## 1. To find the value of $T(2)$ for the recurrence relation $T(n)=3T(n-1)+12n$ with the initial condition $T(0)=5$, you can follow these steps:

**Compute $T(1)$**

Using the recurrence relation: $T(n)=3T(n-1)+12n$

Substitute $n=1$: $T(1)=3T(0)+12\times1T(1) =3\times5+12$ $T(1)=15+12T(1)= 27$

**Compute $T(2)$**

Using the recurrence relation: $T(n)=3T(n-1)+12nT(n)$

Substitute $n=2$: $T(2)=3T(1)+12\times2T(2) =3\times27+24$ $T(2)=81+24T(2) = 81 + 24T(2)=81+24$ $T(2)= 105$

**Summary**

The value of $T(2)$ is 105.

### 2. a. $T(n)=T(n-1)+c$

**Assumption:** Let's assume $T(n)=T(0)+cn$. We will prove this by induction.

**Base Case:** For $n=0$: $T(0)=T(0)+c\times0T(0) = T(0)$ So, the base case holds.

**Inductive Step:** Assume $T(k)=T(0)+ck$ holds for $k$. We need to show that $T(k+1)=T(0)+c(k+1)$.

Using the recurrence relation: $T(k+1)=T(k)+c$

By the inductive hypothesis: $T(k)=T(0)+ckT(k+1) = (T(0)+ck)+cT(k+1) =T(0)+c(k+1)$

Thus, the assumption holds, and the solution is: $T(n)= T(0)+cn$

## 2. b. $T(n)=2T(n/2)+n$

**Goal:** Solve for $T(n)$using the substitution method.

**Solution:**

1. **Assumption (Guess):** We assume $T(n)$ has a solution of the form $T(n)=anlogn+bn$

   $anlog\ n + b\ n=2[an/2logn/2+bn/2]+n$

Simplify:

$anlog(n)+bn=an\ log(n/2)+bn+n$

$anlog(n)+bn = an(logn-log2)+bn+n$

$anlog(n)+bn= anlogn-anlog2+bn+n$

Equate coefficients:

$anlog(n)= anlog(n)$

and

$bn=bn+n-anlog2$

So,

$0=n-anlog2$

To satisfy this, $a=1/log2$, and we can $b=0$.

Thus,

$T(n)=nlog(n)$

## 2.c. T(n)=2T(n/2)+c

**Goal:** Solve for T(n).

Solution:

1. **Assumption (Guess):** Assume $T(n)=an$ for some constant a.
2. Substitute the Guess:
   Substitute $T(n)=an$ into the recurrence relation:

   $T(n)=2T(n/2)+c$
   $an=2[an/2]+c$
   $an=an+c$

   For this to hold, we need:

   $0=c$
   Therefore, $T(n)=an$ and a can be any constant.
   For a specific solution, considering $T(n)=clogn$ works.

   So,
   $T(n)=clogn$

## 3.d $T(n)=T(n/2)+c$

**Goal:** Solve for $T(n)$.

Solution:

**Assumption (Guess):** Assume $T(n)=a\log n+b$.

Substitute the Guess:

Substitute $T(n)=a\log n+b$ into the recurrence relation:

$T(n)=T(n/2)+c$

$a\log n+b=a\log n/2+b+c$

$a\log n+b=a(\log n-\log 2)+b+c$

$a\log n+b=a\log n-a\log 2+b+c$

For this to hold, the constants must satisfy:

$0=-a\log 2+c$

Thus,

$a=c/\log 2$

The constant b cancels out, so we can set b to 0.

Thus,

$T(n)=(c/\log 2)\log n+b$

For simplicity, if b=0, then:

$T(n)= c\log n/ \log 2$