Starting equations

$$\frac{D_{NO}}{r}\frac{d}{dr}\left(r\frac{dC_{NO}}{dr}\right)+R_{NO}=0 \tag{1}$$

Simplify

$$D_{NO}\frac{d^2C_{NO}}{dr^2}+\frac{D_{NO}}{r}\frac{dC_{NO}}{dr}=-R_{NO} \tag{2}$$

Compare to form

$$u''+P(r)u'=F(r) \tag{3}$$

$$u=C_{NO},\ P(r)=\frac{1}{r},\ F(r)=-\frac{R_{NO}}{D_{NO}} \tag{4}$$

Taylor expansion

$$\begin{aligned} u_{i+1}&=u_i+u_i'\Delta r+\frac{1}{2}u_i''\Delta r^2,\\ u_{i-1}&=u_i-u_i'\Delta r+\frac{1}{2}u_i''\Delta r^2 \end{aligned}$$

Central difference

$$\begin{aligned} u_i'&=\frac{u_{i+1}-u_{i-1}}{2\Delta r},\\ u_i''&=\frac{u_{i+1}-2u_i+u_{i-1}}{\Delta r^2} \end{aligned}$$

Sub back into Eq. (3)

$$\frac{u_{i+1}-2u_i+u_{i-1}}{h^2}+P_i\frac{u_{i+1}-u_{i-1}}{2h}=F_i,\ h=\Delta r \tag{5}$$

where index i starts at 1.
Rearrange

$$\left(1-\frac{h}{2}P_i\right)u_{i-1}+(-2)u_i+\left(1+\frac{h}{2}P_i\right)u_{i+1}=h^2F_i \tag{6}$$

$$(-2)u_i=-\left(1-\frac{h}{2}P_i\right)u_{i-1}-\left(1+\frac{h}{2}P_i\right)u_{i+1}+h^2F_i \tag{7}$$

$$u_i=-\frac{1}{2}\left(-\left(1-\frac{h}{2}P_i\right)u_{i-1}-\left(1+\frac{h}{2}P_i\right)u_{i+1}+h^2F_i\right) \tag{8}$$

Use imaginary node to approximate Neumann BCs:

$$\frac{u_2-u_0}{2h}=0\rightarrow u_0=u_2 \tag{9}$$

$$\frac{u_{end+1}-u_{end-1}}{2h}=0\rightarrow u_{end+1}=u_{end-1} \tag{10}$$

Substitute into Eq. (3) when $i=1$

$$(-2)u_1=-2u_2+h^2F_1 \tag{11}$$

Substitute into Eq. (3) when $i=end$

$$(-2)u_{end}=-\left(1-\frac{h}{2}P_{end}\right)u_{end-1}-\left(1+\frac{h}{2}P_{end}\right)u_{end+1}+h^2F_{end} \tag{12}$$

$$(-2)u_{end}=-2u_{end-1}+h^2F_{end} \tag{13}$$

Solving over iterations

$$u_i^{k+1}=-\frac{1}{2}\left(-\left(1-\frac{h}{2}P_i\right)u_{i-1}^{k+1}-\left(1+\frac{h}{2}P_i\right)u_{i+1}^{k}+h^2F_i\right) \tag{14}$$

Split into 5 sections. Solutions are in the general form $A\tilde{\mathbf{u}}=B$.
Let $a_i=\frac{h}{2}P_i$.
RBC core $r_0<r<r_1$
Governing equation:

$$\frac{D_{NO}}{r}\frac{d}{dr}\left(r\frac{dC_{NO}}{dr}\right)-\lambda_{core}C_{NO}=0$$

$$F_i=\frac{\lambda_{core}u_i}{D_{NO}}$$

Boundary condition:

$$\begin{aligned} u'(0)=0&\rightarrow u_1=u_2,\\ u_{RBC}'(r_1)=\sigma,&\ \sigma=u_{CFL}'(r_1) \end{aligned}$$

Using second-order accurate one-sided difference approximation, j represents indexes in CFL domain.

$$\sigma = \frac{-3u_j + 4u_{j+1} - u_{j+2}}{2h},$$
$$\frac{3u_i - 4u_{i-1} + u_{i-2}}{2h} = \sigma,$$
$$3u_i - 4u_{i-1} + u_{i-2} = 2h\sigma$$

$$A = \begin{bmatrix} -2 & 1+a_2 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 1-a_3 & -2 & 1+a_3 & \ddots & & & & \vdots \\ 0 & 1-a_4 & -2 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & -2 & 1+a_{n-3} & 0 & 0 \\ \vdots & & & \ddots & 1-a_{n-2} & -2 & 1+a_{n-2} & 0 \\ \vdots & \cdots & \cdots & \cdots & 0 & 1-a_{n-1} & -2 & 1+a_{n-1} \\ 0 & \cdots & \cdots & \cdots & \cdots & 1 & -4 & 3 \end{bmatrix}, \tilde{\mathbf{u}} = \begin{bmatrix} u_2 \\ u_3 \\ \vdots \\ u_i \\ \vdots \\ u_n \end{bmatrix}, B = \begin{bmatrix} h^2F_2 - (1-a_2)u_1 \\ h^2F_3 \\ \vdots \\ h^2F_i \\ \vdots \\ h^2F_{n-1} \\ 2h\sigma \end{bmatrix} \tag{15}$$

CFL $r_1 < r < r_2$
Governing equation

$$\frac{D_{NO}}{r}\frac{d}{dr}\left(r\frac{dC_{NO}}{dr}\right) = 0$$
$$F_i = \frac{0}{D_{NO}}$$

Boundary condition:

$$u_{CFL}(r_1) = u_{RBC}(r_1),$$
$$u'_{CFL}(r_2) = \phi,\ \phi = u'_{EC}(r_2),$$

Using one-sided difference approximation again, i is CFL, j is RBC.

$$\phi = \frac{-3u_k + 4u_{k+1} - u_{k+2}}{2h},$$
$$\frac{3u_i - 4u_{i-1} + u_{i-2}}{2h} = \phi,$$
$$3u_i - 4u_{i-1} + u_{i-2} = 2h\phi$$

$$A = \begin{bmatrix} -2 & 1+a_2 & 0 & & & & & 0 \\ 1-a_3 & -2 & 1+a_3 & \ddots & & & & \vdots \\ 0 & 1-a_4 & -2 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & -2 & 1+a_{n-3} & 0 & 0 \\ \vdots & & & \ddots & 1-a_{n-2} & -2 & 1+a_{n-2} & 0 \\ \vdots & \cdots & \cdots & \cdots & 0 & 1-a_{n-1} & -2 & 1+a_{n-1} \\ 0 & \cdots & \cdots & \cdots & \cdots & 1 & -4 & 3 \end{bmatrix}, \tilde{\mathbf{u}} = \begin{bmatrix} u_2 \\ u_3 \\ \vdots \\ u_i \\ \vdots \\ u_n \end{bmatrix}, B = \begin{bmatrix} h^2F_2 - (1-a_2)u_1 \\ h^2F_3 \\ \vdots \\ h^2F_i \\ \vdots \\ h^2F_{n-1} \\ 2h\phi \end{bmatrix} \tag{16}$$

EC $r_2 < r < r_3$ is same method as CFL, only difference is in the R_{NO}term.
Governing equation

$$\frac{D_{NO}}{r}\frac{d}{dr}\left(r\frac{dC_{NO}}{dr}\right)+R_{NO}=0$$

$$F_i=-\frac{R_{NO_{max}}}{D_{NO}}\frac{P_{O_2}}{P_{O_2}+K_{m,eNOS}}$$

Boundary condition:

$$\begin{aligned}&u_{EC}(r_2)=u_{CFL}(r_2),\\&u'_{EC}(r_3)=\gamma,\ \gamma=u'_{VW}(r_3)\end{aligned}$$

VW $r_3 < r < r_4$ is same method as CFL, only difference is in the R_{O_2} term.
Governing equation

$$\frac{D_{NO}}{r}\frac{d}{dr}\left(r\frac{dC_{NO}}{dr}\right)-\lambda_{vw,t}C_{NO}=0$$

$$F_i=\frac{\lambda_{vw,t}u_i}{D_{NO}}$$

Boundary condition:

$$\begin{aligned}&u_{VW}(r_3)=u_{EC}(r_3),\\&u'_{VW}(r_4)=\delta,\ \delta=u'_T(r_4)\end{aligned}$$

T $r_4 < r < r_5$. Combined since same governing equation. Governing equation

$$\frac{D_{NO}}{r}\frac{d}{dr}\left(r\frac{dC_{NO}}{dr}\right)-\lambda_{vw,t}C_{NO}=0$$

$$F_i=\frac{\lambda_{vw,t}u_i}{D_{NO}}$$

$$\begin{aligned}&u_T(r_4)=u_{VW}(r_4),\\&u'(r_5)=0\rightarrow u_n=u_{n-1}\end{aligned}$$

$$A=\begin{bmatrix}-2 & 1+a_2 & 0 & \cdots & \cdots & \cdots & \vdots\\ 1-a_3 & -2 & 1+a_3 & \ddots & & & \vdots\\ 0 & 1-a_4 & -2 & \ddots & \ddots & & \vdots\\ \vdots & \ddots & \ddots & \ddots & \ddots & & \vdots\\ \vdots & & \ddots & \ddots & -2 & 1+a_{n-3} & 0\\ \vdots & & & \ddots & 1-a_{n-2} & -2 & 1+a_{n-2}\\ 0 & \cdots & \cdots & \cdots & 0 & 1-a_{n-1} & -2\end{bmatrix},\ \tilde{\mathbf{u}}=\begin{bmatrix}u_2\\u_3\\\vdots\\u_i\\\vdots\\u_{n-1}\end{bmatrix},\ B=\begin{bmatrix}h^2F_2-(1-a_2)u_1\\h^2F_3\\\vdots\\h^2F_i\\\vdots\\h^2F_{n-1}-(1+a_{n-1})u_n\end{bmatrix}\tag{17}$$

O_2 is solved using the same matrix as NO except in RBC where it has a constant partial pressure.
RBC core $r_0 < r < r_1$

$$P_{O_2}=70$$

CFL $r_1 < r < r_2$
Governing equation

$$\alpha\frac{D_{O_2}}{r}\frac{d}{dr}\left(r\frac{dP_{O_2}}{dr}\right)=0$$

$$F_i=\frac{0}{\alpha D_{O_2}}$$

Boundary condition:

$$\begin{aligned}&v_{CFL}(r_1)=v_{RBC}(r_1),\\&v'_{CFL}(r_2)=\phi,\ \phi=v'_{EC}(r_2)\end{aligned}$$

EC $r_2 < r < r_3$
Governing equation

$$\alpha \frac{D_{O_2}}{r}\frac{d}{dr}\left(r\frac{dP_{O_2}}{dr}\right) - R_{NO} = 0$$
$$F_i = \frac{R_{NO_{max}}}{\alpha D_{O_2}}\frac{P_{O_2}}{P_{O_2}+K_{m,eNOS}}$$

Boundary condition:

$$v_{EC}(r_2) = v_{CFL}(r_2),$$
$$v'_{EC}(r_3) = \gamma,\ \gamma = v'_{VW}(r_3)$$

VW $r_3 < r < r_4$
Governing equation

$$\begin{gathered}\alpha \frac{D_{O_2}}{r}\frac{d}{dr}\left(r\frac{dP_{O_2}}{dr}\right) - Q_{O_2\,max\,VW}\frac{P_{O_2}}{P_{O_2}+appK_m} = 0\\ appK_m = K_m\left(1+\frac{C_{NO}}{C_{ref}}\right)\\ F_i = \frac{Q_{O_2\,max\,VW}}{\alpha D_{O_2}}\frac{P_{O_2}}{P_{O_2}+K_m\left(1+\frac{u_i}{C_{ref}}\right)}\end{gathered} \tag{18}$$

Boundary condition:

$$v_{VW}(r_3) = v_{EC}(r_3),$$
$$v'_{VW}(r_4) = \delta,\ \delta = v'_T(r_4)$$

T $r_4 < r < r_5$
Governing equation

$$\begin{gathered}\alpha \frac{D_{O_2}}{r}\frac{d}{dr}\left(r\frac{dP_{O_2}}{dr}\right) - Q_{O_2\,max\,T}\frac{P_{O_2}}{P_{O_2}+appK_m} = 0\\ appK_m = K_m\left(1+\frac{C_{NO}}{C_{ref}}\right)\\ F_i = \frac{Q_{O_2\,max\,T}}{\alpha D_{O_2}}\frac{P_{O_2}}{P_{O_2}+K_m\left(1+\frac{u_i}{C_{ref}}\right)}\end{gathered} \tag{19}$$

Boundary condition:

$$v_T(r_4) = v_{VW}(r_4),$$
$$v'_T(r_5) = 0 \to v_n = v_{n-1}$$

$$A = \begin{bmatrix} -2 & 1+a_2 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 1-a_3 & -2 & 1+a_3 & \ddots & & & & \vdots \\ 0 & 1-a_4 & -2 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & -2 & 1+a_{n-3} & 0 & 0 \\ \vdots & & & \ddots & 1-a_{n-2} & -2 & 1+a_{n-2} & 0 \\ \vdots & \cdots & \cdots & \cdots & 0 & 1-a_{n-1} & -2 & 1+a_{n-1} \\ 0 & \cdots & \cdots & \cdots & \cdots & 1 & -4 & 3 \end{bmatrix},\ \tilde{\mathbf{v}} = \begin{bmatrix} v_2 \\ v_3 \\ \vdots \\ v_i \\ \vdots \\ v_n \end{bmatrix},\ B = \begin{bmatrix} h^2G_2 - (1-a_2)v_1 \\ h^2G_3 \\ \vdots \\ h^2G_i \\ \vdots \\ h^2G_{n-1} \\ 0 \end{bmatrix} \tag{20}$$

$$M=\begin{bmatrix} 1 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 1-a_2 & -2 & 0 & \ddots & & & & \vdots \\ 0 & 1-a_3 & -2 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & 1-a_{n-3} & -2 & 0 & 0 & 0 \\ \vdots & & & \ddots & 1-a_{n-2} & -2 & 0 & 0 \\ \vdots & \cdots & \cdots & \cdots & 0 & 1-a_{n-1} & -2 & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix}, \tag{21}$$

$$N=\begin{bmatrix} 0 & -2 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & -(1+a_2) & \ddots & & & & \vdots \\ 0 & 0 & 0 & -(1+a_3) & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & 0 & -(1+a_{n-3}) & 0 & 0 \\ \vdots & & & \ddots & 0 & 0 & -(1+a_{n-2}) & 0 \\ \vdots & \cdots & \cdots & \cdots & 0 & 0 & 0 & -(1+a_{n-1}) \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & -2 & 0 \end{bmatrix},\ \tilde{\mathbf{u}}=\begin{bmatrix} u_2 \\ u_3 \\ \vdots \\ u_i \\ \vdots \\ u_n \end{bmatrix},\ B=\begin{bmatrix} h^2F_2-(1-a_2)u_1 \\ h^2F_3 \\ \vdots \\ h^2F_i \\ \vdots \\ h^2F_{n-1} \\ 2h\sigma \end{bmatrix} \tag{22}$$