

Variable and Value Ordering When Solving Balanced Academic Curriculum Problems^{*}

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Abstract. In this paper we present the use of Constraint Programming for solving balanced academic curriculum problems. We discuss the important role that heuristics play when solving a problem using a constraint-based approach. We also show how constraint solving techniques allow to very efficiently solve combinatorial optimization problems that are too hard for integer programming techniques.

Key words: Combinatorial Optimization, Variable Ordering, Value Ordering.

1 Introduction

A key factor to take into account when evaluating the academic success of students is the academic load they have in each academic period. The usual way to measure this load is to assign, to each course, a number of credits representing the amount of effort required to successfully follow the course. In this way, the academic load of each period is given by the sum of the credits of each course taken in the period. Generally, some explicit restrictions are imposed when developing a curriculum. For example, a maximum load per period could be allowed in order to prevent overload, and some precedence relationships could be established among some courses. Assuming that a balanced load favors academic habits and facilitates the success of students, since 1994 we have been involved in designing balanced academic curricula at the Federico Santa María Technical University.

The problem of designing balanced academic curricula consist in assigning courses to periods in such a way that the academic load of each period will be balanced, i.e., as similar as possible. In this work, we consider as academic load the notion of credit that represents the effort in time needed to successfully follow the course. In this work, we concentrate on the Informatics careers offered by the Federico Santa María Technical University at Valparaíso. A first attempt in this direction was done on the curriculum of a four-year career (8 academic periods) considering 48 courses [8]. In that work, Integer Programming techniques allowed

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to solve only 6 academic periods. However, it was not possible to solve the complete model. Considering the success of Constraint Programming for solving combinatorial search problems, we were interested in using this technique as an alternative.

This paper is organized as follows: section 2 describes the combinatorial optimization problem we are interested in. Section 3 presents an Integer Programming model for the balanced academic curriculum problem. In section 4, we model the same problem using a constraint-based approach. In section 5, we present experimental results when solving the problem using both approaches. Finally, in section 7, we conclude the paper and give some perspectives for further works.

2 The Balanced Academic Curriculum Problem

In this work, we concentrate on three particular instances of the balanced academic curriculum problem: the three Informatics careers offered by the Federico Santa María Technical University. As a general framework we consider administrative as well as academic regulations of this university.

Academic Curriculum An academic curriculum is defined by a set of courses and a set of precedence relationships among them.

Number of periods Courses must be assigned within a maximum number of academic periods.

Academic load Each course has associated a number of credits or units that represent the academic effort required to successfully follow it.

Prerequisites Some courses can have other courses as prerequisites.

Minimum academic load A minimum amount of academic credits per period is required to consider a student as full time.

Maximum academic load A maximum amount of academic credits per period is allowed in order to avoid overload.

Minimum number of courses A minimum number of courses per period is required to consider a student as full time.

Maximum number of courses A maximum number of courses per period is allowed in order to avoid overload.

3 Integer Programming Model

In this section, we present an Integer Programming model for the balanced academic curriculum problem.

– Parameters

Let

m : Number of courses

n : Number of academic periods

α_i : Number of credits of course i ; $\forall i = 1, \dots, m$

β : Minimum academic load allowed per period

γ : Maximum academic load allowed per period

δ : Minimum amount of courses per period

ϵ : Maximum amount of courses per period

– Decision variables

Let

$$x_{ij} = \begin{cases} 1 & \text{if course } i \text{ is assigned to period } j; \forall i = 1, \dots, m \forall j = 1, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

c_j : academic load of period j ; $\forall j = 1, \dots, n$

c : maximum academic load for all periods

– Objective function

$$\text{Min } c = \text{Max}\{c_1, \dots, c_n\}$$

– Constraints

- The academic load of period j is defined by:

$$c_j = \sum_{i=1}^m \alpha_i \times x_{ij} \quad \forall j = 1, \dots, n$$

- All courses i must be assigned to some period j :

$$\sum_{j=1}^n x_{ij} = 1 \quad \forall i = 1, \dots, m$$

- Course b has course a as prerequisite:

$$x_{bj} \leq \sum_{r=1}^{j-1} x_{ar} = 1 \quad \forall j = 2, \dots, n$$

- The maximum academic load is defined by:

$$c = \text{Max}\{c_1, \dots, c_n\}$$

This can be represented by the following set of linear constraints:

$$c_j \leq c \quad \forall j = 1, \dots, n$$

- The academic load of period j must be greater than or equal to the minimum required:

$$c_j \geq \beta \quad \forall j = 1, \dots, n$$

- The academic load of period j must be less than or equal to the maximum allowed:

$$c_j \leq \gamma \quad \forall j = 1, \dots, n$$

- The number of courses of period j must be greater than or equal to the minimum allowed:

$$\sum_{i=1}^m x_{ij} \geq \delta \quad \forall j = 1, \dots, n$$

- The number of courses of period j must be less than or equal to the maximum allowed:

$$\sum_{i=1}^m x_{ij} \leq \epsilon \quad \forall j = 1, \dots, n$$

4 Constraint-Based Model

In this section, we present our constraint-based model using the Oz language.

4.1 Variables

The domain constraints are the following:

Variable	Description	Oz Implementation
C	Maximum academic load (upper bound for all periods)	{FD.int 0#FD.sup C}
Assignment	Binary vector of assignments course-period (size $N \times K$)	{FD.tuple assignment NK 0#1 Assignment}

4.2 Basic constraints

Constraints involving only one variable and that allow to narrow the domains are:

Description	Oz Implementation
Maximum academic load $C \leq \text{Gamma}$	
Minimum academic load $C \geq \text{Beta}$	

4.3 Propagators

The academic load of each period must be less than or equal to C . The product of the *load vector* by each column of the *assignment matrix* must be less than or equal to C .

```
% Academic load for period J
{For 1 K 1
  proc {$ J}
    {FD.sumC
      {Map L1N fun {$ I} {Nth I LoadVector} end}
      {Map L1N fun {$ I} {Field I J} end}
      '=<:' C}
  end}
```

Any course must be included only once. The sum of any column of the *assignment matrix* must be 1.

```
% Any course must be done only once
{For 1 N 1
  proc {$ I}
    {FD.sum
      {Map L1K fun {$ J} {Field I J} end}
      '=: ' 1}
  end}
```

The number of courses assigned to any period must be less than or equal to the maximum allowed. The sum of any row of the *assignment matrix* must be less than or equal to the maximum allowed ($Epsilon$).

```
% Maximum number of courses in a period
{For 1 K 1
  proc {$ J}
    {FD.sum
      {Map L1N fun {$ I} {Field I J} end}
      '=<:' Epsilon}
  end}
```

Each course must be assigned after its prerequisites. If the course to be assigned has any precedence constraint, then the simple constraint `{Field A 1} =: 0` is inserted because it could not be assigned to the first period. Then, for each ancestor, the propagators are implemented.

```

% Course Precedence Relationship
{ForAll Courses
  proc {$ Course}
    A P
  in
    A = {Order Course Courses}
    P = {Nth A PrecedenceRelations}
    if P \= nil then
      {Field A 1} =: 0
      {ForAll P
        proc {$ Preced}
          B
        in
          B = {Order Preced Courses}
          {For 2 K 1
            proc {$ J}
              {FD.sum
                {Map {List.number 1 J-1 1} fun {$ R} {Field B R} end}
                '>=: ' {Field A J}}
              end}
            end}
          end}
        end}
      end}
    end}
  end}
end}

```

5 Experimental Results

In this section, we detail the results obtained when solving the balanced academic curriculum problem using integer programming and constraint programming techniques. We consider three careers involving 8, 10 and 12 academic periods, respectively.

5.1 lp_solve: The Integer Programming Approach

Several methods are available for solving Integer Programming problems. Explicit enumeration and cutting planes algorithms provide satisfactory results in some cases. However, Branch and Bound algorithms are the most successful techniques for solving Integer Programming models and are included in almost all mathematical programming packages.

Specifically, when solving Integer Programming models that involve only binary variables, Branch and Bound algorithms work in the following way: the original problem P is relaxed by eliminating constraints that impose integer values for the variables. Then, the relaxed problem is solved by using linear programming techniques and, in case the solution does not satisfy the requirement of integer values for the variables, two subproblems are created. The first one corresponds to the original problem P plus the additional constraint $x = 0$,

and the second one corresponds to the original problem P plus the additional constraint $x = 1$. The variable x is called the branching variable. Each time subproblems are created only worse solution can be obtained and this fact is used for bounding the search tree [6].

Table 1 presents the results obtained using `lp_solve`¹ when solving the 8-period problem. In this case, the optimum plan is obtained in 1460 seconds.

Solution quality	Time	Solution quality	Time
C [credits]	[seconds]	C [credits]	[seconds]
54	1.69	33	9.35
52	1.80	32	11.63
50	2.09	30	11.94
48	2.41	29	53.64
47	2.97	27	54.04
45	3.28	26	136.45
44	3.91	24	137.08
42	4.24	23	218.23
41	4.92	21	218.43
39	5.20	20	712.84
38	6.18	19	1441.98
36	6.53	18	1453.75
35	9.04	17 (optimum)	1459.73

Table 1. Partial and final solutions obtained by `lp_solve` for the 8-period problem.

Table 2 presents the results obtained when solving the 10-period problem. In this case, we obtained no optimum result after 5 hours, and the last result was logged before 1700 seconds.

Solution quality	Time	Solution quality	Time
C [credits]	[seconds]	C [credits]	[seconds]
48	5.88	35	8.84
46	5.89	33	9.11
44	5.93	32	25.38
42	6.00	30	25.65
41	6.14	29	1433.18
39	6.33	27	1433.48
38	6.72	26	1626.49
36	7.00	24 (not optimum)	1626.84

Table 2. Partial solutions obtained by `lp_solve` for the 10-period problem.

For the 12-period problem we did not get any log after a “turn-around-time” of 1 day.

¹ `lp_solve`, a software for solving integer linear programming models, is available free of charge at ftp://ftp.ics.ele.tue.nl/pub/lp_solve/

5.2 Oz: The Constraint-Based Approach

Constraint Programming deals with optimization problems using the same basic idea of verifying the satisfiability of a set of constraints. Assuming one is dealing with a minimization problem, the idea is to use an upper bound that represents the best possible solution obtained so far. Then we solve a sequence of CSPs each one giving a better solution with respect to the optimization function. More precisely, we compute a solution α to the problem P and we add the constraint $f < \alpha(f)$ that restricts the set of possible solutions to those that give better values for the optimization function². When, after adding such a constraint, the problem becomes unsatisfiable, the last possible solution so far obtained represents the optimal solution.

When solving our constraint-based model implemented in Oz, we first used a naive heuristic over the *assignment matrix* to find a feasible solution. We did not have any other choice because the representation was a binary matrix involving $n \times m$ variables which are instantiated (with a 0 or 1 value) or not instantiated (with a 0#1 domain) at any time. Because the other heuristics implemented by Oz (**first fail**, **split**) only apply to variables with bigger domains (e.g. **first fail** chooses the variable with minimal domain size) we only use the **naive** heuristic varying the order and interpretation of the vector of assignments. The way the program treats the assignment vector involves the *variable ordering*, and we could reverse the variable ordering by changing a function that is used along the model:

```
fun {Field I J}
    Assignment.((J-1)*N + I)
end
```

If we change the order of the matrix, then the variables are instantiated in a left-right manner always in the assignment vector but in a different way along the assignment matrix (interpretation of that vector). The Oz distributor is:

```
{FD.distribute generic(order:naive value:max) Assignment}
```

The first model we tested was the same model solved by **lp_solve** but now implemented as a CSP problem in Oz. The results for successive iterations reducing **C** (the upper bound of the number of credits) are shown in figure 1, having obtained solutions before 5 hours only for the 8- and 10-period problems.

The Oz performance was better than **lp_solve** giving poor quality solutions faster. But when we incremented **C** the problem got tighter and the latter still gave solutions (see tables 1 and 2) while Oz either ran out of memory³ or did not log any solution after 5 hours.

² Bockmayr and Kasper work on optimization functions f that always take integer values and so they add the constraint $f \leq \alpha(f) - 1$ [1].

³ That was an Oz internal error sent to bugs@mozart-oz.org. In fact, the entire search space (considering only distribution, not propagation) is rather big (2^{420} for the 10-period problem in 42 courses, 10 periods).

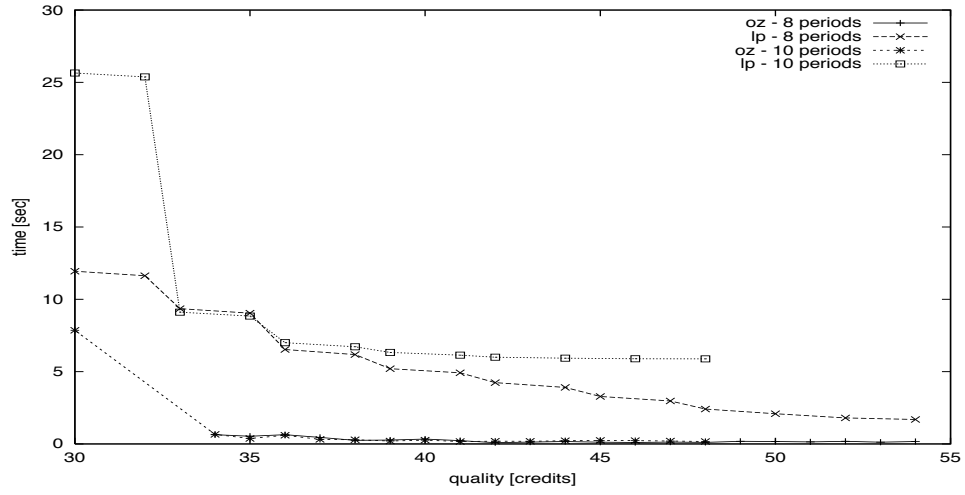


Fig. 1. Results obtained by the first implementation of the model in Oz compared with the `lp_solve` performance

Inspecting the implementation, we noticed that the way Oz chooses the next variable to instantiate (left to right) from the binary vector is relevant to the way we interpretate that vector as a matrix. Varying then the variable ordering changed the results as shown in figure 2.

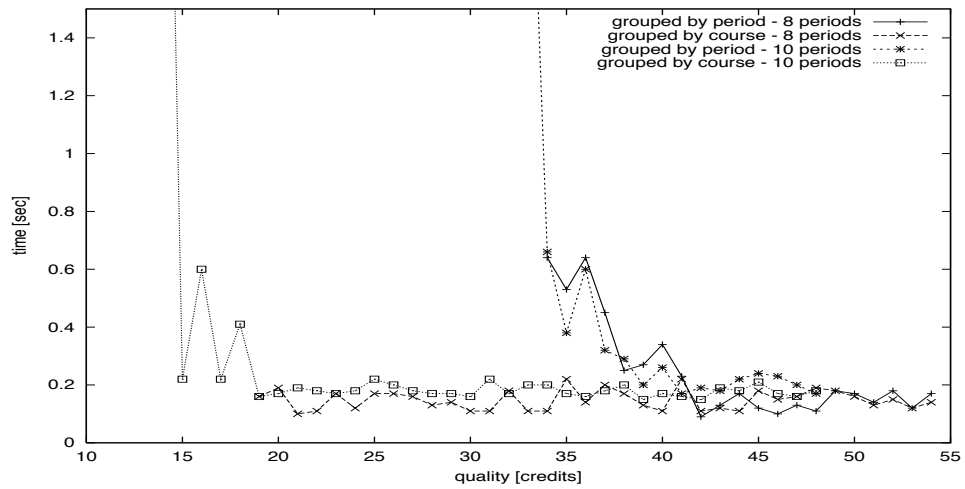


Fig. 2. Results obtained varying the variable ordering in the Oz model

We got the optimal solution for the 10-period problem, but still have problems reaching the 8-period problem optimum⁴. The 12-period problem reports partial solutions with $C \geq 28$, but still far away from the optimum.

Looking for the reason why Oz could not solve the 8-period problem, we tried changing another parameter, the *value ordering*. The way Oz was instantiating the variables was not natural because it was first trying the value 0 which means not to assign the course to a period. Then it first prohibits any assignment until it gets a *failed space*, and backtracks searching for solutions. Given that for instantiating the courses we were using the variable ordering specified by the University's original academic curriculum, a good approach seemed to be to assign the courses in that order. Varying this value ordering we were able to get solutions for any quality requirement in a constant time⁵ and get the optimum for every problem. The results are presented in figure 3.

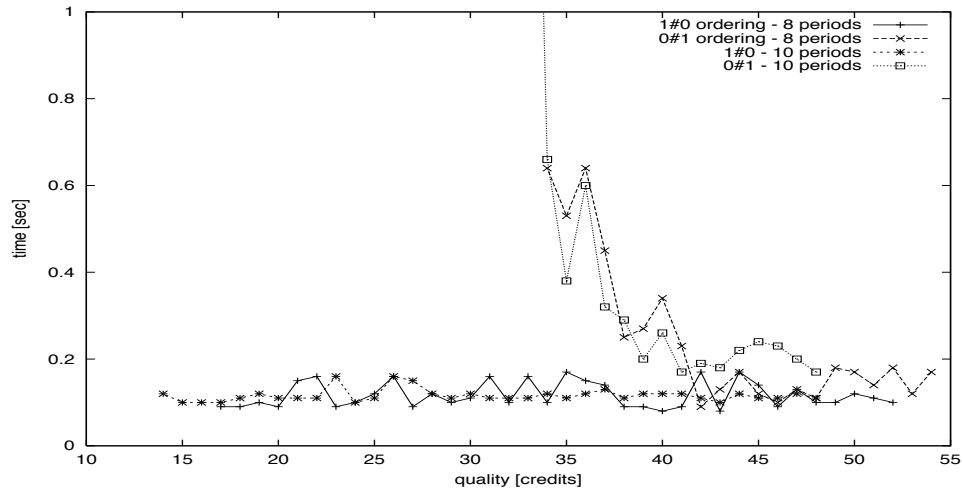


Fig. 3. Results obtained varying the value ordering in the Oz model with the assignment vector grouped by period

Finally, we compared the variable ordering with the new value ordering and got no significant differences as shown in figure 4.

⁴ The optimum is reached at $C = 17$ in the 8-period problem, we did not have any response after 5 hours for $C \leq 18$ with the assignment vector grouped by course.

⁵ The fluctuations are due to machine noise and low precision in the Oz measurement tool used.

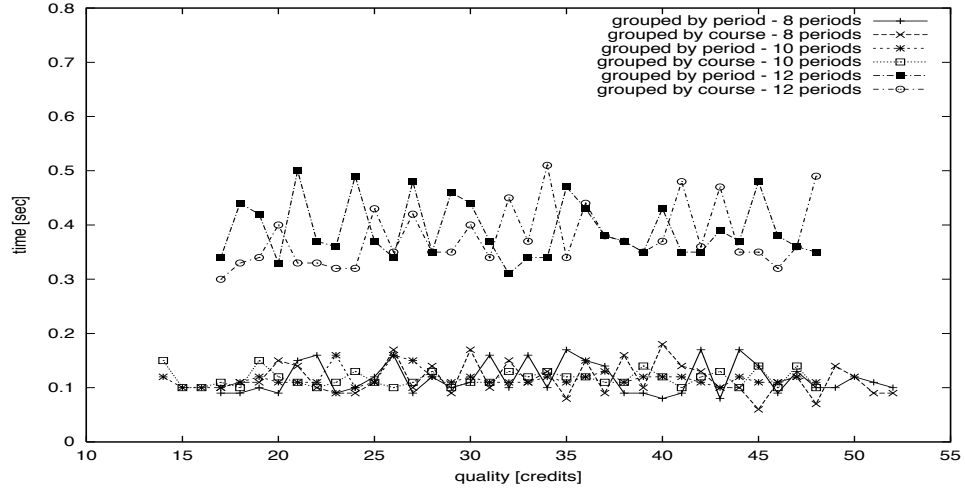


Fig. 4. Results obtained varying the variable ordering in the Oz model with the value order reversed

A comparative summary of the experimental results is given in table 3.

8-period problem					
	lp_solve	Oz			
		grouped by course		grouped by period	
		naive	reverse	naive	reverse
Time in seconds for first solution (worst quality)	1.7	0.1	0.1	0.2	0.1
Time in seconds for optimum solution (best quality)	1459.7	∞	0.1	∞	0.1
10-period problem					
Time in seconds for first solution (worst quality)	5.9	0.2	0.1	0.2	0.1
Time in seconds for optimum solution (best quality)	∞	3.6	0.1	∞	0.1
12-period problem					
Time in seconds for first solution (worst quality)	∞	0.4	0.5	∞	0.3
Time in seconds for optimum solution (best quality)	∞	∞	0.3	∞	0.3

Table 3. Comparative summary of performance between lp_solve and Oz

All details about the Integer Programming model and the implementation in Oz of the constraint-based model, as well as the results obtained using both approaches, can be obtained at <http://www.labsc.inf.utfsm.cl/~smanzano>.

6 Related Work

The works by Henz [5] and by Curtis, Smith, and Wren [2] are the most recent references on the use of Constraint Programming and Integer Programming for solving combinatorial optimization problems. Dincbas, Simonis and Van Hentenryck discuss a case in which the expressive power of Constraint Programming allows to reformulate an Integer Programming model giving a much smaller constraint-based model and reducing in this way the size of the search space [3]. Van Hentenryck and Carillon [4] deal with a warehouse location problem and Smith, Brailsford, Hubbard and Williams [7] apply both techniques for solving the progressive party problem. In both cases, Constraint Programming does better than Integer Programming mainly due to the use of appropriated variables. In our case, we do use the same variables and the same set of constraints. The very high efficiency of Constraint Programming was obtained thanks to the suitable heuristics used for enumerating the variables.

7 Conclusions

We have presented the use of constraint programming techniques for solving a combinatorial optimization problem. Using constraint programming we have been able to solve simple problems that cannot be solved by integer programming techniques. Moreover, we can solve medium size problems very efficiently. An important variability can be obtained when using constraint programming: on one hand, using naive heuristics we can not solve even simple problems, on the other hand, using clever heuristics we can solve very efficiently some problems that are too hard for integer programming techniques. Of course, more work is needed to understand when to apply each of the heuristics used in this paper.

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