Geometric Distribution

Definition

The Geometric Distribution is a discrete probability distribution that models the number of trials required to achieve the first success in a series of independent Bernoulli trials. Each trial has only two possible outcomes: success (with probability p) or failure (with probability 1 - p).

Characteristics

1. Each trial is independent.
2. The probability of success, p, remains the same for all trials.
3. The random variable X represents the number of trials until the first success.

Probability Mass Function (PMF)

The probability that the first success occurs on the k-th trial is given by the formula:

P(X = k) = (1 - p)^(k - 1) * p for k = 1, 2, 3, ...

Key Formulas

1. Mean (Expected Value): E(X) = 1/p
2. Variance: Var(X) = (1 - p) / p^2
3. Cumulative Distribution Function (CDF): P(X <= k) = 1 - (1 - p)^k

Example Problem

Problem Statement: A factory machine produces defective items with a probability of 0.1 (p = 0.1). What is the probability that the first defective item is found on the 4th item inspected?

Solution: Given p = 0.1 and k = 4

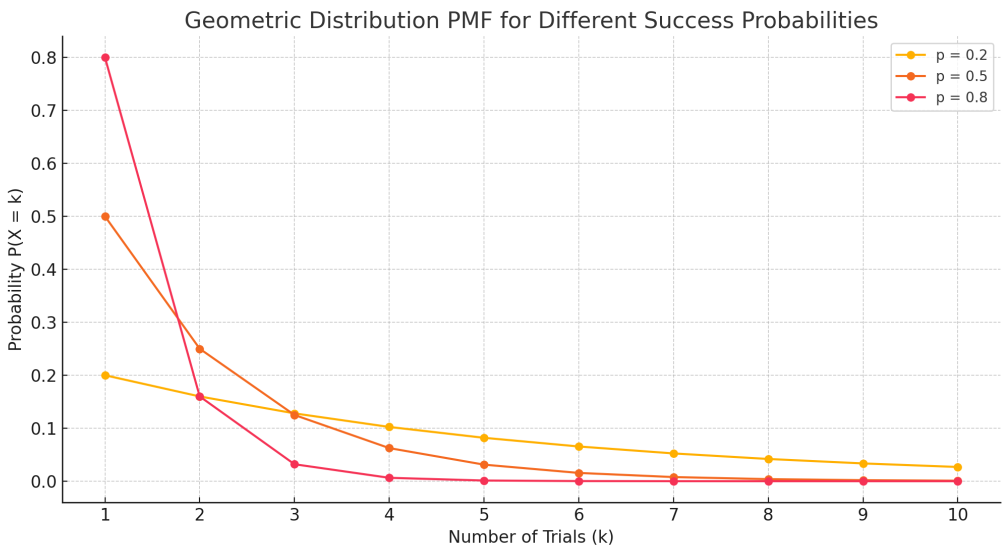
Geometric Distribution

Using the PMF formula: $P(X = 4) = (1 - p)^{(k - 1)} * p$

Substitute the values: $P(X = 4) = (0.9)^3 * 0.1 = 0.0729$

Interpretation: The probability of finding the first defective item on the 4th inspection is 0.0729 (or 7.29%).

Geometric Distribution



Negative Binomial Distribution

Definition

The Negative Binomial Distribution is a discrete probability distribution that models the number of trials required to achieve a fixed number of successes in a series of independent Bernoulli trials. Each trial has only two possible outcomes: success (with probability p) or failure (with probability 1 - p).

Characteristics

1. Each trial is independent.
2. The probability of success, p, remains the same for all trials.
3. The random variable X represents the number of trials required to achieve r successes.

Probability Mass Function (PMF)

The probability of achieving the r-th success on the x-th trial is given by the formula:

P(X = x) = binomial(x - 1, r - 1) * p^r * (1 - p)^(x - r) for x >= r.

Key Formulas

1. Mean (Expected Value): E(X) = r / p
2. Variance: Var(X) = r * (1 - p) / p^2
3. Cumulative Distribution Function (CDF): Calculated using the cumulative binomial sum.

Example Problem

Problem Statement: A basketball player has a 40% chance (p = 0.4) of making a shot. What is the probability that the player makes their 3rd successful shot on the 7th attempt?

Solution: Given p = 0.4, r = 3, and x = 7

Negative Binomial Distribution

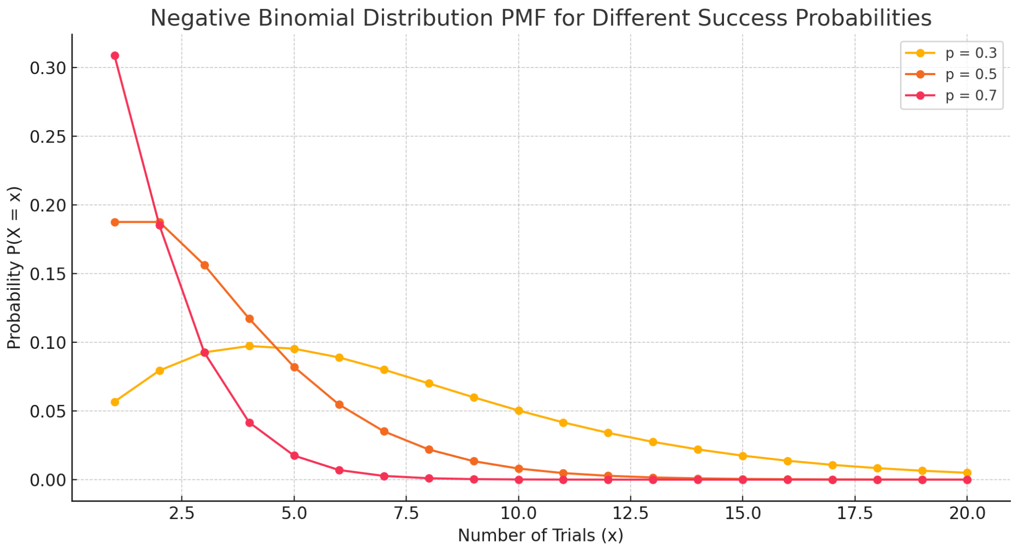
Using the PMF formula: P(X = 7) = binomial(6, 2) * (0.4)^3 * (0.6)^4

Compute: P(X = 7) = 15 * 0.064 * 0.1296

Result: P(X = 7) approximately 0.124

Interpretation: The probability of making the 3rd successful shot on the 7th attempt is approximately 0.124 (or 12.4%).

Negative Binomial Distribution



Negative Binomial Distribution

Hypergeometric Distribution

The Hypergeometric Distribution is a discrete probability distribution that describes the probability of obtaining a specific number of successes in a fixed number of draws, from a finite population without replacement. Unlike the Binomial distribution, where each trial is independent, in the Hypergeometric distribution, the draws are dependent.

Characteristics of Hypergeometric Distribution

1. Finite Population: Total population size N

2. Successes in Population: Number of successes in the population K

3. Sample Size: Number of draws from the population n

4. Random Variable: The number of observed successes X in the sample

Probability Mass Function (PMF) of Hypergeometric Distribution

The probability mass function of the Hypergeometric distribution is defined as:

P(X = k) = (binomial(K, k) * binomial(N - K, n - k)) / binomial(N, n)

Where:

- N = Population size

- K = Number of successes in the population

- n = Number of draws

- k = Number of observed successes

- binomial(a, b) = Binomial coefficient (a choose b)

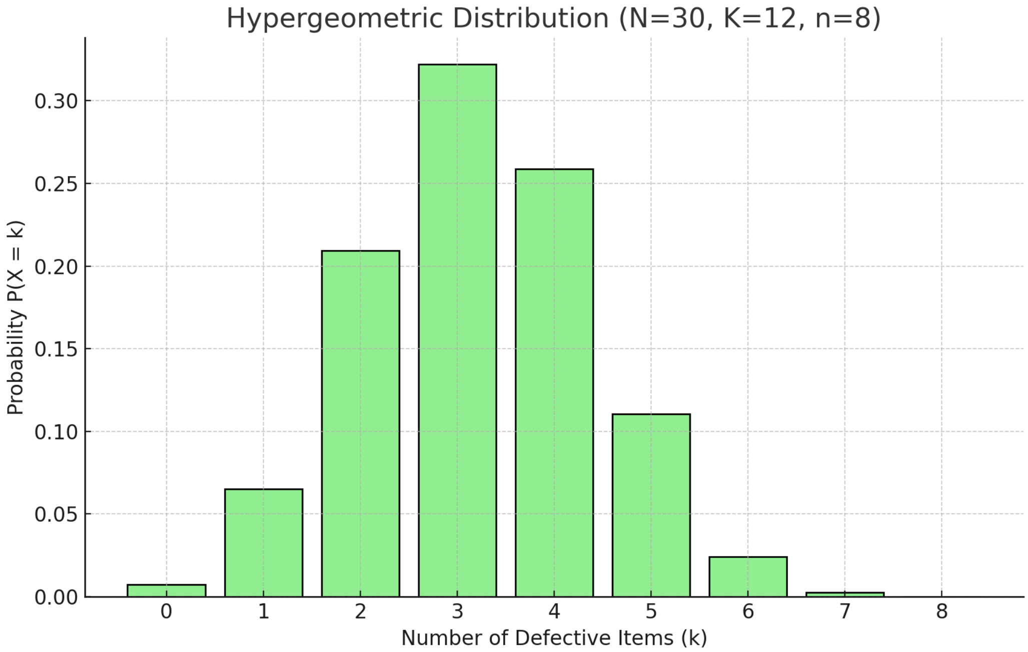
Key Formulas for Hypergeometric Distribution

1. Mean (Expected Value): E(X) = n * (K / N)

2. Variance: Var(X) = n * (K / N) * ((N - K) / N) * ((N - n) / (N - 1))

Graph of Hypergeometric Distribution

The shape of the Hypergeometric distribution varies depending on the parameters N, K, and n. Below is a typical visualization of the distribution.



Example Problem on Hypergeometric Distribution

Problem Statement: A batch of 30 items contains 12 defective items. A quality control inspector randomly selects 8 items from the batch. What is the probability that exactly 3 of the selected items are defective?

Solution: Given N = 30, K = 12, n = 8, and k = 3

Using the PMF formula: P(X = 3) = (binomial(12, 3) * binomial(18, 5)) / binomial(30, 8)

Compute: P(X = 3) = 220 * 8568 / 5852925

Result: P(X = 3) approximately 0.322

Interpretation: The probability of selecting exactly 3 defective items in a sample of 8 is approximately 0.322

(or 32.2%).