Proportional-Differential and Low-Chattering Sliding Mode Controllers for Quadrotor Drone

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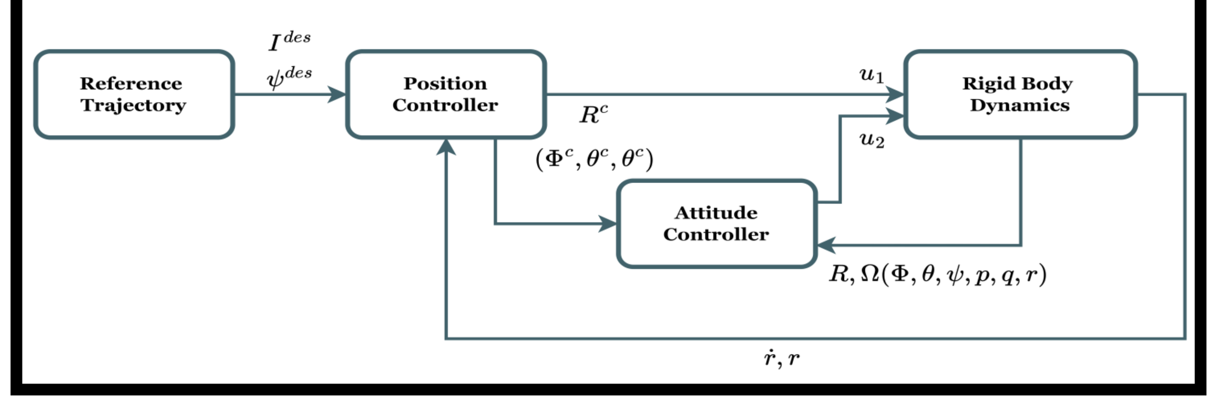
Objective

Analyze and compare the effectiveness of Proportional-Derivative (PD) Control and different sliding surface equations, such as soft sign and other squashing functions, in reducing chattering effects in Sliding Mode Control (SMC) for quadrotor drones.

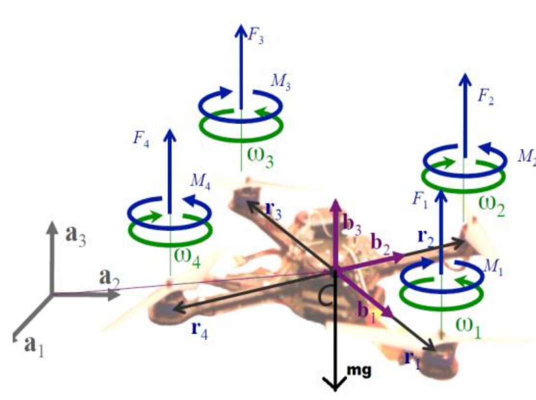
Background and Motivation

Quadrotor drones are increasingly utilized in critical applications such as disaster response, aerial photography, and infrastructure inspection, where precise and stable flight control is paramount. Traditional control systems like **Proportional-Derivative (PD) control**, although straightforward and widely adopted, often struggle with performance degradation due to environmental disturbances. Meanwhile, more robust approaches like **Sliding Mode Control (SMC)** effectively handle such disturbances but can induce chattering, leading to potential mechanical wear and reduced efficiency. This study aims to enhance the control mechanisms of quadrotors by refining these advanced control methods, specifically focusing on **reducing chattering effects** while maintaining operational precision and energy efficiency.

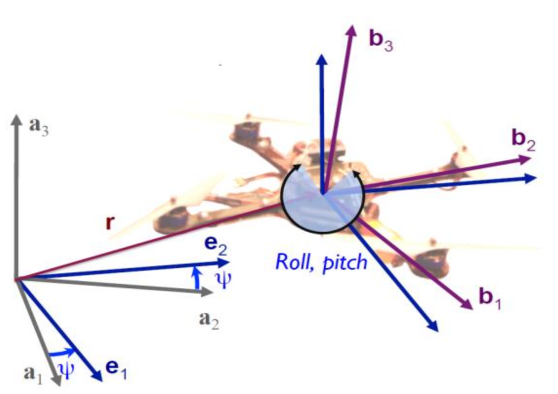
Block Diagram



System Model of Drone



Free Body Diagram of the Quadrotor



World and body frames

Project Features and Specifications

Dynamics of the Drone [1, 4]

The dynamics of a quadrotor drone are governed by interactions between its propulsion system, aerodynamics, and inertial properties. These dynamics encompass both linear motion, which involves the drone's translation in space, and angular motion, which involves its rotation about different axes.

Linear motion equation in world frame:

$$m\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + {}^{W}R_{B} \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

Angular motion equation in body frame:

$$\mathbf{I}\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \tau_{bx} \\ \tau_{by} \\ \tau_{bz} \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbf{I}\begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \mathbf{I}\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_2 + M_4 - M_1 - M_3 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbf{I}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Conversion between two frames of reference:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \rightarrow \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} c\theta & 0 & s\theta \\ s\theta t\phi & 1 & -c\theta t\phi \\ -\frac{s\theta}{c\phi} & 0 & \frac{c\theta}{c\phi} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Control inputs and their relationship with thrusts or torques of the rotors:

$$u_1 = F_1 + F_2 + F_3 + F_4 \text{ and } u_2 = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_2 + M_4 - M_1 - M_3 \end{bmatrix}$$

Control Mechanism

Proportional-Derivative (PD) control and Sliding Mode Control are the two techniques used. PD control offers stability by adjusting control signals proportionally to error and its rate of change. Sliding Mode Control ensures robust performance in the presence of disturbances by driving the system states onto a predefined sliding surface.

Equations for Proportional Derivative (PD) Control

$$\ddot{r}_{1,c} = \ddot{r}_{1,\text{des}} + k_{d,1}(\dot{r}_{1,\text{des}} - \dot{r}_1) + k_{p,1}(r_{1,\text{des}} - r_1)$$
$$\ddot{r}_{2,c} = \ddot{r}_{2,\text{des}} + k_{d,2}(\dot{r}_{2,\text{des}} - \dot{r}_2) + k_{p,2}(r_{2,\text{des}} - r_2)$$
$$\ddot{r}_{3,c} = \ddot{r}_{3,\text{des}} + k_{d,3}(\dot{r}_{3,\text{des}} - \dot{r}_3) + k_{p,3}(r_{3,\text{des}} - r_3)$$
$$\phi_c = \frac{1}{g}(\ddot{r}_{1,c}\sin(\psi_{\text{des}}) - \ddot{r}_{2,c}\cos(\psi_{\text{des}}))$$
$$\theta_c = \frac{1}{g}(\ddot{r}_{1,c}\cos(\psi_{\text{des}}) + \ddot{r}_{2,c}\sin(\psi_{\text{des}}))$$
$$\psi_c = \psi_{\text{des}}$$
$$u_1 = m(g + \ddot{r}_{3,c})$$
$$u_2 = I\begin{bmatrix} k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p) \\ k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q) \\ k_{p,\psi}(\psi_c - \psi) + k_{d,\theta}(q_c - q) \end{bmatrix}$$

Equations for Sliding Mode Control

$$\begin{aligned} \dot{s} &= \ddot{e} + \lambda\dot{e} \\ &= \ddot{\omega} - \ddot{\omega}_c + \lambda(\dot{\omega} - \dot{\omega}_c) \\ &= -\mathbf{I}^{-1}\dot{\omega} \times \mathbf{I}\dot{\omega} + \mathbf{I}^{-1}u_2 - \ddot{\omega}_c + \lambda(\dot{\omega} - \dot{\omega}_c) \\ &= -\mathbf{I}^{-1}\dot{\omega} \times \mathbf{I}\dot{\omega} - \ddot{\omega}_c + \lambda(\dot{\omega} - \dot{\omega}_c) + \mathbf{I}^{-1}u_2 \\ &= \alpha_{SM} + \beta_{SM}u_2 \\ &= v := -K\,\text{sign}(s) \quad \text{[virtual input]} \end{aligned}$$

$$u_2 = \mathbf{I}\left(-\left(-\mathbf{I}^{-1}\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \mathbf{I}\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} \ddot{\phi}_c \\ \ddot{\theta}_c \\ \ddot{\psi}_c \end{bmatrix} + \lambda\left(\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} \dot{\phi}_c \\ \dot{\theta}_c \\ \dot{\psi}_c \end{bmatrix}\right) - K\begin{bmatrix} \text{sign}(s_1) \\ \text{sign}(s_2) \\ \text{sign}(s_3) \end{bmatrix}\right)\right)$$

Conclusion and Future Work

This study looked at using two different types of controllers, the Sliding Mode controller and the conventional Proportional-Derivative controller, to keep a quadrotor drone on a predefined course in both normal and disturbed situations.

Furthermore, we demonstrated the application of several (smooth) squashing techniques to reduce chattering in sliding mode controllers.

In the future, we hope to broaden our research to include actual testing with real drones and assess the amount of energy required to install these controllers while maintaining the drones' functionality and efficiency.

This research will make drone operations more accurate and durable, increasing their use in difficult situations.

References

[1] C. Powers, D. Mellinger, and V. Kumar, “Quadrotor Kinematics and Dynamics,” Springer eBooks, pp. 307–328, Aug. 2014.

[2] MATLAB, “Control Methodologies for a Simple Quadrotor,” www.mathworks.com, Feb. 19, 2024.

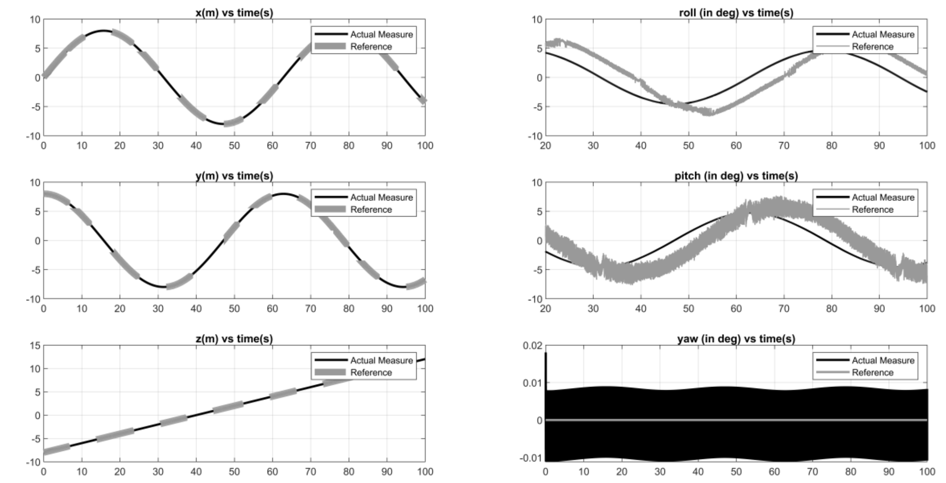
[3] V. Kunc and J. Kléma, “Three Decades of Activations: a Comprehensive Survey of 400 Activation Functions for Neural Networks,” arXiv.org, Feb. 14, 2024.

[4] Q. Quan, “Introduction to Multicopter Design and Control,” *SpringerLink*, 2017.

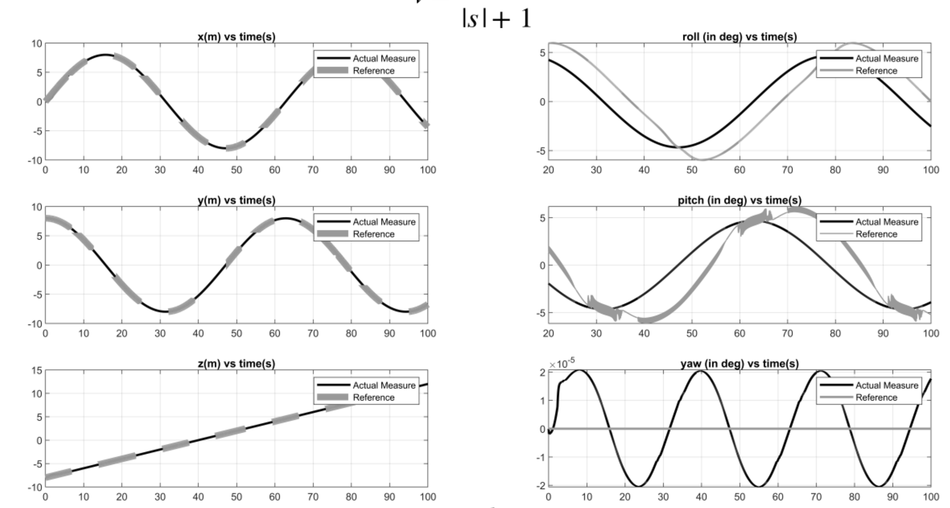
Results

The following plots show how different sliding surface equations impact chattering suppression in quadrotor drone control systems. These plots demonstrate the performance of the drone in following a helical trajectory, including control of position (x, y, z) and orientation (roll, pitch, yaw) angles [2, 3].

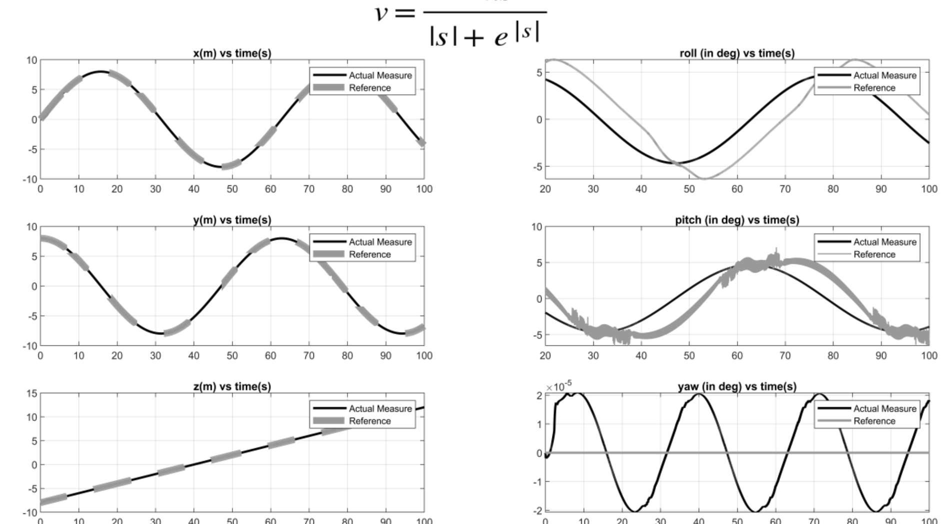
$$v = -K * sign(s)$$



$$v = \frac{-ks}{|s| + 1}$$



$$v = \frac{-ks}{|s| + e^{|s|}}$$



$$v = \frac{-ks}{|s| + 2 - e^{-|s|}}$$