① we know, that if the rental ski problem is offline, the optimal cost of the algorithm is $OPT(ALGO) = \min\left(\sum_{i=1}^{d} x_i , P\right)$

here; d is the number of day that the skiir went for skiing, and x_i is the rental price of a day. here in the given problem, $\sum_{i=1}^{d} x_i = d$ as rental price per day is \$1. and in the dynaic pricing, we dont know $\sum_{i=1}^{d} x_i$.

According to the "better-late-than-never" suggests the adversary rents the skii equipment for t days and then buy on $(t+1)$th day.

$$cost(A_t, I_d) = \begin{cases} \sum_{i=1}^{d} x_i & \text{if } d \leq t. \\ \sum_{i=0}^{t} x_i + p & \text{otherwise} \end{cases}$$

Now lets examine some cases of dynamic pricing,

case 1: if the skier baught the equipment in the first day he went for skiing. then the competitive ratio,

$$c = \max_{d,x} \frac{P}{\min\left(\sum_{i=1}^{d} x_i , P\right)}$$ [as cost $= P$ and opt $= \min(\sum x_i , P)$

$$= \max_{d,x} \frac{P}{1}$$ [since, day 1, he could simply rent for \$1, so OPT = 1.

$$= P.$$

case 2: Lets consider the case when the skier rented the ski for $t \geq 1$ days and then bought.

At this scenario,

the competitive ratio,

$$c = \max_{d,x} \frac{\sum_{i=1}^{d} x_i + p}{\min\left(\sum_{i=1}^{d} x_i , p\right)}$$

as x_i is unbounded, the competitive ratio in this case can be very high/large.

So, as min c is the best competitive ratio for any deterministic online algorithm, case 1 is considered best (dynamic.