

Assignment - 16:

1) Two tailed test for difference b/w 2 populations.

Population 1: Bangalore to Chennai

$$n_1 = 1200$$

$$\bar{x}_1 = 452$$

$$s_1 = 212$$

Population 2: Bangalore to Hosur

$$n_2 = 800$$

$$\bar{x}_2 = 523$$

$$s_2 = 185$$

Solu

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(452 - 523) - 0}{\sqrt{\frac{212^2}{1200} + \frac{185^2}{800}}}$$
$$= \frac{-71}{\sqrt{80.234}} = \frac{-71}{8.96} = -7.926$$

P-value: $p(z < -7.926) \approx 0$

H_0 is rejected at any level.

2) Population 1: Duracell

$$n_1 = 100$$

$$\bar{x}_1 = 308$$

$$s_1 = 84$$

Population 2: Energizer

$$n_2 = 100$$

$$\bar{x}_2 = 254$$

$$s_2 = 67$$

Solu $H_0: \mu_1 - \mu_2 \leq 45$

$$H_1: \mu_1 - \mu_2 > 45$$

$$t = \frac{(308 - 254) - 45}{\sqrt{\frac{84^2}{100} + \frac{67^2}{100}}} = \frac{9}{\sqrt{115.45}} = 0.838$$

P-value: $p(t > 0.838) = 0.201$

H_0 may not be rejected

3) Population 1: Price of sugar = Rs 27.50

$$n_1 = 14$$

$$\bar{x}_1 = 0.317\%$$

$$s_1 = 0.12\%$$

Population 2:

Price of sugar = Rs 20.0

$$n_2 = 9$$

$$\bar{x}_2 = 0.21\%$$

$$s_2 = 0.11\%$$

Solu $df = (n_1 + n_2 - 2)$

$$= (14 + 9 - 2)$$

$$= 21$$

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_1: \mu_1 - \mu_2 \neq 0$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\left[\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \right] \left[\frac{1}{n_1} + \frac{1}{n_2} \right]}}$$

$$= \frac{0.107}{\sqrt{0.00247}} = 2.154$$

$$t_c = 2.080$$

H_0 is rejected at 95% level.

4) Population 1: Before education

$$n_1 = 15$$

$$\bar{x}_1 = \text{Rs } 6598$$

$$s_1 = \text{Rs } 844$$

Population 2: After reduction

$$n_2 = 12$$

$$\bar{x}_2 = \text{Rs } 6870$$

$$s_2 = 669$$

Solu $df = (n_1 + n_2) - 2$

$$= (15 + 12) - 2$$

$$df = 25$$

$$H_0: \mu_2 - \mu_1 \leq 0$$

$$H_1: \mu_2 - \mu_1 > 0$$

$$t = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)_0}{\sqrt{\left[\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \right] \left[\frac{1}{n_1} + \frac{1}{n_2} \right]}}$$

$$= \frac{(6870 - 6598) - 0}{\sqrt{\left[\frac{(15 - 1)844^2 + (12 - 1)669^2}{15 + 12 - 2} \right] \left[\frac{1}{15} + \frac{1}{12} \right]}}$$

$$= \frac{272}{\sqrt{89375.25}} = 0.91$$

$$= \frac{272}{\sqrt{89375.25}} = 0.91$$

$$t_c = t_{0.10} = 1.316$$

H_0 is rejected at 10% level.

Comparisons of two population proportions when the hypothesised diff is 0

5) Population 1 : 1980

$$n_1 = 1000$$

$$x_1 = 53$$

$$\hat{p}_1 = 0.53$$

Soln

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{53 + 43}{1000 + 100} = \frac{96}{1100} = 0.0872$$

Population 2 : 1985

$$n_2 = 100$$

$$x_2 = 43$$

$$\hat{p}_2 = 0.43$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - D}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.53 - 0.43}{\sqrt{(0.0872)(0.9128)\left(\frac{1}{1000} + \frac{1}{100}\right)}} = \frac{0.1}{\sqrt{0.002496(0.01)}} = \frac{0.1}{\sqrt{0.00793 \times 0.01}} = 0.02816$$

$$Z_c = Z_{0.05} = 1.645$$

H_0 may not be rejected at 10%.

6) with sweepstakes

$$n_1 = 300$$

$$x_1 = 120$$

$$\hat{p}_1 = 0.40$$

no sweepstakes

$$n_2 = 700$$

$$x_2 = 140$$

$$\hat{p}_2 = 0.20$$

Soln

$$H_0: p_1 - p_2 \leq 0.1$$

$$H_1: p_1 - p_2 > 0.10$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - D}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$$

$$Z = \frac{(0.40 - 0.20) - 0.10}{\sqrt{\frac{(0.40)(0.60)}{300} + \frac{(0.20)(0.80)}{700}}} = \frac{0.10}{0.03207} = 3.118$$

$$Z_c = Z_{0.001} = 3.09$$

H_0 is accepted.

7) A die is thrown 132 times
No. turned up: 1, 2, 3, 4, 5, 6

: 16, 20, 25, 14, 29, 28

Freq

Is the die unbiased? Consider the df as -1

Soln Unbiased die, $\frac{132}{6} = 22$ times turns up

Observed (O)	Expected (E)	$(O-E)^2$	$\frac{(O-E)^2}{E}$
16	22	36	1.64
20	22	4	0.18
25	22	9	0.41
14	22	64	2.91
29	22	49	2.23
28	22	36	1.64
			$\sum \frac{(O-E)^2}{E} = 9.01$

$$df = n-1 = 6-1 = 5$$

At 5% significance, $\chi^2 = 11.07$
 $\therefore \chi^2_0 < \chi^2_c$. There is no evidence against hypothesis that the die is biased.

8) Sample mean = 10,000

	Men	Women	Total
Voted	2792	3591	6383
Not voted	1486	2131	3617
Observed	4278	5722	10,000

Is "gender and voting independent?"

$$\sum \frac{(O-E)^2}{E} = 6.58$$

$$\chi^2 = 6.58$$

From table 10% = 2.71

5% = 3.84

1% = 6.64

$$3.84 < \chi^2 < 6.64$$

$$1\% < p\text{-value} < 5\%$$

Expected	Men	Women	Total
Voted	2731	3652	6383
Not voted	1547	2070	3617
Total	4278	5722	10000

H_0 is rejected

\therefore Sex and voting are dependent in this town.

- 9) A sample of 100 voters which are 4 candidates they would vote in election.

Higgins Reardon White Charlton
41 19 24 16

Is all candidates are equally popular?

[Chi-Square = 14.96, with 3 df, $\alpha < 0.05$]

Solu

O	E	O - E	(O - E) ²	$\frac{(O - E)^2}{E}$
41	25	16	256	10.24
19	25	-6	36	1.44
24	25	-1	1	0.04
16	25	-9	81	3.24
$\sum \frac{(O - E)^2}{E} = 14.96$				

$df = n - 1$

$4 - 1 = 3$

χ^2_c at 0.05 level 3 df is 7.82
 χ^2_e is $> \chi^2_c$ 95% $> 14.96 > 99\%$

Candidates are not equally preferred

- 10) Is there a significant ~~pre~~ relationship b/w age and photograph preference? Chi-Square = 29.6
 $df = 4$
 $\alpha^2 = 0.05$

		A	B	C	
Age of child	5-6 yrs	18	22	20	60
	7-8 yrs	2	28	40	70
	9-10 yrs	20	10	40	70
		40	60	100	200

$E = \frac{\text{row total} \times \text{column total}}{\text{Grand Total}}$

Solu

O	E	O - E	(O - E) ²	$\frac{(O - E)^2}{E}$
18	12	6	36	3
22	18	4	16	0.89
20	30	-10	100	3.33
2	14	-12	144	10.29
28	21	7	49	2.33
40	35	5	25	0.71
20	14	6	36	2.57
10	21	11	121	5.76
40	35	5	25	0.71

$\chi^2 = 29.60$

χ^2_c is 18.46 at 0.001

$df = 4$

χ^2_e is $> \chi^2_c$

H_0 is accepted

\therefore There is significant relationship b/w age of child & photograph.

11) Asch paradigm

	Support	No Support	Total
Confirm	18	40	58
Not Confirm	32	10	42
Total	50	50	100

Is there a significant diff b/w "the support" and "no support" cond?

$$\chi^2 = 19.87$$

$$df = 1 \quad p < 0.05$$

Solu

O	E	O-E	(O-E) ²	$\frac{(O-E)^2}{E}$
18	29	-11	121	4.17
40	29	11	121	4.17
32	21	11	121	5.76
10	21	-11	121	5.76

$$\Sigma = 19.86$$

$$\therefore \chi^2 = 19.86$$

There is significant diff b/w support and no support conditions.

12) The following table shows the freq with which 43 short people and 52 tall people were categorized as "leaders", "followers" or "unclassifiable". Is there diff b/w height and leadership qualities? [$\chi^2 = 10.71$, with 2 df, $p < 0.01$]

	Height short	Tall	Total
Leaders	12	32	44
Followers	22	14	36
Unclassifiable	9	6	15
Total	43	52	95

$\therefore 10.712$ is bigger than χ^2 at 0.01 significance level.

There is a relationship b/w height and leadership qualities.

Solu

O	E	O-E	(O-E) ²	$\frac{(O-E)^2}{E}$
12	19.92	-7.92	62.72	3.148
32	24.08	7.92	62.72	2.60
22	16.29	5.71	32.6	2
14	19.71	-5.71	32.6	1.654
9	6.79	2.21	4.88	0.720
6	8.21	-2.21	4.88	0.6

$$\Sigma = 10.712$$

13) The results for men in California age 35-44 can be cross-tabulated by marital status

	Married	Widow	Never married	Total
Employed	679	103	114	896
Unemployed	63	10	20	93
Not in force	42	18	25	85
Total	784	131	159	1074

<u>Obs</u>	O	E	O - E	(O - E) ²	$\frac{(O - E)^2}{E}$
	679	656	23	529	0.80
	103	109	-6	36	0.33
	114	133	-19	361	2.71
	63	68	-5	25	0.36
	42	11	31	961	8.73
	20	14	6	36	2.57
	42	62	-20	400	6.45
	18	10	8	64	6.4
	25	13	12	144	11.07

$$\Sigma = 30.95$$

Chi square dist
with $(3-1)(3-1)$
 $= 2 \times 2 = 4$ df

Since $30.96 > 13.28$ conclude $P < 0.01$
and reject all confidence. Marital Status seems
to be related to job status in this town.