Pumping Lemma for regular ~~__~~ Grammers

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Theorem : Let L be a R.L. then there exists a constant 'p' such that every strong w in L -

$$|w| \geq p$$

We can treat w into three strings, $w = xyz$ such that

$$|y| > 0$$

$$|xy| \leq p$$

For all $k \geq 0$, the string xy^kz is also in L.

Applications of Pumping Lemma

Pumping lemma is to be applied to show that certain languages are not regular. It should never be used to show a language is regular.

- ~~__~~ If L is regular, it satisfies ~~pumptin~~ pumping lemma

- If L does not satisfy pumping lemma it is non regular.

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⇛ To prove that a language is not Regular using pumping lemma we have to follow the steps.

→ At first, we have to assume that L is reguar

→ So, the pumping lemma should hold for L.

→ It has to have a pumping length (say P)

→ All strings longer than P can be pumped $|W| \geq P$

→ Now find a string 'w' in L such that $|W| \geq P$

→ Divide w into xyz

→ Show that $xy^kz \notin L$ for some k

→ Then consider all ways that w can be divided into xyz.

→ Show that none of these can satisfy all the 3 pumping conditions at the same time.

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→ W cannot be pummed ⇒ contradiction

Example 1 : Prove that
$L = \{ a^i b^i \mid i \geq 0 \}$ is not regular

Solution

1. We assume that L is regular and n is the no. of states.

2. Let $w = a^n b^n$ Thus $|w| = 2n \geq n$

3. By pumping lemma, let $w = xyz$ where $|xy| \leq n$

4. Let $x = a^p$
$y = a^q$
$z = a^r b^n$
where $p + q + r = n$ &
$p \neq 0$ $q \neq 0$, $r \neq 0$

* [we have to show $xy^n z \notin L$]

5. Let $k = 2$
Then $xy^2 z = a^p a^{2q} a^r b^n$

6. No. of a's $= (p + 2q + r)$
$= (p + q + r) + q = n + q$

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7. Hence $xy^2z = a^{n+q}b^n$
since $q \neq 0$ xy^2z is not of the for $a^n b^n$

8. Thus xy^2z is not in L
Hence L is not regular

$a^n b^n \mid n \geq 1\}$, $n = 1, 2 \ldots$

$w = \underline{a}\,\underline{a\ b}\,\underline{b}$
$w = x\ y\ z$

$|y| \geq 1$ n is the string length
$|xy| \leq n$

$w = xyz$
$w = xy^iz$ $i \geq 0$
$i = 2$
$w = xy^2yz$ $i = 1$

$\therefore$ w

$i = 2$ $w = x.y^2z$
$= a(ab)^2 b$
$= a\,\underline{abab}\,b$

So the language is not regular. Because we need a pattern of aabb i.e. all a's then all b's.

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