Lecture 2
ADTs, Stacks, Queues

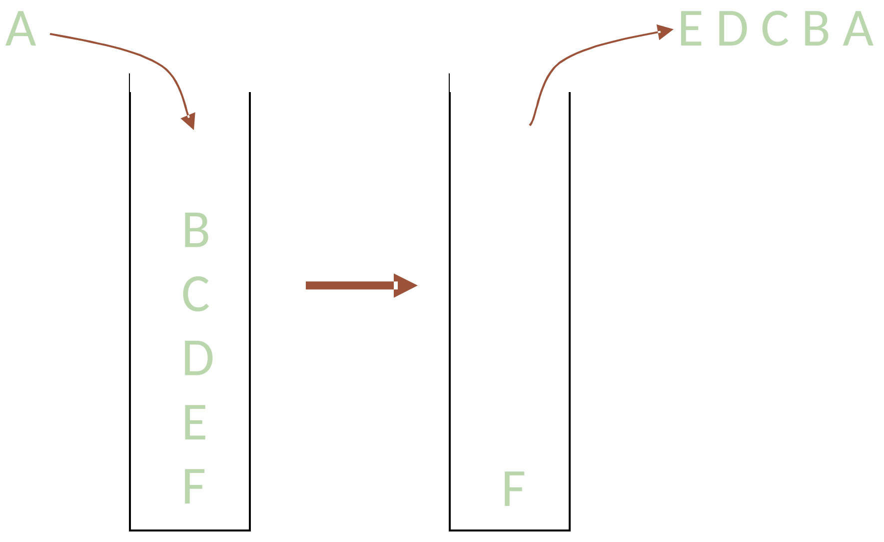
Stacks

- The ***Stack*** ADT supports operations:
  - **`isEmpty`**: have there been same number of pops as pushes
  - **`push`**: takes an item
  - **`pop`**: raises an error if empty, else returns most-recently pushed item not yet returned by a pop
  - … (possibly more operations)

- A Stack data structure could use a linked-list or an array or something else, and associated algorithms for the operations

- One implementation is in the library **`java.util.Stack`**

The Stack ADT



Operations:

`create`
`destroy`
`push`
`pop`
`top`
`is_empty`

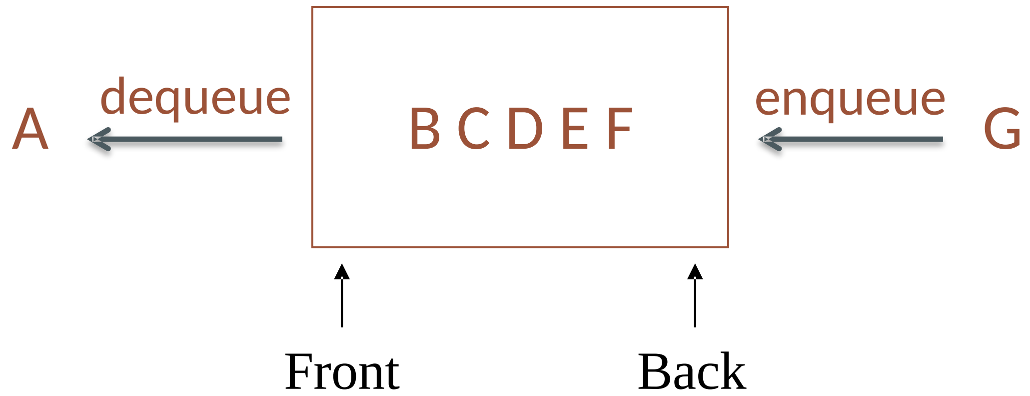
Can also be implemented with an array or a linked list

Why Stack ADT is useful

- It arises all the time in programming
  - Recursive function calls
  - Balancing symbols (parentheses)
  - Evaluating postfix notation: 3 4 + 5 *
  - Clever: Infix ((3+4) * 5) to postfix conversion (see text)
- We can code up a reusable library
- We can communicate in high-level terms
  - "Use a stack and push numbers, popping for operators…"
  - Rather than, "create a linked list and add a node when…"

The Queue ADT

- Operations
  `create`
  `destroy`
  `enqueue`
  `dequeue`
  `is_empty`



- Just like a stack except:
  - Stack: LIFO (last-in-first-out)
  - Queue: FIFO (first-in-first-out)
- Just as useful and ubiquitous

Data Structure Analysis Practice

For each of the following, pick the best Data Structure (**Stack, Queue, either, or neither**) and Implementation (**Array, Linked List, either, or neither**):

- Maintain a collection of customers at a store with a relatively constant stream of customers at all times
- Keep track of a ToDo list
- Maintain a sorted student directory
- Manage the history of webpages visited to be used by the “back” button
- Store data and access the *kth* element often

Amortized Runtime Complexity

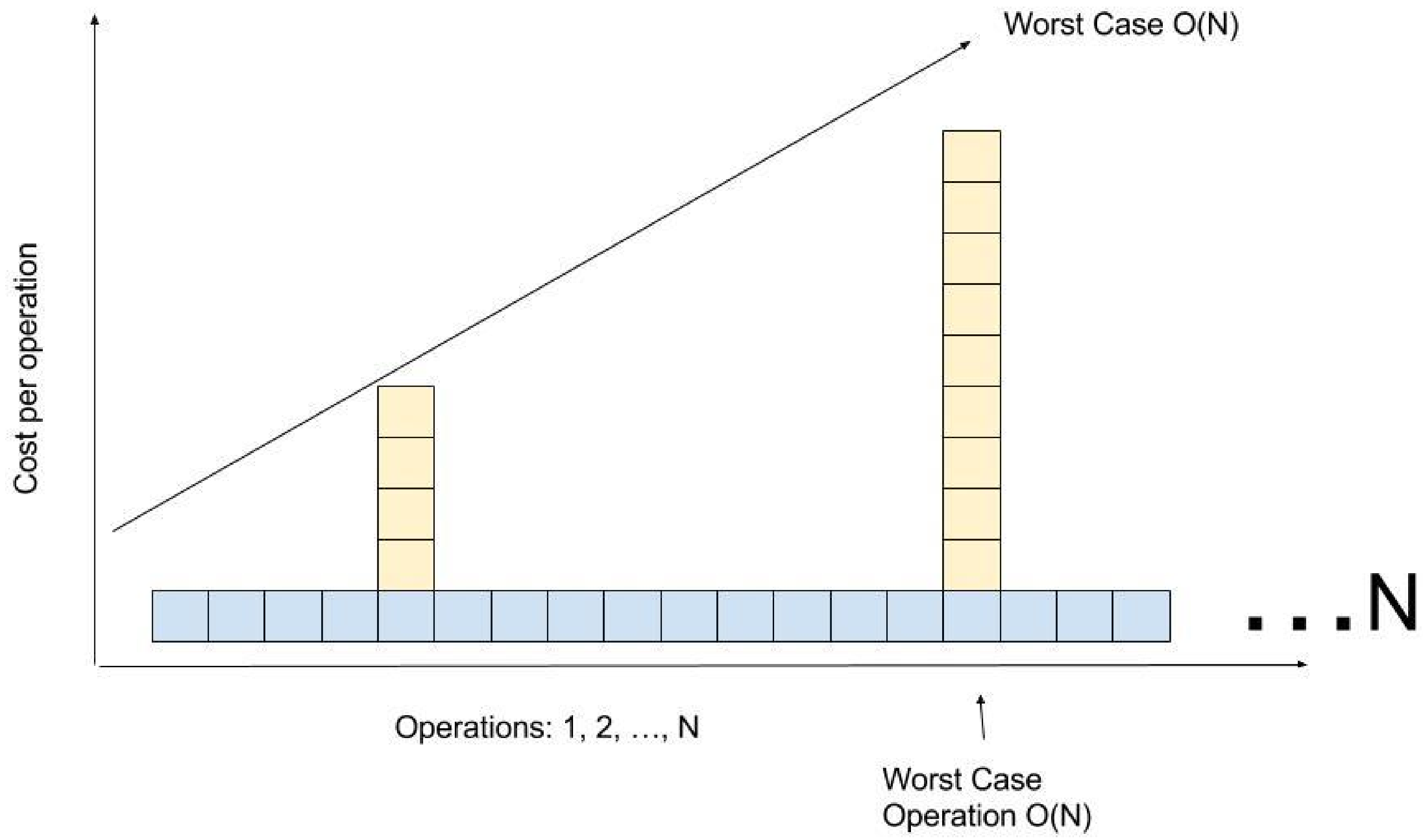
- Think about a stack implemented as an array that doubles its size any time it runs out of room
  - How can we claim **push** is $O(\mathbf{1})$ time if resizing is $O(\mathbf{n})$ time?
  - We *can't*, but we *can* claim it's an $O(\mathbf{1})$ amortized operation
- What does amortized mean?
- When are amortized bounds good enough?
- How can we prove an amortized bound?

Will just do two simple examples

- Text has more sophisticated examples and proof techniques
- *Idea* of how amortized describes average cost is essential

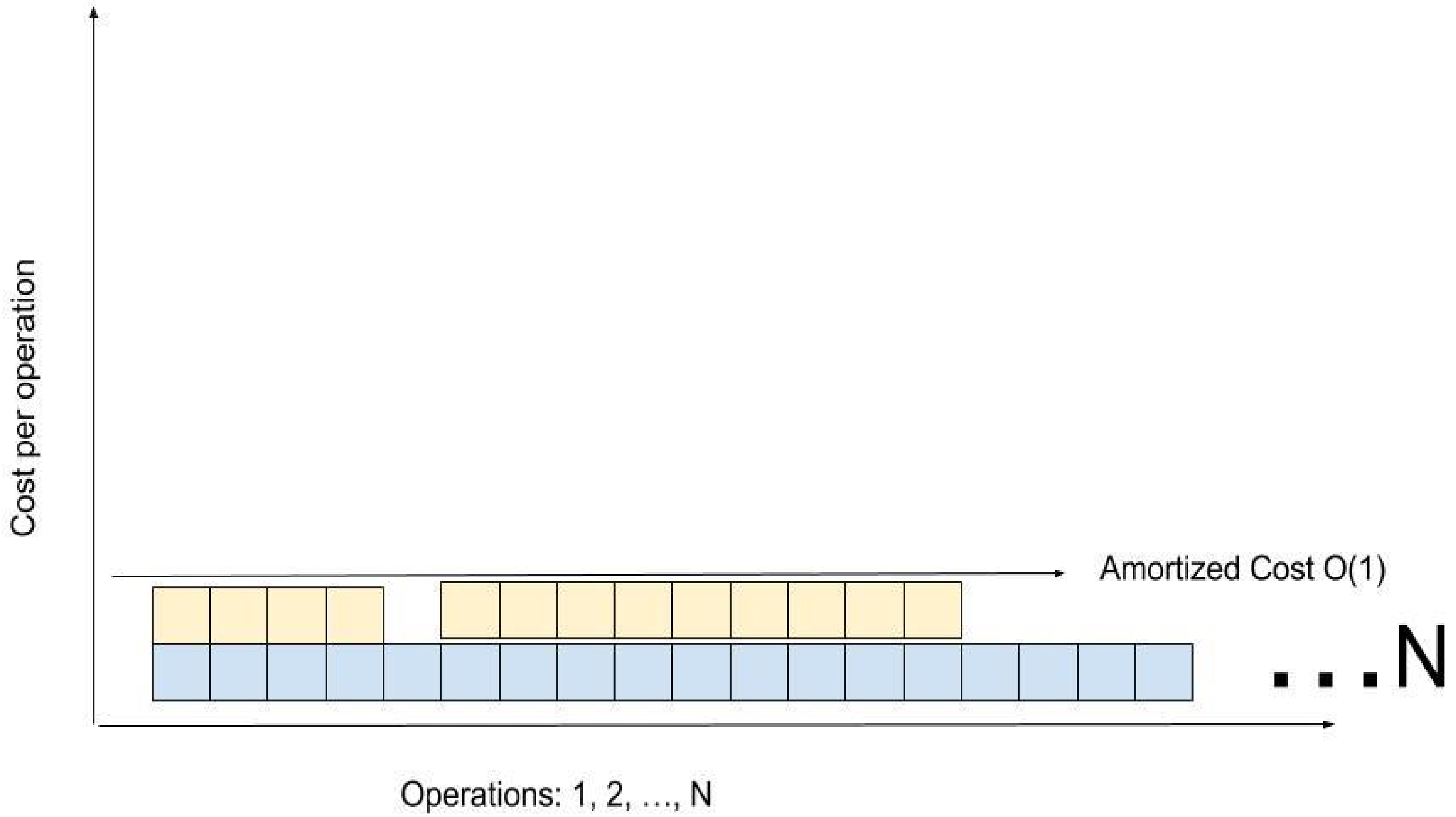
Amortized Runtime Intuition

Consider implementing a Stack with an array. What if we had initially 5 empty slots, and every time it gets full, we add an additional size * 2 slots and have to copy over all the old data? What is the **worst case runtime for the `add(element)`** operation?



Amortized Runtime Intuition

Consider implementing a Stack with an Array. What if we had initially 5 empty slots, and every time it gets full, we add an additional size * 2 slots and have to copy over all the old data? What is the **amortized runtime for the `add(element)`** operation?



“Building Up Credit” Intuition

- Can think of preceding “cheap” operations as building up “credit” that can be used to “pay for” later “expensive” operations

- Because any sequence of operations must be under the bound, enough “cheap” operations must come *first*
  - Else a prefix of the sequence, which is also a sequence, would violate the bound

Amortized Runtime Complexity

If a sequence of $\mathbf{M}$ operations takes $O(\mathbf{M\,f(n)})$ time,
we say the amortized runtime is $O(\mathbf{f(n)})$

Amortized bound: worst-case guarantee over sequences of operations

- Example: If any $\mathbf{n}$ operations take $O(\mathbf{n})$, then amortized $O(\mathbf{1})$
- Example: If any $\mathbf{n}$ operations take $O(\mathbf{n}^3)$, then amortized $O(\mathbf{n}^2)$

- The worst case time per operation can be larger than $\mathbf{f(n)}$
  - As long as the worst case is *always* “rare enough” in *any* sequence of operations

Amortized guarantee ensures the average time per operation for any sequence is $O(\mathbf{f(n)})$

Example: Resizing stack

A stack implemented with an array where we double the size of the array if it becomes full

Claim: Any sequence of **`push`**/**`pop`**/**`isEmpty`** is amortized $O(\mathbf{1})$

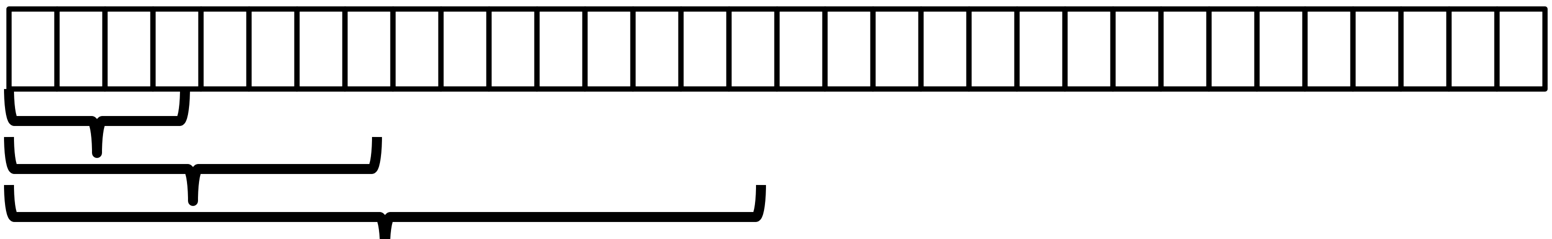
Need to show any sequence of **`M`** operations takes time $O(\mathbf{M})$

- Recall the non-resizing work is $O(\mathbf{M})$ (i.e., $\mathbf{M}*O(\mathbf{1})$)
- The resizing work is proportional to the total number of element copies we do for the resizing
- So it suffices to show that:

After **`M`** operations, we have done **`< 2M`** total element copies

(So average number of copies per operation is bounded by a constant)

Amount of copying



After **`M`** operations, we have done **`< 2M`** total element copies

Let **`n`** be the size of the array after **`M`** operations

- Then we have done a total of:

  `n/2 + n/4 + n/8 + … INITIAL_SIZE < n`

  element copies
- Because we must have done at least enough **`push`** operations to cause resizing up to size **`n`**:

  $$\mathbf{M} \geq \mathbf{n/2}$$
- So

  $\mathbf{2M} \geq \mathbf{n} >$ *number of element copies*