Bangabandhu Sheikh Mujibur Rahman

Science & Technology University, Gopalganj-8100

1st Year 1st Semester B.Sc. Engg. Examination – 2013

Department of Computer Science and Engineering

Course No: MAT104 Course Title: Algebra, Trigonometry and Vector

Full Marks: 70 Time: 4 Hours

N.B.: i) Answer **SIX** questions, taking any **THREE** from each section.

ii) All questions are of equal values.

iii) Used separate answer script for each section.

Section –A

1. a) Define union and intersection of two sets with example. Let P be the set of all positive integers and given by A=$\{x: x^3 - 2x^2 - 19x + 20 = 0\}$, B = $\{x: x \in P \text{ and } x \text{ is an even integer and } x \leq 10\}$ and C = $\{x: x \in P \text{ and } x \text{ is an integer divisible by } 5 \text{ and } x < 15\}$. Find $(A \cup B) \cap C$ and $(A \cup B) - C$. $3\frac{2}{3}$

   b) Define equivalence relation with example. If R be the relation on the set real numbers is given by $R = \left\{(a, b): a, b \in \Re, |a - b| \leq \frac{1}{2}\right\}$. Prove that R is not an equivalence relation. 4

   c) What is function? If f: $\Re \rightarrow \Re$, defined by f(t) = $t^2 + 2$ and g:$\Re \rightarrow \Re$,defined by g(t) = 2t+1, find the composite functions (fog)(t) and (gof)(t). Hence find the value (fog)(3) and (gof)(-2). 4

2. a) What is polynomial equation of nth degree? Find the relation between roots and coefficients of nth degree polynomial equation. 4

   b) If a, b, c be the roots of equation $x^3 + px^2 + qx + r = 0$ then form an equation, whose roots are $bc - a^2$, $ca - b^2$ and $ab - c^2$.

   c) State Discate's rule of sign and determine the nature of the roots of the equation $x^{10} - 4x^6 + x^4 - 2x - 3 = 0$. $3\frac{2}{3}$

3. a) What is determinant? Write down the five fundamental properties of determinants. $3\frac{2}{3}$

   b) Solve the following linear equations by using Cramer's rule: 4

   $x - 2y + 3z = 11.$
   2x+y+2z=10
   3x+2y+z=9

   c) Establish 4

   $$\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \cdots\cdots\cdots + \cos\{\alpha + (n-1)\beta\} = \cos\left\{\alpha + \frac{1}{2}(n-1)\beta\right\}.\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$$

4. a) State De-Moiver's theorem. Prove De- Moiver's theorem for positive integers. 4

   b) Solve $x^7 + x^4 + x^3 + 1 = 0$ by using De- Moiver's theorem. 4

   c) If α and β are the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$ form an equation whose roots are α^n and β^n. $3\frac{2}{3}$

Section - B

1. a) Define linearly dependent and independent vectors. Examine whether the vectors $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$, $3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $2\hat{\imath} - 4\hat{\jmath} - \hat{k}$ are linearly dependent or independent $3\frac{2}{3}$

   b) Find the unit vector perpendicular to each of the vector $\underline{a} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\underline{b} = 2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$. Also determine the sine angle between $\underline{a}$ and $\underline{b}$. 4

   c) What is vector product of two vectors? If $\underline{a} + \underline{b} + \underline{c} = \underline{0}$ and $|\underline{a}| = 3$, $|\underline{b}| = 5$ and , $|\underline{c}| = 7$. Find the angle between $\underline{a}$ and $\underline{b}$. 4

2. a) Find the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t$, $y = \cancel{2\cos}\,2 cos3t$ and $z = 8t$ at any time $t > 0$. Find the magnitude of the velocity and acceleration. $3\frac{2}{3}$

   b) Show that any vector $\underline{r}$ can be expressed as a linear combination $\underline{r} = (\underline{r}.\underline{\acute{a}})\underline{a} + (\underline{r}.\underline{\acute{b}})\underline{b} + (\underline{r}.\underline{\acute{c}})\underline{c}$, where $\underline{\acute{a}}$, $\underline{\acute{b}}$ and $\underline{\acute{c}}$ are reciprocal system of vectors of $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively. 4

   c) What do you mean by divergence of a vector function? If $\underline{f} = f_1\hat{\imath} + f_2\hat{\jmath} + f_3\hat{k}$ be a continuously differentiable vector function, then prove that $\nabla.\underline{f} = \frac{\delta f_1}{\delta x} + \frac{\delta f_2}{\delta y} + \frac{\delta f_3}{\delta z}$. 4

3. a) Define surface integral. If $\underline{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$, Evaluate $\iint_S \underline{F}.\underline{n}\, ds$ where S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1. 7

   b) Define line integral. Evaluate $\int_C \underline{F}.d\underline{r}$, where $\underline{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$, curve C is the rectangle in xy plane bounded by y = 0, x = a, y = b and x = 0. $4\frac{2}{3}$

4. a) Define vector product of three vectors. Prove that $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a}.\underline{c})\underline{b} - (\underline{a}.\underline{b})\underline{c}$ $4\frac{2}{3}$

   b) Determine the constants a, b and c so that the vector $\underline{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. 3

   c) If $\underline{F} = 2xz\hat{\imath} - x\hat{\jmath} + y^2\hat{k}$, evaluate $\iiint_V \underline{F} dv$, where V is the region bounded by the planes x = y = z = 0 and x = y = z =1 4