Governing equation:

$$\frac{D_{NO}}{r} \frac{d}{dr}\left(r\frac{dC_{NO}}{dr} \right) = R_{NO}$$

$$\frac{\alpha D_{O_2}}{r} \frac{d}{dr}\left(r\frac{dP_{O_2}}{dr} \right) = R_{O_2}$$

$$u'' + P(r)u' = F(r) \tag{1}$$

$$\frac{d^2 C_{NO}}{dr^2} + \frac{1}{r}\frac{dC_{NO}}{dr} = \frac{R_{NO}}{D_{NO}}$$

$$\frac{d^2 P_{O_2}}{dr^2} + \frac{1}{r}\frac{dP_{O_2}}{dr} = \frac{R_{O_2}}{\alpha D_{O_2}}$$

$$v'' + P(r)v' = G(r) \tag{2}$$

RBC: $r_0 < r < r_1$

$R_{NO} = \lambda_{core} u$

$$\int \omega \left[\frac{d}{dr}\left(\frac{du}{dr} \right) + P(r)\frac{du}{dr} - \frac{\lambda_{core}}{D_{NO}}u \right] dr = 0 \tag{3}$$

Integration by parts: $\int f\frac{dg}{dx}dx = fg - \int g\frac{df}{dx}dx$. Choose $f = \omega$ and $g = \frac{du}{dr}$

Weak form:

$$\int -\frac{du}{dr}\frac{d\omega}{dr} + \omega\left[P(r)\frac{du}{dr} - \frac{\lambda_{core}}{D_{NO}}u \right] dr = -\omega\frac{du}{dr} \tag{4}$$

Discretization:

$$u_n \int_0^1 \left(-\frac{d\Psi_n}{d\xi}\frac{d\xi}{dr}\frac{d\Psi_m}{d\xi}\frac{d\xi}{dr} + \Psi_m \Psi_n P_n \frac{d\Psi_n}{d\xi}\frac{d\xi}{dr} - \frac{\lambda_{core}}{D_{NO}}\Psi_m \Psi_n \right) J\, d\xi = -\omega\frac{du}{dr} \tag{5}$$

where

$$J = \left| \frac{dr}{d\xi} \right|,$$
$$u(\xi) = \Psi_n u_n,$$
$$P(\xi) = \Psi_n P_n,$$
$$\omega(\xi) = \Psi_m$$