Application of Input to State Stability to ecological reservoir models

Department Biogeochemical Processes

Challenge

Many models in ecology and biogeochemistry, in particular models of the global carbon cycle, can be generalized as systems of non-autonomous ordinary differential equations (ODEs). For many applications, it is important to determine the stability properties for this type of systems, but most methods available for autonomous systems are not necessarily applicable for the non-autonomous case. We discuss here stability notions for non-autonomous nonlinear models represented by systems of ODEs explicitly dependent on time and a time-varying input signal. Is there a stability concept that is:

1. broad enough to encompass these models
2. rigorous enough to be proved analytically
3. interpretable in ecologically meaningful terms ?

Example: general soil model

$$
\begin{aligned}
\mathrm{C} &= \boldsymbol{I}(t) + \mathrm{T}(\mathrm{C}, t) \cdot \mathrm{N}(t, \boldsymbol{C}) \cdot \boldsymbol{C}(t) \\
N_{i,i}(\mathrm{C}, t) &\geq 0 \quad \forall i \\
T_{i,i}(\mathrm{C}, t) &= -1 \quad \forall i \\
T_{i,j}(\mathrm{C}, t) &\geq 0 \quad \forall i \neq j \\
\sum_i T_{i,j}(\mathrm{C}, t) &= 1 \quad \forall j
\end{aligned}
$$

This model structure generalizes most SOM decomposition models with any arbitrary number of pools, including those describing nonlinear interactions among state variables. It enforces mass balance and substrate dependence of decomposition, and it is flexible enough to describe:

1. Heterogeneity of decomposition rates
2. Transformations of organic matter
3. Environmental variability effects
4. Organic matter interactions

Examples for nonlinear models are:

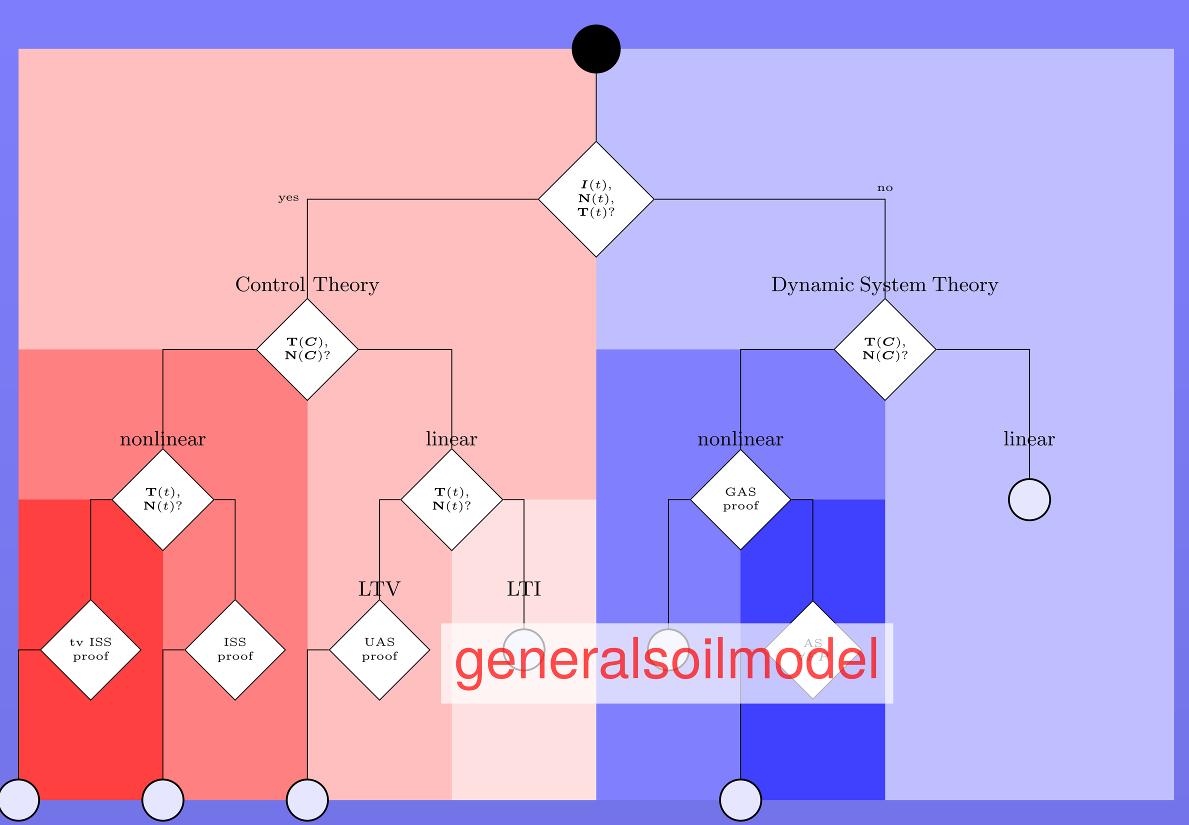
1. Exoenzyme models (??)
2. AWB (?)
3. Bacwave (?)
4. MEND (?)
5. Manzoni (?)

Also linear models fit into the general framework

1. Henin's model (??)
2. ICBM (?)
3. RothC (??)
4. Century (?)
5. Fontaine 1-4 (?)

One general concept to encompass especially these nonlinear models is clearly desirable.

Results I, ISS as generalization of available stability concepts



The graph shows different stability concepts one could try to establish for the general soil model mentioned above depending on properties of its components I, T and N. The hardest to prove is Input to State Stability for time varying systems (ISStv) in the lower left corner. It turns out that ISStv also generalizes all the other concepts mentioned:

- In the case of Linear Time Invariant (LTI) systems ISS follows from the properties of the matrix.(eigenvalues)
- In the case of Linear Time Variant (LTV) systems it can be established if sufficient information about the state transition operator allows to prove uniform asymptotic stability UAS.
- For input free system (on the blue right-hand side) ISS reduces to Global Asymptotic Stability (GAS)
- ...

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