

CFG

Context Free Grammar

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CFG stands for context-free grammar. It is a formal grammar which is used to generate all possible patterns of strings in a given FL.

Context free grammar G can be defined by 4 tuple

$$G = \{V, T, P, S\}$$

→ G is Grammar which consists of a set of production rule. It is used to generate the string of a language.

→ T is the final set of Terminal symbol. It is denoted by lower case letters.

→ V is the final set of non-Terminal symbol. It is represented by Capital letters.

→ P is set of production rules which is used for replacing non terminal symbols on left side of production in a string with other terminals of non-terminal symbols (on right side of production).

→ S is the start symbol used to derive the string.

Ex 1

Construct a CFG for the language having any no. of a 's over the set $\Sigma = \{a\}$

Solⁿ

$$\Sigma = \{a\}$$

$$L = \{\epsilon, a, aa, aaa, \dots\}$$

$$R.E. = a^*$$

~~suppose~~

Production Rule :

$$S \rightarrow as - ①$$

$$S \rightarrow \epsilon - ②$$

Suppose I need to derive a i/p string = 'aaaaaa'

⇒ ~~as~~ S (According to the rule we have to start with S , start symbol)

S is replaced with $as \Rightarrow aas$

$$\Rightarrow aas \quad | \quad S \rightarrow as$$

$$\Rightarrow aaaS \quad | \quad S \rightarrow$$

$$\Rightarrow aaaaS$$

$$\Rightarrow aaaaaS$$

$$\Rightarrow aaaaaaS$$

$$\Rightarrow aaaaaa\epsilon \quad | \quad S \rightarrow \epsilon$$

Ex 2. Construct a CFG for language
 $L = \{ WCWR \mid \text{where } W \in (a,b)^*$

Solⁿ

$WCWR$
 ↓ ↓
 string Reverse string

$L = \{ aacaa, bcb, abcb, abbcbb, \dots \}$
 (Note: $abcb$ and $abbcbb$ are shown with arrows indicating they are reverses of each other.)

The grammar could be

$S \rightarrow asa - (1)$

$S \rightarrow bsb - (2)$

$S \rightarrow \epsilon - (3)$

Let assume
 i/p =

"abbcbb"

$S \rightarrow asa$
 $S \rightarrow a b s b a$
 $S \rightarrow a b b s b b a$
 $S \rightarrow abbcbb a$

Ex 3. Construct a CFG for the language

$L = a^n b^{2n}$ where $n \geq 1$

Solⁿ

$L = \{ a^1 b^2, a^2 b^4, a^3 b^6, \dots \}$
 (Note: $a^1 b^2$, $a^2 b^4$, and $a^3 b^6$ are shown below the corresponding terms in the set.)

The grammar could be

$S \rightarrow asbb$

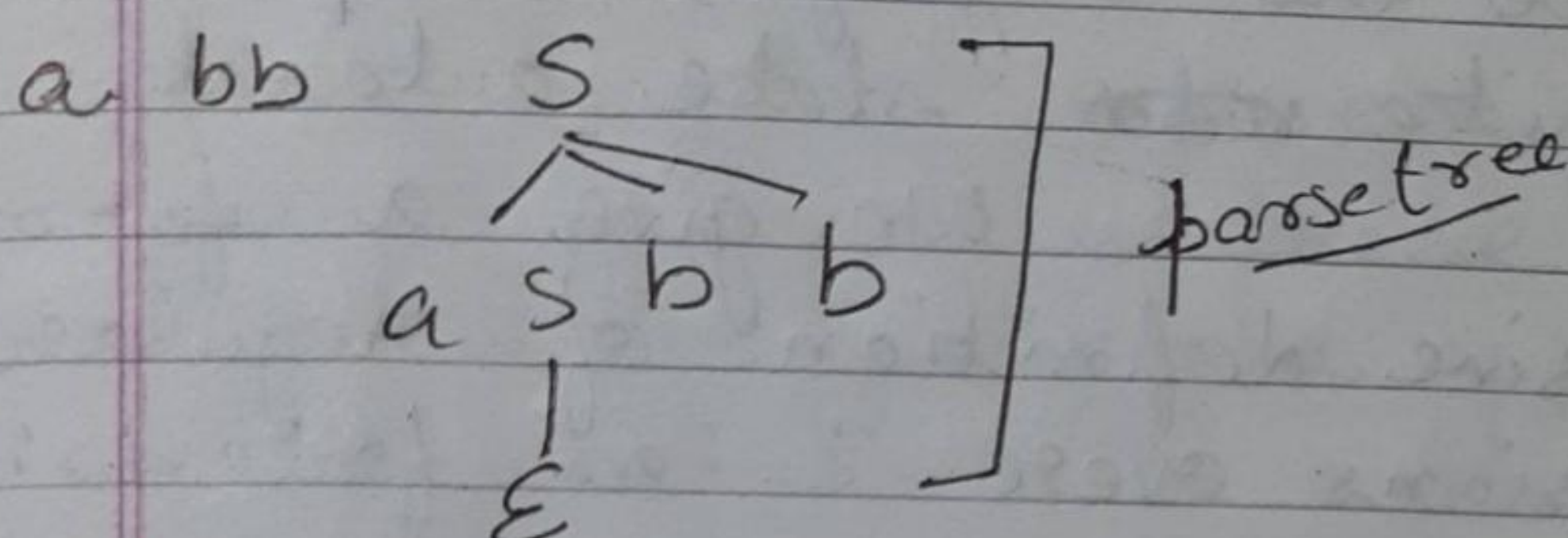
$S \rightarrow abb$

i/p

aaabbbbbb

$S \rightarrow asbb$
 $\Rightarrow asbbbb (s \rightarrow asbb)$
 ~~$\Rightarrow aasbbbbb$~~
 $\Rightarrow aaaa bbbbbb (s \rightarrow abb)$

parse tree



Ex. Find a grammar generating
 $L = \{ a^n b^n c^i \mid n \geq 1, i \geq 0 \}$

Soln $S \rightarrow A$, $A \rightarrow ab$, $A \rightarrow aAb$, $S \rightarrow Sc$

Ex. Let $G = (\{S, A_1\}, \{0, 1, 2\}, P, S)$
 where P consists of $S \rightarrow 0SA_12$,
 $S \rightarrow 012$, $2A_1 \rightarrow A_12$, $1A_1 \rightarrow 11$

show that
 $L(G) = \{ 0^n 1^n 2^n \mid n \geq 1 \}$