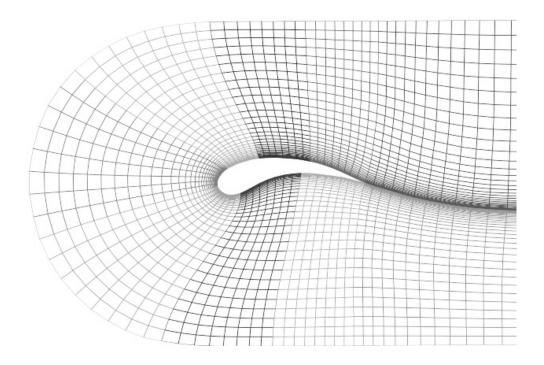
# MECH<sub>70015</sub> Computational Fluid Dynamics

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# Introduction

**Equations of Motion for Fluid Flow, Heat and Mass Transfer** 

### **Finite Volume Solution**

# The Navier-Stokes equations

### **Best Practises**

#### **Random Examples**

#### Definition 5.0.1: Limit of Sequence in $\mathbb{R}$

Let  $\{s_n\}$  be a sequence in  $\mathbb{R}$ . We say

$$\lim_{n\to\infty} s_n = s$$

where  $s \in \mathbb{R}$  if  $\forall$  real numbers  $\varepsilon > 0$   $\exists$  natural number N such that for n > N

$$s - \epsilon < s_n < s + \epsilon$$
 i.e.  $|s - s_n| < \epsilon$ 

#### Question 1

Is the set x-axis $\Origin$  a closed set

**Solution:** We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls

#### Note:-

We will do topology in Normed Linear Space (Mainly  $\mathbb{R}^n$  and occasionally  $\mathbb{C}^n$ )using the language of Metric Space

Claim 5.0.1 Topology

Topology is cool

#### Example 5.0.1 (Open Set and Close Set)

Open Set: • φ

•  $_{x \in X} B_r(x)$  (Any r > 0 will do)

•  $B_r(x)$  is open

Closed Set:

• X, φ

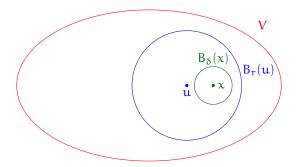
•  $\overline{B_r(x)}$ 

x-axis  $\cup y$ -axis

#### Theorem 5.0.1

If  $x \in \text{open set } V \text{ then } \exists \ \delta > 0 \text{ such that } B_{\delta}(x) \subset V$ 

**Proof:** By openness of  $V, x \in B_r(u) \subset V$ 



Given  $x \in B_r(u) \subset V$ , we want  $\delta > 0$  such that  $x \in B_\delta(x) \subset B_r(u) \subset V$ . Let d = d(u,x). Choose  $\delta$  such that  $d + \delta < r$  (e.g.  $\delta < \frac{r-d}{2}$ )

If  $y \in B_{\delta}(x)$  we will be done by showing that d(u, y) < r but

$$d(u,y) \leqslant d(u,x) + d(x,y) < d + \delta < r$$

(2)

#### Corollary 5.0.1

By the result of the proof, we can then show...

#### Lenma 5.0.1

Suppose  $v_1, ..., v_n \in \mathbb{R}^n$  is subspace of  $\mathbb{R}^n$ .

#### Proposition 5.0.1

1 + 1 = 2.

#### Random

#### Definition 5.0.2: Normed Linear Space and Norm $\|\cdot\|$

Let V be a vector space over  $\mathbb{R}$  (or  $\mathbb{C}$ ). A norm on V is function  $\|\cdot\| V \to \mathbb{R}_{\geq 0}$  satisfying

- (2)  $\|\lambda x\| = |\lambda| \|x\| \ \forall \ \lambda \in \mathbb{R}(\text{or } \mathbb{C}), \ x \in V$
- (3)  $||x + y|| \le ||x|| + ||y|| \ \forall \ x, y \in V$  (Triangle Inequality/Subadditivity)

And V is called a normed linear space.

• Same definition works with V a vector space over C (again  $\|\cdot\| \to \mathbb{R}_{\geqslant 0}$ ) where ② becomes  $\|\lambda x\| = |\lambda| \|x\|$   $\forall \ \lambda \in \mathbb{C}, \ x \in V$ , where for  $\lambda = a + ib$ ,  $|\lambda| = \sqrt{a^2 + b^2}$ 

#### Example 5.0.2 (p—Norm)

 $V=\mathbb{R}^m$ ,  $\mathfrak{p}\in\mathbb{R}_{\geqslant 0}$ . Define for  $x=(x_1,x_2,\cdots,x_m)\in\mathbb{R}^m$ 

$$||x||_p = (|x_1|^p + |x_2|^p + \dots + |x_m|^p)^{\frac{1}{p}}$$

(In school p = 2)

**Special Case p = 1:**  $||x||_1 = |x_1| + |x_2| + \cdots + |x_m|$  is clearly a norm by usual triangle inequality.

**Special Case p**  $\to \infty$  ( $\mathbb{R}^m$  with  $\|\cdot\|_{\infty}$ ):  $\|x\|_{\infty} = \max\{|x_1|, |x_2|, \cdots, |x_m|\}$  For m = 1 these p—norms are nothing but |x|. Now exercise

#### Ouestion 2

Prove that triangle inequality is true if  $p \ge 1$  for p—norms. (What goes wrong for p < 1?)

Solution: For Property (3) for norm-2

#### When field is $\mathbb{R}$ :

We have to show

$$(x_i + y_i)^2 \le \left(\sqrt{x_i^2} + \sqrt{y_i^2}\right)^2$$

$$\Longrightarrow_i (x_i^2 + 2x_iy_i + y_i^2) \le x_i^2 + 2\sqrt{\left[x_i^2\right]\left[y_i^2\right]} + y_i^2$$

$$\Longrightarrow_i \left[x_iy_i\right]^2 \le \left[x_i^2\right]\left[y_i^2\right]$$

So in other words prove  $\langle x, y \rangle^2 \le \langle x, x \rangle \langle y, y \rangle$  where

$$\langle x, y \rangle = x_i y_i$$

#### Note:-

- $||x||^2 = \langle x, x \rangle$
- $\langle x, y \rangle = \langle y, x \rangle$
- $\langle \cdot, \cdot \rangle$  is  $\mathbb{R}$ -linear in each slot i.e.

 $\langle rx + x', y \rangle = r \langle x, y \rangle + \langle x', y \rangle$  and similarly for second slot

Here in  $\langle x, y \rangle$  x is in first slot and y is in second slot.

Now the statement is just the Cauchy-Schwartz Inequality. For proof

$$\langle x,y\rangle^2\leqslant \langle x,x\rangle\langle y,y\rangle$$

expand everything of  $\langle x - \lambda y, x - \lambda y \rangle$  which is going to give a quadratic equation in variable  $\lambda$ 

$$\begin{split} \langle x - \lambda y, x - \lambda y \rangle &= \langle x, x - \lambda y \rangle - \lambda \langle y, x - \lambda y \rangle \\ &= \langle x, x \rangle - \lambda \langle x, y \rangle - \lambda \langle y, x \rangle + \lambda^2 \langle y, y \rangle \\ &= \langle x, x \rangle - 2\lambda \langle x, y \rangle + \lambda^2 \langle y, y \rangle \end{split}$$

Now unless  $x = \lambda y$  we have  $\langle x - \lambda y, x - \lambda y \rangle > 0$  Hence the quadratic equation has no root therefore the discriminant is greater than zero.

#### When field is $\mathbb{C}$ :

Modify the definition by

$$\langle x, y \rangle = \overline{x_i} \overline{y_i}$$

Then we still have  $\langle x, x \rangle \ge 0$ 

#### **Algorithms**

```
Algorithm 1: what
   Input: This is some input
   Output: This is some output
   /* This is a comment */
 1 some code here;
 _{2} \chi\leftarrow0;
y \leftarrow 0;
4 if x > 5 then
 5 | x is greater than 5;
                                                                                          // This is also a comment
 _7 | x is less than or equal to 5;
 9 foreach y in 0..5 do
10 y \leftarrow y + 1;
11 end
12 for y in 0..5 do
13 y \leftarrow y - 1;
14 end
15 while x > 5 do
16 x \leftarrow x - 1;
17 end
18 return Return something here;
```