Polygon Partitioning

Vidit Goel 16IE33002
Manad Mishra 16CS30019

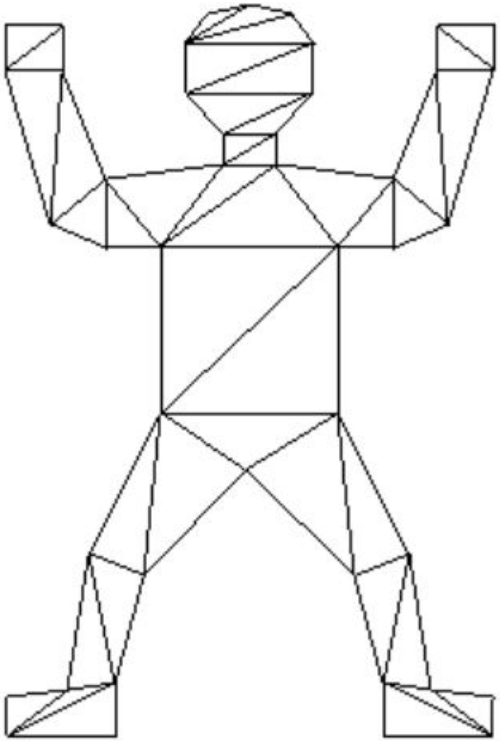
Triangulation by Ear Clipping

Time complexity: $O(n^2)$

Space complexity: $O(n)$

Support holes: Yes

Quality of Solution: Optimal/Satisfactory



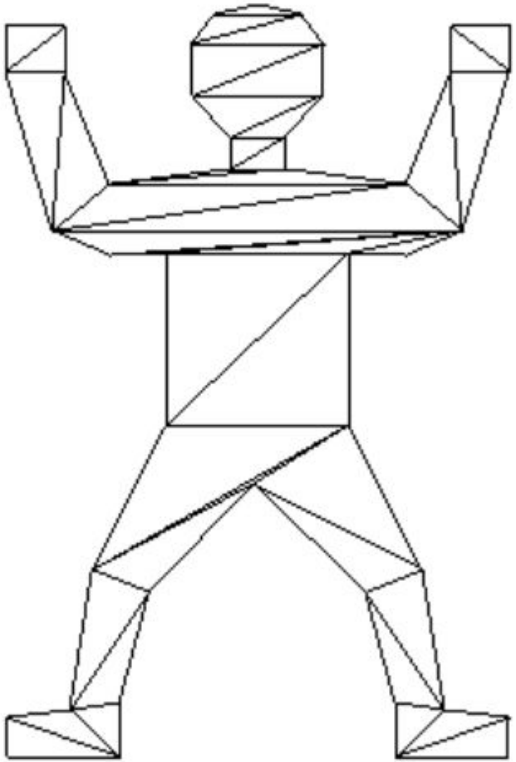
Triangulation by Partition into Monotone Polygons

Time complexity: O(nlogn)

Space complexity: O(n)

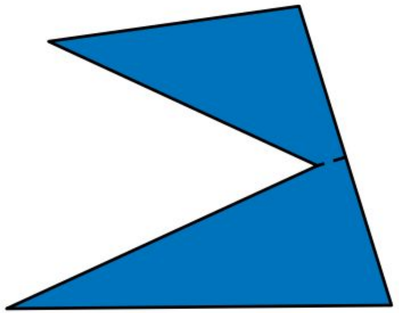
Support Holes: Yes, by design

Quality of Solution:
Poor, many thin triangles created

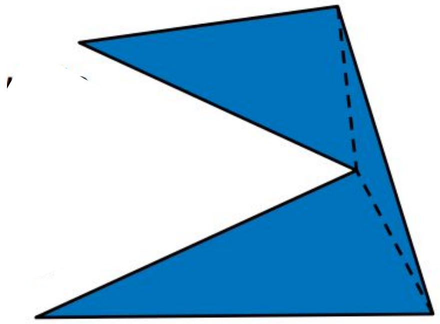


Convex Partitioning

A convex partition by **segments** of a polygon P is a decomposition of P into convex polygons obtained by introducing arbitrary segments.



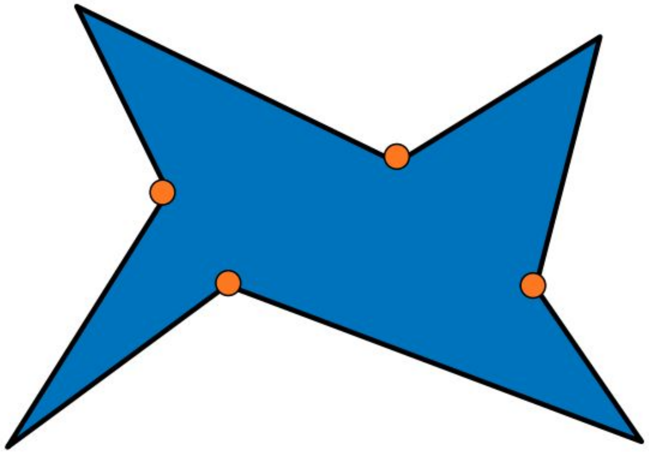
A convex partition by **diagonals** of a polygon P is a decomposition of P into convex polygons obtained by only introducing diagonals.



Convex Partitions (by Segments)

Claim (Chazelle): Assume the polygon P has r reflex vertices. If Φ is the fewest number of polygons required for a convex partition by segments of P then:

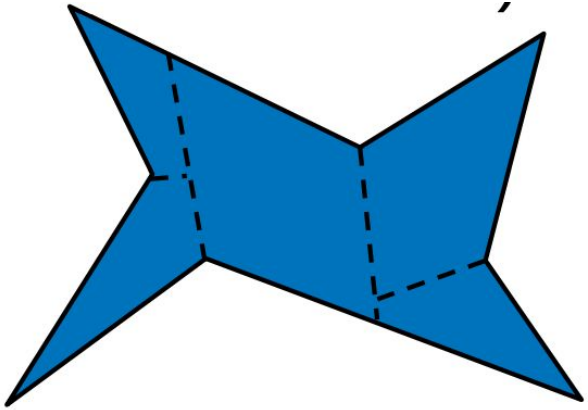
$\lceil r/2 \rceil + 1 \leq \Phi \leq r + 1$



Convex Partitions (by Segments)

Proof ($\Phi \leq r + 1$):

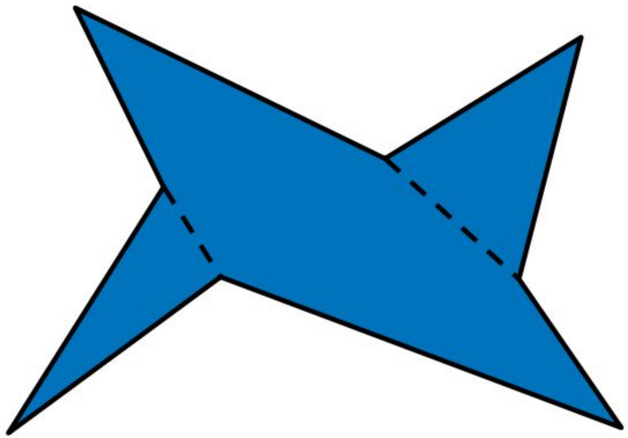
For each reflex vertex, add the bisector. Because the segment bisects, the reflex angle splits into two convex angles. (Angles at the new vertices have to be $< \pi$) Doing this for each reflex vertices, gives a convex partition with $r + 1$ pieces.



Convex Partitions (by Segments)

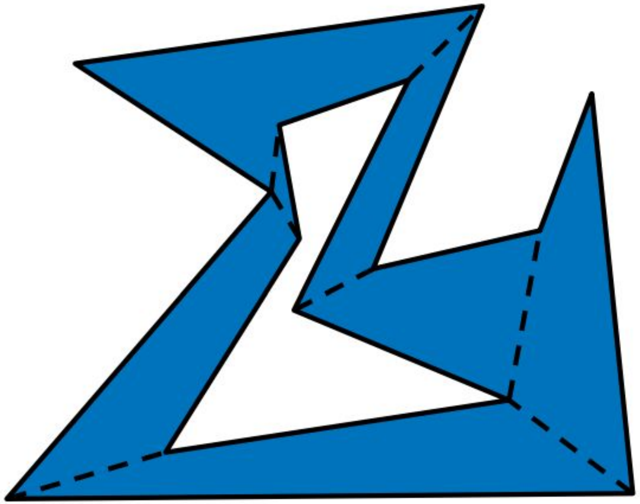
Proof ($\lceil r/2 \rceil \leq \Phi$):

Each reflex vertex needs to be split and each introduced segment can split at most two reflex vertices.



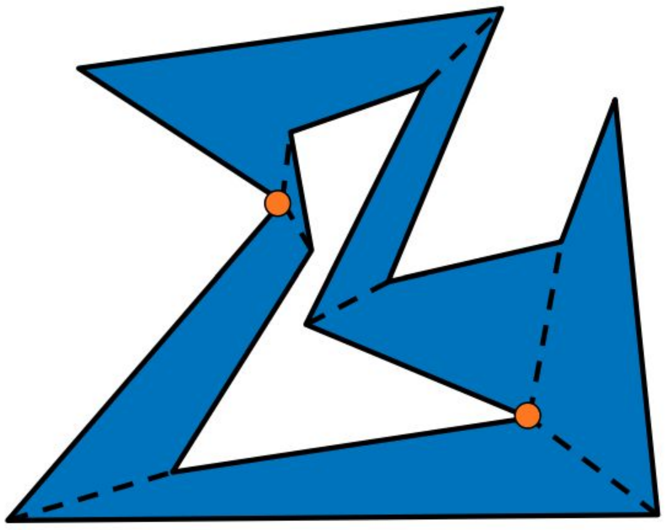
Convex Partitions (by Diagonals)

A diagonal in a convex partition is **essential** for vertex $v \in P$ if removing the diagonal creates a piece that is not convex at v.



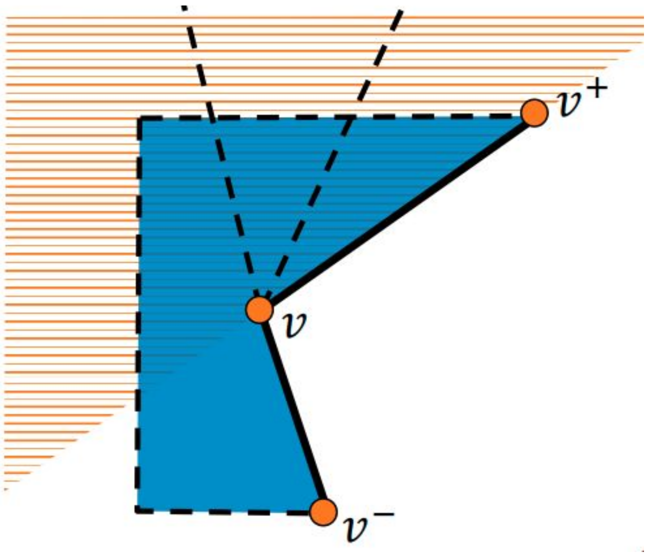
Convex Partitions (by Diagonals)

Claim: If v is a reflex vertex, it can have at most two essential diagonals.



Convex Partitions (by Diagonals)

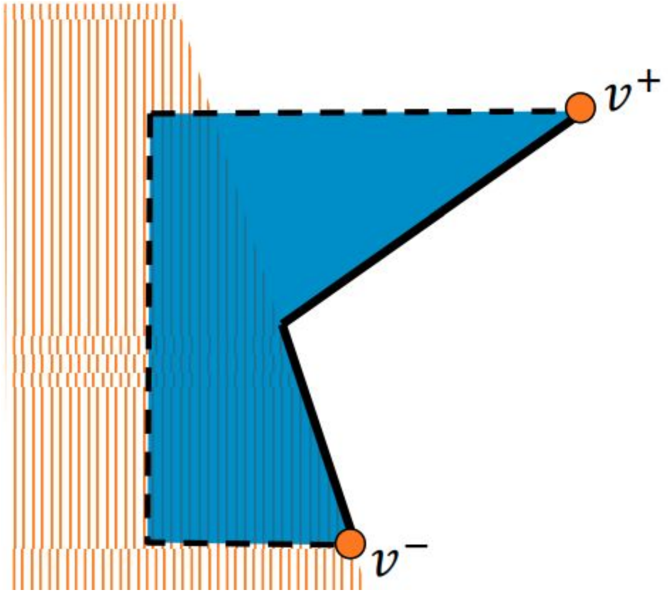
Proof: Given a reflex vertex v, let $v-$ and $v+$ be the vertices immediately before and after v in P. There can be at most one essential segment in the half-space to the right of $vv+$. (If there were two, we could remove the one closer to $vv+$ without creating a non-convexity).



Convex Partitions (by Diagonals)

Proof: Given a reflex vertex v, let $v-$ and $v+$ be the vertices immediately before and after v in P. Similarly, there can be at most one essential segment in the half-space to the right of $vv-$.

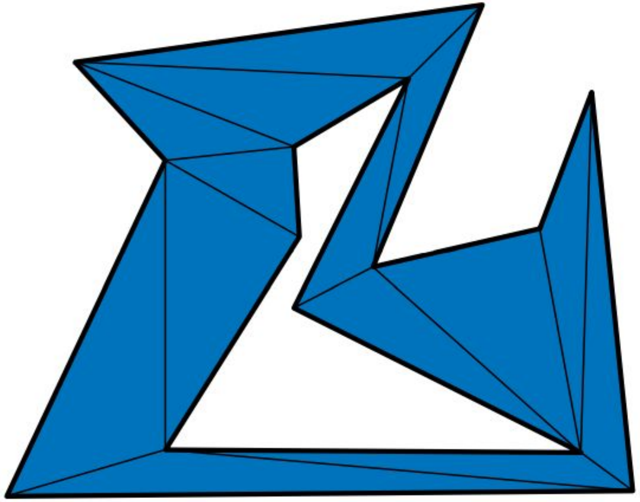
Since the two half-spaces cover the interior of the vertex there are at most two essential vertices at v.



Convex Partition using Hertel-Mehlhorn Algorithm

Start with a triangulation and remove inessential diagonals.

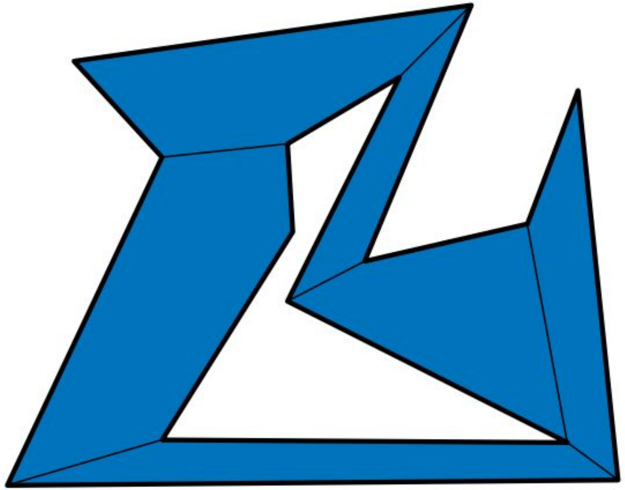
Claim: This algorithm is never worse than 4 × optimal in the number of convex pieces.



Convex Partition using Hertel-Mehlhorn Algorithm

Proof: When the algorithm terminates, every remaining diagonal is essential for some (reflex) vertex. Each reflex vertex can have at most two essential diagonals.

⇒ There can be at most $2r + 1$ pieces in the partition. Since at least $\lceil r/2 \rceil + 1$ are required, the result is within 4×optimal.



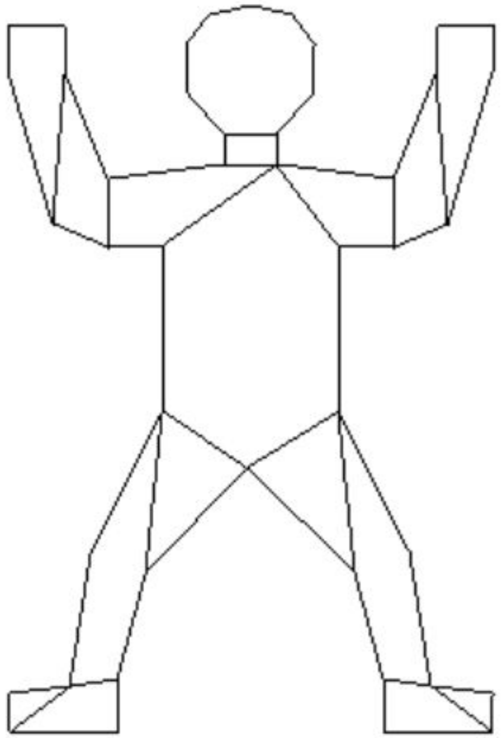
Convex Partition using Hertel-Mehlhorn Algorithm

Time complexity: $O(n^2)$

Space complexity: $O(n)$

Support Holes: Yes

Quality of Solution:
Mostly optimal, sometimes up to four times the optimal number of polygons created



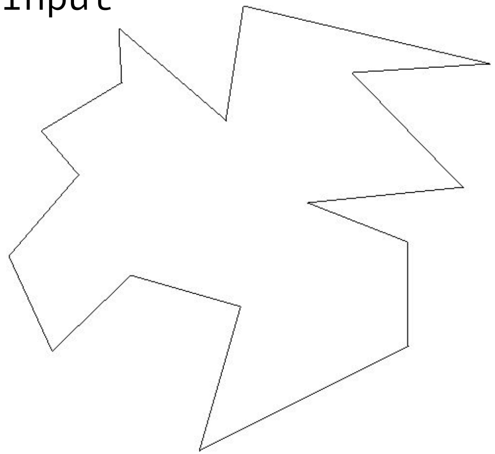
Experimentation Setup

- We implemented the Hertel-Mehlhorn algorithm using C++.
- The algorithm takes a triangulated polygon as an input, for which we used Ear-Clipping Triangulation.
- Our code is available at the link below:

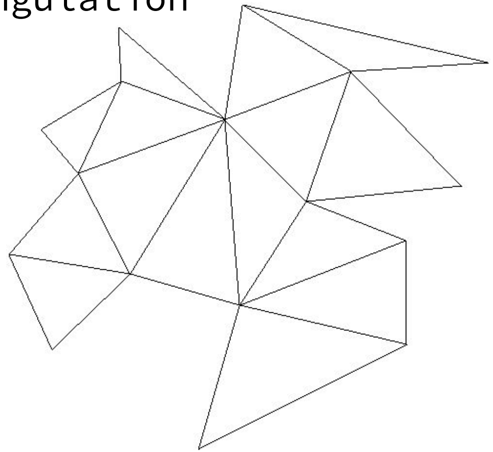
https://github.com/manadmishra/Polygon-Partitioning

Results

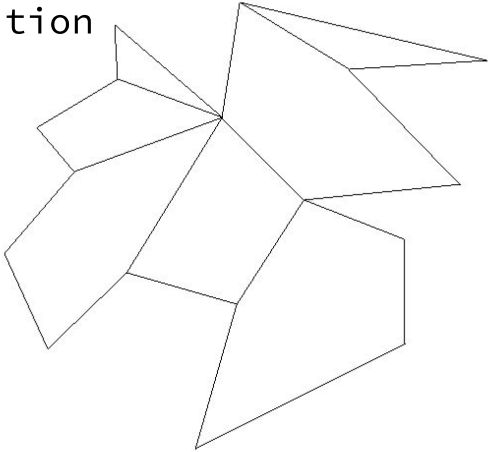
Input



Triangulation



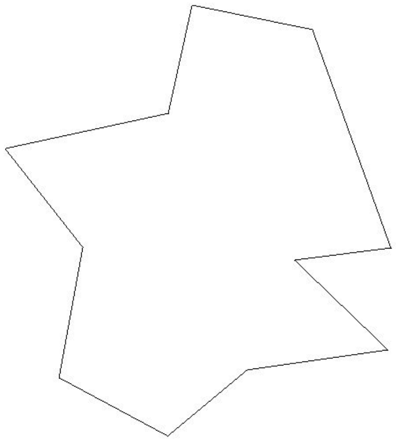
Polygon Partition



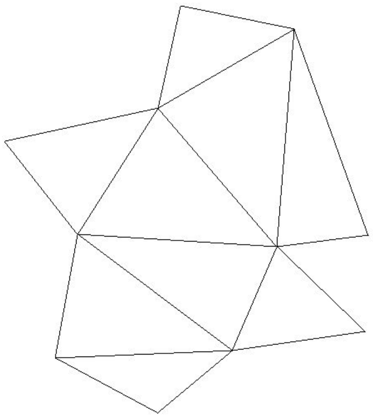
Number of Sides = 17
Number of reflex angles = 7
Number of convex partitions obtained = 7

Results

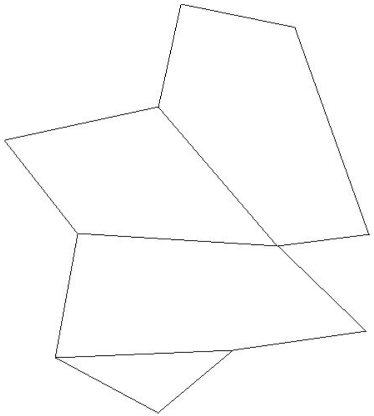
Input



Triangulation



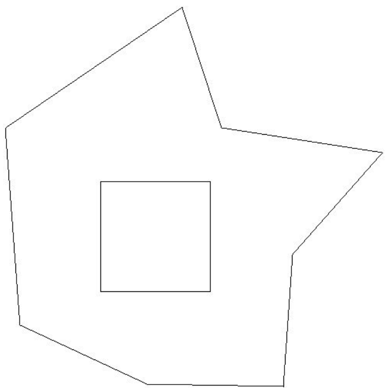
Polygon
Partition



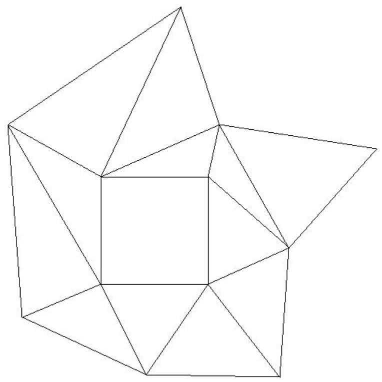
Number of Sides = 11
Number of reflex angles = 4
Number of convex partitions obtained = 4

Results

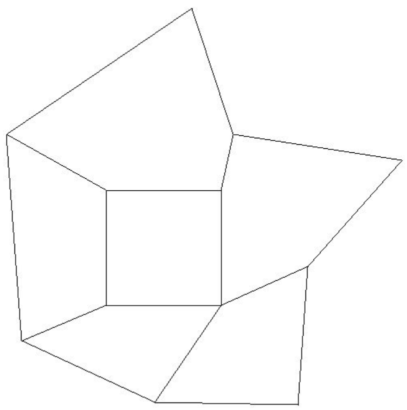
Input



Triangulation



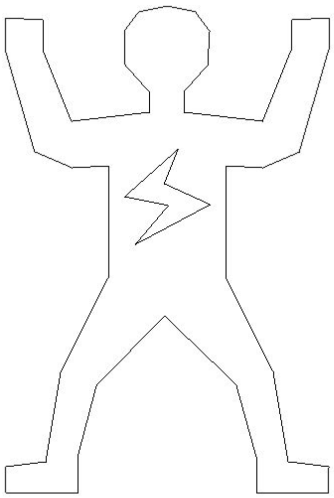
Polygon
Partition



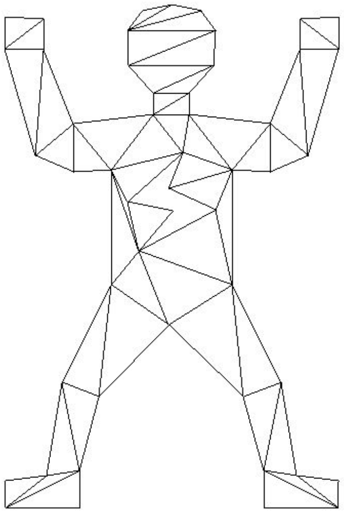
Number of Sides = 8
Number of Holes = 1
Number of reflex angles = 6
Number of convex partitions obtained = 5

Results

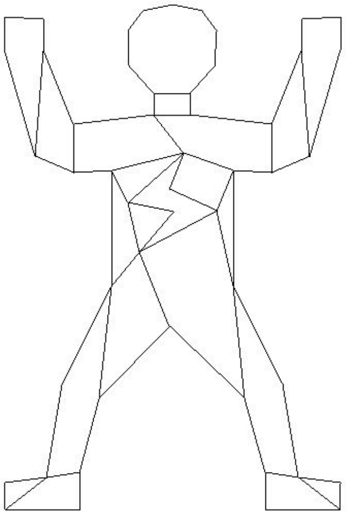
Input



Triangulation



Polygon
Partition



Number of Sides = 44
Number of Holes = 1
Number of reflex angles = 22
Number of convex partitions obtained = 21

Observation

We observed that the Hertel-Mehlhorn Algorithm divided the input polygons into the optimal number of convex segments (as defined by Chazelle’s claim) while also supporting large polygons with holes.

References

[1]H. Y. F. Feng and T. Pavlidis. Decomposition of polygons into simpler components: feature generation for syntactic pattern recognition. IEEE Trans. Comput., C-24:636{650, 1975.

[2] Armaselu, Bogdan, and Ovidiu Dăescu. "Algorithms for fair partitioning of convex polygons." *Theoretical Computer Science* 607 (2015): 351-362.

[3] Struzyna, Markus. Flow-based partitioning and position constraints in VLSI placement. IEEE, 2011.

[4] G.H. Meisters, Polygons have ears, Amer. Math. Monthly, vol. 82, pp. 648-651, 1975

[5] Mark de Berg, Marc van Kreveld, Mark Overmars, and Otfried Schwarzkopf (2000), Computational Geometry (2nd revised ed.), Springer-Verlag, ISBN 3-540-65620-0 Chapter 3: Polygon Triangulation: pp.45–61.

[6] Hertel, Stefan, and Kurt Mehlhorn. "Fast triangulation of simple polygons." International Conference on Fundamentals of Computation Theory. Springer, Berlin, Heidelberg, 1983.

[7] Keil, Mark, and Jack Snoeyink. "On the time bound for convex decomposition of simple polygons." *International Journal of Computational Geometry & Applications* 12.03 (2002): 181-192.