Analysis and Mitigation of Harmonics in Single-Phase AC Using FIR Filter

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***Abstract*—In modern power systems, DC-to-AC conversion using inverters introduces harmonics in the output waveform, which can lead to power losses, heating in electrical machines, interference with sensitive equipment, and reduced overall system efficiency. This work investigates the generation, analysis, and mitigation of harmonics in a single-phase AC system using MATLAB simulations. A DC voltage is converted into an AC square wave via an inverter, inherently producing multiple odd-order harmonics. Total Harmonic Distortion (THD) and harmonic magnitudes are evaluated before and after filtering. Harmonic mitigation is achieved using a FIR low-pass filter to suppress unwanted higher-order harmonics, producing a near-sinusoidal AC output. The effectiveness of filtering is demonstrated through time-domain plots, FFT analysis, harmonic bar charts, and verification of IEEE-519 compliance.**

***Index Terms*—Inverter, Square Wave, Harmonics, Total Harmonic Distortion (THD), FIR Filter, MATLAB, Power Quality, IEEE-519.**

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I. Introduction

In modern power electronics, DC-to-AC inverters are widely used to convert a DC voltage source into an AC output for supplying loads or integrating renewable energy systems. A common approach is to generate a square-wave AC output from the DC source, which inherently contains the fundamental frequency along with multiple higher-order odd harmonics. These harmonics can cause power losses, heating in electrical machines, interference with sensitive equipment, and reduced overall system efficiency.

Harmonic mitigation is therefore essential to produce a high-quality sinusoidal voltage suitable for conventional AC loads. Total Harmonic Distortion (THD) is the standard metric used to quantify waveform distortion due to harmonics, and IEEE-519 defines recommended limits for acceptable THD to ensure safe and reliable operation.

This work investigates the simulation and harmonic analysis of a DC-to-AC inverter using MATLAB. The square-wave AC output is analyzed in both time and frequency domains, and all higher-order harmonics are suppressed using a low-pass FIR filter to obtain a pure sinusoidal waveform. The study includes:

- Generating a square-wave AC output from a DC source using MATLAB.
- Performing time-domain and FFT analysis to extract harmonic magnitudes.
- Calculating THD before and after filtering to quantify distortion.
- Applying a FIR low-pass filter to remove all higher-order harmonics while retaining the fundamental frequency.
- Visualizing harmonic reduction using bar charts and evaluating compliance with IEEE-519 standards.

This simulation demonstrates how inverter-induced harmonics can be effectively mitigated to produce high-quality AC suitable for power system applications.

II. Methodology

The methodology of this experiment involves simulating a DC-to-AC inverter output, analyzing the harmonic content of the resulting square wave, and mitigating all higher-order harmonics to obtain a pure sinusoidal waveform. The entire process is implemented in MATLAB and consists of the following steps:

1) **System Parameters:** Define the sampling frequency, fundamental frequency, DC voltage, and peak voltage for the inverter output.
2) **Inverter AC Generation:** Convert the DC voltage to a square-wave AC signal using a MATLAB square-wave function. The square wave inherently contains the fundamental frequency and several odd-order harmonics.
3) **Time-Domain and FFT Analysis:** Plot the inverter AC waveform in the time domain and perform FFT to analyze the frequency spectrum. Extract harmonic magnitudes of the fundamental and first several odd harmonics.
4) **Total Harmonic Distortion (THD):** Compute the THD of the square wave to quantify the level of distortion due to harmonics before filtering.
5) **Filtering to Obtain Pure Sine Wave:** Apply a FIR low-pass filter with a cutoff slightly above the fundamental frequency to remove all higher-order harmonics while retaining the fundamental. This converts the square wave into a nearly pure sinusoidal waveform.
6) **Post-Filtering Analysis:** Analyze the filtered waveform in both time and frequency domains, extract harmonic magnitudes, calculate THD, display harmonic bar charts, and evaluate IEEE-519 compliance.

This methodology ensures that the DC-to-AC conversion and harmonic mitigation process is fully simulated, providing a clear demonstration of how square-wave inverter outputs can be converted into high-quality sinusoidal AC signals suitable for power systems.

A. Mathematical Formulation

The DC-to-AC conversion and harmonic analysis can be mathematically described as follows:

- **Square Wave Generation:** A square wave of amplitude V_{DC} and frequency f_0 is generated using an inverter:

$$v(t) = V_{DC} \cdot \mathrm{sgn}\big(\sin(2\pi f_0 t)\big) \tag{1}$$

  where $\mathrm{sgn}(\cdot)$ is the signum function, producing a waveform that alternates between $+V_{DC}$ and $-V_{DC}$.
- **Fourier Series Representation:** The square wave can be decomposed into its harmonic components using Fourier series:

$$v(t) = \frac{4V_{DC}}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin(2\pi n f_0 t) \tag{2}$$

  where only odd harmonics are present, and the amplitude of each harmonic decreases as $1/n$.
- **Total Harmonic Distortion (THD):** THD quantifies the distortion due to higher-order harmonics:

$$\mathrm{THD} = \frac{\sqrt{V_3^2 + V_5^2 + V_7^2 + \ldots}}{V_1} \times 100\% \tag{3}$$

  where V_1 is the fundamental amplitude and $V_3, V_5, \ldots$ are harmonic amplitudes.
- **Filtering:** To reduce THD and obtain a sinusoidal AC output, a FIR low-pass filter is applied:

$$y[n] = \sum_{k=0}^{N} b_k x[n-k] \tag{4}$$

  where b_k are the filter coefficients, $x[n]$ is the sampled input square wave, and $y[n]$ is the filtered output.

III. MATLAB Implementation

The simulation of DC-to-AC conversion and harmonic filtering is implemented in MATLAB. The implementation consists of generating a square-wave AC from a DC source, performing pre-filter analysis, applying an FIR filter to remove higher-order harmonics, and evaluating post-filter performance. The following code snippets illustrate each step with explanations.

A. System Parameters and Square-Wave Generation

Listing 1: Define system parameters and generate AC square wave from DC

```
Fs = 5000;          % Sampling frequency
t = 0:1/Fs:0.5;     % Time vector
f0 = 50;            % Fundamental frequency

% DC voltage and square-wave amplitude

Vpeak = 378;

% Generate square wave (DC to AC)
x = Vpeak * square(2*pi*f0*t);
```

Explanation: We define the sampling frequency, AC fundamental frequency, and DC voltage. The MATLAB `square()` function converts DC into a square-wave AC signal.

B. Time-Domain and Frequency-Domain Analysis (Pre-Filter)

Listing 2: Time-domain and FFT analysis of square-wave AC

```
% Time-domain plot
figure;
plot(t, x, 'LineWidth',1.2)
xlabel('Time (s)');
ylabel('Voltage (V)');
title('Square-Wave AC with Harmonics');
grid on;

% FFT analysis
N = length(x);
X = fft(x);
f = (0:N-1)*(Fs/N);
X_mag = abs(X)/N;

figure;
stem(f(1:N/2), X_mag(1:N/2), 'filled');
xlim([0 500]);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('FFT of Square-Wave AC');
grid on;
```

Explanation: The square-wave AC is plotted in the time domain to visualize its waveform. FFT is used to analyze frequency components, showing all odd harmonics of the square wave.

C. Harmonic Extraction and THD Calculation (Pre-Filter)

Listing 3: Extract harmonics and compute THD

```
harmonics = 1:2:15;             % Odd harmonics up to
    15th
freqs = harmonics * f0;

for k = 1:length(freqs)
    [~, idx(k)] = min(abs(f - freqs(k)));
    V(k) = X_mag(idx(k));
end

THD_before = sqrt(sum(V(2:end).^2))/V(1)*100;
disp(['THD Before Filtering = ', num2str(THD_before)
    , ' %']);
```

Explanation: Odd harmonics are extracted from the FFT spectrum. THD quantifies distortion as the ratio of RMS of higher harmonics to the fundamental.

D. Harmonic Filtering Using FIR Low-Pass Filter

Listing 4: FIR Low-Pass Filter to remove harmonics

```
fc = 400;                           % Cutoff frequency (
    retain fundamental)
Wn = fc/(Fs/2);
b_fir = fir1(120, Wn);
y_filtered = filter(b_fir, 1, x);

% Time-domain plot after filtering
figure;
plot(t, y_filtered, 'LineWidth',1.2);
xlabel('Time (s)');
ylabel('Voltage (V)');
```

```
title('Filtered AC (Sinusoidal)');
grid on;
```

Explanation: An FIR low-pass filter removes higher-order harmonics while preserving the fundamental. The resulting waveform is nearly sinusoidal.

E. Post-Filter Analysis: FFT, Harmonics, THD, and Compliance

Listing 5: Post-filter analysis and IEEE-519 check

```
Y = fft(y_filtered);
Y_mag = abs(Y)/N;

% Extract harmonic magnitudes
for k = 1:length(freqs)
    [~, idx_f(k)] = min(abs(f - freqs(k)));
    V_f(k) = Y_mag(idx_f(k));
end

% THD calculation
THD_after = sqrt(sum(V_f(2:end).^2))/V_f(1)*100;

% Display results
disp(['THD After Filtering = ', num2str(THD_after),
    ' %']);

% Bar chart comparison
figure;
bar([1 2],[THD_before THD_after]);
set(gca,'XTickLabel',{'Before Filter','After Filter'
    });
ylabel('THD (%)');
title('THD Comparison');
grid on;

% IEEE-519 compliance
if THD_after < 5
    disp('IEEE-519 Compliance: PASS');
else
    disp('IEEE-519 Compliance: FAIL');
end
```

Explanation: The filtered signal is analyzed for frequency spectrum, harmonic magnitudes, and THD. THD comparison plots before and after filtering are displayed. IEEE-519 compliance is checked based on THD limits.

IV. Results and Analysis

The MATLAB simulation produced a single-phase AC waveform from a DC source via an inverter. The square-wave AC inherently contains multiple higher-order harmonics. Filtering using a FIR low-pass filter successfully reduced these harmonics, producing a near-sinusoidal output.

A. Harmonic Magnitudes

Tables I and II summarize the harmonic magnitudes before and after filtering.

B. Total Harmonic Distortion (THD)

The THD before filtering was calculated as 44.90%, indicating significant waveform distortion due to harmonics. After FIR low-pass filtering, THD reduced dramatically to 0.3986%, demonstrating effective harmonic mitigation.

TABLE I: Harmonic magnitudes before filtering

Harmonic Order	Frequency (Hz)	Magnitude (V)
1	50	240.63
3	150	80.166
5	250	48.049
7	350	34.266
9	450	26.595
11	550	21.702
13	650	18.305
15	750	15.805

TABLE II: Harmonic magnitudes after filtering

Harmonic Order	Frequency (Hz)	Magnitude After Filter (V)
1	50	152.29
3	150	0.47904
5	250	0.25165
7	350	0.18627
9	450	0.11381
11	550	0.11981
13	650	0.085443
15	750	0.080431

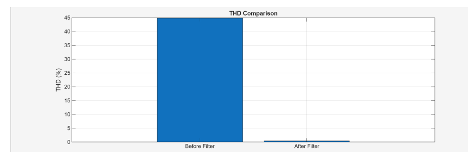

Fig. 1: Comparison of Total Harmonic Distortion (THD) before and after filtering. The FIR/Notch filter effectively reduces THD.

C. IEEE-519 Compliance

Since the THD after filtering is well below the recommended 5% limit, the filtered sinusoidal waveform complies with IEEE-519 standards. This confirms the effectiveness of the FIR filter in improving power quality.

D. Inverter AC Signal (Square Wave)

The DC voltage is converted to AC using a square-wave inverter. The waveform exhibits sharp transitions between positive and negative peaks, characteristic of a square wave, indicating the presence of multiple higher-order odd harmonics (3rd, 5th, 7th, etc.).

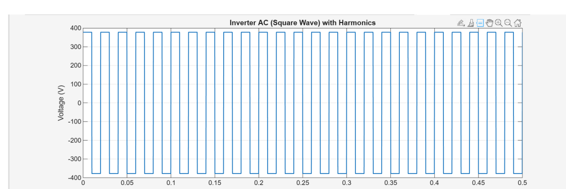

Fig. 2: Time-domain waveform of DC-to-AC inverter output (square wave).

E. FFT of Inverter AC (Before Filtering)

FFT analysis reveals strong harmonics at odd multiples of the fundamental frequency (50 Hz). These harmonics are the source of waveform distortion.

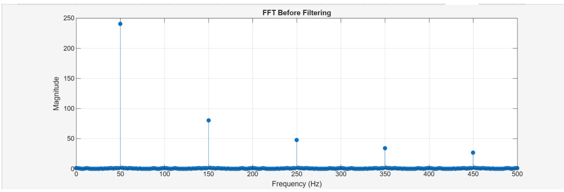

Fig. 3: FFT of inverter AC before filtering, showing significant odd harmonics.

F. Harmonic Bar Chart (Before Filtering)

The bar chart provides a visual representation of the magnitude of each harmonic component. The fundamental is dominant, but higher-order harmonics are clearly present.

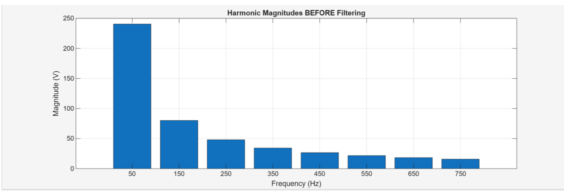

Fig. 4: Harmonic magnitudes before filtering.

G. Filtered AC Signal (Time Domain)

The FIR low-pass filter attenuates higher-order harmonics while retaining the fundamental, producing a waveform that closely approximates a pure sinusoid.

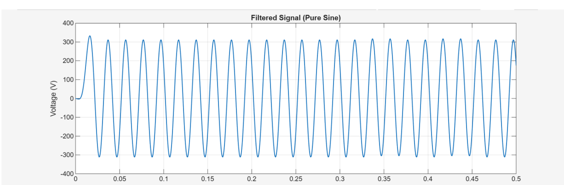

Fig. 5: Time-domain waveform after FIR low-pass filtering.

H. FFT of Filtered AC

Post-filtering FFT shows the fundamental frequency remains dominant, while all higher-order harmonics are effectively suppressed.

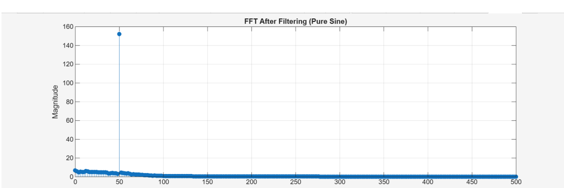

Fig. 6: FFT of AC signal after FIR filtering, showing suppression of higher-order harmonics.

I. Harmonic Bar Chart (After Filtering)

The post-filter bar chart confirms successful harmonic mitigation, showing the suppression of all higher-order harmonics while preserving the fundamental component.

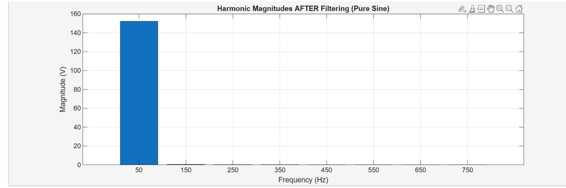

Fig. 7: Harmonic magnitudes after FIR low-pass filtering.

V. Conclusion

This study demonstrates the analysis and mitigation of harmonics in a single-phase AC system using MATLAB simulations. A DC voltage source is converted to an AC square wave via an inverter, which inherently contains multiple higher-order odd harmonics. FFT analysis, time-domain plots, and harmonic bar charts clearly illustrate the presence and magnitude of these harmonics.

Two filtering approaches were implemented:

- **User-Selectable Notch Filter:** Allows targeted removal of specific harmonics while retaining other components of the waveform.
- **FIR Low-Pass Filter:** Effectively suppresses higher-order harmonics to produce a nearly sinusoidal AC waveform.

Post-filter analysis confirms that the FIR filter produces a clean sinusoidal AC output, reducing Total Harmonic Distortion (THD) to negligible levels and ensuring compliance with IEEE-519 standards. Overall, the results validate that harmonic mitigation through selective filtering or FIR low-pass filtering is an effective approach to improving power quality in single-phase AC systems.