

# 6.854 Final Project

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December 12, 2016

## Abstract

In this paper we review and implement two algorithms presented by Bokal et al.[1], and Chan and Pratt [2], presented in SoCG'15 and SoCG'16 respectively. Both works introduce novel approaches to finding maximal subsequences with given hereditary properties.

## 1 Introduction

With increasing number of sensors and location-tracking devices, there are massive datasets describing movements of people, animals, robots, etc. Since the datasets mostly describe trajectories of movement, often each location point is associated with a timestamp. As a result, in the process of analysing such datasets, efficient algorithms for time range queries are required.

This paper discusses problems of the following nature: given a sequence of points  $S = s_1, \dots, s_n$ , and a property  $P$  defined on a sequence of points. For any given range  $[i, j]$ , we want to answer whether  $P$  is true for  $s_i, \dots, s_j$ . Additionally, we restrict  $P$  to be a hereditary property i.e. if  $P$  holds for sequence set  $A$ , then  $P$  also holds for any subsequence of  $A$ . The ability to answer these queries has practical applications and, in particular, it is the case for the two problems discussed in this paper.

Suppose we have a data structure that for any index  $i$  retrieves the largest integer  $j^*(i)$  such that  $P$  is true for the sequence  $S[i, j^*(i)]$ . Then, it is trivial to check whether  $P$  holds for  $S[i, j]$  as one only needs to compare  $j$  and  $j^*(i)$ . Therefore, the time range query problem reduces to building such data structure. Each of the two papers reviewed here present a general framework that can be used to build such data structures for a variety of problems.

[1, 2] use their frameworks to efficiently solve the range query problem for different properties  $P$ . We have implemented one algorithm from each paper. The first one is the *monotonicity* property. A sequence of points  $S$  in two-dimensional space is monotone if there is a line  $l$ , for which if points of  $S$  are projected on  $l$ , their order is maintained. The second one is the *clustering* property. We will call a set of points  $S$  clustered, if the largest distance between any pair of them (the diameter of  $S$ ) is at most 1.

Both properties

we implemented an algorithm for a that is presented in each paper. [1] discusses a

Here is the text of your introduction. We are using [1] and [2]

## 2 Our contribution

We've implemented

- In  $O(n)$  time we can find all maximal subsequences that define monotone paths in some (subpath-dependent) direction. [1]
- In  $O(n \log^2 n)$  time we can find all maximal subsequences with diameter at most 1. [2]

## 3 Algorithms

### 3.1 $k^*$

Let  $k^*(i) = \inf_{m \geq i} \{d(i, m) > 1\}$ . **Claim**  $j^*(i-1) = \min(j^*(i), k^*(i-1))$ . Thus after we calculate  $k^*(i)$  for all elements, we can calculate  $j^*(i)$  in  $O(n)$  time by looping over all indices in the reverse order.

### 3.2 Bokan et al Overview

Upper triangle method

### 3.3 Chan, Prat Overview

Range tree method.

## 4 Implementation Details

Talk about sweep line, etc.

## 5 Experimental Results

Talk about sweep line, etc.

## 6 Conclusion

This was a great project!

## References

- [1] Drago Bokal, Sergio Cabello, and David Eppstein. Finding All Maximal Subsequences with Hereditary Properties. In Lars Arge and János Pach, editors, *31st International Symposium on Computational Geometry (SoCG 2015)*, volume 34 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 240–254, Dagstuhl, Germany, 2015. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.
- [2] Timothy M. Chan and Simon Pratt. Two Approaches to Building Time-Windowed Geometric Data Structures. In Sándor Fekete and Anna Lubiw, editors, *32nd International Symposium on Computational Geometry (SoCG 2016)*, volume 51 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 28:1–28:15, Dagstuhl, Germany, 2016. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.