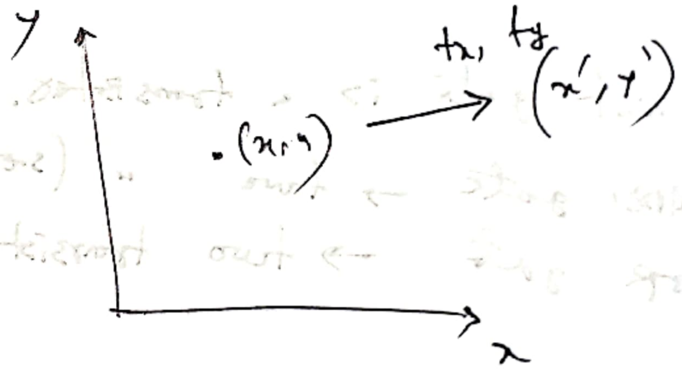
computer grofies # Raihan Ullah

* ① Two Dimensional Transformation;

3 basic operation;

① Translation
② Rotation
③ Scaling

① Translation;



IV

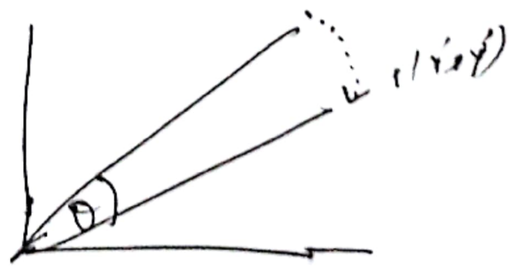
Iv

$$\begin{pmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x+tx \\ y+ty \\ 1 \end{pmatrix}$$

Here, $x' = x + tx$

$y' = y + ty$

② Rotation;



$(x', y') = R_\theta (x, y)$

Here, $x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

* $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

Again,

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

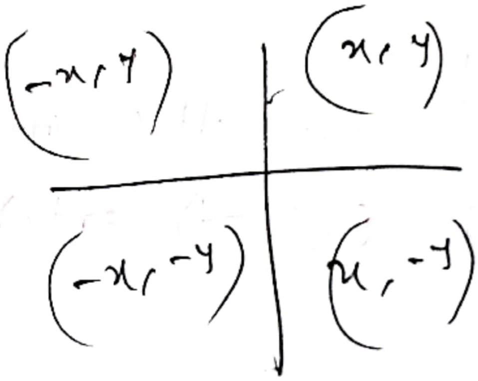
* Scalling

$$(x', y') = S_{xy}(x, y) \quad , \quad \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Mirror Reflection:

$$M_x = \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}$$

$$M_y = \begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix}$$



$\boxed{\text{Problem}}$:

(a) Find the matrix that represents rotation of an object by 30°. about the origin.

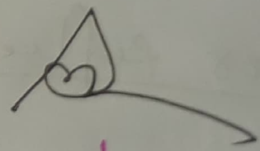
Problem:

(a) Find the matrix that represents rotation of an object by $30°$ about the origin.

Soln:

$$R_{30°} = \begin{pmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$



(b) What are the new coordinates of the point $P(2,-4)$ after the rotation?

Soln:

So, the new coordinate can be found by multiplying:

$$= \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{3}+2 \\ 1-2\sqrt{3} \end{pmatrix}$$

(4.) Derive the transformation that rotates an object point ϕ about the origin. Write the matrix representation for this rotation.

Soln:

Definition of the trigonometric functions sin and cos yields.

$$x' = r\cos(\theta+\phi)$$
$$y' = r\sin(\theta+\phi)$$

$$\boxed{\begin{matrix} x = r\cos\phi \\ y = r\sin\phi \end{matrix}}$$

→ By applying trigonometric identities, we obtain,

$$r\cos(\theta+\phi)$$
$$= r(\cos\theta\cos\phi - \sin\theta\sin\phi)$$
$$= x\cos\theta - y\sin\theta$$

Again,

$$r\sin(\theta+\phi)$$
$$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$$
$$= x\sin\theta + y\cos\theta$$

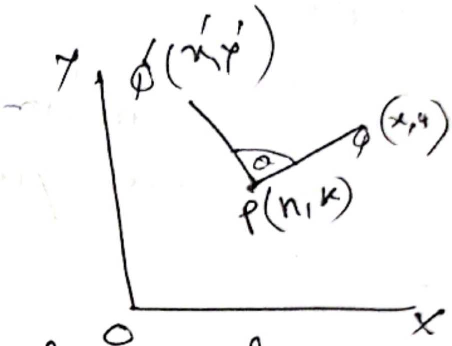
now,

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

writing, $P' = \begin{pmatrix} x' \\ y' \end{pmatrix}$, $P = \begin{pmatrix} x \\ y \end{pmatrix}$

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$P' = R_\theta . P$$



Q.3 Describe the transformation that rotates an object point, $Q(x,y)$, θ° about a fixed ~~point~~ center of rotation $P(h,k)$.

Soln: We determine the transformation $R_\theta . P$ in ~~there~~ three steps :-

1. Translate so that the center of rotation P is at the origin.

2. Perform a rotation of θ degree about the origin.

③ translate P back to (h, k)

Now, the transformation becomes,

$$R_\theta \cdot P = T_v \cdot R_\theta \cdot T_{-v}$$

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RAIHAN ULLAH

Q.4 write the general form of the matrix for rotation about a point p(h, k).

Sol^n: Applying the the transformation formula, we can write,

$$R_\theta \cdot P = T_v \cdot R_\theta \cdot T_{-v}$$

$$= \begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{matrix} \cancel{\cos\theta \quad 0} \quad 0 & \cos\theta & \cos\theta \\ \cancel{0 \quad \sin\theta}, 0 & \sin\theta & \\ \cancel{0 \quad 0 \quad 0} & 0 & \end{matrix}$$

(4.9)

Applying transformation method we can get,

$R_{\theta,P} = T_v \cdot R_\theta \cdot T_{-v}$

$$= \begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \cancel{\begin{matrix} \cos\theta + 0 + 0 \\ 0 + \cos\theta + 0 \\ 0 + 0 + 1 \end{matrix}} = \cancel{\begin{matrix} \cos\theta + 0 + 0 & -\sin\theta \\ 0 + \sin\theta + 0 & \\ 0 + 0 + 0 & \end{matrix}}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \\ 2 & \end{pmatrix} = 1 + 4$$

$$= \begin{pmatrix} \cos\theta + 0 + 0 & -\sin\theta + 0 + 0 & 0 + 0 + h \\ 0 + \sin\theta + 0 & 0 + \cos\theta + 0 & 0 + 0 + k \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta & h \\ \sin\theta & \cos\theta & k \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta + 0 + 0 & 0 - \sin\theta + 0 & -h\cos\theta + k\sin\theta + h \\ \sin\theta + 0 + 0 & 0 + \cos\theta + 0 & -\sin\theta + k\cos\theta + k \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta & -h\cos\theta + k\sin\theta + h \\ \sin\theta & \cos\theta & -\sin\theta + k\cos\theta + k \\ 0 & 0 & 1 \end{pmatrix}$$

Q.5 perform a 45° rotation of triangle A(0,0), B(1,1), C(5,2)

(a) about the origin

(b) about (-1,1)

We can represent the triangle by a matrix,

$$\begin{array}{ccc} A & B & C \\ \end{array}$$

$$\begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{(\sqrt{2})^2} = \frac{\sqrt{2}}{2}$

Solⁿ: a.

The matrix rotation is,

$$R_{45^\circ} = \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So, the coordinates A'B'C' of the rotated triangle ~~~~ ABC can be found

$A'B'C' = R_{45^\circ} \cdot [A B C]$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & \frac{3\sqrt{2}}{2} \\ 0 & \sqrt{2} & \frac{7\sqrt{2}}{2} \\ 1 & 1 & 1 \end{pmatrix}$$

We can represent,

$$\begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{pmatrix} 0 & 0 & \frac{3\sqrt{2}}{2} \\ 0 & \sqrt{2} & \frac{7\sqrt{2}}{2} \end{pmatrix} \end{array}$$

So, $A = (0, 0)$

$B = (0, \sqrt{2})$

$C = (3\sqrt{2}/2, 7\sqrt{2}/2)$

(b)

Sol^n:

From previous problem,

the rotation matrix is given by

$$R_{45°, P} = T_V \cdot R_{45°} \cdot T_{-V}$$

$$= \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & (\sqrt{2}-1) \\ 0 & 0 & 1 \end{pmatrix}$$

Now,

$$R_{45°, P} \cdot [ABC]$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & (\sqrt{2}-1) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 & \frac{3}{2}(\sqrt{2}-1) \\ \sqrt{2}-1 & 2\sqrt{2}-1 & \frac{9}{2}(\sqrt{2}-1) \\ 1 & 1 & 1 \end{pmatrix}$$

So, $A' = (-1, \sqrt{2}-1)$, $B = (-1, 2\sqrt{2}-1)$, $C = \frac{3}{2}(\sqrt{2}-1), \frac{9}{2}(\sqrt{2}-1)$

9.6

(a) The scaling transformation applied to a point $P(x, y)$, we can write,

$$\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ y \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ by \end{pmatrix}$$

(c) $x' = ax$, $y = bx$.

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ by \end{pmatrix}$$

9.7 We can write the matrix as follows:

$T_v \cdot S_{ab} \cdot T_{-v}$

$$= \begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} a & 0 & -ah + h \\ 0 & b & -bk + k \\ 0 & 0 & 1 \end{pmatrix}$$

(Q-8) Magnify the triangle with vertices A(0,0), B(1,1), & C(5,2) to twice it's size while keeping C(5,2) fixed.

Sol^n, We can write,

$$S_{2,2,C} = T_v \cdot S_{2,2} \cdot T_{-v}$$

$$= \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

Representing a point P with coordinates (x,y) by the column vector $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ we have,

$$S_{2,2,C} \cdot A = \begin{pmatrix} 2 & 0 & -5 \\ 2 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -2 \\ 1 \end{pmatrix}$$

$$S_{2,2,C} \cdot B = \begin{pmatrix} 2 & 0 & -5 \\ 2 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

$$S_{2,2,C} \cdot C = \begin{pmatrix} 2 & 0 & -5 \\ 2 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$$

Ans

So, $S_{2,2} \cdot C \cdot [ABC]$

$$= \begin{pmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -3 & 5 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

Now, $A' = (-5, -2)$

$B' = (-3, 0)$

$C' = (5, 2)$