Imperial College London

Probability and Stochastic Processes

Lecture 1: Random Variables

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Course Information

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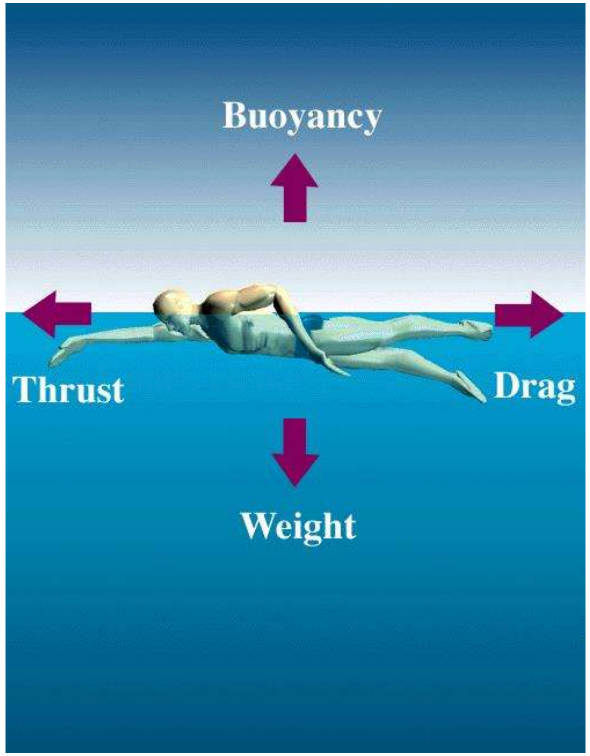
- Course notes are available in Blackboard
- Desirable knowledge: Elementary probability
- Grading
  - 3-hour exam
  - Coursework accounts for 15% (deadline: **end of term**)
- Non-assessed problem sheets (for problem classes)

About the Classes

- Our responsibility is to facilitate you to learn. You have to make the effort.
- Spend time reviewing lecture notes afterwards.
- If you have a question on the lecture material after a class, then
  - Look up a book! Be resourceful.
  - Try to work it out yourself.
  - Ask during classes or by email.



Lectures

Probability

1. Random variables
2. Joint distributions
3. Sequences of random variables
4. Parameter estimation

Stochastic processes

5. Stochastic processes
6. Power spectrum
7. Mean-square estimation
8. Markov chains
9. Continuous-time processes
10. Martingales

1 lecture = 2 hours

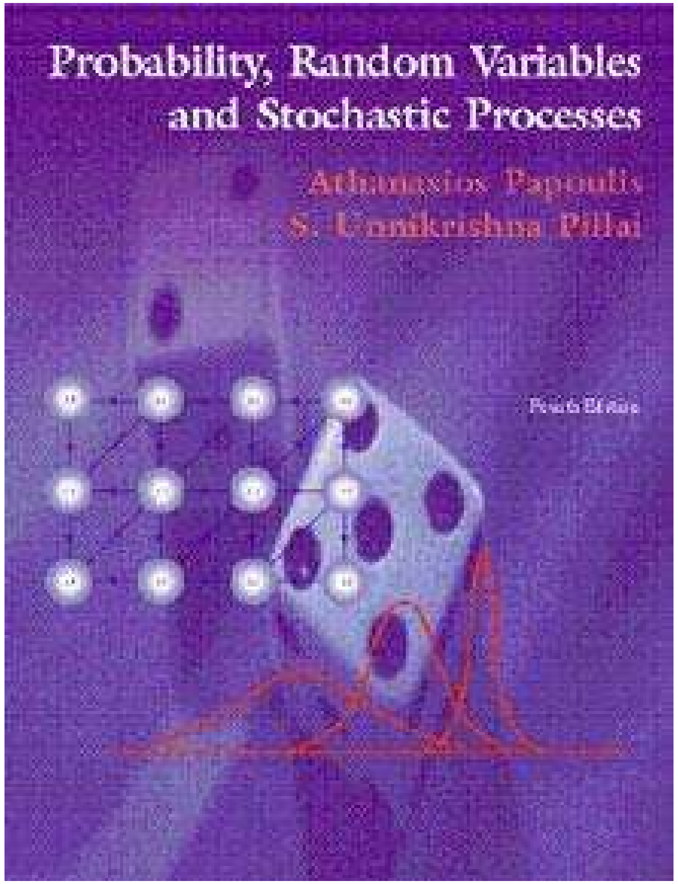
Please watch 1 lecture/week

Why This Course?

- This is a foundational course for many areas of EEE (e.g., automatic control, statistical signal processing, machine learning, network traffic/queuing theory, digital communications, information theory and coding) and beyond (e.g., finance, quantum).
- This is intended to be a medium-level course on probability and random processes, biased towards EEE applications (but not a survey of applications).
- Main objectives:
  - To develop the main ideas of probability theory in a systematic way;
  - To study randomly-varying functions of time, known as stochastic processes or random processes;
  - To demonstrate how to set up probabilistic models for engineering problems.

Textbook

- A. Papoulis and S. Pillai, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, 4th edition (old editions are ok)
- Book website including slides, problem hints etc.: http://www.mhhe.com/papoulis/
- These lecture notes (except the later lectures) are largely adapted from the slides on the book website
- Other books:
  - Grimmett & Stirzaker, *Probability and Random Processes*, Oxford
  - Stark & Woods, *Probability, Random Processes, and Estimation Theory for Engineers*, Prentice Hall
  - Leon-Garcia, *Probability, Statistics, and Random Processes for Electrical Engineering*, Addison-Wesley



PROBABILITY THEORY

Basics

Probability theory deals with the study of random phenomena, which under repeated experiments yield different outcomes that have certain underlying patterns about them. The notion of an experiment assumes a set of repeatable conditions that allow any number of identical repetitions. When an experiment is performed under these conditions, certain elementary events ξ_i occur in different but *uncertain* ways. We can assign nonnegative number as the probability $P(\xi_i)$, of the event ξ_i in various ways:

Laplace's Classical Definition: The probability of an event A is defined a-priori without actual experimentation as

$$P(A) = \frac{\text{Number of outcomes favorable to } A}{\text{Total number of possible outcomes}},$$

provided all these outcomes are *equally likely*.

Relative Frequency Definition: The probability of an event A is defined as

$$P(A) = \lim_{n \to \infty} \frac{n_A}{n}$$

where n_A is the number of occurrences of A and n is the total number of trials.

Axioms of Probability

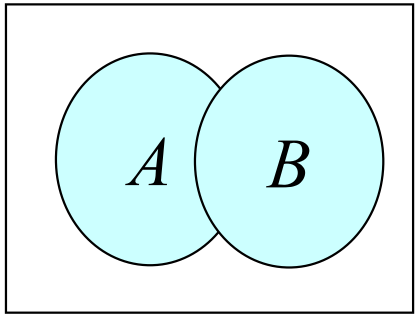
The axiomatic approach to probability, due to Kolmogorov, developed through a set of axioms is generally recognized as superior to the above definitions, as it provides a solid foundation for complicated applications.

For any event A, we assign a number $P(A)$, called the probability of the event A. This number satisfies the following three conditions that act the axioms of probability.

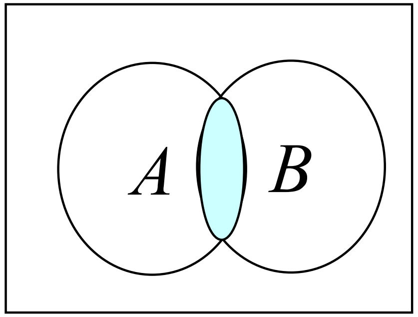
(i) $P(A) \geq 0$ (Probability is a nonnegative number)

(ii) $P(\Omega) = 1$ (Probability of the whole set is unity)

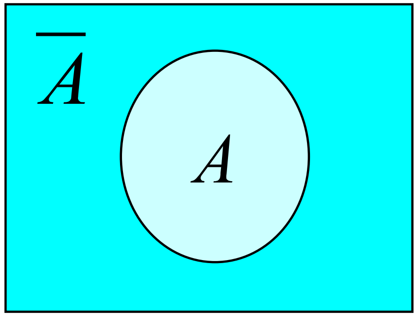
(iii) If $A \cap B = \varphi$, then $P(A \cup B) = P(A) + P(B)$.



$A \cup B$



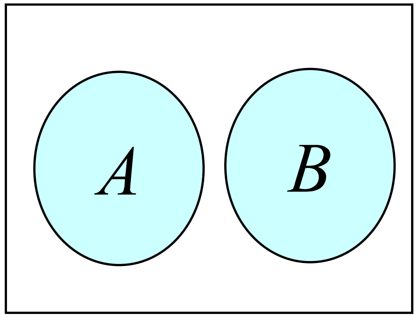
$A \cap B$



$\overline{A}$

- If $A \cap B = \phi$, the empty set, then A and B are said to be mutually exclusive (M.E).
- A partition of Ω is a collection of mutually exclusive subsets of Ω such that their union is Ω.

$$A_i \cap A_j = \phi, \quad \text{and} \quad \bigcup_{i=1} A_i = \Omega.$$



$A \cap B = \phi$