221975582686
Mean error from LASSO: 0.44783573806881244
X \text{ subset} = 9 , i = 8 , j = 4
Mean error from standard least squares: 0.45754716981132076
Mean error from LASSO: 0.45005549389567145
X \text{ subset} = 9 , i = 8 , j = 5
Mean error from standard least squares: 0.4514428412874584
Mean error from LASSO: 0.4439511653718091
X \text{ subset} = 9, i = 8, j = 6
Mean error from standard least squares: 0.4442286348501665
Mean error from LASSO: 0.44034406215316313
X \text{ subset} = 9, i = 8, j = 7
Mean error from standard least squares: 0.44977802441731407
Mean error from LASSO: 0.44339622641509435
X \text{ subset} = 9 , i = 8 , j = 9
Mean error from standard least squares: 0.413984461709212
Mean error from LASSO: 0.40732519422863483
X \text{ subset} = 9 , i = 8 , j = 10
Mean error from standard least squares: 0.40482796892341844
Mean error from LASSO: 0.40316315205327413
X \text{ subset} = 9 , i = 9 , j = 0
Mean error from standard least squares: 0.4295227524972253
Mean error from LASSO: 0.42924528301886794
X \text{ subset} = 9, i = 9, j = 1
Mean error from standard least squares: 0.419811320754717
Mean error from LASSO: 0.40011098779134296
X \text{ subset} = 9, i = 9, j = 2
Mean error from standard least squares: 0.4542175360710322
Mean error from LASSO: 0.40149833518312983
X \text{ subset} = 9, i = 9, j = 3
Mean error from standard least squares: 0.46059933407325193
Mean error from LASSO: 0.40011098779134296
X \text{ subset} = 9, i = 9, j = 4
Mean error from standard least squares: 0.4328523862375139
Mean error from LASSO: 0.4267480577136515
X \text{ subset} = 9, i = 9, j = 5
Mean error from standard least squares: 0.4109322974472808
Mean error from LASSO: 0.408157602663707
X \text{ subset} = 9, i = 9, j = 6
Mean error from standard least squares: 0.40038845726970035
Mean error from LASSO: 0.4009433962264151
X \text{ subset} = 9, i = 9, j = 7
Mean error from standard least squares: 0.416204217536071
Mean error from LASSO: 0.41481687014428414
X \text{ subset} = 9 , i = 9 , j = 8
Mean error from standard least squares: 0.3981687014428413
Mean error from LASSO: 0.39678135405105436
X \text{ subset} = 9 , i = 9 , j = 10
```

```
Mean error from standard least squares: 0.3809655937846837
Mean error from LASSO: 0.37264150943396224
X \text{ subset} = 9, i = 10, j = 0
Mean error from standard least squares: 0.4306326304106548
Mean error from LASSO: 0.4009433962264151
X subset = 9 , i = 10 , j = 1
Mean error from standard least squares: 0.4178690344062153
Mean error from LASSO: 0.4053829078801332
X \text{ subset} = 9 , i = 10 , j = 2
Mean error from standard least squares: 0.4589345172031077
Mean error from LASSO: 0.4020532741398446
X \text{ subset} = 9, i = 10, j = 3
Mean error from standard least squares: 0.46392896781354054
Mean error from LASSO: 0.4056603773584906
X subset = 9 , i = 10 , j = 4
Mean error from standard least squares: 0.4239733629300777
Mean error from LASSO: 0.4020532741398446
X \text{ subset} = 9, i = 10, j = 5
Mean error from standard least squares: 0.4134295227524972
Mean error from LASSO: 0.39844617092119866
X \text{ subset} = 9 , i = 10 , j = 6
Mean error from standard least squares: 0.4095449500554939
Mean error from LASSO: 0.3992785793562708
X \text{ subset} = 9 , i = 10 , j = 7
Mean error from standard least squares: 0.4100998890122087
Mean error from LASSO: 0.39872364039955605
X \text{ subset} = 9 , i = 10 , j = 8
Mean error from standard least squares: 0.4017758046614872
Mean error from LASSO: 0.3831853496115427
X \text{ subset} = 9 , i = 10 , j = 9
Mean error from standard least squares: 0.38873473917869034
Mean error from LASSO: 0.3690344062153163
X \text{ subset} = 10 , i = 0 , j = 1
Mean error from standard least squares: 0.5013873473917869
Mean error from LASSO: 0.4975027746947836
X subset = 10 , i = 0 , j = 2
Mean error from standard least squares: 0.5044395116537181
Mean error from LASSO: 0.4894561598224195
X \text{ subset} = 10 , i = 0 , j = 3
Mean error from standard least squares: 0.4980577136514983
Mean error from LASSO: 0.49889012208657046
X \text{ subset} = 10 , i = 0 , j = 4
Mean error from standard least squares: 0.4958379578246393
Mean error from LASSO: 0.5011098779134295
X subset = 10 , i = 0 , j = 5
Mean error from standard least squares: 0.4936182019977802
Mean error from LASSO: 0.4955604883462819
X \text{ subset} = 10 , i = 0 , j = 6
```

```
Mean error from standard least squares: 0.49694783573806883
Mean error from LASSO: 0.4980577136514983
X subset = 10 , i = 0 , j = 7
Mean error from standard least squares: 0.5074916759156493
Mean error from LASSO: 0.5
X subset = 10 , i = 0 , j = 8
Mean error from standard least squares: 0.47863485016648166
Mean error from LASSO: 0.4794672586015538
X \text{ subset} = 10 , i = 0 , j = 9
Mean error from standard least squares: 0.4553274139844617
Mean error from LASSO: 0.4572697003329634
X \text{ subset} = 10 , i = 0 , j = 10
Mean error from standard least squares: 0.4542175360710322
Mean error from LASSO: 0.45172031076581576
X \text{ subset} = 10 , i = 1 , j = 0
Mean error from standard least squares: 0.5338512763596004
Mean error from LASSO: 0.5357935627081021
X \text{ subset} = 10 , i = 1 , j = 2
Mean error from standard least squares: 0.5294117647058824
Mean error from LASSO: 0.5316315205327414
X \text{ subset} = 10 , i = 1 , j = 3
Mean error from standard least squares: 0.5194228634850167
Mean error from LASSO: 0.5274694783573807
X \text{ subset} = 10 , i = 1 , j = 4
Mean error from standard least squares: 0.5227524972253053
Mean error from LASSO: 0.5263596004439511
X \text{ subset} = 10 , i = 1 , j = 5
Mean error from standard least squares: 0.5382907880133185
Mean error from LASSO: 0.5385682574916759
X \text{ subset} = 10 , i = 1 , j = 6
Mean error from standard least squares: 0.529134295227525
Mean error from LASSO: 0.5302441731409545
X subset = 10 , i = 1 , j = 7
Mean error from standard least squares: 0.5269145394006659
Mean error from LASSO: 0.529134295227525
X \text{ subset} = 10 , i = 1 , j = 8
Mean error from standard least squares: 0.5091564927857936
Mean error from LASSO: 0.5141509433962265
X \text{ subset} = 10 , i = 1 , j = 9
Mean error from standard least squares: 0.4916759156492786
Mean error from LASSO: 0.49250832408435075
X \text{ subset} = 10 , i = 1 , j = 10
Mean error from standard least squares: 0.47780244173140957
Mean error from LASSO: 0.4811320754716981
X subset = 10 , i = 2 , j = 0
Mean error from standard least squares: 0.44755826859045506
Mean error from LASSO: 0.39900110987791343
X subset = 10 , i = 2 , j = 1
```

```
Mean error from standard least squares: 0.4364594894561598
Mean error from LASSO: 0.40011098779134296
X \text{ subset} = 10 , i = 2 , j = 3
Mean error from standard least squares: 0.46642619311875694
Mean error from LASSO: 0.40843507214206437
X subset = 10 , i = 2 , j = 4
Mean error from standard least squares: 0.4586570477247503
Mean error from LASSO: 0.4009433962264151
X \text{ subset} = 10 , i = 2 , j = 5
Mean error from standard least squares: 0.42563817980022195
Mean error from LASSO: 0.39678135405105436
X \text{ subset} = 10 , i = 2 , j = 6
Mean error from standard least squares: 0.4225860155382908
Mean error from LASSO: 0.39456159822419534
X subset = 10 , i = 2 , j = 7
Mean error from standard least squares: 0.4303551609322974
Mean error from LASSO: 0.3973362930077691
X \text{ subset} = 10 , i = 2 , j = 8
Mean error from standard least squares: 0.3998335183129856
Mean error from LASSO: 0.38235294117647056
X \text{ subset} = 10 , i = 2 , j = 9
Mean error from standard least squares: 0.41453940066592676
Mean error from LASSO: 0.36736958934517205
X \text{ subset} = 10 , i = 2 , j = 10
Mean error from standard least squares: 0.40593784683684797
Mean error from LASSO: 0.36598224195338513
X subset = 10 , i = 3 , j = 0
Mean error from standard least squares: 0.44561598224195337
Mean error from LASSO: 0.41148723640399554
X \text{ subset} = 10 , i = 3 , j = 1
Mean error from standard least squares: 0.4264705882352941
Mean error from LASSO: 0.40899001109877914
X \text{ subset} = 10 , i = 3 , j = 2
Mean error from standard least squares: 0.4672586015538291
Mean error from LASSO: 0.42563817980022195
X \text{ subset} = 10 , i = 3 , j = 4
Mean error from standard least squares: 0.4583795782463929
Mean error from LASSO: 0.4117647058823529
X \text{ subset} = 10 , i = 3 , j = 5
Mean error from standard least squares: 0.451165371809101
Mean error from LASSO: 0.40677025527192007
X \text{ subset} = 10 , i = 3 , j = 6
Mean error from standard least squares: 0.4370144284128746
Mean error from LASSO: 0.4053829078801332
X subset = 10 , i = 3 , j = 7
Mean error from standard least squares: 0.44311875693673697
Mean error from LASSO: 0.40843507214206437
X \text{ subset} = 10 , i = 3 , j = 8
```

```
Mean error from standard least squares: 0.4225860155382908
Mean error from LASSO: 0.3940066592674806
X \text{ subset} = 10 , i = 3 , j = 9
Mean error from standard least squares: 0.4078801331853496
Mean error from LASSO: 0.3737513873473918
X \text{ subset} = 10 , i = 3 , j = 10
Mean error from standard least squares: 0.4078801331853496
Mean error from LASSO: 0.3748612652608213
X \text{ subset} = 10 , i = 4 , j = 0
Mean error from standard least squares: 0.4700332963374029
Mean error from LASSO: 0.4625416204217536
X \text{ subset} = 10 , i = 4 , j = 1
Mean error from standard least squares: 0.4692008879023307
Mean error from LASSO: 0.46448390677025525
X subset = 10 , i = 4 , j = 2
Mean error from standard least squares: 0.47225305216426194
Mean error from LASSO: 0.4714206437291898
X \text{ subset} = 10 , i = 4 , j = 3
Mean error from standard least squares: 0.4916759156492786
Mean error from LASSO: 0.48251942286348504
X \text{ subset} = 10 , i = 4 , j = 5
Mean error from standard least squares: 0.46004439511653716
Mean error from LASSO: 0.4630965593784684
X \text{ subset} = 10 , i = 4 , j = 6
Mean error from standard least squares: 0.4625416204217536
Mean error from LASSO: 0.46642619311875694
X subset = 10 , i = 4 , j = 7
Mean error from standard least squares: 0.4700332963374029
Mean error from LASSO: 0.4700332963374029
X \text{ subset} = 10 , i = 4 , j = 8
Mean error from standard least squares: 0.45033296337402884
Mean error from LASSO: 0.44783573806881244
X \text{ subset} = 10 , i = 4 , j = 9
Mean error from standard least squares: 0.43729189789123196
Mean error from LASSO: 0.4325749167591565
X \text{ subset} = 10 , i = 4 , j = 10
Mean error from standard least squares: 0.43923418423973365
Mean error from LASSO: 0.4370144284128746
X \text{ subset} = 10 , i = 5 , j = 0
Mean error from standard least squares: 0.49056603773584906
Mean error from LASSO: 0.5
X \text{ subset} = 10 , i = 5 , j = 1
Mean error from standard least squares: 0.5041620421753608
Mean error from LASSO: 0.49944506104328523
X subset = 10 , i = 5 , j = 2
Mean error from standard least squares: 0.49056603773584906
Mean error from LASSO: 0.4986126526082131
X \text{ subset} = 10 , i = 5 , j = 3
```

```
Mean error from standard least squares: 0.5033296337402886
Mean error from LASSO: 0.5055493895671476
X subset = 10 , i = 5 , j = 4
Mean error from standard least squares: 0.4938956714761376
Mean error from LASSO: 0.49056603773584906
X subset = 10 , i = 5 , j = 6
Mean error from standard least squares: 0.48473917869034405
Mean error from LASSO: 0.48640399556048836
X \text{ subset} = 10 , i = 5 , j = 7
Mean error from standard least squares: 0.49250832408435075
Mean error from LASSO: 0.49472807991120976
X \text{ subset} = 10 , i = 5 , j = 8
Mean error from standard least squares: 0.4877913429522753
Mean error from LASSO: 0.48335183129855713
X \text{ subset} = 10 , i = 5 , j = 9
Mean error from standard least squares: 0.4583795782463929
Mean error from LASSO: 0.459211986681465
X \text{ subset} = 10 , i = 5 , j = 10
Mean error from standard least squares: 0.45199778024417314
Mean error from LASSO: 0.45394006659267483
X \text{ subset} = 10 , i = 6 , j = 0
Mean error from standard least squares: 0.5008324084350722
Mean error from LASSO: 0.5005549389567148
X \text{ subset} = 10 , i = 6 , j = 1
Mean error from standard least squares: 0.4961154273029967
Mean error from LASSO: 0.4916759156492786
X subset = 10 , i = 6 , j = 2
Mean error from standard least squares: 0.5024972253052165
Mean error from LASSO: 0.49528301886792453
X \text{ subset} = 10 , i = 6 , j = 3
Mean error from standard least squares: 0.4972253052164262
Mean error from LASSO: 0.5008324084350722
X \text{ subset} = 10 , i = 6 , j = 4
Mean error from standard least squares: 0.49667036625971145
Mean error from LASSO: 0.4997225305216426
X \text{ subset} = 10 , i = 6 , j = 5
Mean error from standard least squares: 0.4858490566037736
Mean error from LASSO: 0.49223085460599336
X \text{ subset} = 10 , i = 6 , j = 7
Mean error from standard least squares: 0.4869589345172031
Mean error from LASSO: 0.4869589345172031
X \text{ subset} = 10 , i = 6 , j = 8
Mean error from standard least squares: 0.46503884572697
Mean error from LASSO: 0.46587125416204217
X subset = 10 , i = 6 , j = 9
Mean error from standard least squares: 0.46004439511653716
Mean error from LASSO: 0.4608768035516093
X \text{ subset} = 10 , i = 6 , j = 10
```

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Mean error from standard least squares: 0.4531076581576027
Mean error from LASSO: 0.45005549389567145
X subset = 10 , i = 7 , j = 0
Mean error from standard least squares: 0.47253052164261933
Mean error from LASSO: 0.4716981132075472
X subset = 10 , i = 7 , j = 1
Mean error from standard least squares: 0.4630965593784684
Mean error from LASSO: 0.4653163152053274
X \text{ subset} = 10, i = 7, j = 2
Mean error from standard least squares: 0.47891231964483905
Mean error from LASSO: 0.4739178690344062
X \text{ subset} = 10 , i = 7 , j = 3
Mean error from standard least squares: 0.4758601553829079
Mean error from LASSO: 0.47197558268590456
X subset = 10 , i = 7 , j = 4
Mean error from standard least squares: 0.4736403995560488
Mean error from LASSO: 0.4697558268590455
X \text{ subset} = 10 , i = 7 , j = 5
Mean error from standard least squares: 0.4653163152053274
Mean error from LASSO: 0.4597669256381798
X \text{ subset} = 10 , i = 7 , j = 6
Mean error from standard least squares: 0.4716981132075472
Mean error from LASSO: 0.46059933407325193
X \text{ subset} = 10 , i = 7 , j = 8
Mean error from standard least squares: 0.4422863485016648
Mean error from LASSO: 0.44145394006659266
X \text{ subset} = 10 , i = 7 , j = 9
Mean error from standard least squares: 0.427857935627081
Mean error from LASSO: 0.42147613762486125
X \text{ subset} = 10 , i = 7 , j = 10
Mean error from standard least squares: 0.42758046614872364
Mean error from LASSO: 0.42508324084350724
X \text{ subset} = 10 , i = 8 , j = 0
Mean error from standard least squares: 0.46809100998890124
Mean error from LASSO: 0.4694783573806881
X \text{ subset} = 10 , i = 8 , j = 1
Mean error from standard least squares: 0.4572697003329634
Mean error from LASSO: 0.4547724750277469
X \text{ subset} = 10 , i = 8 , j = 2
Mean error from standard least squares: 0.4692008879023307
Mean error from LASSO: 0.46448390677025525
X \text{ subset} = 10 , i = 8 , j = 3
Mean error from standard least squares: 0.46503884572697
Mean error from LASSO: 0.46476137624861263
X subset = 10 , i = 8 , j = 4
Mean error from standard least squares: 0.48057713651498335
Mean error from LASSO: 0.47225305216426194
X \text{ subset} = 10 , i = 8 , j = 5
```

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Mean error from standard least squares: 0.4525527192008879
Mean error from LASSO: 0.4542175360710322
X \text{ subset} = 10 , i = 8 , j = 6
Mean error from standard least squares: 0.448945615982242
Mean error from LASSO: 0.44755826859045506
X subset = 10 , i = 8 , j = 7
Mean error from standard least squares: 0.4589345172031077
Mean error from LASSO: 0.45449500554938954
X \text{ subset} = 10 , i = 8 , j = 9
Mean error from standard least squares: 0.41897891231964485
Mean error from LASSO: 0.42480577136514985
X \text{ subset} = 10 , i = 8 , j = 10
Mean error from standard least squares: 0.4134295227524972
Mean error from LASSO: 0.4053829078801332
X subset = 10 , i = 9 , j = 0
Mean error from standard least squares: 0.44034406215316313
Mean error from LASSO: 0.4309100998890122
X \text{ subset} = 10 , i = 9 , j = 1
Mean error from standard least squares: 0.4200887902330744
Mean error from LASSO: 0.4076026637069922
X \text{ subset} = 10 , i = 9 , j = 2
Mean error from standard least squares: 0.459211986681465
Mean error from LASSO: 0.4447835738068812
X \text{ subset} = 10 , i = 9 , j = 3
Mean error from standard least squares: 0.4611542730299667
Mean error from LASSO: 0.4481132075471698
X \text{ subset} = 10 , i = 9 , j = 4
Mean error from standard least squares: 0.4553274139844617
Mean error from LASSO: 0.44617092119866814
X \text{ subset} = 10 , i = 9 , j = 5
Mean error from standard least squares: 0.42758046614872364
Mean error from LASSO: 0.4153718091009989
X \text{ subset} = 10 , i = 9 , j = 6
Mean error from standard least squares: 0.4125971143174251
Mean error from LASSO: 0.3948390677025527
X \text{ subset} = 10 , i = 9 , j = 7
Mean error from standard least squares: 0.4303551609322974
Mean error from LASSO: 0.3992785793562708
X \text{ subset} = 10 , i = 9 , j = 8
Mean error from standard least squares: 0.42119866814650386
Mean error from LASSO: 0.40593784683684797
X \text{ subset} = 10 , i = 9 , j = 10
Mean error from standard least squares: 0.4051054384017758
Mean error from LASSO: 0.36403995560488345
X subset = 10 , i = 10 , j = 0
Mean error from standard least squares: 0.4408990011098779
Mean error from LASSO: 0.4397891231964484
X \text{ subset} = 10 , i = 10 , j = 1
```

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Mean error from standard least squares: 0.42730299667036625
Mean error from LASSO: 0.408157602663707
X \text{ subset} = 10 , i = 10 , j = 2
Mean error from standard least squares: 0.4630965593784684
Mean error from LASSO: 0.4009433962264151
X subset = 10 , i = 10 , j = 3
Mean error from standard least squares: 0.46448390677025525
Mean error from LASSO: 0.4012208657047725
X \text{ subset} = 10 , i = 10 , j = 4
Mean error from standard least squares: 0.44145394006659266
Mean error from LASSO: 0.4303551609322974
X \text{ subset} = 10 , i = 10 , j = 5
Mean error from standard least squares: 0.42896781354051056
Mean error from LASSO: 0.42591564927857933
X \text{ subset} = 10 , i = 10 , j = 6
Mean error from standard least squares: 0.42119866814650386
Mean error from LASSO: 0.4134295227524972
X \text{ subset} = 10 , i = 10 , j = 7
Mean error from standard least squares: 0.42064372918978915
Mean error from LASSO: 0.4200887902330744
X \text{ subset} = 10 , i = 10 , j = 8
Mean error from standard least squares: 0.4156492785793563
Mean error from LASSO: 0.3981687014428413
X \text{ subset} = 10 , i = 10 , j = 9
Mean error from standard least squares: 0.40260821309655936
Mean error from LASSO: 0.38207547169811323
X subset = 11 , i = 0 , j = 1
Mean error from standard least squares: 0.505826859045505
Mean error from LASSO: 0.505826859045505
X subset = 11 , i = 0 , j = 2
Mean error from standard least squares: 0.4980577136514983
Mean error from LASSO: 0.4980577136514983
X \text{ subset} = 11 , i = 0 , j = 3
Mean error from standard least squares: 0.5099889012208657
Mean error from LASSO: 0.5108213096559379
X \text{ subset} = 11 , i = 0 , j = 4
Mean error from standard least squares: 0.508046614872364
Mean error from LASSO: 0.5072142064372919
X \text{ subset} = 11 , i = 0 , j = 5
Mean error from standard least squares: 0.4944506104328524
Mean error from LASSO: 0.49334073251942284
X \text{ subset} = 11 , i = 0 , j = 6
Mean error from standard least squares: 0.5077691453940066
Mean error from LASSO: 0.5074916759156493
X subset = 11 , i = 0 , j = 7
Mean error from standard least squares: 0.49944506104328523
Mean error from LASSO: 0.49944506104328523
X \text{ subset} = 11 , i = 0 , j = 8
```

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Mean error from standard least squares: 0.5180355160932297
Mean error from LASSO: 0.5177580466148723
X \text{ subset} = 11 , i = 0 , j = 9
Mean error from standard least squares: 0.5055493895671476
Mean error from LASSO: 0.505826859045505
X \text{ subset} = 11 , i = 0 , j = 10
Mean error from standard least squares: 0.5169256381798002
Mean error from LASSO: 0.517480577136515
X \text{ subset} = 11 , i = 1 , j = 0
Mean error from standard least squares: 0.5360710321864595
Mean error from LASSO: 0.5357935627081021
X \text{ subset} = 11 , i = 1 , j = 2
Mean error from standard least squares: 0.5238623751387348
Mean error from LASSO: 0.5238623751387348
X \text{ subset} = 11 , i = 1 , j = 3
Mean error from standard least squares: 0.5363485016648168
Mean error from LASSO: 0.5366259711431742
X \text{ subset} = 11 , i = 1 , j = 4
Mean error from standard least squares: 0.5258046614872364
Mean error from LASSO: 0.5258046614872364
X \text{ subset} = 11, i = 1, j = 5
Mean error from standard least squares: 0.5416204217536071
Mean error from LASSO: 0.5416204217536071
X \text{ subset} = 11 , i = 1 , j = 6
Mean error from standard least squares: 0.5277469478357381
Mean error from LASSO: 0.5288568257491676
X \text{ subset} = 11 , i = 1 , j = 7
Mean error from standard least squares: 0.5321864594894562
Mean error from LASSO: 0.5324639289678136
X \text{ subset} = 11 , i = 1 , j = 8
Mean error from standard least squares: 0.5413429522752498
Mean error from LASSO: 0.5435627081021087
X \text{ subset} = 11 , i = 1 , j = 9
Mean error from standard least squares: 0.5288568257491676
Mean error from LASSO: 0.529134295227525
X \text{ subset} = 11 , i = 1 , j = 10
Mean error from standard least squares: 0.532741398446171
Mean error from LASSO: 0.5307991120976693
X \text{ subset} = 11 , i = 2 , j = 0
Mean error from standard least squares: 0.4553274139844617
Mean error from LASSO: 0.4236958934517203
X \text{ subset} = 11 , i = 2 , j = 1
Mean error from standard least squares: 0.43174250832408434
Mean error from LASSO: 0.41120976692563815
X subset = 11 , i = 2 , j = 3
Mean error from standard least squares: 0.4583795782463929
Mean error from LASSO: 0.4583795782463929
X \text{ subset} = 11 , i = 2 , j = 4
```

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Mean error from standard least squares: 0.4525527192008879
Mean error from LASSO: 0.4264705882352941
X \text{ subset} = 11 , i = 2 , j = 5
Mean error from standard least squares: 0.45172031076581576
Mean error from LASSO: 0.427857935627081
X subset = 11 , i = 2 , j = 6
Mean error from standard least squares: 0.4345172031076582
Mean error from LASSO: 0.41453940066592676
X \text{ subset} = 11 , i = 2 , j = 7
Mean error from standard least squares: 0.43895671476137627
Mean error from LASSO: 0.4298002219755827
X \text{ subset} = 11 , i = 2 , j = 8
Mean error from standard least squares: 0.43312985571587126
Mean error from LASSO: 0.4342397336293008
X \text{ subset} = 11 , i = 2 , j = 9
Mean error from standard least squares: 0.4514428412874584
Mean error from LASSO: 0.43562708102108766
X \text{ subset} = 11 , i = 2 , j = 10
Mean error from standard least squares: 0.45172031076581576
Mean error from LASSO: 0.43118756936736957
X \text{ subset} = 11 , i = 3 , j = 0
Mean error from standard least squares: 0.4550499445061043
Mean error from LASSO: 0.4367369589345172
X \text{ subset} = 11 , i = 3 , j = 1
Mean error from standard least squares: 0.4422863485016648
Mean error from LASSO: 0.4223085460599334
X subset = 11 , i = 3 , j = 2
Mean error from standard least squares: 0.46642619311875694
Mean error from LASSO: 0.45588235294117646
X \text{ subset} = 11 , i = 3 , j = 4
Mean error from standard least squares: 0.45643729189789123
Mean error from LASSO: 0.4406215316315205
X \text{ subset} = 11 , i = 3 , j = 5
Mean error from standard least squares: 0.46032186459489455
Mean error from LASSO: 0.42702552719200887
X \text{ subset} = 11 , i = 3 , j = 6
Mean error from standard least squares: 0.44200887902330743
Mean error from LASSO: 0.4345172031076582
X \text{ subset} = 11 , i = 3 , j = 7
Mean error from standard least squares: 0.45338512763596006
Mean error from LASSO: 0.4386792452830189
X \text{ subset} = 11 , i = 3 , j = 8
Mean error from standard least squares: 0.4528301886792453
Mean error from LASSO: 0.44367369589345174
X \text{ subset} = 11 , i = 3 , j = 9
Mean error from standard least squares: 0.459211986681465
Mean error from LASSO: 0.44589345172031075
X \text{ subset} = 11 , i = 3 , j = 10
```

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Mean error from standard least squares: 0.459211986681465
Mean error from LASSO: 0.44922308546059936
X \text{ subset} = 11 , i = 4 , j = 0
Mean error from standard least squares: 0.4733629300776915
Mean error from LASSO: 0.4733629300776915
X subset = 11 , i = 4 , j = 1
Mean error from standard least squares: 0.4902885682574917
Mean error from LASSO: 0.4900110987791343
X \text{ subset} = 11 , i = 4 , j = 2
Mean error from standard least squares: 0.48612652608213097
Mean error from LASSO: 0.4730854605993341
X \text{ subset} = 11 , i = 4 , j = 3
Mean error from standard least squares: 0.48473917869034405
Mean error from LASSO: 0.48501664816870144
X \text{ subset} = 11 , i = 4 , j = 5
Mean error from standard least squares: 0.4741953385127636
Mean error from LASSO: 0.47447280799112096
X \text{ subset} = 11 , i = 4 , j = 6
Mean error from standard least squares: 0.4755826859045505
Mean error from LASSO: 0.47613762486126526
X \text{ subset} = 11 , i = 4 , j = 7
Mean error from standard least squares: 0.4728079911209767
Mean error from LASSO: 0.4733629300776915
X \text{ subset} = 11 , i = 4 , j = 8
Mean error from standard least squares: 0.47780244173140957
Mean error from LASSO: 0.47780244173140957
X subset = 11 , i = 4 , j = 9
Mean error from standard least squares: 0.4875138734739179
Mean error from LASSO: 0.46781354051054386
X \text{ subset} = 11 , i = 4 , j = 10
Mean error from standard least squares: 0.4811320754716981
Mean error from LASSO: 0.46503884572697
X \text{ subset} = 11 , i = 5 , j = 0
Mean error from standard least squares: 0.4955604883462819
Mean error from LASSO: 0.4972253052164262
X \text{ subset} = 11 , i = 5 , j = 1
Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.49472807991120976
X \text{ subset} = 11 , i = 5 , j = 2
Mean error from standard least squares: 0.49528301886792453
Mean error from LASSO: 0.4955604883462819
X \text{ subset} = 11 , i = 5 , j = 3
Mean error from standard least squares: 0.5083240843507214
Mean error from LASSO: 0.5119311875693674
X subset = 11 , i = 5 , j = 4
Mean error from standard least squares: 0.5052719200887902
Mean error from LASSO: 0.5052719200887902
X \text{ subset} = 11 , i = 5 , j = 6
```

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Mean error from standard least squares: 0.5049944506104328
Mean error from LASSO: 0.5086015538290788
X \text{ subset} = 11 , i = 5 , j = 7
Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.49639289678135406
X \text{ subset} = 11 , i = 5 , j = 8
Mean error from standard least squares: 0.5011098779134295
Mean error from LASSO: 0.5005549389567148
X \text{ subset} = 11 , i = 5 , j = 9
Mean error from standard least squares: 0.5086015538290788
Mean error from LASSO: 0.5083240843507214
X \text{ subset} = 11 , i = 5 , j = 10
Mean error from standard least squares: 0.5083240843507214
Mean error from LASSO: 0.5072142064372919
X \text{ subset} = 11 , i = 6 , j = 0
Mean error from standard least squares: 0.49889012208657046
Mean error from LASSO: 0.4983351831298557
X \text{ subset} = 11 , i = 6 , j = 1
Mean error from standard least squares: 0.5063817980022197
Mean error from LASSO: 0.505826859045505
X \text{ subset} = 11 , i = 6 , j = 2
Mean error from standard least squares: 0.494173140954495
Mean error from LASSO: 0.48834628190899
X \text{ subset} = 11 , i = 6 , j = 3
Mean error from standard least squares: 0.48834628190899
Mean error from LASSO: 0.48834628190899
X \text{ subset} = 11 , i = 6 , j = 4
Mean error from standard least squares: 0.4958379578246393
Mean error from LASSO: 0.4997225305216426
X \text{ subset} = 11 , i = 6 , j = 5
Mean error from standard least squares: 0.4875138734739179
Mean error from LASSO: 0.48668146503884574
X \text{ subset} = 11 , i = 6 , j = 7
Mean error from standard least squares: 0.5113762486126526
Mean error from LASSO: 0.5133185349611543
X \text{ subset} = 11 , i = 6 , j = 8
Mean error from standard least squares: 0.5033296337402886
Mean error from LASSO: 0.5063817980022197
X \text{ subset} = 11 , i = 6 , j = 9
Mean error from standard least squares: 0.49639289678135406
Mean error from LASSO: 0.4938956714761376
X \text{ subset} = 11 , i = 6 , j = 10
Mean error from standard least squares: 0.4958379578246393
Mean error from LASSO: 0.4975027746947836
X subset = 11 , i = 7 , j = 0
Mean error from standard least squares: 0.4836293007769145
Mean error from LASSO: 0.48335183129855713
X \text{ subset} = 11 , i = 7 , j = 1
```

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Mean error from standard least squares: 0.47475027746947834
Mean error from LASSO: 0.4758601553829079
X \text{ subset} = 11 , i = 7 , j = 2
Mean error from standard least squares: 0.47669256381798003
Mean error from LASSO: 0.47669256381798003
X \text{ subset} = 11 , i = 7 , j = 3
Mean error from standard least squares: 0.4797447280799112
Mean error from LASSO: 0.4797447280799112
X \text{ subset} = 11 , i = 7 , j = 4
Mean error from standard least squares: 0.48057713651498335
Mean error from LASSO: 0.47197558268590456
X \text{ subset} = 11 , i = 7 , j = 5
Mean error from standard least squares: 0.4714206437291898
Mean error from LASSO: 0.47031076581576026
X \text{ subset} = 11 , i = 7 , j = 6
Mean error from standard least squares: 0.47669256381798003
Mean error from LASSO: 0.4772475027746948
X \text{ subset} = 11 , i = 7 , j = 8
Mean error from standard least squares: 0.4700332963374029
Mean error from LASSO: 0.4700332963374029
X \text{ subset} = 11 , i = 7 , j = 9
Mean error from standard least squares: 0.47835738068812433
Mean error from LASSO: 0.4714206437291898
X \text{ subset} = 11 , i = 7 , j = 10
Mean error from standard least squares: 0.4836293007769145
Mean error from LASSO: 0.470865704772475
X \text{ subset} = 11 , i = 8 , j = 0
Mean error from standard least squares: 0.4692008879023307
Mean error from LASSO: 0.46365149833518315
X \text{ subset} = 11 , i = 8 , j = 1
Mean error from standard least squares: 0.4655937846836848
Mean error from LASSO: 0.46420643729189787
X \text{ subset} = 11 , i = 8 , j = 2
Mean error from standard least squares: 0.4852941176470588
Mean error from LASSO: 0.4855715871254162
X \text{ subset} = 11 , i = 8 , j = 3
Mean error from standard least squares: 0.4669811320754717
Mean error from LASSO: 0.4692008879023307
X \text{ subset} = 11 , i = 8 , j = 4
Mean error from standard least squares: 0.4669811320754717
Mean error from LASSO: 0.46781354051054386
X \text{ subset} = 11 , i = 8 , j = 5
Mean error from standard least squares: 0.46004439511653716
Mean error from LASSO: 0.4608768035516093
X \text{ subset} = 11 , i = 8 , j = 6
Mean error from standard least squares: 0.4697558268590455
Mean error from LASSO: 0.4697558268590455
X \text{ subset} = 11 , i = 8 , j = 7
```

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Mean error from standard least squares: 0.4730854605993341
Mean error from LASSO: 0.4733629300776915
X \text{ subset} = 11 , i = 8 , j = 9
Mean error from standard least squares: 0.4772475027746948
Mean error from LASSO: 0.4772475027746948
X \text{ subset} = 11 , i = 8 , j = 10
Mean error from standard least squares: 0.4653163152053274
Mean error from LASSO: 0.4653163152053274
X \text{ subset} = 11 , i = 9 , j = 0
Mean error from standard least squares: 0.45033296337402884
Mean error from LASSO: 0.42480577136514985
X \text{ subset} = 11 , i = 9 , j = 1
Mean error from standard least squares: 0.43146503884572696
Mean error from LASSO: 0.43146503884572696
X \text{ subset} = 11 , i = 9 , j = 2
Mean error from standard least squares: 0.4481132075471698
Mean error from LASSO: 0.4483906770255272
X \text{ subset} = 11 , i = 9 , j = 3
Mean error from standard least squares: 0.4542175360710322
Mean error from LASSO: 0.45394006659267483
X \text{ subset} = 11 , i = 9 , j = 4
Mean error from standard least squares: 0.4630965593784684
Mean error from LASSO: 0.42452830188679247
X \text{ subset} = 11 , i = 9 , j = 5
Mean error from standard least squares: 0.4325749167591565
Mean error from LASSO: 0.43340732519422864
X \text{ subset} = 11 , i = 9 , j = 6
Mean error from standard least squares: 0.4267480577136515
Mean error from LASSO: 0.4253607103218646
X \text{ subset} = 11 , i = 9 , j = 7
Mean error from standard least squares: 0.4428412874583796
Mean error from LASSO: 0.42064372918978915
X \text{ subset} = 11 , i = 9 , j = 8
Mean error from standard least squares: 0.44034406215316313
Mean error from LASSO: 0.4381243063263041
X \text{ subset} = 11 , i = 9 , j = 10
Mean error from standard least squares: 0.4464483906770255
Mean error from LASSO: 0.42758046614872364
X \text{ subset} = 11 , i = 10 , j = 0
Mean error from standard least squares: 0.45033296337402884
Mean error from LASSO: 0.41870144284128746
X \text{ subset} = 11 , i = 10 , j = 1
Mean error from standard least squares: 0.4483906770255272
Mean error from LASSO: 0.4159267480577136
X subset = 11 , i = 10 , j = 2
Mean error from standard least squares: 0.45588235294117646
Mean error from LASSO: 0.4553274139844617
X \text{ subset} = 11 , i = 10 , j = 3
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Mean error from standard least squares: 0.46392896781354054
Mean error from LASSO: 0.46392896781354054
X \text{ subset} = 11 , i = 10 , j = 4
Mean error from standard least squares: 0.4556048834628191
Mean error from LASSO: 0.42480577136514985
X \text{ subset} = 11 , i = 10 , j = 5
Mean error from standard least squares: 0.43479467258601556
Mean error from LASSO: 0.4109322974472808
X \text{ subset} = 11 , i = 10 , j = 6
Mean error from standard least squares: 0.41870144284128746
Mean error from LASSO: 0.41897891231964485
X \text{ subset} = 11 , i = 10 , j = 7
Mean error from standard least squares: 0.43174250832408434
Mean error from LASSO: 0.4125971143174251
X \text{ subset} = 11 , i = 10 , j = 8
Mean error from standard least squares: 0.43340732519422864
Mean error from LASSO: 0.4339622641509434
X \text{ subset} = 11 , i = 10 , j = 9
Mean error from standard least squares: 0.4442286348501665
Mean error from LASSO: 0.422031076581576
Mean Error for pure least squares, X subset 1 =
Mean Error for LASSO, X subset 1 = [0.52573656]
Mean Error for pure least squares, X subset 2 =
                                                  [0.46912521]
Mean Error for LASSO, X subset 2 = [0.46006962]
Mean Error for pure least squares, X subset 3 = [0.47820099]
Mean Error for LASSO, X subset 3 = [0.47896277]
Mean Error for pure least squares, X subset 4 = [0.49562355]
Mean Error for LASSO, X subset 4 = [0.49523509]
Mean Error for pure least squares, X subset 5 = [0.47164262]
Mean Error for LASSO, X subset 5 = [0.46202704]
Mean Error for pure least squares, X subset 6 = [0.413258]
Mean Error for LASSO, X subset 6 = [0.413258]
Mean Error for pure least squares, X subset 7 = [0.44235698]
Mean Error for LASSO, X subset 7 = [0.4423595]
Mean Error for pure least squares, X subset 8 = [0.43358137]
Mean Error for LASSO, X subset 8 = [0.43358137]
Mean Error for pure least squares, X subset 9 = [0.49982595]
Mean Error for LASSO, X subset 9 = [0.48122793]
Mean Error for pure least squares, X subset 10 = [0.4582585]
Mean Error for LASSO, X subset 10 = [0.44681162]
Mean Error for pure least squares, X subset 11 = [0.46232469]
Mean Error for LASSO, X subset 11 = [0.4515841]
```