# Negative Binomial Distribution

## Erlang Distribution

The Erlang distribution is a continuous probability distribution used to model waiting times in systems with multiple phases or stages. It is a special case of the Gamma distribution where the shape parameter (k) is a positive integer. The distribution is commonly used in queueing theory, telecommunications, and reliability engineering.

## Characteristics of Erlang Distribution

1. Shape Parameter (k): The number of phases or stages

2. Rate Parameter (lambda): The rate at which events occur

3. Random Variable: The time required for k events to occur

## Probability Density Function (PDF) of Erlang Distribution

The probability density function of the Erlang distribution is defined as:

f(x; k, lambda) = (lambda^k * x^(k - 1) * e^(-lambda * x)) / (k - 1)!, x >= 0

Where:

- k is the shape parameter (positive integer)

- lambda is the rate parameter (positive real number)

- x is the random variable (time)

## Key Formulas for Erlang Distribution

1. Mean (Expected Value): E(X) = k / lambda

2. Variance: Var(X) = k / lambda^2

## Graph of Erlang Distribution

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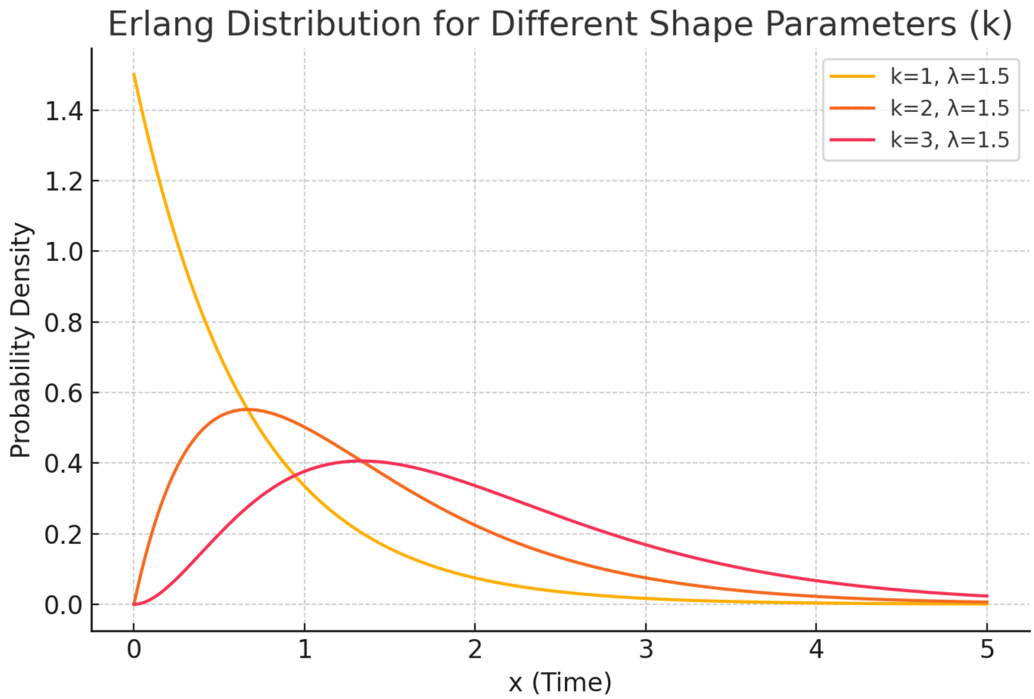
The shape of the Erlang distribution depends on the values of k and lambda. Below is a typical visualization of the distribution for different parameter values.




## Example Problem on Erlang Distribution

Problem Statement: Suppose calls arrive at a call center at a rate of 3 calls per minute. The time between calls follows an Erlang distribution with two phases (k = 2). What is the probability that the waiting time for two calls is less than 1 minute?

Solution: Given k = 2, lambda = 3 (calls per minute), and x = 1 minute

Using the PDF formula: f(x; 2, 3) = (3^2 * 1^(2 - 1) * e^(-3 * 1)) / (2 - 1)!

Compute: f(1; 2, 3) = 9 * e^(-3)

Result: f(1; 2, 3) approximately 0.448

Interpretation: The probability that the waiting time for two calls is less than 1 minute is approximately 0.448 (or 44.8%).

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## Lognormal Distribution

The Lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. In other words, if a random variable X follows a Lognormal distribution, then Y = ln(X) follows a Normal distribution.

This distribution is commonly used in financial modeling, reliability analysis, and natural phenomena where values cannot be negative and tend to cluster around a positive mean.

## Characteristics of Lognormal Distribution

1. Support: The distribution is defined for positive real numbers (X > 0).

2. Parameters:

- mu (mean of the underlying normal distribution)

- sigma (standard deviation of the underlying normal distribution)

3. Skewed Distribution: The lognormal distribution is right-skewed.

## Probability Density Function (PDF)

The probability density function of the lognormal distribution is defined as:

f(x; mu, sigma) = (1 / (x * sigma * sqrt(2 * pi))) * exp(-(ln(x) - mu)^2 / (2 * sigma^2)), for x > 0

Where:

- x is the random variable

- mu is the mean of the log of the variable

- sigma is the standard deviation of the log of the variable

## Key Formulas for Lognormal Distribution

1. Mean (Expected Value): E(X) = exp(mu + sigma^2 / 2)

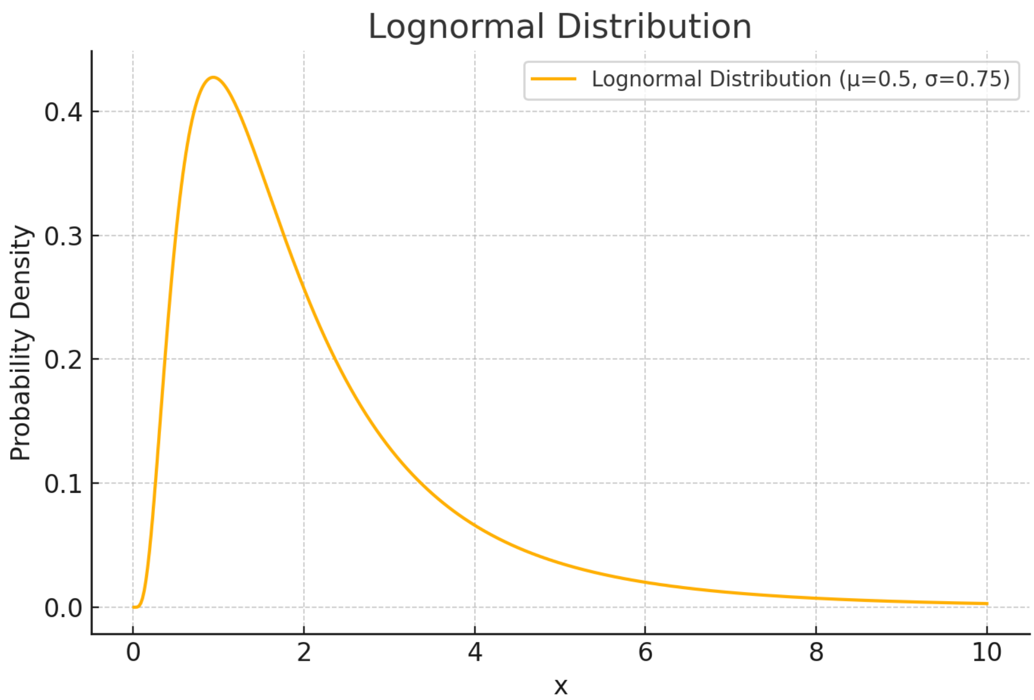
2. Variance: Var(X) = (exp(sigma^2) - 1) * exp(2 * mu + sigma^2)

3. Median: exp(mu)

4. Mode: exp(mu - sigma^2)

## Graph of Lognormal Distribution

The shape of the lognormal distribution depends on the values of mu and sigma. Below is a typical visualization of the distribution.




## Example Problem on Lognormal Distribution

Problem Statement: Suppose the natural logarithm of the lifetime of a certain type of electronic component follows a normal distribution with a mean of 2.5 and a standard deviation of 0.5. Find the probability that the lifetime of the component is less than 20 units.

Solution: Given mu = 2.5, sigma = 0.5, and x = 20

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First, transform the problem using the natural logarithm:

$P(X < 20) = P(\ln(X) < \ln(20))$

$P(\ln(X) < 2.9957)$

Standardizing using the Z-score formula:

Z = (2.9957 - mu) / sigma approximately  (2.9957 - 2.5) / 0.5 approximately  0.991

Using standard normal tables, $P(Z < 0.991)$ approximately  0.84

Interpretation: The probability that the lifetime of the component is less than 20 units is approximately 0.84 (or 84%).

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## Beta Distribution

The Beta distribution is a continuous probability distribution defined on the interval [0, 1]. It is characterized by two positive shape parameters alpha and beta, which determine the shape of the distribution. The Beta distribution is commonly used to model random variables that represent proportions or probabilities.

## Characteristics of Beta Distribution

1. Support: The distribution is defined for values in the interval [0, 1].

2. Parameters:

- alpha (shape parameter 1)

- beta (shape parameter 2)

3. Flexible Shape: Depending on the values of alpha and beta, the Beta distribution can take on a variety of shapes, including uniform, U-shaped, and bell-shaped.

## Probability Density Function (PDF)

The probability density function of the Beta distribution is defined as:

f(x; alpha, beta) = (x^(alpha - 1) * (1 - x)^(beta - 1)) / B(alpha, beta), for 0 < x < 1

Where B(alpha, beta) is the Beta function:

B(alpha, beta) = integral from 0 to 1 of [t^(alpha - 1) * (1 - t)^(beta - 1)] dt

## Key Formulas for Beta Distribution

1. Mean (Expected Value): E(X) = alpha / (alpha + beta)

2. Variance: Var(X) = (alpha * beta) / ((alpha + beta)^2 * (alpha + beta + 1))

3. Mode (when alpha, beta > 1): Mode(X) = (alpha - 1) / (alpha + beta - 2)

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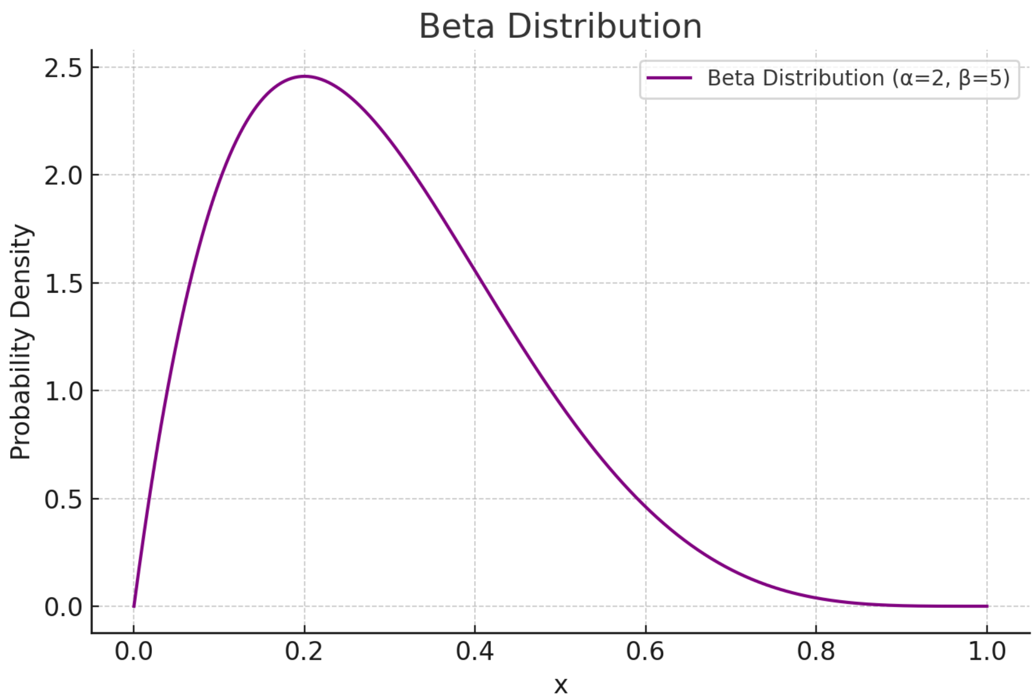
## Graph of Beta Distribution

The shape of the Beta distribution depends on the values of alpha and beta. Below is a typical visualization of the distribution.




## Example Problem on Beta Distribution

Problem Statement: Suppose a Beta distribution has parameters alpha = 2 and beta = 5. Find the mean and variance of the distribution.

Solution:

Given:

- alpha = 2

- beta = 5

Mean:

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E(X) = alpha / (alpha + beta) = 2 / (2 + 5) = 2 / 7 approximately  0.286

Variance:

Var(X) = (alpha * beta) / ((alpha + beta)^2 * (alpha + beta + 1))

= (2 * 5) / ((2 + 5)^2 * (2 + 5 + 1))

= 10 / (49 * 8) = 10 / 392 approximately  0.0255

Interpretation: The mean of the distribution is approximately 0.286, and the variance is approximately 0.0255.