Starting equations

$$\frac{D_{NO}}{r}\frac{d}{dr}\left(r\frac{dC_{NO}}{dr}\right)+R_{NO}=0 \tag{1}$$

Simplify

$$D_{NO}\frac{d^2C_{NO}}{dr^2}+\frac{D_{NO}}{r}\frac{dC_{NO}}{dr}=-R_{NO} \tag{2}$$

Compare to form

$$u''+P(r)u'=F(r) \tag{3}$$

$$u=C_{NO},\ P(r)=\frac{1}{r},\ F(r)=-\frac{R_{NO}}{D_{NO}} \tag{4}$$

Taylor expansion

$$\begin{aligned} u_{i+1}&=u_i+u_i'\Delta r+\frac{1}{2}u_i''\Delta r^2,\\ u_{i-1}&=u_i-u_i'\Delta r+\frac{1}{2}u_i''\Delta r^2 \end{aligned}$$

Central difference

$$\begin{aligned} u_i'&=\frac{u_{i+1}-u_{i-1}}{2\Delta r},\\ u_i''&=\frac{u_{i+1}-2u_i+u_{i-1}}{\Delta r^2} \end{aligned}$$

Sub back into Eq. (3)

$$\frac{u_{i+1}-2u_i+u_{i-1}}{h^2}+P_i\frac{u_{i+1}-u_{i-1}}{2h}=F_i,\ h=\Delta r \tag{5}$$

where index i starts at 1.
Rearrange

$$\left(1-\frac{h}{2}P_i\right)u_{i-1}+(-2)u_i+\left(1+\frac{h}{2}P_i\right)u_{i+1}=h^2F_i \tag{6}$$

Split into 4 sections since equations for vascular wall and tissue are the same. Solutions are in the general form $A\tilde{\mathbf{u}}=B$. Let $a_i=\frac{h}{2}P_i$.

RBC core $r_0<r<r_1$

$$\begin{aligned} u'(0)=0&\rightarrow u_1=u_2,\\ u'_{RBC}(r_1)=\sigma,&\ \sigma=u'_{CFL}(r_1) \end{aligned}$$

Using second-order accurate one-sided difference approximation, j represents indexes in CFL domain.

$$\begin{aligned} \sigma&=\frac{-3u_j+4u_{j+1}-u_{j+2}}{2h},\\ \frac{3u_i-4u_{i-1}+u_{i-2}}{2h}&=\sigma \end{aligned}$$

$$A=\begin{bmatrix} -2-(1-a_2) & 1+a_2 & 0 & \cdots & \cdots & \cdots & \cdots & 0\\ 1-a_3 & -2 & 1+a_3 & \ddots & & & & \vdots\\ 0 & 1-a_4 & -2 & \ddots & \ddots & & & \vdots\\ \vdots & \ddots & \ddots & \ddots & \ddots & & & \vdots\\ \vdots & & \ddots & \ddots & -2 & 1+a_{n-3} & 0 & 0\\ \vdots & & & \ddots & 1-a_{n-2} & -2 & 1+a_{n-2} & 0\\ \vdots & \cdots & \cdots & \cdots & 0 & 1-a_{n-1} & -2 & 1+a_{n-1}\\ 0 & \cdots & \cdots & \cdots & \cdots & \frac{h}{2} & -2h & \frac{3h}{2} \end{bmatrix},\ \tilde{\mathbf{u}}=\begin{bmatrix} u_2\\ u_3\\ \vdots\\ u_i\\ \vdots\\ u_n \end{bmatrix},\ B=\begin{bmatrix} h^2F_2\\ h^2F_3\\ \vdots\\ h^2F_i\\ \vdots\\ h^2F_{n-1}\\ \sigma \end{bmatrix} \tag{7}$$

CFL $r_1 < r < r_2$

$$u'_{CFL}(r_1) = \sigma,\ \sigma = u'_{RBC}(r_1),$$
$$u'_{CFL}(r_2) = \phi,\ \phi = u'_{EC}(r_2),$$

Using one-sided difference approximation again, i is CFL, j is RBC and k is EC.

$$\begin{aligned}\sigma &= \frac{3u_j - 4u_{j-1} + u_{j-2}}{2h},\\ \frac{-3u_i + 4u_{i+1} - u_{i+2}}{2h} &= \sigma,\\ \phi &= \frac{-3u_k + 4u_{k+1} - u_{k+2}}{2h},\\ \frac{3u_i - 4u_{i-1} + u_{i-2}}{2h} &= \phi\end{aligned}$$

$$A = \begin{bmatrix} -\frac{3h}{2} & 2h & -\frac{h}{2} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 1-a_2 & -2 & 1+a_2 & 0 & & & & & \vdots \\ \vdots & 1-a_3 & -2 & 1+a_3 & \ddots & & & & \vdots \\ \vdots & 0 & 1-a_4 & -2 & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & & & \vdots \\ \vdots & & & \ddots & \ddots & -2 & 1+a_{n-3} & 0 & 0 \\ \vdots & & & & \ddots & 1-a_{n-2} & -2 & 1+a_{n-2} & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & 0 & 1-a_{n-1} & -2 & 1+a_{n-1} \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{h}{2} & -2h & \frac{3h}{2} \end{bmatrix},\ \tilde{\mathbf{u}} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_i \\ \vdots \\ u_n \end{bmatrix},\ B = \begin{bmatrix} \sigma \\ h^2F_2 \\ h^2F_3 \\ \vdots \\ h^2F_i \\ \vdots \\ h^2F_{n-1} \\ \phi \end{bmatrix} \tag{8}$$

EC $r_2 < r < r_3$ is same method as CFL, only difference is in the R_{NO}term.
VW $r_3 < r < r_4$ is same method as CFL, only difference is in the R_{O_2} term.

T $r_4 < r < r_5$. Combined since same governing equation.

$$u'_T(r_4) = \delta,\ \delta = u'_{VW}(r_4),$$
$$u'(r_5) = 0 \to u_n = u_{n-1}$$

Using one-sided difference approximation again, i is T, j is VW.

$$\delta = \frac{3u_j - 4u_{j-1} + u_{j-2}}{2h},$$
$$\frac{-3u_i + 4u_{i+1} - u_{i+2}}{2h} = \delta$$

$$A = \begin{bmatrix} -\frac{3h}{2} & 2h & -\frac{h}{2} & 0 & \cdots & \cdots & \cdots & 0 \\ 1-a_2 & -2 & 1+a_2 & 0 & & & & \vdots \\ \vdots & 1-a_3 & -2 & 1+a_3 & \ddots & & & \vdots \\ \vdots & 0 & 1-a_4 & -2 & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & -2 & 1+a_{n-3} & 0 \\ \vdots & & & & \ddots & 1-a_{n-2} & -2 & 1+a_{n-2} \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1-a_{n-1} & -2+(1+a_{n-1}) \\ & & & & & & & \end{bmatrix}, \tilde{\mathbf{u}} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_i \\ \vdots \\ u_{n-1} \end{bmatrix}, B = \begin{bmatrix} \delta \\ h^2F_2 \\ h^2F_3 \\ \vdots \\ h^2F_i \\ \vdots \\ h^2F_{n-1} \end{bmatrix} \quad (9)$$