# Additional Exercise _ 12·11



1) A company mfgs LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the avg value of this process. Also evaluate the std dev associated with it.

Soln $n = 6$

prob of 2 wrong LED's

$$P(x) = \frac{n!}{x!(n-x)!} \cdot p^x \cdot (1-p)^{n-x}$$

$$P(2) = \frac{6!}{2!\,4!} (0.3)^2 (0.7)^{6-2}$$

$$= 15 \times (0.3)^2 (0.7)^4$$

$$= 0.3241$$

Avg value of the processes $= np = 6 \times 0.3 = 1.8$

Std dev $= \sigma = \sqrt{np(1-p)} = \sqrt{6 \times 0.3 \times 0.7} = 1.12$

---

2) Gaurav attempts 8 questions with a correction rate of 75% while Barakha avg around 12 questions / day with correctn rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solns? What do you infer from it?

Soln

| | Gaurav | Barakha |
|---|---|---|
| $n=$ | 8 | 12 |
| $p$ | 0.75 | 0.45 |
| $1-p$ | 0.25 | 0.55 |
| $x$ | 5 | 5 |

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

a) probability of each of them solving 5 questions correctly

Gamav: $^8C_5 (0.75)^5 (0.25)^3 = 0.20 \Rightarrow 20\%$

Barakha: $^{12}C_5 (0.45)^5 (0.55)^7 = 0.22 \Rightarrow 22\%$

b) 4 questions correctly

Gamav: $^8C_4 (0.75)^4 (0.25)^4 = 9\%$

Barakha: $^{12}C_4 (0.45)^4 (0.55)^8 = 17\%$

c) 6 questions correctly

Gamav: $^8C_6 (0.75)^6 (0.25)^2 = 31\%$

Barakha: $^{12}C_6 (0.45)^6 (0.55)^6 = 21\%$

Inference: Barakha has ability to answer 12 questions per day. But the correction rate is less. when the correctness of answer increases, then the probability of Gamav increases & Barakha decreases.

3) Customer arrive at a rate of 72/hr to my shop. what is the probability of customers arriving in 4 mins a) 5 customers, b) not more than 3 customers, c) than 3 customers.

Soln $P(x) = \dfrac{e^{-\mu} \mu^x}{x!}$ $\qquad \mu (1\,hr) = 72$

$\mu (4\,min) = \dfrac{72}{60} \times 4 = 4.8/4\,mints$

$P(x=4) = \dfrac{e^{-4.8}(4.8)^4}{4!} = 0.181$

$P(x=5) = \dfrac{e^{-4.8}(4.8)^5}{5!} = 0.174$

$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= \dfrac{e^{-4.8} \times (4.8)^0}{0!} + \dfrac{e^{-4.8}(4.8)^1}{1!} + \dfrac{e^{-4.8}(4.8)^2}{2!} + \dfrac{e^{-4.8}(4.8)^3}{3!}$

$= e^{-4.8}\left[1 + 4.8 + \dfrac{4.8^2}{2!} + \dfrac{(4.8)^3}{3!}\right]$

$P(x \leq 3) = 0.2931$

$P(x>3) = 1 - P(x \leq 3)$

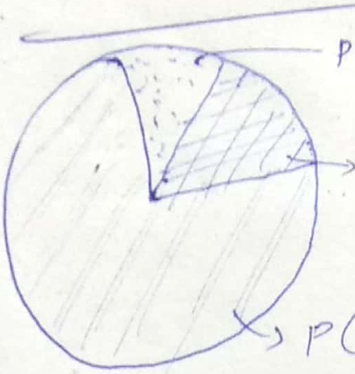
$= 1 - 0.2931$

$= 0.7069$

$\therefore P(x \geq 3) = 0.7069$

$P(x=5) = 0.174$

$P(x \leq 3) = 0.2931$




4) Efficiency of entering 77 words / min with 6 errors / hr. what is the probability 2 errors in a 455-word report? what happens when no., of words increases / decreases (in case of 1000 words, 255 words)? How is $\lambda$ affected? How does it influence the PMF?

Soln 77 words / min. $\mu = \frac{6}{60} \times \frac{1}{77} = 0.0013$

6 errors / hr

$\mu_{455} = 0.0013 \times 455 = 0.59$

$P(x=2) = \frac{e^{-0.59} \times (0.59)^2}{2!} = \frac{0.554 \times 0.3481}{2} = 0.096$

$\mu_{1000} = 0.0013 \times 1000 = 1.3$

$\mu_{255} = 0.0013 \times 255 = 0.3315$

$P(x=2)_{\mu=1000} = \frac{e^{1.3} \times 1.3^2}{2!} = \frac{0.277 \times 1.69}{2} = 0.229$

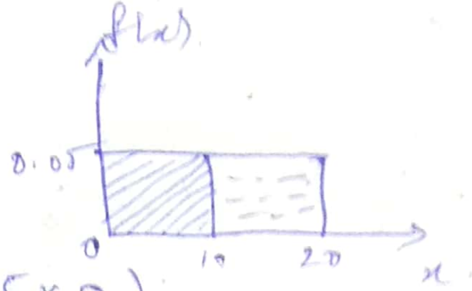
$P(x=2)_{\mu=255} = \frac{e^{-0.35} \times (0.315)^2}{2!} = \frac{0.729 \times 0.109}{2} = 0.039$

$\therefore$ $\mu$ increases as words increases and along with that probability also increases.

5) Assume that the range of X is [0, 20 mA]. The probability PDF is given by $f(x) = 0.05$ for $0 \le x \le 20$. What is the probability that a current measurement is less than 10 mA? Draw PDF & CDF.

Soln. PDF $f(x) = 0.05$ for $0 \le x \le 20$

$$P(x \le 10) = \int_0^{10} x\, f(x)\, dx.$$




$$= \int_0^{10} 0.05x\, dx.$$

$$= (0.05 \times 10) - (0.05 \times 0)$$

$$P(x < 10) = 0.5$$

CDF

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 0.05x & \text{for } 0 \le x \le 20 \\ 1 & \text{for } x \ge 20. \end{cases}$$