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Introduction

Equations of Motion for Fluid Flow, Heat and Mass Transfer

Finite Volume Solution

The Navier-Stokes equations

Best Practises

Random Examples

Definition 5.0.1: Limit of Sequence in \mathbb{R}

Let $\{s_n\}$ be a sequence in \mathbb{R} . We say

$$\lim_{n\to\infty} s_n = s$$

where $s \in \mathbb{R}$ if \forall real numbers $\epsilon > 0$ \exists natural number N such that for n > N

$$s - \epsilon < s_n < s + \epsilon$$
 i.e. $|s - s_n| < \epsilon$

Question 1

Is the set x-axis\{Origin} a closed set

Solution: We have to take its complement and check whether that set is a open set i.e. if it is a union of open

Note:-

We will do topology in Normed Linear Space (Mainly \mathbb{R}^n and occasionally \mathbb{C}^n) using the language of Metric Space

Claim 5.0.1 Topology

Topology is cool

Example 5.0.1 (Open Set and Close Set)

Open Set: $\bullet \phi$

- $\bullet \bigcup_{x \in X} B_r(x) \text{ (Any } r > 0 \text{ will do)}$
- $B_r(x)$ is open

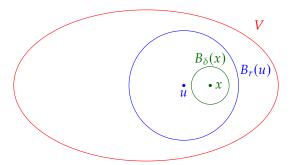
- Closed Set: X, ϕ
 - \bullet $\overline{B_r(x)}$

x-axis $\cup y$ -axis

Theorem 5.0.1

If $x \in \text{open set } V \text{ then } \exists \ \delta > 0 \text{ such that } B_{\delta}(x) \subset V$

Proof: By openness of $V, x \in B_r(u) \subset V$



Given $x \in B_r(u) \subset V$, we want $\delta > 0$ such that $x \in B_\delta(x) \subset B_r(u) \subset V$. Let d = d(u, x). Choose δ such that $d + \delta < r$ (e.g. $\delta < \frac{r-d}{2}$)

If $y \in B_{\delta}(x)$ we will be done by showing that d(u, y) < r but

$$d(u, y) \le d(u, x) + d(x, y) < d + \delta < r$$

⊜

Corollary 5.0.1

By the result of the proof, we can then show...

Lenma 5.0.1

Suppose $\vec{v_1}, \dots, \vec{v_n} \in \mathbb{R}^n$ is subspace of \mathbb{R}^n .

Proposition 5.0.1

1 + 1 = 2.

Random

Definition 5.0.2: Normed Linear Space and Norm $\|\cdot\|$

Let V be a vector space over \mathbb{R} (or \mathbb{C}). A norm on V is function $\|\cdot\|$ $V \to \mathbb{R}_{\geq 0}$ satisfying

- (2) $\|\lambda x\| = |\lambda| \|x\| \ \forall \ \lambda \in \mathbb{R}(\text{or } \mathbb{C}), \ x \in V$
- (3) $||x + y|| \le ||x|| + ||y|| \ \forall \ x, y \in V$ (Triangle Inequality/Subadditivity)

And V is called a normed linear space.

• Same definition works with V a vector space over \mathbb{C} (again $\|\cdot\| \to \mathbb{R}_{\geq 0}$) where ② becomes $\|\lambda x\| = |\lambda| \|x\|$ $\forall \lambda \in \mathbb{C}, x \in V$, where for $\lambda = a + ib$, $|\lambda| = \sqrt{a^2 + b^2}$

Example 5.0.2 (*p*-Norm)

 $V = \mathbb{R}^m, p \in \mathbb{R}_{\geq 0}$. Define for $x = (x_1, x_2, \dots, x_m) \in \mathbb{R}^m$

$$||x||_p = (|x_1|^p + |x_2|^p + \dots + |x_m|^p)^{\frac{1}{p}}$$

(In school p = 2)

Special Case p = 1: $||x||_1 = |x_1| + |x_2| + \cdots + |x_m|$ is clearly a norm by usual triangle inequality. Special Case $p \to \infty$ (\mathbb{R}^m with $||\cdot||_{\infty}$): $||x||_{\infty} = \max\{|x_1|, |x_2|, \cdots, |x_m|\}$

For m = 1 these p-norms are nothing but |x|. Now exercise

Question 2

Prove that triangle inequality is true if $p \ge 1$ for p-norms. (What goes wrong for p < 1?)

Solution: For Property (3) for norm-2

When field is \mathbb{R} :

We have to show

$$\sum_{i} (x_i + y_i)^2 \le \left(\sqrt{\sum_{i} x_i^2} + \sqrt{\sum_{i} y_i^2} \right)^2$$

$$\implies \sum_{i} (x_i^2 + 2x_i y_i + y_i^2) \le \sum_{i} x_i^2 + 2\sqrt{\left[\sum_{i} x_i^2\right] \left[\sum_{i} y_i^2\right]} + \sum_{i} y_i^2$$

$$\implies \left[\sum_{i} x_i y_i \right]^2 \le \left[\sum_{i} x_i^2 \right] \left[\sum_{i} y_i^2 \right]$$

So in other words prove $\langle x,y\rangle^2 \leq \langle x,x\rangle \langle y,y\rangle$ where

$$\langle x, y \rangle = \sum_{i} x_i y_i$$

Note:-

- $\bullet \ \, ||x||^2 = \langle x, x \rangle$
- $\langle x, y \rangle = \langle y, x \rangle$
- $\langle \cdot, \cdot \rangle$ is $\mathbb{R}\text{--linear}$ in each slot i.e.

 $\langle rx + x', y \rangle = r \langle x, y \rangle + \langle x', y \rangle$ and similarly for second slot

Here in $\langle x, y \rangle$ x is in first slot and y is in second slot.

Now the statement is just the Cauchy-Schwartz Inequality. For proof

$$\langle x, y \rangle^2 \leq \langle x, x \rangle \langle y, y \rangle$$

expand everything of $\langle x - \lambda y, x - \lambda y \rangle$ which is going to give a quadratic equation in variable λ

$$\langle x - \lambda y, x - \lambda y \rangle = \langle x, x - \lambda y \rangle - \lambda \langle y, x - \lambda y \rangle$$

$$= \langle x, x \rangle - \lambda \langle x, y \rangle - \lambda \langle y, x \rangle + \lambda^2 \langle y, y \rangle$$

$$= \langle x, x \rangle - 2\lambda \langle x, y \rangle + \lambda^2 \langle y, y \rangle$$

Now unless $x = \lambda y$ we have $\langle x - \lambda y, x - \lambda y \rangle > 0$ Hence the quadratic equation has no root therefore the discriminant is greater than zero.

When field is \mathbb{C} :

Modify the definition by

$$\langle x, y \rangle = \sum_{i} \overline{x_i} y_i$$

Then we still have $\langle x, x \rangle \ge 0$

Algorithms

```
Algorithm 1: what
   Input: This is some input
   Output: This is some output
   /* This is a comment */
 1 some code here;
 \mathbf{z} \ x \leftarrow 0;
\mathbf{3} \ \mathbf{y} \leftarrow 0;
4 if x > 5 then
 5 x is greater than 5;
                                                                                          // This is also a comment
 6 else
 7 x is less than or equal to 5;
8 end
9 foreach y in 0..5 do
10 y \leftarrow y + 1;
11 end
12 for y in 0..5 do
13 y \leftarrow y - 1;
14 end
15 while x > 5 do
16 x \leftarrow x - 1;
17 end
18 return Return something here;
```