Chapter 0

Mathematical preliminaries

0.1 Even and odd functions

Definition 1 *The function $f : \mathbb{R} \to \mathbb{R}$ is an even function iff*

$$f(-x) = f(x), \quad \forall x \in \mathbb{R}. \tag{1}$$

The graph of any even function is symmetric with respect to the y-axis.

Lemma 1 *Let $f(x)$ be an integrable even function. Then,*

$$\int_{-a}^{0} f(x)dx = \int_{0}^{a} f(x)dx, \quad \forall a \in \mathbb{R}, \tag{2}$$

and therefore

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx, \quad \forall a \in \mathbb{R}. \tag{3}$$

Moreover, if $\int_0^\infty f(x)dx$ exists, then

$$\int_{-\infty}^{0} f(x)dx = \int_{0}^{\infty} f(x)dx, \tag{4}$$

and

$$\int_{-\infty}^{\infty} f(x)dx = 2\int_{0}^{\infty} f(x)dx. \tag{5}$$

Definition 2 *The function $f : \mathbb{R} \to \mathbb{R}$ is an odd function iff*

$$f(-x) = -f(x), \quad \forall x \in \mathbb{R}. \tag{6}$$

If we let $x = 0$ in (6), we find that $f(0) = 0$ for any odd function $f(x)$. Also, the graph of any odd function is symmetric with respect to the point (0, 0).

Lemma 2 *Let $f(x)$ be an integrable odd function. Then,*

$$\int_{-a}^{a} f(x)dx = 0, \quad \forall a \in \mathbb{R}. \tag{7}$$

Moreover, if $\int_0^\infty f(x)dx$ exists, then

$$\int_{-\infty}^{\infty} f(x)dx = 0. \tag{8}$$