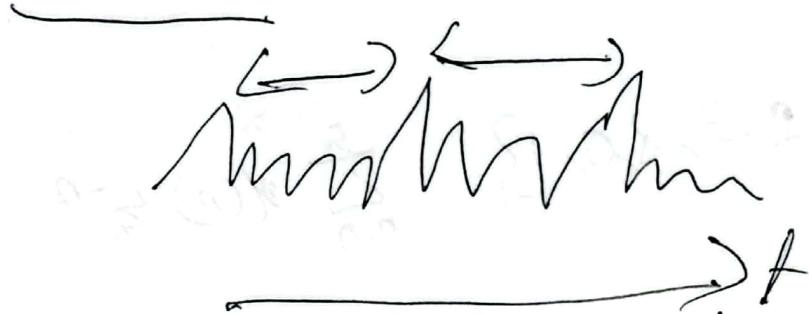


\rightarrow - Transform:



$$\cancel{z} = n \cdot i\omega$$

$$z = n e^{j\omega} \rightarrow \cancel{\text{BUT}}$$

The \rightarrow transform of a DT. signal $x(n)$ is defined as the power series,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

z is a complex variable.

$$x(n) \xleftrightarrow{z} X(z)$$

\rightarrow transform exists only for those values of z for which the series converges. The region of convergence (ROC) of $X(z)$ is the set of all values of z for which $X(z)$ attains a finite value.

Ex: 3.1.1

$$h(n) = \{1, 2, 5, 10, 13\} \quad \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$= h(0)z^0 + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4}$$

$$= 1 + 2z^{-1} + 5z^{-2} + 10z^{-3} + 13z^{-4} + h(5)z^{-5}$$

ROC: entire z -plane.

$$\text{enclosed by } z=0$$

$$h_2(n) = \{1, 2, 5, 10, 13\}$$

$$= z^2 + 2z + 5 + 10z^{-1} + 13z^{-2}, \quad \underline{\text{ROC: entire } z}$$

$$h_3(n) = f(n)$$

$$f(z) = 1$$

R OC: 1. Entire z -plane.

Plan. except

$z=0$ and

$z=\infty$

$$x(n) = \delta(n+K)$$

$$k(?) = \geq K.$$

m) - n th one

15/03/2022

Ques: $\mathcal{Z} \sim$ Transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}, \quad z = re^{j\omega}$$

Ex. $x(n) = \{1, 2, 5, 2, 0, 1\}$

$$\begin{matrix} & & & \uparrow \\ n = -2 & -1 & 0 & 1 & 2 \end{matrix}$$

$$X(z) = x(-2) \cdot z^2 + x(-1) \cdot z^1 + x(0) \cdot z^0 + x(1) \cdot z^{-1} + x(2) \cdot z^{-2}$$

$$X(z) = z^2 + 2z + 5 + 2z^{-1} + z^{-2} + x(3)z^{-3}$$

ROC: all values of z except $z=0, \infty$.

Region of convergence

Ex. 3.1.2

Determine the \mathcal{Z} -transform of the signal $x(n) = \left(\frac{1}{2}\right)^n x(n)$ $|x(n)| \leq 1, 1, \dots$

$$x(n) = \left\{1, \frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \dots\right\}$$

$$n = 0, 1, 2, 3, \dots$$

P.t.n

$$H(z) = 1 + \frac{1}{2}z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \left(\frac{1}{2}\right)^3 z^{-3} + \dots$$

$$\approx 1 + A + A^2 + A^3 + \dots$$

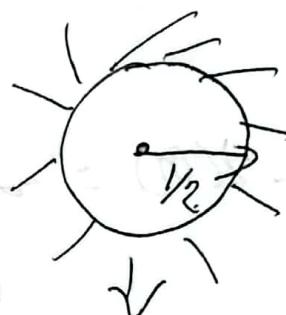
$$= \frac{1}{1-A}, \text{ for } A < 1.$$

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$|A| < 1$$

$$\approx \left|\frac{1}{2}z^{-1}\right| < 1$$

$$= z > \frac{1}{2} \rightarrow \text{ROC}$$



ROC Figure (Most Important)

[For a right-hand signal the ROC is outside the circle.]

* Ex. 3-1.3 Determine the z -transform of the signal...

$$x(n) = \alpha^n u(n) \left\{ \begin{array}{ll} \text{odd phq.} \\ 0, \quad n > 0 \\ \alpha^n, \quad n \leq 0 \end{array} \right\} \rightarrow \text{right hand side signal.}$$

$$X(z) = \sum_{n=0}^{\infty} \alpha^n z^{-n} = \sum_{n=0}^{\infty} (\alpha z^{-1})^n$$

P.T.O

[R.H.S 2nd रेखा Power(n) ग्राम outside
2nd ROC] Suppose α^n

$$X(z) = \frac{1}{1 - \alpha z^{-1}}$$

$$|\alpha z^{-1}| < 1 \rightarrow |z| > |\alpha|: [ROC]$$



ROC [outer side of alpha]

$$\text{Ex. 3-10.9} \quad X(n) = \begin{cases} \omega^n u(-n-1) & n < 0 \\ \omega^n, n \geq -1 & \end{cases}$$

Left-hand side, signs!

$$X(z) = \sum_{n=-\infty}^{-1} (-\omega^n) z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} (\omega^{-1} z)^n$$

$$= \sum_{n=1}^{\infty} (\omega^{-1} z)^n \boxed{dz = -n}$$

$$= A + A^2 + A^3 + \dots$$

$$= A(1 + A + A^2 + A^3 + \dots)$$

$$\begin{aligned}
 h(z) &= \frac{A}{1-A} \\
 &= \frac{\omega^{-1}z}{1-\omega^{-1}z} = \frac{1}{\frac{1}{\omega^{-1}z}-1} \\
 &= \frac{1}{1-\omega z^{-1}} \quad | \omega^{-1}z | < 1
 \end{aligned}$$

ROC:



[ROC of a left-hand side signal
is the interior of a circle]

Ex: 3.1.5

Ans

$$h(n) = \omega^n u(n) + b^n u(-n-1)$$

$$h(z) = \frac{1}{1-\omega z^{-1}} - \frac{1}{1-b z^{-1}}$$

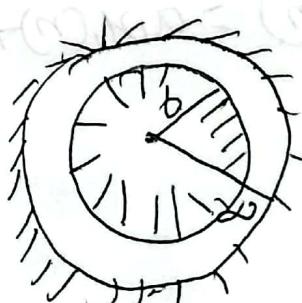
\downarrow
ROC: $|z| > \omega$

\downarrow
ROC: $|z| < b$

Case 1: $|b| < |\omega|$

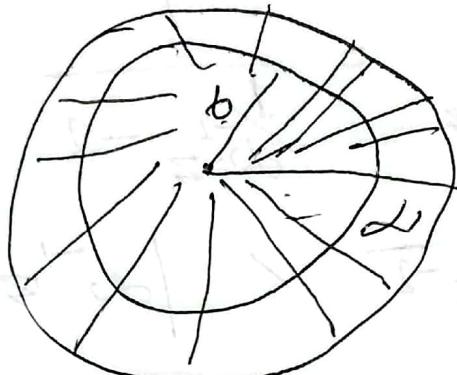
ROC does not exist!

P.T.O



ROC doesn't exist.

Case 2 :- $|b| > |d|$



Roc :- $|d| < |z| < |b|$

✳ Table - 3.1 [বেক্টর দ্বারা হ্যাপ্পেনিং অনুসৃত]

DSP-3rd
Final-class

04/04/23

3.3 Properties of \mathcal{Z} -transform.

Linearity :- If $\mathcal{H}_1(n) \xleftrightarrow{\mathcal{Z}} \mathcal{H}_1(z)$
 $\mathcal{H}_2(n) \xleftrightarrow{\mathcal{Z}} \mathcal{H}_2(z)$

Then, $\mathcal{H}(n) = a_1\mathcal{H}_1(n) + a_2\mathcal{H}_2(n) \xleftrightarrow{\mathcal{Z}} a_1\mathcal{H}_1(z) + a_2\mathcal{H}_2(z)$

পুনর মিহিরের summation এর coruation $\Rightarrow \frac{1-a^N}{1-a}$
 Infinite $\Rightarrow \frac{1}{1-a}$. Finite হলে

$$\text{Ex: } 3 \cdot 2 \cdot 1$$

$$x(n) = [3(2^n) - 4(3^n)] v(n)$$

$$= 3(2^n) v(n) - 4(3^n) v(n)$$

$$X(z) = \frac{3}{1 - 2z^{-1}} - \frac{4}{1 - 3z^{-1}}$$

④ Time shifting:-

$$f(x(n)) \xrightarrow{z} X(z)$$

$$x(n-k) \xrightarrow{z} z^{-k} X(z)$$

$$Ex: 3 \cdot 2 \cdot 1 \Rightarrow H.W$$

$$Ex: 3 \cdot 2 \cdot 1: x(n) = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & \text{else where.} \end{cases}$$

$$X(z) = \sum_{n=0}^{N-1} 1 \cdot z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \dots + z^{-N}$$

$$= \frac{1 - z^{-N}}{1 - z^{-1}}$$

↑ formula

Find the z-transform:

$$x(n) = u(n) - u(n-N)$$

$$X(z) = z \left[u(n) - u(n-N) \right]$$

$$= z \left[u(n) - z^{-N} u(n-N) \right]$$

$$= \frac{1}{1-z^{-1}} - z^{-N} \cdot \frac{1}{1-z^{-1}}$$

$$= (1-z^{-N}) \cdot \frac{1}{1-z^{-1}}$$

④ Differentiation in the z-domain:

if $x(n) \xrightarrow{z} X(z)$

then $n x(n) \longleftrightarrow -z \frac{dX(z)}{dz}$ \Rightarrow (Proof book)

Ex. 3.2. x_i: $X(z) = n a^n u(n) \Rightarrow z \text{ transform. } \frac{1}{1-a z^{-1}}$

$$\begin{aligned} n a^n u(n) &= -z \frac{d}{dz} \left[\frac{1}{1-a z^{-1}} \right] = -z \frac{(1-a z^{-1})' - a z^{-2}}{(1-a z^{-1})^2} \\ &= \frac{a z^{-1}}{(1-a z^{-1})^2} \quad \text{Ans.} \end{aligned}$$

Differentiate
 $(1-a z^{-1})^{-1}$
 $-a z^{-2}$
 $(1-a z^{-1})^2$

* Convolution of two sequences:- [Most Important Property]

$$\text{If } x_1(n) \rightarrow X_1(z)$$

$$x_2(n) \rightarrow H_2 X_2(z) \xrightarrow{\text{Inverses}} x_1(n) * x_2(n)$$

$$x_1(n) * x_2(n) \xleftarrow{z} X_1(z) \cdot X_2(z)$$

$$X(z) \rightarrow x(n)$$

* Ex. - 3.2.9 :-

$$H(z) = \sum x(n) z^{-n}$$

$$x_1(n) = \{1, -2, 1\}, x_2(n) = \{1, 1, 1, 1, 1\}$$

Compute the convolution $x(n)$ of the signals.

$$H_1(z) = 1 - 2z^{-1} + z^{-2}$$

$$H_2(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}$$

$$H(z) = H_1(z) \cdot H_2(z) = 1 - z^{-1} - z^{-6} + z^{-7}$$

$$x(n) = \{1, -1, 0, 0, 0, 0, -1, 1\}$$

Ans.

Steps for finding Convolution Using z-transform

- ① Compute the z-tran. of the signals to be convolved.

$$X_1(z) = \mathcal{Z}\{x_1(n)\}$$

$$X_2(z) = \mathcal{Z}\{x_2(n)\}$$

- ② Multiply the two z-transform:

$$X(z) = X_1(z) X_2(z)$$

- ③ Find the inverse z-transform of $X(z)$

$$x(n) = \mathcal{Z}^{-1}\{X(z)\}$$

④ 3. 3 : Rational z -Transform:

$$H(z) = \frac{B(z)}{A(z)}$$

$$= \frac{b_0 + b_1 z^{-1} + \dots + b_m z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}} \quad [\text{polynomial}]$$

$$= \frac{(z - z_1)(z - z_2) \dots (z - z_m)}{(z - P_1)(z - P_2) \dots (z - P_N)} \quad (z_1, P_1 \dots \text{DSP Tech Roots})$$

⑤ Poles and zeros:

(this is roots)
↓ (this not roots) or

The zeros of z -transform $H(z)$
are the values of z for which $H(z) = 0$
[zeros: $z_1, z_2, z_3, \dots, z_m$].

The poles of a z -transform are the values
of z for which $H(z) = \infty$.

[Poles: $P_1, P_2, P_3, \dots, P_N$].

↓
(Roots of the denominator) \Rightarrow

Ex: 3.3.1

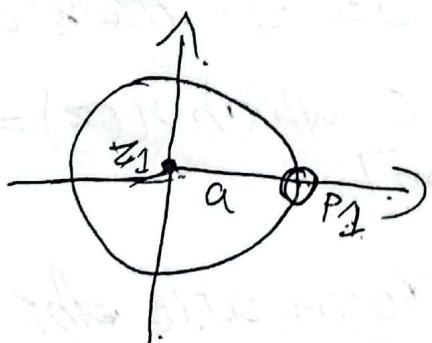
Determine the Pole-zero Plot for the signal

$$x(n) = a^n u(n)$$

$$X(z) = \frac{1}{1 - az^{-1}}$$

$$\begin{array}{l} \text{Zero: } z_1 = 0 \\ \text{Pole: } P_1 = a \end{array} \quad = \frac{1}{1 - \frac{a}{z}}$$

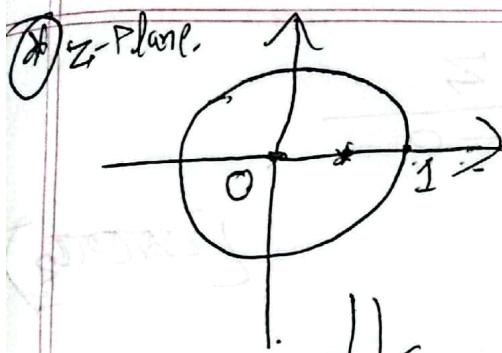
$$= \frac{1}{\frac{z-a}{z}} = \frac{(z-a)}{(z-a)}$$



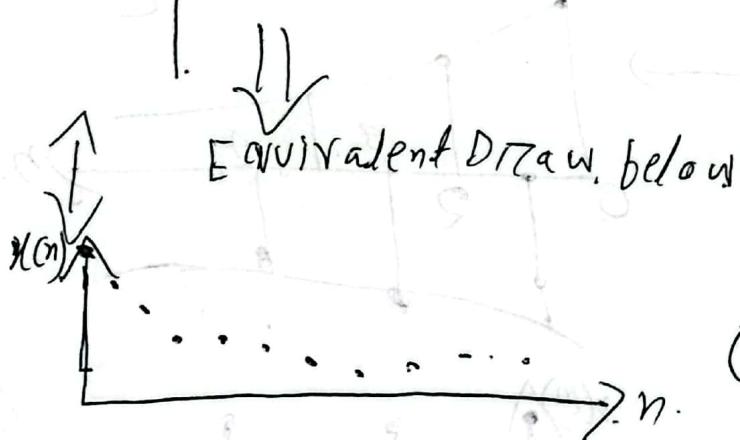
Ex 3.3.2: Pole-zero location and time-domain behaviour for causal signals.

$$x(n) = a^n u(n) \longleftrightarrow X(z) = \frac{1}{1 - az^{-1}}$$

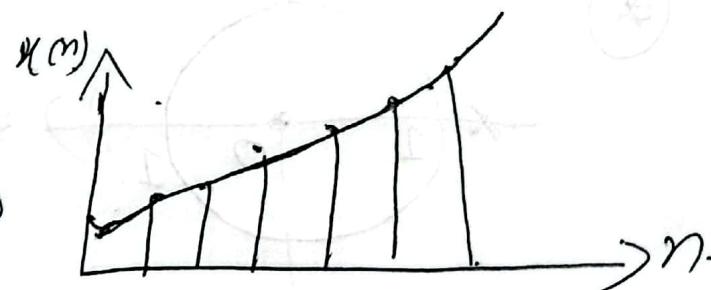
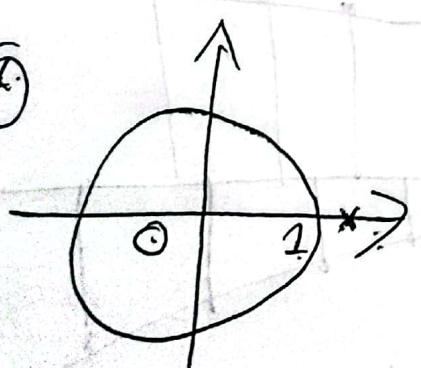
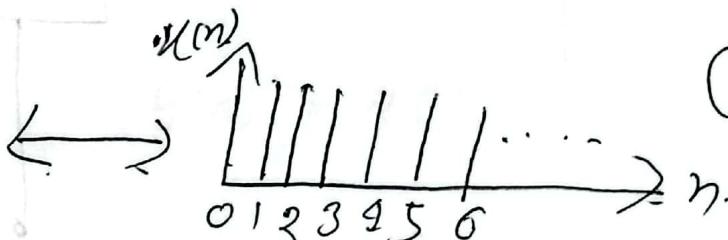
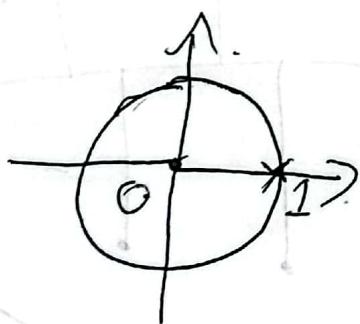
$$= \frac{z}{z - a}$$



[Exponential Z-transform]
~~z-transform~~

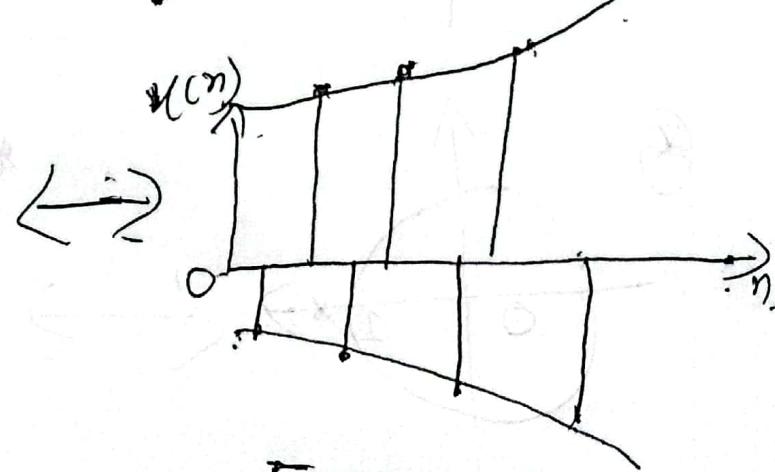
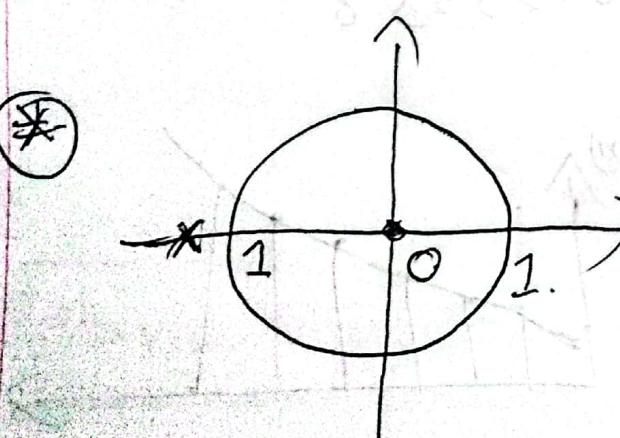
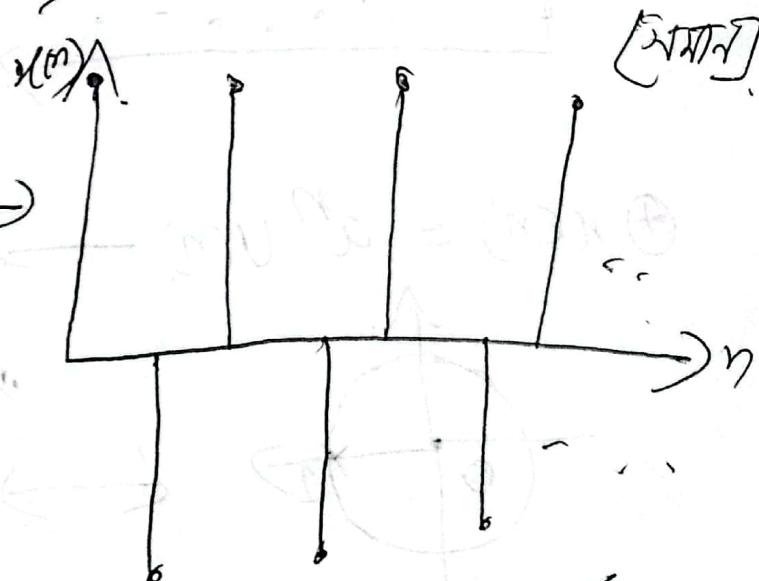
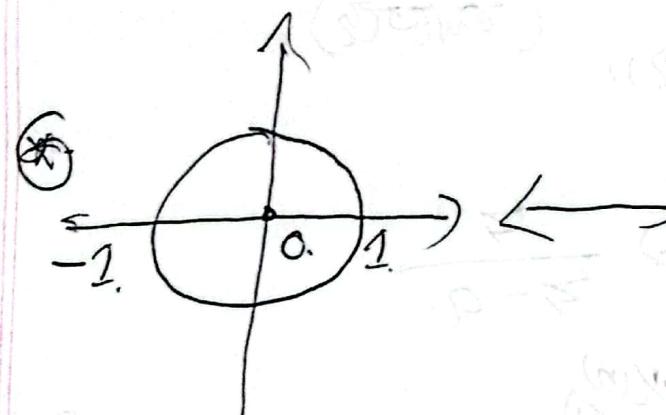
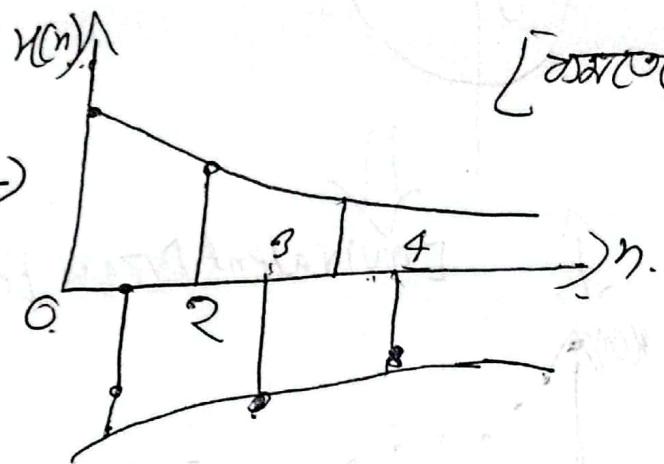
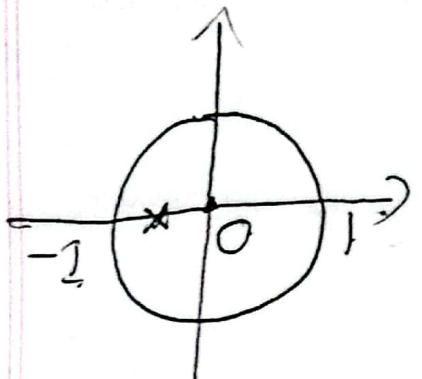


(b) $x(n) = a^n \cdot u(n) \longrightarrow \frac{z}{z-a}$



(বাহ্যিক).

$$\textcircled{*} \quad h(n) = a^n u(n) \longrightarrow \frac{z}{z-a}$$



[বাস্তুত

Time domain C.R. convolution
↳ " C.R. Multiplication

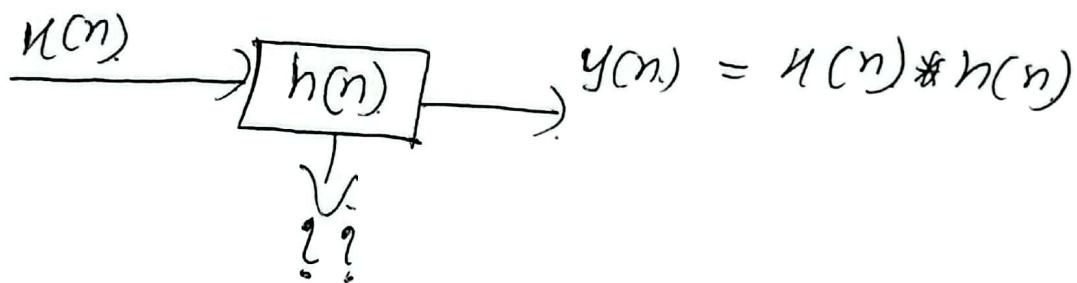
Q3-3.3 Function
The system of an LTI system.

$$y(n) = x(n) * h(n)$$

$$\begin{aligned} y(z) &= \cancel{x(z)} \cdot H \\ &= X(z) \cdot H(z) \end{aligned}$$

$$H(z) = \frac{Y(z)}{X(z)} \rightarrow \text{Output}$$

↳ System Function.



* To determine the system function and unit sample response:-

$$y(n) = \frac{1}{2} y(n-1) + 2x(n)$$

$$y(z) = \frac{1}{2} z^{-1} y(z) + 2x(z)$$

$$2x(z) = y(z) \left[1 - \frac{1}{2} z^{-1} \right]$$

$$\begin{cases} H(z) = ? \\ h(n) = ? \\ H(z) = \frac{y(z)}{x(z)} \end{cases}$$

p.t.o

$$\frac{Y(z)}{H(z)} = \frac{2}{1 - \frac{1}{2}z^{-1}} \Rightarrow H(z) = \frac{2}{1 - \frac{1}{2}z^{-1}}$$

\Rightarrow System Function

$$h(n) = 2 \cdot \left(\frac{1}{2}\right)^n u(n)$$

[\Rightarrow করলে $H(z)$ আসে, Inverse করলে $h(n)$ আসে]

Ans.

$$(a) K + (1-a)L = 0.05$$



$$(a)K + (1-a)L - \frac{1}{S} = (a)N$$

$$\frac{Y(z)}{H(z)} = \frac{2}{1 - \frac{1}{2}z^{-1}} \Rightarrow H(z) = \frac{2}{1 - \frac{1}{2}z^{-1}}$$

System Function

$$h(n) = 2 \left(\frac{1}{2}\right)^n u(n)$$

[ই-ক্যাল $H(z)$ আর, Inverse ক্যাল $h(n)$ আর]

Ans-

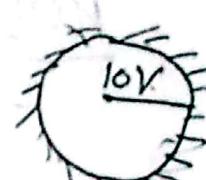
DSP - 5th Class
Final

10/09/23

* Inverse z-transform :-

Ex. $H(z) = \frac{1}{1 - az^{-1}}, |z| > |a|$

$h(n) = a^n u(n) / a^n u(-n-1)$


Scalable signal.

\times ↳ anti-causal signal

* The inverse z-transform by Power series expansion:

Ex 3.4.2 Determine the inverse z transform.

P.T.O

$$H(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

where - ④ ROC: $|z| > 1 \rightarrow H(n)$ should be causal.

⑤ ROC: $|z| < 0.5 \rightarrow H(n)$ is anti-causal.

$$\begin{aligned} H(n) &= \{1, 2, 3\} \Rightarrow R.H. \text{ signal.} \\ H(z) &= 1 + 2z^{-1} + 3z^{-2} \end{aligned}$$

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$\begin{cases} H(n) = \{1, 2, 3\} \\ H(z) = z^2 + 2z + 3 \end{cases} \Rightarrow L.H. \text{ signal.}$$

$$\underline{E[X = 8 \cdot 9 \cdot 2]}: H(z) = \frac{1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}} \rightarrow z^{-1}z^{-2}z^{-3} \dots$$

$$\begin{aligned} &\left(1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right) \left(1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right) \left(1 + \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right) \\ &\frac{\frac{3}{2}z^{-1} - \frac{1}{2}z^{-2}}{\frac{3}{2}z^{-1} - \frac{9}{4}z^{-2} + \frac{3}{4}z^{-3}} \\ &\frac{\frac{3}{2}z^{-1} - \frac{9}{4}z^{-2} + \frac{3}{4}z^{-3}}{\frac{8}{4}z^{-2} - \frac{3}{4}z^{-3}} \end{aligned}$$

$$h(n) = \left\{ 1, \frac{3}{2}, \frac{7}{4}, \dots \right\}$$

④ 3.9.8: $H(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$ $H(z) = ?$

⑤ $\text{ROC}_{H(z)}(z) > 1 \rightarrow \text{causal.}$

⑥ $\text{ROC}_{H(z)}(z) < 0.5 \rightarrow \text{anti causal.}$

⑦ $\text{ROC}_{H(z)}(z) \in 0.5 < |z| < 1 \rightarrow \text{contain both causal.}$
& anti causal part.

Factorize: $H(z) = \frac{z}{1 - z^{-1}} - \frac{1}{1 - 0.5z^{-1}}$

$$\begin{cases} H(z) = \frac{1}{1 - az^{-1}} \\ h(n) = a^n u(n) \end{cases}$$

⑧ $h(n) = 2(1)^n u(n) - (0.5)^n u(n)$

⑨ $h(n) = 2(1)^n u(-n-1) - (0.5)^n u(-n-1)$

⑩ $h(n) = -2(1)^n u(-n-1) - (0.5)^n u(n)$

⊗ - transformation

$$\textcircled{2} \quad X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$x(n) = \{1, 2, 5, 2, 0, 1\}$$

$$X(z) = 1 + 2z^{-1} + 5z^{-2} + 2z^{-3} + z^{-5}$$

$$X(z) = 1 + 2z^{-1} + 5 + 2z^{-2} + z^{-3}$$

~~①~~
$$\textcircled{3} \quad x(n) = \delta(n)$$

$$X(z) = 1 \cdot z^0$$

$$= 1$$

~~②~~
$$\textcircled{4} \quad x(n) = \delta(n-k)$$

$$X(z) = z^{-k}$$

~~③~~
$$\textcircled{5} \quad x(n) = \delta(n+k)$$

$$X(z) = z^k$$

$$\textcircled{5} \quad h(n) = a^n u(n)$$

$$h(z) = \sum h(n) z^{-n}$$

$$= \sum a^n z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (az^{-1})^n = \frac{1}{1 - az^{-1}}$$

$$|az^{-1}| < 1$$

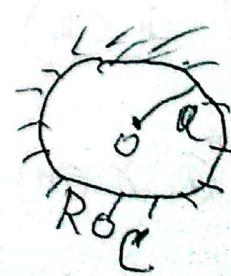
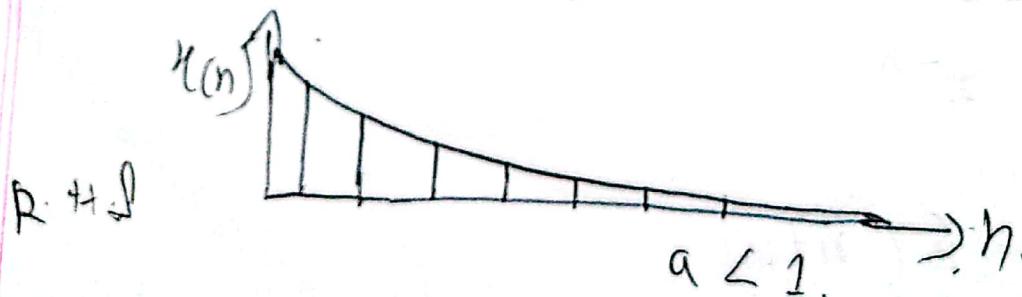
$$\frac{z > |a|}{\text{R.O.C. } \underline{\text{Ans.}}}$$

$$\textcircled{6} \quad h(n) = a^n u(-n-1)$$

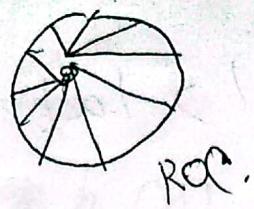
$$\frac{1}{1 - az^{-1}}, \quad |z| < a.$$

Ans.

$$\textcircled{7} \quad h(n) = a^n u(n) \quad a < 1 \quad (\text{Signal Draw})$$

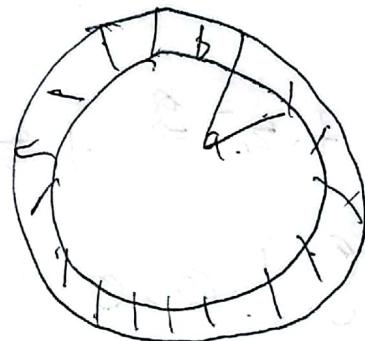


L-H-f :



① $n(z) = a^n u(z) \quad \text{for } z > 1$

a. b.



② Convolution property:

$$n_1(z) \xrightarrow{\star} n_1(z)$$

$$n_2(z) \xrightarrow{\star} n_2(z)$$

$$n_1(z) * n_2(z) \xrightarrow{\star} n_1(z) \cdot n_2(z)$$

$$H(z) = \frac{1}{1 - az^{-2}} \quad (|z| > |a|)$$

$$h(n) = a^n u(n)$$

$$H(z) = \cancel{t + z^{-3}} \quad / + z^{-2} + z^{-6}$$

$$h(z) = 1 + z^{-2} + z^{-6}$$

$$h(n) = \{1, 0, 0, 1; 0, 0, 1\}$$

3.5.3: Causality & Stability: A causal.

LTI system is one whose unit impulse

responds satisfies the condition
 $b(n) = 0, \quad n < 0$

A stable LTI system is one whose

Unit - impulse response satisfies the condition

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

~~Ex. 2.5.2~~ An LTI system is characterized by the system function

$$H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$$
$$= \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{2}{1 - 5z^{-1}}$$

Specify the ROC of $H(z)$ and determine $h(n)$ for the following conditions:

- (a) The system is stable.
- (b) " " causal
- (c) " " anti-causal

if causal:

$$\textcircled{5} \quad h(n) = \left(-\frac{1}{2}\right)^n u(n) + 2 \cdot (3)^n u(n)$$

④ if anti causal:

$$h(n) = -\left(\frac{1}{2}\right)^n u(-n-1) - 2 \cdot (3)^n u(-n-1)$$

⑤ $\frac{1}{2} < |z| < 3$

$$h(n) = \left(\frac{1}{2}\right)^n u(n) - 2 \cdot (3)^n u(-n-1)$$

Q. Ex 3.5.8. Determine the response of the

system, $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2)$

The input signal $x(n) = \delta(n) = \sum_{k=0}^{\infty} \delta(n-k)$

$$y(z) = h(z) * (z)$$

$$y(n)$$

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$$

$$y(z) = \frac{5}{6}z^{-1}y(z) - \frac{1}{6}z^{-2}y(z) + x(z)$$

$$Y(z) = \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = X(z)$$

$$Y(z) \cdot \left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right) = X(z)$$

$$H(z) = \frac{1}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)}$$

$$H(z) = \left(1 - \frac{1}{3}z^{-1}\right)^{-1}$$

$$y(z) = H(z)x(z) \quad x(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$y(n) = \left(\frac{1}{2}\right)^n u(n)$$

④ one-sized \Rightarrow fraction n.

$$N(z) = \sum_{n=0}^{\infty} N(n) z^n$$

OS
Final - 1st class.

03/05

⑤ Deadlock: A deadlock is a situation where each of the computer processes are holding a resource which is being assigned to some other process.

In this situation, none of the

Previous question

① Write down the z-transforms of $\delta(n)$, $\delta(n-k)$ and $\delta(n+k)$.

$$\text{Ans: } h(n) = \delta(n)$$

$$H(z) = 1 \cdot z^{-0} \\ = 1$$

$$h(n) = \delta(n-k)$$

$$H(z) = z^{-k}$$

$$h(n) = \delta(n+k)$$

$$H(z) = z^k. \quad \underline{\text{Ans}}$$

② Determine the z-transforms of $x(n) = [u(n) - u(n-10)]$.

$$\begin{aligned} \text{Ans: } H(z) &= z [u(n) - u(n-10)] \\ &= z [u(n) - z^{-10} [u(n-10)]] \\ &= \frac{1}{1 - z^{-1}} - z^{-10} \cdot \frac{1}{1 - z^{-1}} \end{aligned}$$

$$= (1 - z^{-10}) \cdot \frac{1}{1 - z^{-1}}$$

Ans.

- ③ Determine dhp convolution of the following pairs of signals by means of the z-transform.

$$x(n) = \{1, -2, 1\} \text{ and } h(n) = \{1, 1, 1, 1, 1\}$$

Compute the convolution $x(n)$ of the signals:

$$H_1(z) = 1 - 2z^{-1} + z^{-2}$$

$$H_2(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}$$

$$H(z) = H_1(z) \cdot H_2(z) = 1 - z^{-1} - z^{-6} + z^{-2}$$

$$x(n) = \{1, -1, 0, 0, 0, 0, -1, 1\}$$

Ans.

④ An LTI system is characterized by the system function $H(z) = \frac{3 - 2z^{-1}}{1 - 3 \cdot 5 z^{-1} + 1 \cdot 5 z^{-2}}$. Determine

the ROC and $h(n)$ if the system is causal and if the system is anti-causal.

Ans: $H(z) = \frac{3 - 2z^{-1}}{1 - 3 \cdot 5 z^{-1} + 1 \cdot 5 z^{-2}}$

$$= \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{\frac{2}{3}}{1 - 3 z^{-2}}$$

Specify the ROC of $H(z)$ and determine $h(n)$ for the following condition.

- ① The system is stable.
- ② " " " causal.
- ③ " " " anti- " .

④ $\frac{1}{2} < |z| < 3$

$h(n) = \left(\frac{1}{2}\right)^n u(n) - 2 \cdot (3)^n u(-n-1)$

⑤ $h(n) = \left(\frac{1}{2}\right)^n u(n) + 2 \cdot (3)^n u(n)$

⑥ $h(n) = -\left(\frac{1}{2}\right)^n u(-n-1) - 2 \cdot (3)^n u(-n-1)$

Ans:

⑤ Determine the system function $H(z)$ and unit impulse response of the system described by $y(n) = 0.5y(n-1) - 2x(n)$

Ans: $y(n) = 0.5y(n-1) - 2x(n)$

$$y(z) = 0.5z^{-1} y(z) - 2x(z)$$

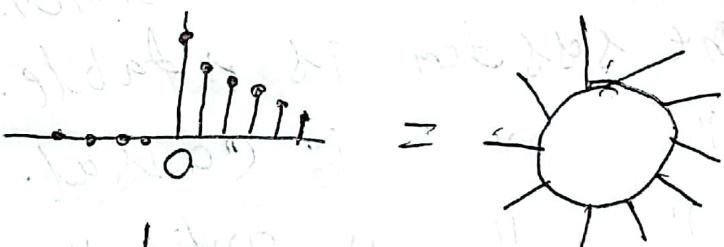
$$y(z)(1 - 0.5z^{-1}) = -2x(z)$$

$$\frac{y(z)}{x(z)} = \frac{-2}{1 - 0.5z^{-1}}$$

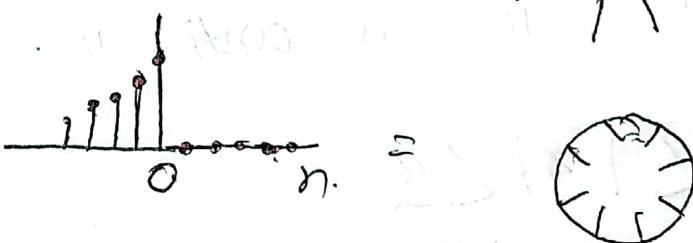
$$H(z) = -\frac{2}{1 - 0.5z^{-1}}$$

⑥ Plot the ROC of the following infinite-duration signals.

Causal:



Anticausal:



Two-sided:

