Bangabandhu Sheikh Mujibur Rahman Science and Technology University

Department of Computer Science and Engineering

1st Year 2nd Semester B.Sc. Engineering Examination-2013

Course No: MAT-156 Course Title: Differential and Integral Calculus

Total marks: 70 Time: 4 hours.

N.B.

i. Answer **SIX** questions, taking any **THREE** from each section.

ii. All questions are of equal values

iii. Use separate answer script for each section

SECTION - A

1. (a) Define domain and range of a real valued function. 2

(b) Draw the graph of the following function, $3\frac{2}{3}$

$$f(x) = \begin{cases} 0, \text{ if } |x| > 1 \\ 1+x, \text{ if } -1 \leq x \leq 0 \\ 1-x, \text{ if } 0 < x < 1 \end{cases}$$

Find the domain and range of the function.

(c) Define limit of a function. Using the (ε, δ) definition of limit show that 3

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 3$$

(d) Sketch the graph of the function: $y = x^3$, and its inverse function. 3

2. (a) Test the continuity of the function: 4

$$f(x) = \begin{cases} x^2, & x \leq 0 \\ 2x, & x > 0 \end{cases}$$

at $x=0$.

(b) Prove that if f is differentiable at a point c, then f is continuous at c, but the converse however not always true. 4

(c) Find the differential co-efficient of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$ $3\frac{2}{3}$

3. (a) A function $f(x)$ is defined as: 3

$$f(x) = \begin{cases} x^2, if\ x \leq 0 \\ x, if\ 0 < x < 1 \\ \frac{1}{x}, if\ x \geq 1 \end{cases}$$

Show that $f(x)$ is not differentiable at $x=0$. What is about $f'(x)$ at $x=1$?

(b) Find $\frac{dy}{dx}$ where : 6

i. $y = (sinx)^{lnx}$ ii. $y = \cos^{-1}\left(\sqrt{1+x^2} - x\right)$ iii. $x^y y^x = 1$

(c) If $y = (x^2-1)^n$ then prove that $(x^2-1)y_{n+2}+2xy_{n+1}-n(n+1)y_n=0$. Hence if $u=y_n$, show that this equation can be written as: $2\frac{2}{3}$

$$(1-x^2)\frac{d^2u}{dx^2} - 2x\frac{du}{dx} + n(n+1)u = 0$$

4. (a) State and prove the Mean Value Theorem. 4

(b) Find the maximum and minimum values of the function: 4

$f(x) = x^3 - 3x^2 + 24x - 12.$

(c) Using Taylor's theorem expand $log_e x$ in power of $(x-2)$. $3\frac{2}{3}$

SECTION-B

5. (a) If $u(x,y) = xcosy + ye^x$, then find : 4

i. $\frac{\partial^2 u}{\partial x^2}$; ii. $\frac{\partial^2 u}{\partial y^2}$; iii. $\frac{\partial^2 u}{\partial x \partial y}$; iv. $\frac{\partial^2 u}{\partial y \partial x}$

(b) Define Homogeneous functions. State and prove Euler's theorem on homogeneous function for three variables. 4

(c) If $u = \tan^{-1}\frac{x^2+y^2}{x-y}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ $3\frac{2}{3}$

6. Evaluate the following integrals:

(a) $\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$ (b) $\int e^{2x}\cos 2x\, dx$ $2\frac{2}{3}$ +3

(c) $\int \sqrt{\frac{x}{a-x}}dx$ (d) $\int \frac{dx}{sinx + cosx}$ 3+ 3

7. (a) State and prove the Fundamental theorem of integral calculus. 4

(b) Evaluate the reduction formula, $I_n = \int_0^{\frac{\pi}{2}} cos^n x dx$ 4

Hence evaluate I_5.

(c) Using the Fundamental theorem of calculus evaluate the following definite integral: $3\frac{2}{3}$

$$\int_0^{\frac{\pi}{3}} \frac{dx}{5+4cosx}$$

8. (a) Find the area above the x-axis, inclined between the curve, $y^2=ax$ and $y^2+x^2=2ax$. 4

(b) Find the area of the cardioids: $r = a(1-\cos\theta)$ 4

(c) Find the area of the region enclosed by the parabola $y = 2-x^2$ and the line $y=-x$. $3\frac{2}{3}$