# 1. Brief Overview:

In our supply chain network, there are two producers of biomass: Harvester 1 and Harvester 2. From Harvester 1 the raw materials are transported to the storage space. It takes 50 miles by mode-1 to transport the raw materials to the storage. Goods from Harvester 2 can be transported using mode-1 which takes 30 miles or by mode-3 which is 90 miles.

Once the raw materials reach the storage facility, it is transported to Pre-Processing Plant 1 and Pre-Processing Plant 2. Raw materials from Harvester 1 can be directly transported to the Pre-Processing Plant 1 or can be transported from the Storage Facility.  It takes 75 miles by mode-1 and 150 miles by mode-2 to reach the Pre-Processing Plan 1 directly from Harvester 1 and 75 miles by mode-1 and 120 miles by mode-2 to reach the plant from Storage facility. For Pre-Processing plant 2 the materials are transported either using mode-1, which takes 80 miles to reach Plant 2 or mode-3, which takes 180 miles to reach Plant 2.

The final step in the network is to transport the goods from Pre- Processing Plant 1 and Pre-Processing plant 2 to Biorefinery 1 and Biorefinery 2. From Pre-Processing Plant 1 the materials are transported to Biorefinery 1 using mode-1 (200 miles) or mode-3 (350 miles). Biorefinery 2 can also receive materials directly from Pre-Processing Plant 1 using mode-1 which is 380 miles apart. Similarly, for Biorefinery 2 the materials are received from Pre-Processing Plant 2 using mode-1 (300 miles) or mode-2 (490 miles). Materials form Pre-Processing Plant 2 can be directly transported to Biorefinery 1 using mode-1 (250 miles) or mode-3 (500 miles).

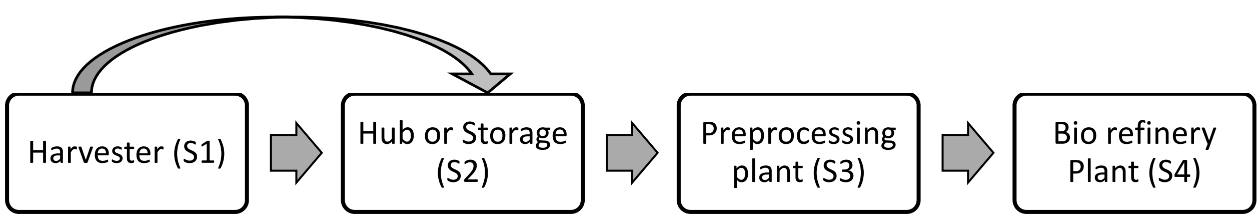


**Figure 1.1:** Four echelon supply chain network for biomass transportation

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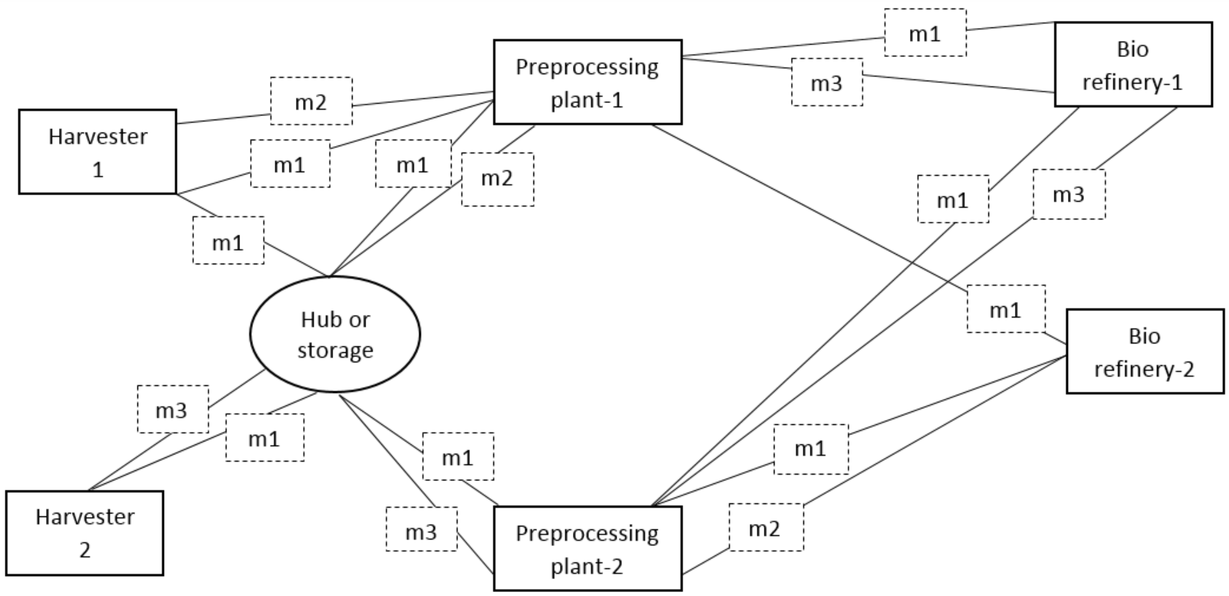


**Figure 1.2:** Transportation network with available modes and routes

## 2. Problem Statement

In the above transportation network two objectives have been considered. One is transportation cost and the other one is amount of GHG emission. Therefore it is multi objective (MO) optimization problem. In the subsequent sections appropriate mathematical model is developed and solved for optimal solutions which clearly shows the tradeoff between transportation cost and GHG emission levels. It might be helpful to the appropriate stakeholders to make their decisions in a more effective and better way.

# 3. Data Used

The necessary data related to demand, distance, capacity, and ordering cost for different transportation modes are taken from the article by Sarder et al. and U.S. department of Energy [1, 2]. Some of the other necessary data are logically assumed. Following tables show all the data used for this class project.

**Table 3.1:** Capacity and demand at different facilities

| Harvester (i) | Capacity |
|---|---|
| 1 | 200 |
| 2 | 150 |

| Storage Hub (j) | Capacity |
|---|---|
| 1 | 250 |

| Preprocessing Plant (k) | Capacity |
|---|---|
| 1 | 180 |
| 2 | 150 |

| Bio refinery (l) | Demand |
|---|---|
| 1 | 150 |
| 2 | 100 |

**Table 3.2:** Fuel and ordering costs for different transport modes

| Transport Mode (m) | Cost/ton-mile ($c_m$) | ordering cost per order ($O_m$) |
|---|---|---|
| 1 | 0.12 | 100 |
| 2 | 0.06 | 300 |
| 3 | 0.015 | 500 |

**Table 3.3:** Greenhouse gas (GHG) emission rate for different transport modes

| Transport Mode (m) | GHG emission rate (gram /ton-mile) ($e_m$) |
|---|---|
| 1 | 3 |
| 2 | 10 |
| 3 | 7 |

**Table 3.4:** Distances among different facilities for different transport modes

| Harvester (i) | Hub storage (j) | Mode (m) | Distance |
|---|---|---|---|
| 1 | 1 | 1 | 50 |
| 1 | 1 | 2 | - |
| 1 | 1 | 3 | - |
| 2 | 1 | 1 | 30 |
| 2 | 1 | 2 | - |
| 2 | 1 | 3 | 90 |

| Harvester (i) | Preprocess plant (k) | Mode (m) | Distance |
|---|---|---|---|
| 1 | 1 | 1 | 75 |
| 1 | 1 | 2 | 150 |

| Hub storage (j) | Preprocess plant (k) | Mode (m) | Distance |
|---|---|---|---|
| 1 | 1 | 1 | 75 |
| 1 | 1 | 2 | 120 |
| 1 | 1 | 3 | - |
| 1 | 2 | 1 | 80 |
| 1 | 2 | 2 | - |
| 1 | 2 | 3 | 180 |

| Preprocess plant (k) | Bio refinery (l) | Mode (m) | Distance |
|---|---|---|---|
| 1 | 1 | 1 | 200 |
| 1 | 1 | 2 | - |
| 1 | 1 | 3 | 350 |
| 1 | 2 | 1 | 380 |
| 1 | 2 | 2 | - |
| 1 | 2 | 3 | - |
| 2 | 1 | 1 | 250 |
| 2 | 1 | 2 | - |
| 2 | 1 | 3 | 500 |
| 2 | 2 | 1 | 300 |
| 2 | 2 | 2 | 490 |
| 2 | 2 | 3 | - |

# 4. Methodology

## 4.1. Modeling Transportation Cost and GHG Emission

**Indices:**

| | |
|---|---|
| $i$ | Harvesters |
| $j$ | Storage hub |
| $k$ | Preprocessing plants |
| $l$ | Bio-refinery plants |
| $m$ | Transport modes |

**Sets:**

| | |
|---|---|
| $S_1$ | Set of harvesters |
| $S_2$ | Set of hub or storage |
| $S_3$ | Set of processing plants |
| $S_4$ | Set of bio-refineries |
| $M$ | Set of available transport modes |

**Parameters:**

| | |
|---|---|
| $d_{ijm}$ | Distance between harvester i & hub storage j for transport mode m |
| $d_{ikm}$ | Distance between harvester i & processing plant k for transport mode m |
| $d_{jkm}$ | Distance between hub storage j & plant k for transport mode m |
| $d_{klm}$ | Distance between processing plant k & bio-refinery l for transport mode m |
| $c_m$ | Per mile transportation cost for one unit of material using mode m |
| $e_m$ | Per mile emission amount for mode m |
| $O_m$ | Ordering cost for mode m |
| $C_i$ | Capacity of harvesters |
| $C_j$ | Capacity of hub storages |
| $C_k$ | Capacity of preprocessing plants |
| $D_l$ | Demand of bio-refineries |

**Continuous Decision Variables:**

| | |
|---|---|
| $x_{ijm}$ | Amount of biomass transported from harvester i to hub storage j using mode m |
| $x_{ikm}$ | Amount of biomass transported from harvester i to processing plant k using mode m |
| $x_{jkm}$ | Amount of biomass transported from hub storage j to processing plant k using mode m |
| $x_{klm}$ | Amount of biomass transported from processing plant k to bio-refinery l using mode m |

**Objective functions**

**1st Objective:**

Minimize transportation cost

$= \sum x_{ijm} d_{ijm} c_m + \sum x_{ikm} d_{ikm} c_m + \sum x_{jkm} d_{jkm} c_m + \sum x_{klm} d_{klm} c_m +$

$\sum z_{ijm} O_m + \sum z_{ikm} O_m + \sum z_{jkm} O_m + \sum z_{klm} O_m$

**2nd Objective:**

Minimize GHG emission amount

$= \sum x_{ijm} d_{ijm} e_m + \sum x_{ikm} d_{ikm} e_m + \sum x_{jkm} d_{jkm} e_m + \sum x_{klm} d_{klm} e_m$

**Constraints:**

$$\sum_{\substack{j\in S_2\\ m\in M}} x_{ijm} + \sum_{\substack{k\in S_3\\ m\in M}} x_{ikm} \le C_i \qquad \forall i \qquad (1)$$

$$\sum_{\substack{i\in S_1\\ m\in M}} x_{ijm} \le C_j \qquad \forall j \qquad (2)$$

$$\sum_{\substack{i\in S_1\\ m\in M}} x_{ikm} + \sum_{\substack{j\in S_2\\ m\in M}} x_{jkm} \le C_k \qquad \forall k \qquad (3)$$

$$\sum_{\substack{k\in S_3\\ m\in M}} x_{klm} \ge D_l \qquad \forall l \qquad (4)$$

$$\sum_{\substack{i\in S_1\\ m\in M}} x_{ijm} = \sum_{\substack{k\in S_3\\ m\in M}} x_{jkm} \qquad \forall j \qquad (5)$$

$$\sum_{\substack{i\in S_1\\ m\in M}} x_{ikm} + \sum_{\substack{j\in S_2\\ m\in M}} x_{jkm} = \sum_{\substack{l\in S_4\\ m\in M}} x_{klm} \qquad \forall k \qquad (6)$$

$$z_{ijm} = \begin{cases} 1, & x_{ijm} > 0 \\ 0, & otherwise \end{cases} \qquad \forall i, j, m \qquad (7)$$

$$z_{ikm} = \begin{cases} 1, & x_{ikm} > 0 \\ 0, & otherwise \end{cases} \qquad \forall i, k, m \qquad (8)$$

$$z_{jkm} = \begin{cases} 1, & x_{jkm} > 0 \\ 0, & otherwise \end{cases} \quad \forall j, k, m \qquad (9)$$

$$z_{klm} = \begin{cases} 1, & x_{klm} > 0 \\ 0, & otherwise \end{cases} \quad \forall k, l, m \qquad (10)$$

$$x_{ijm}, x_{ikm}, x_{jkm}, x_{klm} \geq 0 \quad \forall\, i, j, k, l, m \qquad (11)$$

**Significance of the Constraints**

| | | |
|---|---|---|
| Constraints 1-3 | **:** | Amount of biomass transported does not cross the capacity of the respective facilities. |
| Constraint 4 | **:** | The demand for every bio refinery plant is satisfied. |
| Constraints 5, 6 | **:** | Incoming materials equal to outgoing materials at intermediate facilities. |
| Constraints 7-10 | **:** | Take only binary values to describe whether a link in between two facilities exists or not. If biomass is transported through a specific link then the binary variable takes the value '1', otherwise takes the value '0'. |

## 4.2. Solution Procedure and Pareto Optimality

There are two objectives in this study.

**Objective 1:** Minimization of transportation cost

**Objective 2:** Minimization of greenhouse gas (GHG) emission

The above two objectives are conflicting to each other, that means if we want to minimize the transportation cost it will lead to higher GHG emission and vice versa.

Following are some of the common techniques to solve these types of multi objective optimization problems.

i. Scalarization technique

ii. ε-constraint method
iii. Goal programming
iv. Multilevel programming

### 4.3. ε-constraint method

In our class project **ε-constraint method** [3, 4] is used. In mathematical terms, if we let $f_2(x)$ be the objective function chosen to be minimized, we have the following problem $P(\varepsilon_2)$:

$$\min f_2(x)$$

$$f_i(x) \leq \varepsilon_i, \forall i \in \{1, \ldots, n\} \backslash \{2\}$$

$$x \in S.$$

Where x is the vector of decision variables, $f_1(x),\ldots, f_n(x)$ are the n objective functions and S is the feasible region. By parametrical variation in the RHS of the constrained objective functions ($e_i$) the efficient solutions of the problem are obtained.

In our problem first the 1$^{st}$ objective function, transportation cost, is minimized relaxing the second objective function, GHG emission. The minimum value for the 1$^{st}$ objective is \$7,208 and the corresponding value of the 2$^{nd}$ objective function is 1,026,250 grams. Later on we added a new constrain-

$$2^{nd} \text{ objective: GHG emission} \leq \varepsilon \qquad (17)$$

Where we gradually reduced the value of ε in different iterations and the corresponding value of the 1$^{st}$ objective function is obtained.

## 5. Software Package Used

To formulate the problem IBM ILOG OPL (Version 12.6.3) is used and to solve the problem CPLEX solver (academic version) is used. All the OPL codes and data file is shown in the Appendix section of this report. As CPLEX cannot solve nonlinear problems, the first objective function is linearized which is already discussed in section 4.1.

# 6. Results and Discussion

Table 6.1 shows the transportation cost versus greenhouse gas (GHG) emission tradeoffs for the above problem.

**Table 6.1:** Transportation cost and GHG emission tradeoffs

| Transportation Cost (USD) | GHG Emission Amount (g) |
|---|---|
| 3,315 | 26,640 |
| 3,413 | 15,906 |
| 3,593 | 15,869 |
| 3,623 | 14,578 |
| 3,653 | 14,532 |
| 3,893 | 14,496 |
| 4,163 | 14,009 |
| 4,193 | 13,963 |
| 4,223 | 12,512 |
| 4,403 | 12,476 |
| 4,433 | 11,185 |
| 4,463 | 11,139 |
| 4,703 | 11,103 |
| 4,973 | 10,616 |
| 4,973 | 10,616 |
| 6,750 | 7,526 |
| 7,290 | 6,957 |
| 7,350 | 5,461 |
| 7,560 | 4,133 |
| 8,100 | 3,564 |
| 9,240 | 2,186 |
| 9,780 | 1,617 |
| 9,810 | 1,571 |

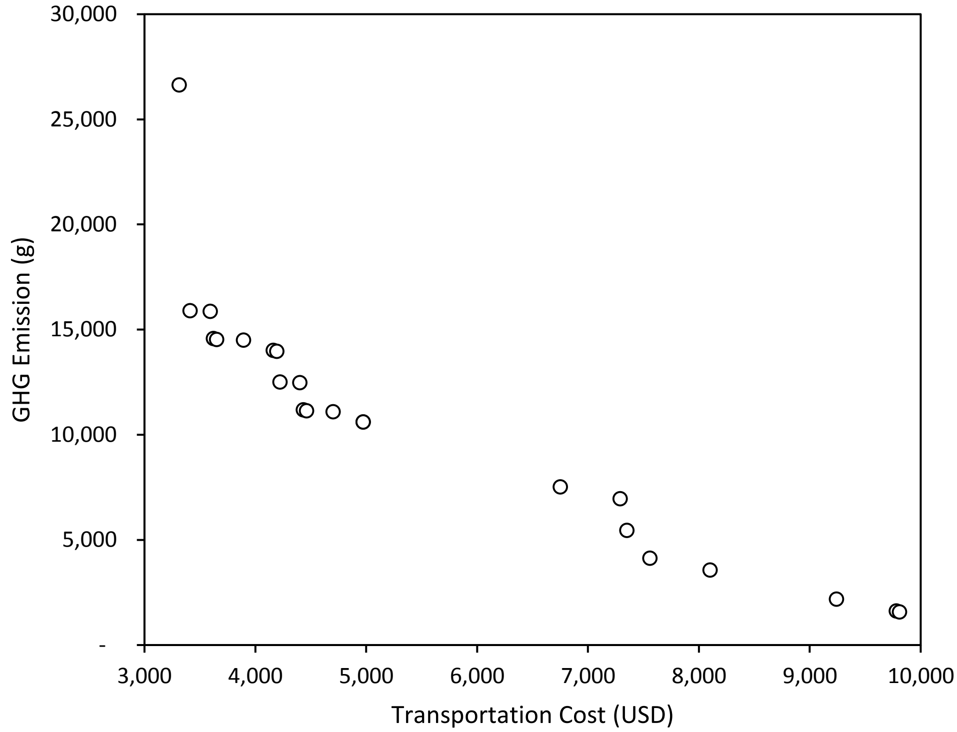


**Figure 6.1:** Pareto frontier for the objective functions

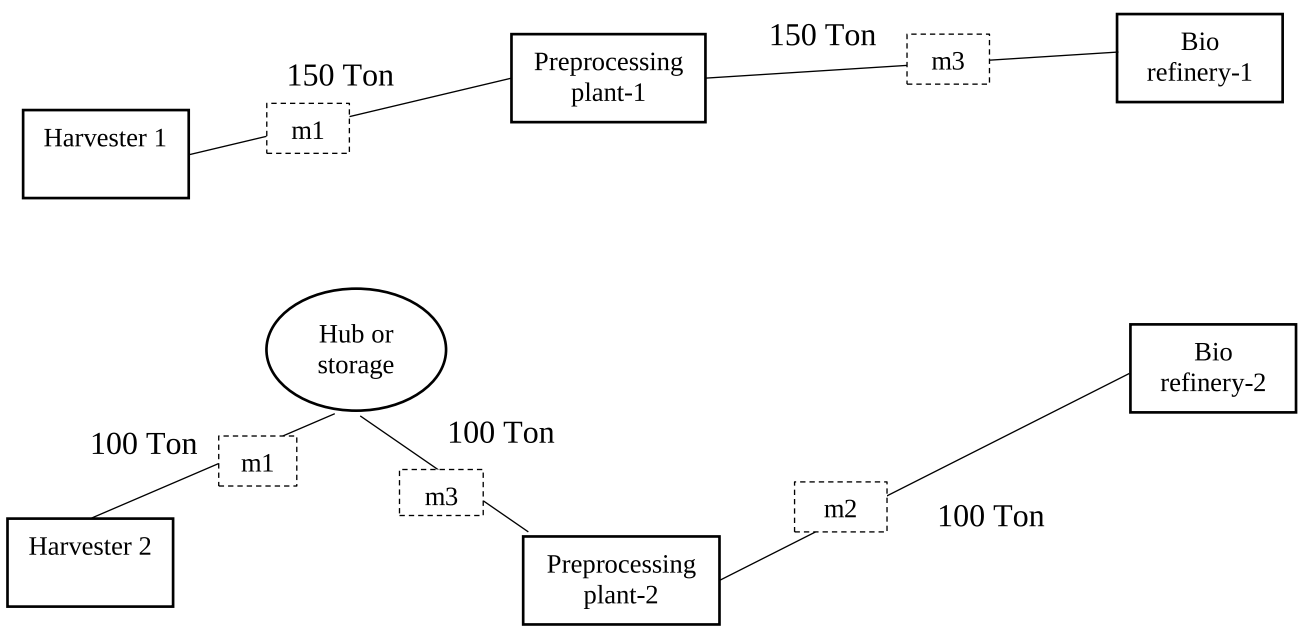


**Figure 6.2:** Transportation network for minimum cost

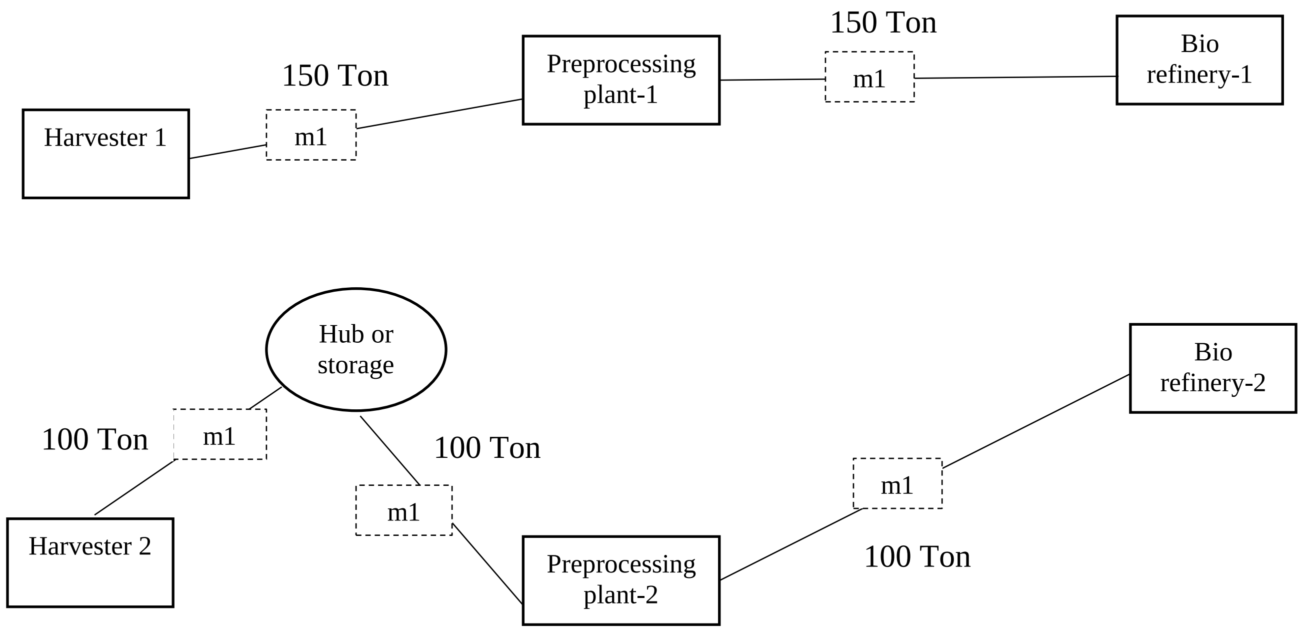


**Figure 6.3:** Transportation network for minimum GHG emission

Figure 6.1 shows the pareto optimality frontier for the two objective functions. Six feasible points connected by dashed line appear in the plot. It is observed that at the left portion the pareto frontier line has got a pretty high slope. So we can reduce the GHG emission significantly by sacrificing small transportation cost. For the middle and right part the slope of the pareto frontier is small. So to improve the value of GHG emission we have to incur relatively larger amount of transportation cost. Figure 6.2 and 6.3 shows the transportation network for minimum transportation cost and minimum GHG emission amount respectively.

# 7. Conclusion

In different fields of business and engineering Multi Objective (MO) optimization has become a common OR problem. Decision makers can get a very clean understanding of these types of MO problems by analyzing the Pareto optimality frontier. It helps to make decisions in a more effective way to determine future strategies.

In this class project we have selected a transportation problem in a four echelon supply chain framework with two important conflicting objectives- cost and GHG emission. A mathematical model has been formulated using proper OR techniques and solved it with an optimization software. Several feasible points have been found in the pareto optimality frontier. These findings might be helpful for appropriate stakeholders for future decision making strategies.

# References


[1] M. D. Sarder, Z. Adnan and C. Miller, "Biomass Transportation Model for Intermodal Network," *International Journal of Supply Chain Management*, vol. 2, no. 2, pp. 7-18, 2013.

[2] "U.S Department of Energy," [Online]. Available: http://energy.gov/articles/release-national-biofuels-action-plan. [Accessed 2 October 2016].

[3] C. Vira and Y. Y. Haimes, "Multiobjective decision making: theory and methodology (No. 8)," North-Holland, 1983.

[4] J. L. Cohon, in *Multiobjective programming and planning*, Courier Corporation, 2013.