Advanced Control

MECH70026

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Contents

Chapter 1 **Analogue Control Systems** **Page 2**

1.1 Mathematical Modelling Signals and Linear Dynamic Systems 2
1.2 Frequency Response Analysis and Design 2
1.3 Complex Frequency Analysis and Design 2

Chapter 2 **Digital Control Systems** **Page 3**

2.1 Design of a Digital Controller Using Continuous System Theory: CNC Controller Case Study 3
2.2 Discrete System Analysis Using Z-Transforms 3

Chapter 3 **Introduction to State Variable Analysis** **Page 4**

3.1 State Variable Analysis of Continuous Systems 4
3.2 State Variable Representation of Discrete Systems 4
3.3 Non-examinable Material 4

Chapter 1

Analogue Control Systems

1.1 Mathematical Modelling Signals and Linear Dynamic Systems

System Models Using Differential Equations

Signal Representation in the Frequency Domain and Transfer Functions

1.2 Frequency Response Analysis and Design

System Models Using Differential Equations

Signal Representation in the Frequency Domain and Transfer Functions

Bode Diagrams

Design of Compensation

1.3 Complex Frequency Analysis and Design

Laplace Transforms and Complex Frequency Concepts

Signal Representation in the Frequency Domain and Transfer Functions

Root Locus Design Method

Chapter 2

Digital Control Systems

2.1 Design of a Digital Controller Using Continuous System Theory: CNC Controller Case Study

CNC System Modelling

CNC Controller Design for Transients, Disturbance Rejection and Multi-Axis Contouring

Effects of Sampling

2.2 Discrete System Analysis Using Z-Transforms

Z-Transforms of Sampled Data Signals, Modified Z-Transforms and Fractional Time Delays

Discrete Transfer Function

Digital Equivalent of a Continuous Transfer Function (Approx. Integration, MPZ, ZOH)

Root Locus Design in the 'Z' Domain

Jury's Stability Test

Sampling Theorem

Chapter 3

Introduction to State Variable Analysis

3.1 State Variable Analysis of Continuous Systems

State Variable Modelling in Relation to Block Diagrams

Eigenvalues, Eigenvectors and Characteristic Equation, Stability of State Variable Models

Conversion Between Transfer Function and State Variable Models

The State Transition Matrix

Closed Loop Systems

State Variable Feedback

Design of a Tracking Controller

Controllability, Observability

3.2 State Variable Representation of Discrete Systems

Discrete State Variable Model from the Time Response of the Continuous Model

Discrete State Variable Model from Discrete Transfer Function G(z)

3.3 Non-examinable Material

Kalman Filtering

Optimal Control

Random Examples

Definition 3.3.1: Limit of Sequence in $\mathbb{R}$

Let $\{s_n\}$ be a sequence in $\mathbb{R}$. We say

$$\lim_{n\to\infty} s_n = s$$

where $s \in \mathbb{R}$ if $\forall$ real numbers $\epsilon > 0$ $\exists$ natural number N such that for $n > N$

$$s - \epsilon < s_n < s + \epsilon \text{ i.e. } |s - s_n| < \epsilon$$

Question 1

Is the set $x-$axis$\setminus\{$Origin$\}$ a closed set

Solution: We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls

Note:-

We will do topology in Normed Linear Space (Mainly $\mathbb{R}^n$ and occasionally $\mathbb{C}^n$)using the language of Metric Space

Claim 3.3.1 Topology

Topology is cool

Example 3.3.1 (Open Set and Close Set)

Open Set:
- ϕ
- $\bigcup_{x\in X} B_r(x)$ (Any $r > 0$ will do)
- $B_r(x)$ is open

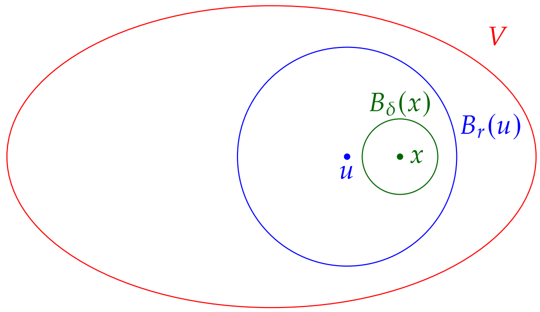
Closed Set:
- $X,\ \phi$
- $\overline{B_r(x)}$

$x-$axis $\cup$ $y-$axis

Theorem 3.3.1

If $x \in$ open set V then $\exists$ $\delta > 0$ such that $B_\delta(x) \subset V$

Proof: By openness of V, $x \in B_r(u) \subset V$



Given $x \in B_r(u) \subset V$, we want $\delta > 0$ such that $x \in B_\delta(x) \subset B_r(u) \subset V$. Let $d = d(u, x)$. Choose δ such that $d + \delta < r$ (e.g. $\delta < \frac{r-d}{2}$)

If $y \in B_\delta(x)$ we will be done by showing that $d(u, y) < r$ but

$$d(u, y) \leqslant d(u, x) + d(x, y) < d + \delta < r$$

😄

Corollary 3.3.1

By the result of the proof, we can then show...

Lenma 3.3.1

Suppose $\vec{v}_1, \ldots, \vec{v}_n \in \mathbb{R}^n$ is subspace of $\mathbb{R}^n$.

Proposition 3.3.1

$1 + 1 = 2$.

Random

Definition 3.3.2: Normed Linear Space and Norm $\|\cdot\|$

Let V be a vector space over $\mathbb{R}$ (or $\mathbb{C}$). A norm on V is function $\|\cdot\|$ $V \longrightarrow \mathbb{R}_{\geqslant 0}$ satisfying

1. $\|x\| = 0 \iff x = 0 \; \forall \; x \in V$
2. $\|\lambda x\| = |\lambda| \|x\| \; \forall \; \lambda \in \mathbb{R}(\text{or } \mathbb{C}),\, x \in V$
3. $\|x + y\| \leqslant \|x\| + \|y\| \; \forall \; x, y \in V$ (Triangle Inequality/Subadditivity)

And V is called a normed linear space.

• Same definition works with V a vector space over $\mathbb{C}$ (again $\|\cdot\| \longrightarrow \mathbb{R}_{\geqslant 0}$) where 2 becomes $\|\lambda x\| = |\lambda| \|x\|$ $\forall \; \lambda \in \mathbb{C}$, $x \in V$, where for $\lambda = a + ib$, $|\lambda| = \sqrt{a^2 + b^2}$

Example 3.3.2 (***p*−**Norm)

$V = \mathbb{R}^m$, $p \in \mathbb{R}_{\geqslant 0}$. Define for $x = (x_1, x_2, \cdots, x_m) \in \mathbb{R}^m$

$$\|x\|_p = \left(|x_1|^p + |x_2|^p + \cdots + |x_m|^p\right)^{\frac{1}{p}}$$

(In school $p = 2$)

Special Case $p = 1$: $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_m|$ is clearly a norm by usual triangle inequality.

Special Case $p \to \infty$ ($\mathbb{R}^m$ with $\|\cdot\|_\infty$): $\|x\|_\infty = \max\{|x_1|, |x_2|, \cdots, |x_m|\}$

For $m = 1$ these $p-$norms are nothing but $|x|$. Now exercise

Question 2

Prove that triangle inequality is true if $p \geqslant 1$ for $p-$norms. (What goes wrong for $p < 1$?)

Solution: **For Property ③ for norm-2**

When field is $\mathbb{R}$:

We have to show

$$\begin{aligned}
&\sum_i (x_i + y_i)^2 \leqslant \left(\sqrt{\sum_i x_i^2} + \sqrt{\sum_i y_i^2}\right)^2 \\
\implies &\sum_i (x_i^2 + 2x_i y_i + y_i^2) \leqslant \sum_i x_i^2 + 2\sqrt{\left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]} + \sum_i y_i^2 \\
\implies &\left[\sum_i x_i y_i\right]^2 \leqslant \left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]
\end{aligned}$$

So in other words prove $\langle x, y\rangle^2 \leqslant \langle x, x\rangle\langle y, y\rangle$ where

$$\langle x, y\rangle = \sum_i x_i y_i$$

Note:-

- $\|x\|^2 = \langle x, x\rangle$
- $\langle x, y\rangle = \langle y, x\rangle$
- $\langle \cdot, \cdot\rangle$ is $\mathbb{R}-$linear in each slot i.e.

$$\langle rx + x', y\rangle = r\langle x, y\rangle + \langle x', y\rangle \text{ and similarly for second slot}$$

  Here in $\langle x, y\rangle$ x is in first slot and y is in second slot.

Now the statement is just the Cauchy-Schwartz Inequality. For proof

$$\langle x, y\rangle^2 \leqslant \langle x, x\rangle\langle y, y\rangle$$

expand everything of $\langle x - \lambda y, x - \lambda y\rangle$ which is going to give a quadratic equation in variable λ

$$\begin{aligned}
\langle x - \lambda y, x - \lambda y\rangle &= \langle x, x - \lambda y\rangle - \lambda\langle y, x - \lambda y\rangle \\
&= \langle x, x\rangle - \lambda\langle x, y\rangle - \lambda\langle y, x\rangle + \lambda^2\langle y, y\rangle \\
&= \langle x, x\rangle - 2\lambda\langle x, y\rangle + \lambda^2\langle y, y\rangle
\end{aligned}$$

Now unless $x = \lambda y$ we have $\langle x - \lambda y, x - \lambda y\rangle > 0$ Hence the quadratic equation has no root therefore the discriminant is greater than zero.

When field is $\mathbb{C}$:

Modify the definition by

$$\langle x, y\rangle = \sum_i \overline{x_i} y_i$$

Then we still have $\langle x, x\rangle \geqslant 0$

Algorithms

Algorithm 1: what

```
Input: This is some input
Output: This is some output
/* This is a comment */

some code here;
x ← 0;
y ← 0;
if x > 5 then
  | x is greater than 5 ;                    // This is also a comment
else
  | x is less than or equal to 5;
end
foreach y in 0..5 do
  | y ← y + 1;
end
for y in 0..5 do
  | y ← y − 1;
end
while x > 5 do
  | x ← x − 1;
end
return Return something here;
```