

Let's just write this plainly.

MM law states:

“In space, given X mission against Y opponents with Z parameters, you will need N ships with K parameters to succeed”.

Okay. So, first, I believe it's important to define parameters.

First, missions.

X Mission types:

1. Offensive

- Attack

An attack is carried out whilst the enemy target is at rest (not in transit). The goal of an attack is to cause as much damage to the target as possible, and turning back to repair and resupply for another attack/engagement.

- Interdiction

Interdiction is an act of chasing the enemy down and forcing them to engage. This is mainly done with ships with higher delta V, more acceleration and weapons that pose a threat to the target.

For interdiction, you want lightly armored vessels with high delta v and acceleration. In addition, you require weapons that will force the enemy to waste their ammunition, power or fuel.

- Engagement

2. Defensive

- Escort

To escort a ship of certain value, escorting ships need to have sufficient point defense value to ensure that the ship is relatively unharmed. However, they must also be able to fire back at the attacking force and render them inoperable, ensuring that the ship is unable to target the high value target.

- Evasion

Evasion is quite simple. Ships simply need enough acceleration and delta V to outrun the chasing opponent.

- Interception

The difference between interdiction and interception lies in the fact that an intercepting force may not need to “chase the enemy” to force an engagement. They would simply need to position themselves in a position where engagement is unavoidable.

3. Support

- Recon

For reconnaissance, you need ships that will have only sensors and great amount of delta v. If possible, acceleration as well.

Reconnaissance is done in 2 ways. First, you simply go in the enemy space, take measurements of their craft and possibly the weapons they have. After which you depart (hopefully without the enemy chasing you).

Second case could classify as attack, but because the mission is recon, it falls into recon. Here, you send a group of fast, lightly armored and lightly armed ships at the enemy fleet as they are docked/in orbit(i.e. not in transit). Here, you “test the enemy in battle”. You want ships with sufficient enough kill threshold and weapon range and damage to pose any threat to said ships, engage their point defense and force them to maneuver to avoid a lateral hit.

- Electronic warfare
- Logistics/resupply

Regardless of size, a Non-FtL ship will have following parameters:

1) Propulsion

a) Acceleration

General acceleration formula:

$$a = \frac{\Delta v}{t}$$

- Combat acceleration

During combat, we assume that the engine is firing at full thrust. Formula:

$$a = \frac{F}{m}$$

- Cruise acceleration

Sustained acceleration over the journey of the spacecraft between two bodies. The time required to complete a maneuver depends on the delta V expenditure and sustained acceleration.

b) ISP

c) Engine mass

d) Required fuel

There is a limit as to how much fuel you can cram. You need for fuel to fall within the lateral surface area of the warship. Otherwise, it's going to be an easy target.

e) Delta V

Delta V can be calculated by the rocket equation:

$$\Delta v = v_e \ln \frac{m_0}{m_f}$$

Where:

v_e : effective exhaust velocity

m_0 : wet mass

m_f : dry mass

f) Angular acceleration (degrees per second):

Angular acceleration is calculated by:

$$\alpha = \frac{\tau}{I}$$

Where:

α : angular acceleration(rads/s²)

τ : applied torque

I : moment of inertia

First, we need to consider applied torque. Formula:

$$\tau = r \times F \times \sin \theta$$

Where:

τ : torque($N \cdot m$)

r : distance from the rotation axis to where the force is applied

F : force applied

θ : angle between the force direction and the rotation axis.

Then, we consider moment of inertia. Generally, we can break ship components down to basic shapes and apply the parallel axis theorem.

Shape	Rotation Axis	Moment of Inertia
Solid cylinder (e.g. fuel tank)	About center, along axis	$I = \frac{1}{2}MR^2$
Solid cylinder	About center, perpendicular to axis	$I = \frac{1}{12}M(3R^2 + H^2)$
Solid sphere	Any axis through center	$I = \frac{2}{5}MR^2$
Thin rod	About center, perpendicular	$I = \frac{1}{12}ML^2$
Thin rod	About end, perpendicular	$I = \frac{1}{3}ML^2$
Box (cuboid)	Through center	$I = \frac{1}{12}M(W^2 + H^2)$

Where:

M : total mass

R : radius

L : length

W, H : width/height of a box

Parallel axis theorem:

$$I_{total} = I_{part} + md^2$$

Where:

m : mass of the part

d : distance from the ship's center of mass to the part's center

I_{part} : moment of inertia of the part around its own center

The sum of all parts:

$$I_{ship} = \sum I_{eachpart}$$

2) Power

- a) Reactor/Generator type
- b) Power output(MW)
- c) Mass(kg)
- d) Required support
- e) Operation time
- f) Energy storage(MJ/kg)

3) Hull

- a) Materials of the hull

This determines the structural integrity, alongside damage absorbtion.

- b) Volume and surface area

4) Radiators

- a) Radiator surface area

Another point to add similar to fuel. Radiators can't be infinitely large. There is only a certain volume they can occupy. In addition, more radiators immensely increase the surface area of the ship, plus the mass and degrade structural integrity, whilst also taking up more time to fold/unfold.

- b) Amount of heat expelled
- c) Time it takes to retract radiators

5) Armor

- A) Material of the armor
- B) Armor durability per cubic cm
- C) Armor thickness
- D) Mass

6) Utility modules(EMC/EMMC, Heat sinks and Repair crew are optional)

a) Sensors

Sensors decrease acquisition time (if no EMC)

b) Life support(if crew)

c) Heat sinks

Absorbs the heat of the ship. Heat density is calculated from J/m^2

d) Repair crew

Repair crew increases the kill threshold value by repairing the ship over time.

e) Electronic warfare modules(EMC, EMMC)

Electronic warfare affects hit probability. Efficiency ranges from [0; 1]

7) Weapons

a) Kinetics

- Electromagnetic slug throwers
- Gun turrets
- Plasma
- Missiles/torpedoes
- Other kinetic weapons(macron guns etc.)

b) Energy

- Lasers
- Particle Beams(Electron, Ion etc.)

c) Range/effective range

d) Damage per shot

e) Rate of fire

f) Ammo capacity

g) Reload time

h) Accuracy

i) Energy requirements

j) Mounting arc

k) Acquisition time

8) Crew

9) Point defense

10) Mass

11) Dimensions

12) Time it can be operational

13) Cost to build

Main ship variables:

Variable	Description
Position	XYZ coords or vector in 3D space
Velocity	Vector magnitude & direction
Orientation	Direction ship is facing (Euler angles or quaternions)
Angular Velocity	How fast it can turn
Remaining Delta-V	Burn potential left
Target Lock	Is it aiming at anything? Acquisition progress
Current Weapon Cooldown	Can it fire now?
Power Buffer	Amount of energy available this frame
Heat Load	Internal thermal state
PD Overlap	Combined PD cone with neighbors
EWAR State	Jammed? Target fuzzy?
Damage Log	Internal module states & thresholds
Kill Value	Estimated impact on battle if killed

At its core, the variable for ship's parameter relative to the enemy is k . It's basically "the amount of enemy killed per second". Now, k is defined by a lot of things. For this reason, let's summarize the formula:

$$k = \frac{P_{hit} R_f D_{ps}}{E_{kt}}$$

P – probability of hit

R – fire rate of weapon/weapons

D – damage per shot/volley

E – Enemy kill threshold

For further use, we will summarize the top side of the equation as d , i.e damage.

To incorporate dynamics: during mission, every ship has a certain relative variable.

Each of these variables (except the fire rate, as it depends on the weapon itself) are derived from:

1) $P_{hit1} = \min(1, \frac{4cv^4}{\pi a^2 D^4})$ for kinetic weapons, where:

c – target minimum cross section

v – projectile velocity

a – target lateral acceleration

D – engagement range

2) $P_{hit2} = \min(1, \frac{Cc^4}{4\pi a^2 D^4})$ for energy weapons, where

C – target cross section

c – speed of light

a – target lateral acceleration

D – distance to target

The min value means that hit probability is the minimum between 1 (i.e. 100% chance of hit) or the value given by the second thing.

- 3) Your weapon isn't this omnidirectional laser beam that wipes everything in a million-mile radius. No. It has a certain distance in which it's effective. Plus, as it is mounted on your ship, it can't simply shoot everywhere. It has a direction. Basically, the volume in which it can hit a target is a cone, and that volume can be calculated by:

$$V_{weapon} = \frac{1}{3} \times \pi(R \times \tan \alpha)^2 \times R$$

R : maximum effective range

α : half the cone angle of the weapon itself (i.e. the weapon can't fire radially or inwards)

To win, you basically have to align the weapon cone with the enemy.

If your enemy is static and within a reasonable range, you simply need to accelerate towards them to finish them off.

If the enemy is static and in some other location other than right in front of you, you simply need to turn your ship for the weapon cone to intersect with the enemy ship's surface area.

The time of acquisition can be calculated with:

$$T_{acquire} = \frac{\theta}{\omega} T_{lock}$$

Where

θ is angular difference between you and the enemy (I.e. "The enemy is 90° on the left")

If the enemy isn't static and is in front of you, but doesn't have weapons: in both directions (forward and backwards), it has a volume in which it can be located within time T . However, if that volume overlaps with your weapon cone, it's killed.

Now, that volume depends on:

- Acceleration
 - Angular acceleration (α)
 - Thrust direction
 - Delta V (m/s)
 - Time window(T)
- a) The angle in which the ship can rotate in a T time window if no RCS(in that case, the rotation angle is omnidirectional) is:

$$\theta_{max} = \alpha \times T$$

- b) If the ship can sustain constant acceleration, then the distance it can travel(\vec{d}) is dependent on acceleration and delta V, as time to cover that distance also depends on these 2 parameters. Time is calculated by:

$$T_{max} = \frac{\Delta v}{a}$$

Therefore, the max distance a ship can travel is:

$$\vec{d} = \frac{1}{2} a T_{max}^2 = \frac{1}{2} a \left(\frac{\Delta v^2}{a^2} \right) = \frac{\Delta v^2}{2a}$$

- c) However, the engagement window may be shorter than T_{max} . Therefore, \vec{d} will equal to:

$$d = \frac{1}{2} a T_{window}^2$$

- d) If $T_{window} > T_{burn}$, the ship will continue to coast at velocity v in $T_{window} - T_{burn}$ time. Therefore, the total distance will be:

$$d_{max} = \vec{d} + v_{coast} \times (T_{window} - T_{burn}) = \frac{\Delta v^2}{a^2} + v_{coast} \times (T_{window} - \frac{\Delta v}{a})$$

Let's use an example, as this is getting incredibly complicated (I'll do this later)

- e) The volume is roughly:

$$V_{maneuver} \approx \frac{1}{3} \pi (r_{base})^2 d_{max}$$

Where:

$$r_{base} = d_{max} \times \tan \theta_{max}$$

I'll use an example later.

- f) Therefore, the hit probability will be:

$P_{geometric} = \min(1, \frac{V_{overlap}}{V_{maneuver}})$; It is preferable for the overlap to be at least equal to maneuver volume. For example, the enemy knows the overlap, therefore, it will try to avoid said zone. However, if the overlap is equal to, or greater than the volume, you achieve a kill. This is the desired quality of your ship.

This has assumed you being stationary, whilst the enemy moved around.

If the enemy isn't static and in an angle against you, and you aren't static, we will take relative velocity and acceleration as means to calculate enemy maneuver volume.

$$\vec{v}_{rel} = \vec{v}_{enemy} - \vec{v}_{you}$$

$$\vec{a}_{rel} = \vec{a}_{enemy} - \vec{a}_{you}$$

Therefore, the time the enemy will burn for is:

$$T_{burn} = T_{engagment}, \min\left(\frac{\Delta v}{|\vec{a}_{rel}|}\right)$$

Distance is then:

$$d = \frac{1}{2} |\vec{a}_{rel}| T_{burn}^2$$

And volume is:

$$V_{Relative_maneuver} \approx \frac{1}{3} \pi (d \times \tan \theta_{max})^2 \times d$$

Okay, so, finally, hit probability for laser weapons is equal to:

$$P_{hit} = \min(1, P_{geometric} \times P_{hit2}) = \min\left(1, \frac{V_{overlap}}{V_{Relative_maneuver}} \times \frac{Cc^4}{4\pi a^2 D^4}\right)$$

For kinetic weapons, it's kinda the same:

$$P_{hit} = \min(1, P_{geometric} \times P_{hit1}) = \min\left(1, \frac{V_{overlap}}{V_{Relative_maneuver}} \times \frac{4cv^4}{\pi a^2 D^4}\right)$$

Doctrine: Kill Assurance via Volume Collapse

Doctrine characteristics:

As mentioned before, the enemy knows the overlap volume, therefore, it will try to avoid said zone. However, if the overlap is equal to, or greater than the volume, you achieve a kill. This is the desired quality of your ship.

As you can move, you can assume that the enemy maneuver volume gets reduced the more velocity and acceleration you have (As per the volume calculation using relative acceleration and velocity). Therefore, your objective is to have fast enough ships to constrict said volume to equal your weapon cone.

$$V_{weapon} \geq V_{relative_maneuver} \Rightarrow P_{geometric} = 1$$

I.e. If the difference between weapon volume and relative maneuver volume is positive, kill is possible. If it's negative, evasion is likely.

Explanation:

First, this doctrine is basically flanking and encirclement.

Generally, the enemy ship isn't going to have a load of delta V and acceleration. They won't be able to move much. Weapons have decent range in space. Plus, their turret rotates, or they can just shoot at something that's 200 km away at a 50 degree angle. Which means that they can COVER A VOLUME(i.e. if you are in that volume, you are getting stung).

And as I said, general ships ain't gonna have insane delta V and acceleration. So, they won't be able to not get hit with your weapon!

But. If they DO have sufficient delta V and/or acceleration, they can just... Evade your weapon. And, simply, if you really want to hit them, just move around that ship so that wherever it chooses to go, your weapon is ready to fry their buttocks.

Of course, you'll need more ships if the enemy can also has good weapons, but, that's about it.

And, you also benefit from the sweet little fact that ships can't put much lateral armor on themselves (because it adds a load of mass). So, you get that nice bonus from flanking.

4) $D_{PS} = (E_{weapon} \times \epsilon_{eff})$, where:

- $E_{weaponKinetic} = \frac{1}{2} m_{projectile} v^2$
 - We need to make an incredibly important point here. For kinetic weapons:
 - $v = v_{projectile} + v_{RelativeVelocity}$
 - As the ship's relative velocity might play a major role in a battle.

$$E_{weaponBeam} = P_{beam} t_{beam}$$

P_{beam} = beam power output (in watts)

t_{beam} = time the beam is held on the target(s)

For lasers, the intensity (i.e. the energy deposited at the target) falls off based on the inverse square law:

$$intensity \propto \frac{1}{distance^2}$$

$$E_{weaponWarhead} = Y_{warhead} \text{ or } E_{weaponWarhead} = Y_{warhead} + E_{warhead}$$

$$Y_{warhead} = \text{warhead/missile yield (in Joules)}$$

$$E_{warhead} = \text{warhead/missile kinetic energy (in Joules)}$$

ϵ_{eff} – effectiveness of weapon (i.e. percentage of the energy transferred to target). Depends on angle of attack, weapon type, ship materials (i.e. certain armor types might grant bonus to kinetic damage?)

Doctrine: Swarming

Is countered by sufficient amount of point defense.

Whilst designing a spacecraft, a tradeoff between mass and velocity has to be made. Armoring the spacecraft requires a logarithmic increase of fuel (per the rocket equation), and it drops acceleration. Therefore, it is possible to have lightly armored craft that maximize acceleration and delta V.

Swarms can not only be used for volume collapse (and the increase of general hit probability thanks to restricting distance to the target), but also to increase the D_{PS} value.

For kinetic weapons (be it slugs or missiles), it is quite simple – the relative velocity of the spacecraft gets added to the projectile, which increases the damage to the target.

For energy weapons (lasers specifically), the power is governed by the inverse square law. Therefore, the closer the laser, the more damage it will do to the target.

5) Kill threshold is a complicated process haha! Let's logically build it.

So, we are interested on how much energy is required to “kill” a ship. For this, we must first understand the components of a ship, as it's not a single unified “blob” of matter that we need to fully annihilate. Here are also the formulas to calculate the amount of energy they can absorb:

a) Frontal armor.

$$E_{armor,front} = t_{front}a_{armor} + t_{hull}a_{hull}$$

Where:

t_{front} – thickness of frontal armor
 a_{armor} – energy absorption of frontal armor
 t_{hull} – thickness of hull
 a_{hull} – energy absorption of the hull

- b) Lateral armor (same as frontal)
- c) Radiators – 0. Simply take surface area of them and plug it inside hit probability.
- d) Reactor

$$E_{reactor} = t_{ReactorShielding} + a_{absorbition}$$

- e) Engines/Propulsion (if armored – same as frontal armor; if not – same as reactor)
- f) Crew/AI. This works on both ships and missiles.
 When a crewed/non-crewed vessel is targeted by energy weapons, two things happen.

1. Heat: electronics/crew have a certain temperature ceiling, above which they degrade/fail. Heat is mainly generated by laser weapons, but it can accumulate due to radiator failure. Values:

C – heat capacity(J/K)
 Q – heat absorbed
 $T_{current}$ – current internal temperature
 T_{max} – maximum safe operating temperature
 T_{fail} – temperature at which total failure occurs.

Electronics damage rate:

$$k_{he} = \frac{1}{T_{fail,e} - T_{max,e}}$$

Crew damage rate:

$$k_{hc} = \frac{1}{T_{fail,c} T_{max,c}^2}$$

Electronics heat degradation:

$$D_e = k_{he} \times \max\left(0, \frac{Q}{C_e} + T_{current} - T_{max,e}\right)$$

Crew heat degradation:

$$D_c = k_{hc} \times \max\left(0, \frac{Q}{C_c} + T_{current} - T_{max,c}\right)^2$$

2. Radiation. Particle weapons deliver radiation upon a ship/missile, which will degrade electronics/crew. Values:

R – Radiation dose received
 R_{max} – Max safe radiation dose
 R_{fail} – Dose at which full failure/death occurs
 A_s – Radiation attenuation (crew only, from suit)

Electronics damage rate:

$$k_{re} = \frac{1}{R_{fail,e} - R_{max,e}}$$

Crew damage rate:

$$k_{rc} = \frac{1}{R_{fail,c} R_{max,c}^2}$$

Electronics radiation degradation:

$$D_e = k_{re} \times \max(0, R - R_{max,e})$$

Crew radiation degradation:

$$D_c = k_{rc} \times \max(0, R(1 - A_s) - R_{max,c})^2$$

Energy required for system failure depends for the heat/radiation to penetrate protective layers, reaches the crew/electronics and degrades them to failure.

For heat, we want $T_{current} \rightarrow T_{fail}$. Use $Q = C\Delta T$, or, in this case:

$$Q_{needed} = C \times (T_{fail} - T_{current})$$

Protection factors include:

α – Armor attenuation factor (0–1)

β - Hull attenuation factor (0–1)

γ – component shielding (0–1)

Factor of total transmission through protection is:

$$f = \alpha \times \beta \times \gamma$$

Therefore, to make the system fail:

$$E_{incoming} = \frac{Q_{needed}}{f}$$

For radiation energy, we account for the same protection factors. The radiation dose required is:

$$R_{needed} = R_{fail} - R_{current}$$

The energy that reaches the system is:

$$E_{incoming} = \frac{R_{needed}}{f}$$

- g) Total annihilation. If the energy deposited by the weapon is the same/exceeds the absorption ability of both armor and hull, ship is killed.

$$E_{weapon} \geq t_{armor}a_{armor} + t_{hull}a_{hull}$$

We now also need to account for critical hits.

Different strategies emerge for attack/engagement if both sides have energy weapons:

- 1) 1:1 engagement. Every ship engages another enemy ship if both sides have same amount of ships. Winner determined by a higher k value.
- 2) Group vs 1 engagement. Z number of ships engage 1 opponent as a group.

First, we will define a cycle. Basically, there are 2 types of cycles:

- A) Cycle α : Your group kills the enemy before they can fire back. Here, you simply need a high k value to finish off the enemy (e.g. multiple long range lasers exploding the enemy reactor before the enemy can shoot its guns).

Simple formula: $N = \frac{E_{kt}}{d}$; basically, divide the kill threshold by the total damage output of a single ship in a single shot, and it'll tell you how many ships you need to one-shot the enemy.

- B) Cycle 1: You need a group that will kill the enemy in 1 cycle (i.e. in a single salvo). In this case, you also have to account for the fact that the enemy can shoot back. Therefore, the amount of ships you lose equates to k_{enemy} , or basically the amount of ships the enemy can kill in a single shot/salvo before it is killed.

- C) Cycle n : Your group fires, but the enemy doesn't die in one shot.

Example scenario:

You start with 3 ships. Enemy has a ship with 1000 HP. Each of your ships can do 100 HP of damage per cycle. Therefore, in the first cycle, the group deals 300 HP of damage to the lone ship.

But, let's assume the enemy can one-shot our ships. In this case, we get cycle 1, where:

Friendly: 2 Ships; Enemy: 700 HP

Now, your side can only do -200 HP of damage. Therefore, in cycle 2, you have:

Friendly: 1 Ship; Enemy: 500 HP

And in cycle 3, the enemy HP is 400 HP, and you have no ships.

Here, we see the following things.

1. In each cycle, the damage of the group was:

$$d_{total} = N_{Ship} d_{perShip}$$

2. The amount of damage the group outputted dropped by:

$$d_{perShip} \times [N + (N - [C + k_{enemy}]) + (N - [C + k_{enemy}]) \dots + 1]$$

Or

$$T_{dmg} = d_{perShip} \times \frac{N(N + 1)}{2}$$

Where:

T_{dmg} – Total damage from our side

N – number of friendly ship

k_{enemy} – enemy k value (For this scenario, it is 1)

C – cycle

Therefore, if we wanted to win, we'd need:

$$T_{dmg} \geq E_{kt}$$

Basically, we'd need to solve for:

$$Z = N^2 + N - \frac{2 \times E_{kt}}{d_{perShip}} \geq 0$$

For this specific case

$$\begin{aligned} Z &= N^2 + N - \frac{2 \times 1000}{100} \geq 0 \\ Z &= N^2 + N - 20 \geq 0 \\ Z &= 4 \end{aligned}$$

Now let's rerun the scenario for 4 ships:

You start with 4 ships. Enemy has 1000 HP ship. Each of your ships can do -100 HP of damage. The enemy can 1 shot 1 of your ship.

Cycle 1.

Friendly ships: 3; Enemy HP: 600 HP

Cycle 2.

Friendly ships: 2; Enemy HP: 300 HP

Cycle 3.

Friendly ships: 1; Enemy HP: 100 HP

Cycle 4.

Friendly ships: 0; Enemy HP: 0 HP

Therefore, we needed 4 ships in a group to win against the enemy. If we need survivors(signified by value M), we calculate:

$$Z = d_{perShip} \times \frac{N(N + 1) + M}{2} - E_{kt}$$

As more ships reduce the amount of cycles as well.

To incorporate the previous hit probability discussion, we also need to account for group interaction.

We will now incorporate Volume Collapse in team dynamics.

Multiple ships can work together to create a combined weapon volume that entirely overlaps the enemy's maneuver volume — even when no single ship can do it alone.

The combined firing volume (i.e. combine the weapon cones of all friendly ships) is calculated by:

$$V_{team} = \bigcup_{i=1}^n V_{weapon}$$

The effective coverage is:

$$V_{coverage} = V_{team} \cap V_{maneuver}$$

Kill occurs if:

$$V_{coverage} \geq V_{maneuver}$$

Therefore, the total geometric hit probability of a team is:

$$P_{geometric,team} = \frac{V_{coverage}}{V_{maneuver}}$$

Where evasion percentage chance of the enemy is equal to:

$$P_{evasion} = 100(1 - P_{geometric,team})$$

If the weapon cone volume overlap is small, the required amount of ships can be calculated by:

$$M = \left\lceil \frac{2}{1 - \cos \alpha} \right\rceil$$

Incorporating this in the cycle logic, M will be the minimum amount of ships that will be necessary for the containment of the enemy.

- 3) Sequencing. Sequencing is at its core “group vs 1” targeting tactic. However, the rate at which our ships die increases due to the number of enemy combatants. Let’s use a simple example.

Let’s say you have 10 ships, and the enemy has 15. In group vs 1 tactic, you do indeed consider the damage the enemy ship causes to your ships. However, the enemy is also running group vs 1 against our ships. Therefore, relative to our ship that’s being targeted, we need to use the full enemy group k value and not of that of a single ship.

Basically, plug the team k value but for the enemy.

Point defense modeling

This is incredibly important, as nearly every ship is going to have some sort of point defense.

Point defense works against kinetic weapons. To understand how point defense works, we need to view it as a “ship” of its own.

For overall k value, point defense counts as an increase to the opposing ship’s kill threshold.

Point defense has qualities such as:

a) Hit probability:

For point defense, hit probability is calculated similarly to ship’s hit probability. In this case, the target is the kinetic impactor, be it a slug or a guided warhead.

The volume in which point defense can operate is:

$$V_{PD} = \frac{1}{3} \times \pi(R \times \tan \alpha)^2 \times R$$

Against a kinetic slug, the formula for hit probability is the same as for kinetic or energy weapons, as in:

$$P_{kPD} = \min(1, \frac{4cv^4}{\pi a^2 D^4})$$

For kinetic point defense, and:

$$P_{ePD} = \min(1, \frac{Cc^4}{4\pi a^2 D^4})$$

For energy point defense (lasers, particle beams).

Against missiles, we again incorporate the volume overlap, where the hit probability for kinetic point defense is:

$$P_{hitPD} = \min(1, \frac{V_{PDoverlap}}{V_{MRelative_maneuver}} \times \frac{4cv^4}{\pi a^2 D^4})$$

And for energy point defense:

$$P_{hitPD} = \min\left(1, \frac{V_{PDoverlap}}{V_{MRelative_maneuver}} \times \frac{Cc^4}{4\pi a^2 D^4}\right)$$

We may incorporate team dynamics. The total volume of multiple point defense modules on a ship/ships is equal to:

$$V_{teamPD} = \bigcup_{i=1}^n V_{PD}$$

This will come again in damage per shot calculations. Against missiles, we can surmise that:

$$V_{PDcoverage} = V_{teamPD} \cap V_{Mmaneuver}$$

The probability of the missile being hit is:

$$P_{geometric,teamPD} = \frac{V_{PDcoverage}}{V_{Mmaneuver}}$$

Point defense penetration percentage chance is calculated by:

$$P_{penetrate} = 100(1 - P_{geometric,teamPD})$$

b) Fire rate

This just depends on the weapon design itself.

c) Damage per shot:

This is similar to cycle based attrition against ships. You can concentrate point defense on target similarly to how you would do with ships. If the point defense module isn't fired at, then we simply need $N = \frac{E_{kt}}{d_{PD}}$ modules (divide enemy projectile kill threshold by the damage caused by point defense).

But, we have to take into consideration the worst case scenarios. First, we need to know the worst case scenario, where the enemy tries to saturate out point defense. Based on the fire rate of the enemy fleet and the kill threshold of the projectiles, we can build the minimum amount of point defense modules that will be required to survive.

$$N_{PD} = \frac{d_{PD}}{A_p \times E_{pkt}}$$

Where:

N_{PD} : number of PD modules

d_{PD} : damage per PD module

A_p : amount of projectiles

E_{pkt} : projectile kill threshold (take the maximum value)

d) Kill threshold

Similar to ship's kill threshold. Simply think of it as "If the point defense module gets hit, it will fail". However, for point defense, we also need to incorporate jamming.

The energy required to "kill" a point defense module is:

$$E_{PD} = t_{PD} a_{PD}$$

Which is basically the amount of damage that the point defense module can sustain until it fails.

However, if the PD module is kinetic based, it has the possibility of jamming, especially at higher fire rates.

Doctrine: Point Defense Overlap

As we have previously discussed, team dynamics can be incorporated in point defense, which raise not only the hit probability (due to more overlap volume with the projectiles), but the overall damage, as multiple point defense modules will be fired sequentially at opposing projectiles.

Therefore, to ensure impenetrability, the volume of the point defense module on the furthest-most ship in the group must cover the volume of the point defense module on the opposing side of the group.

Basically, the group must set up a grid, where the ships are close together to ensure point defense effectiveness.

This is to lower the $P_{penetrate}$ value, and possibly bring it down to 0 to avoid any harm to the group.

Alongside that, we also have to consider that with even a 100% probability of hit, we need to take the projectiles down fast enough. Depending on the pattern of projectiles (i.e. if the volley is targeting one ship or multiple), the constraint becomes the fire rate of your modules, not their damage output.

Therefore, you may create multiple subgroups to target multiple volleys (effectively increasing the fire rate).

Ship designer

Now onto a real spacecraft and a gun that was tested in space.

For the gun (let's get the fun out of the way first), we will use Rikhter R-23. A gun that the soviets fired in space and was proven to be effective (as in, it fired 50 rounds).

We will assume that we mount this weapon at the front of our ship. Mass: 60kg; Fire rate: 2,500 rpm; muzzle velocity: 850 m/s.

Currently, we will exclude the weapon mass influence on delta V. we will suppose that weapon, ammo, targeting and the crew needed to operate the gun fall under the "cargo".

For spacecraft, we will use SpaceX Dragon 2 module. Specifications: