Probability introduction

Randomness

Probability motivation

Outcomes

Sample spaces

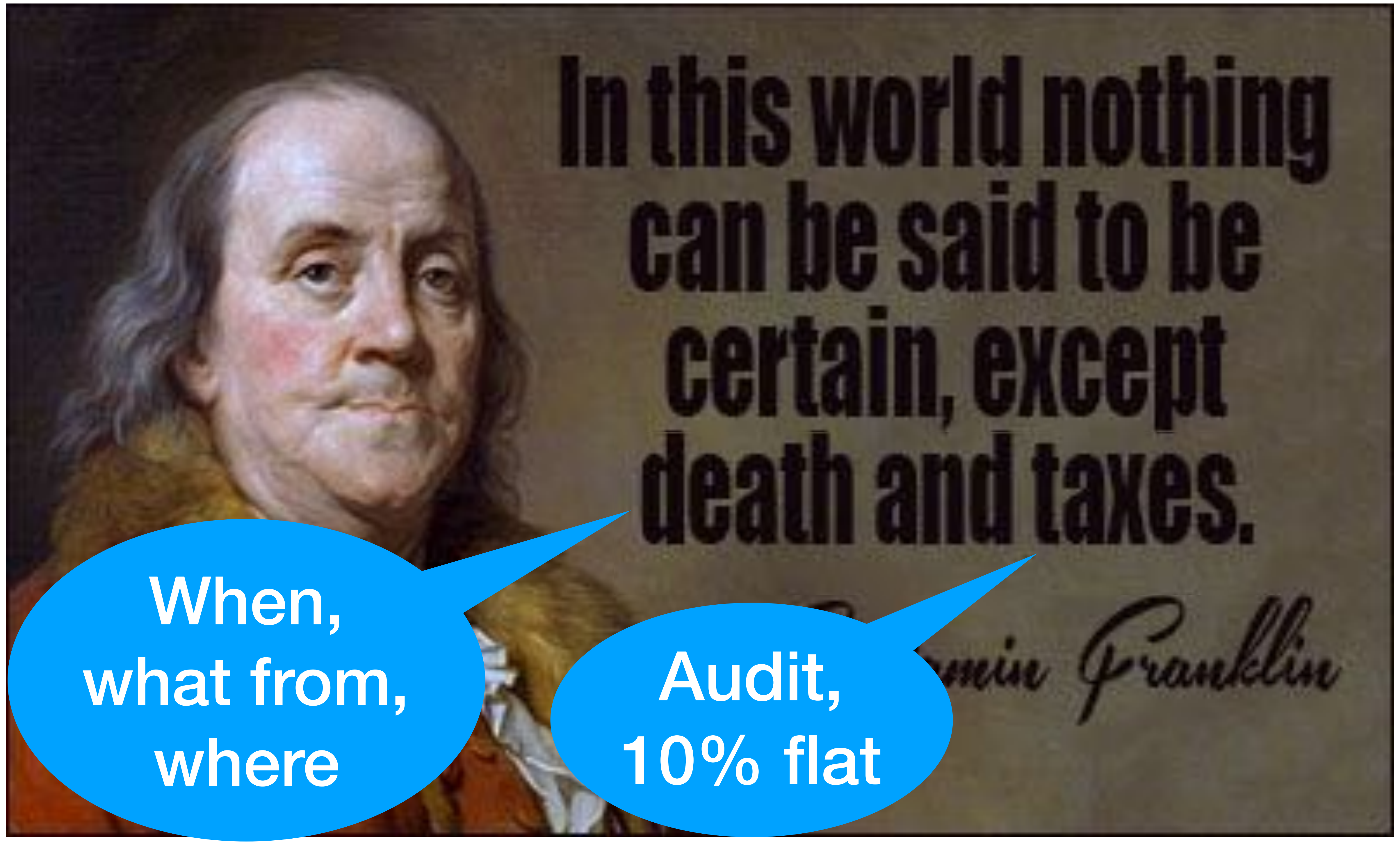
Distributions

Why Probability?

Some things in life are certain

Most are a less predictable

Physicians	illness, medication
Farmers	rain, diet trends
Investors	stock price, economy
Advertisers	views, competition
Consumers	availability, sale
Students	food line, grade, parents, job, date, game



When, what from, where

Audit, 10% flat

Randomness is everyone's business

Random Phenomena

Give up?

Reason intelligently?

Learn	Range	Average	Variability
Infer	Structure	Change	Relations
Predict	Future	Likelihood	Guarantees
Benefit	Compete	Plan	Build

Coming to Terms

As with sets

Need terminology

Discuss

Concisely

Precisely

Effectively

Process of generating and observing data

Individual and collection of observations

Meaning of probability

Several approaches

Intuitive

Axioms

Data

Experiments

Probability developed in part to aid science

Process

- Generate random data
- Observe outcome

Experiment

Unified approach

Applies generally

Biology Engineering Business

… Sociology

Understand Analyze generalize

Our experiments

Start simple

Get very complex

Outcomes and Sample Space

Potential experiment results are called (possible) outcomes

Set of possible outcomes is the sample space, denoted Ω

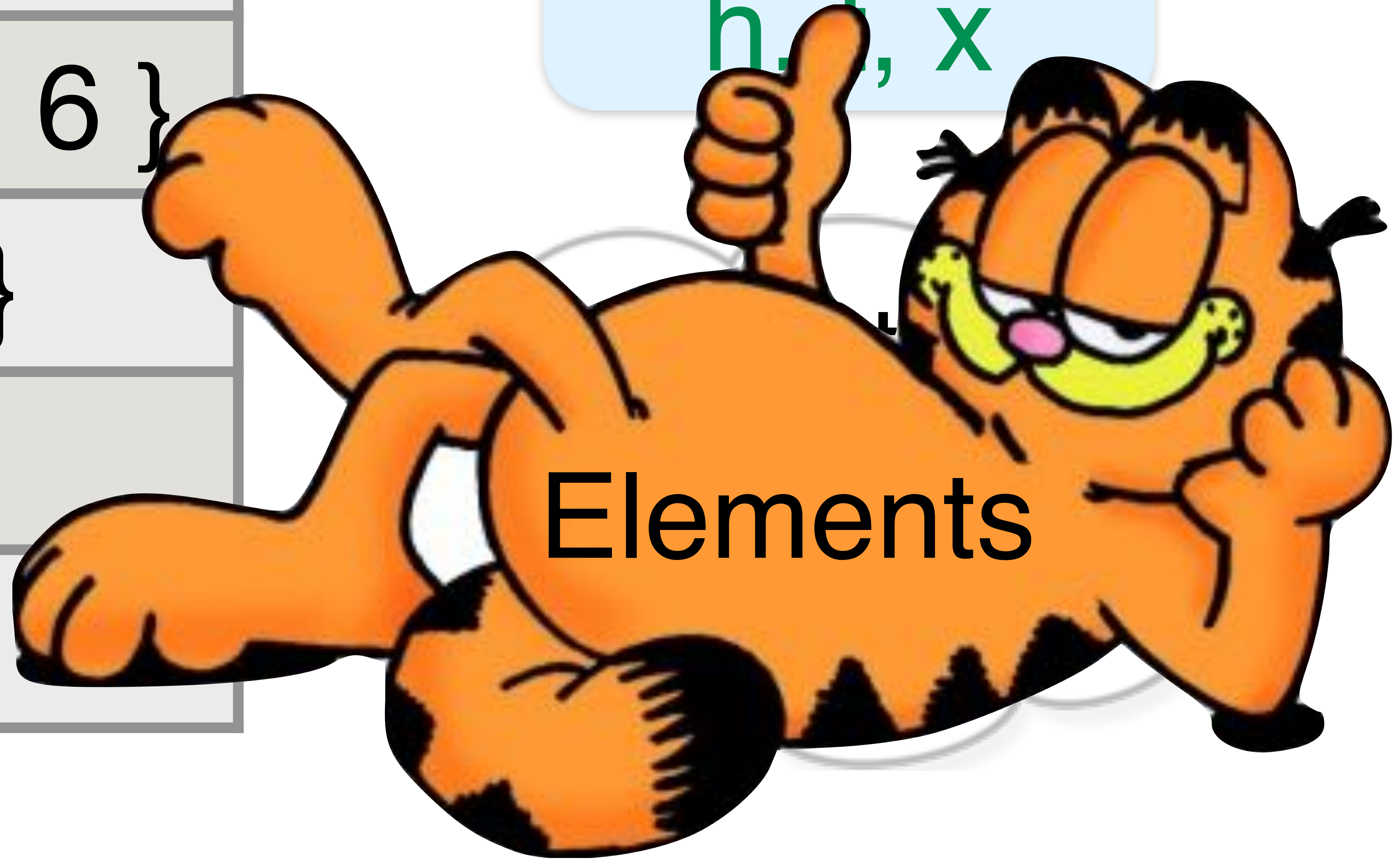
Notetation

Some use S or U

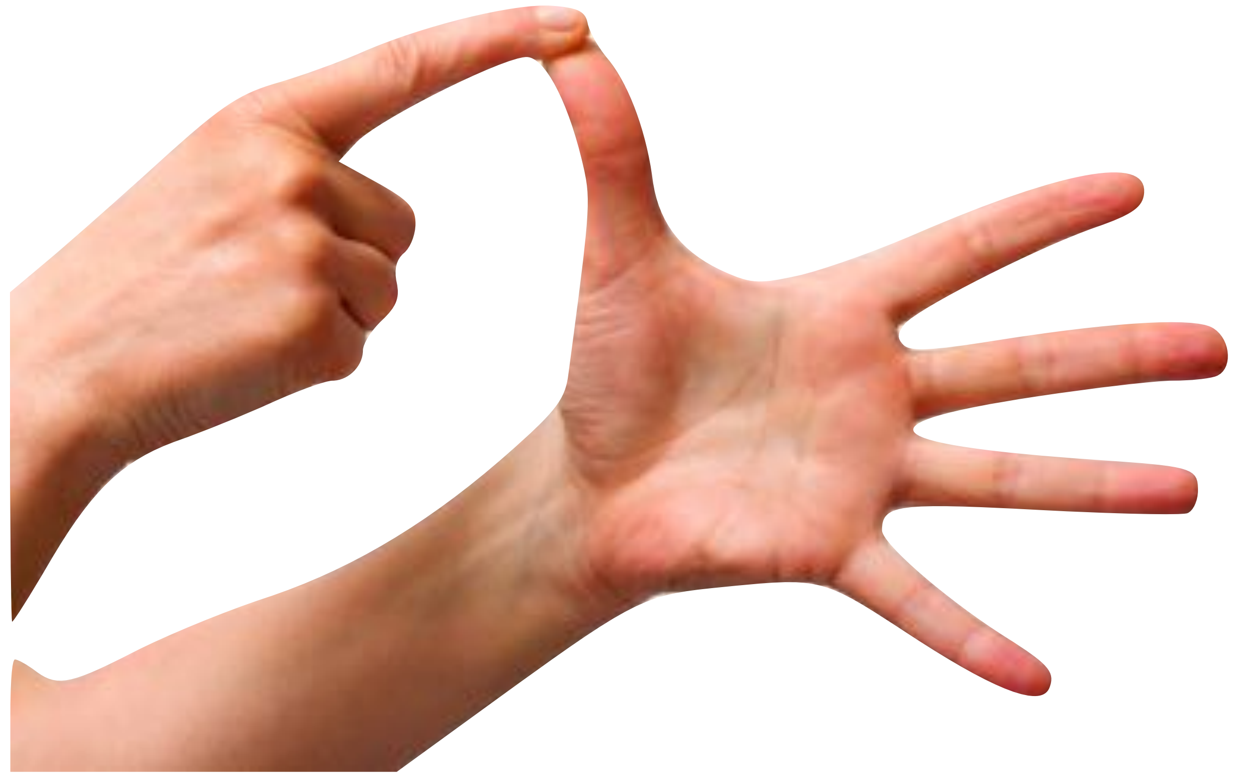
Experiment	Ω
Coin	{ h, t }
Die	{ 1, 2, ..., 6 }
Gender	{ m, f }
Age	$\mathbb{N}$
Temperature	$\mathbb{R}$

typically lower-case

h, t, x



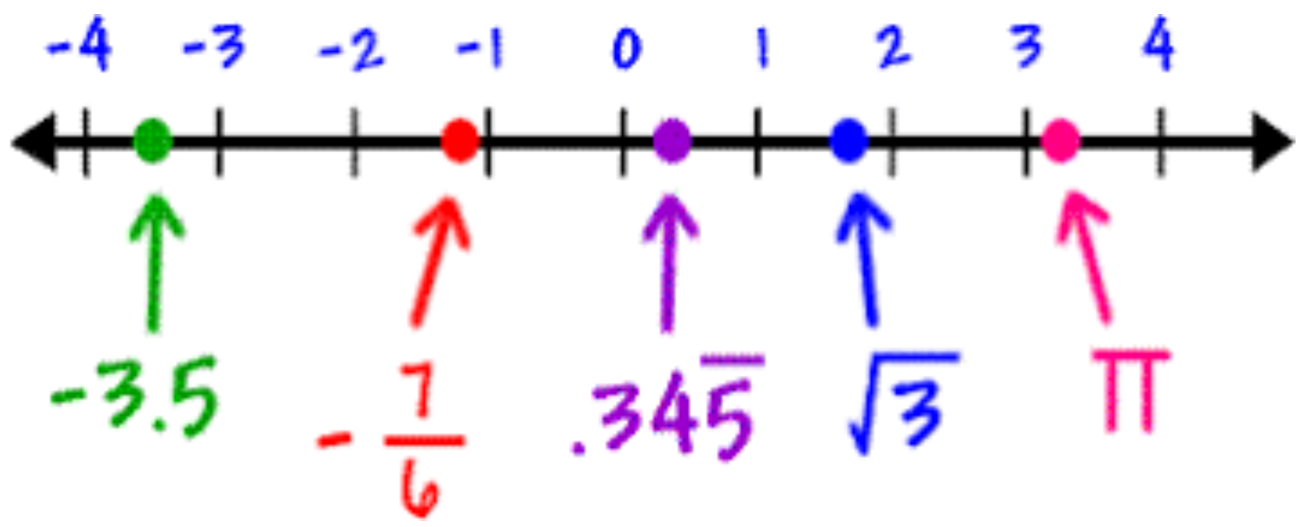
Two Sample-Space Types



Finite or countably infinite sample space is discrete

{ h, t }   { 1, 2, …, 6 } $\mathbb{N}$ $\mathbb{Z}$ { words }   { cites }   { people }

Uncountably infinite sample space - continuous



$\mathbb{R}$ [0,1]   { temperatures }   { salaries }   { prices }

upgraded

Discrete spaces   Easier to understand, visualize, analyze

Important   First

Next few topics: Discrete

Continuous   Important   Conceptually harder   Later

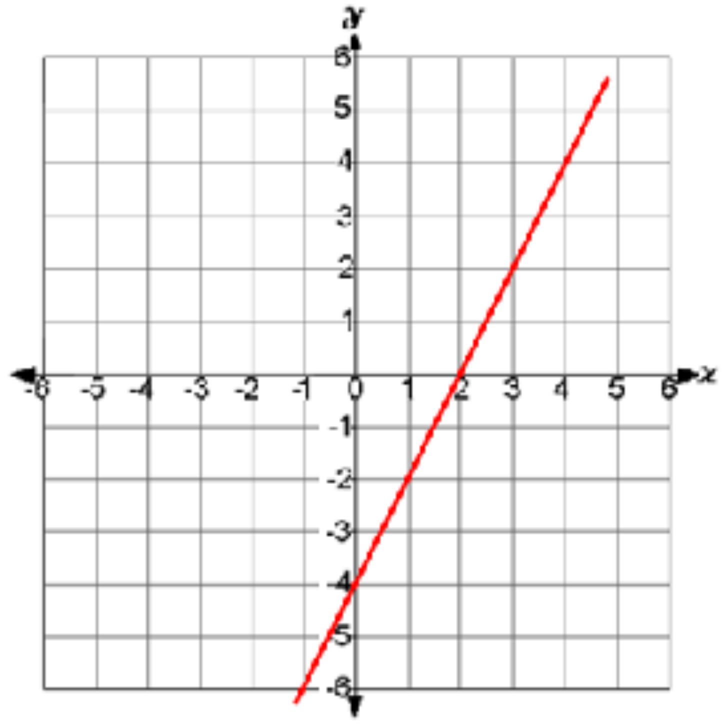
Random Outcomes

Algebra

Unknown value denoted by x

lower case

$2x - 4 = 0$



Solve

Before

After

$x \in \mathbb{R}$

$x = 2$

Solution

Notetation

If needed use y, z, …

UPPER CASE

Probability

Random outcome denoted by X

X - coin flip outcome

Experiment

Before

After

$X \in \Omega$

get h

$X = h$

get t

$X = t$

Outcome seen called observation

Notetation

If needed use Y, Z, …

Probability of Outcome

The probability, or likelihood, of an outcome $x \in \Omega$, denoted $P(x)$, or $P(X=x)$, is the fraction of times x will occur when experiment is repeated many times

Fair coin

As # experiments $\to \infty$, fraction of heads (or tails) $\to \frac{1}{2}$

heads has probability ½	$P(h) = \frac{1}{2}$	$P(X=h) = \frac{1}{2}$
tails has probability ½	$P(t) = \frac{1}{2}$	$P(X=t) = \frac{1}{2}$

Fair die

As # experiments $\to \infty$, fraction of 1's (or 2,…,6) $\to \frac{1}{6}$

1 has probability ⅙	$P(1) = \frac{1}{6}$	$P(X=1) = \frac{1}{6}$

$P(x)$ $P(X=x)$ probability of x fraction of times x will occur

Probability Distribution Function

$P(x)$ is the fraction of times outcome x occurs

$P(h) = ½$

$P(1) = ⅙$

Viewed over the whole sample space, a pattern emerges

Coin $P(h) = ½$ $P(t) = ½$

Die $P(1) = ⅙$ … $P(6) = ⅙$

Rain $P(\text{rain}) = 10\%$ $P(\text{no rain}) = 90\%$

Sample space Ω and distribution P define the whole probability space

P maps outcomes in Ω to nonnegative values that sum to 1

$P: \Omega \rightarrow \mathbb{R}$

$P(x) \geq 0$

$\sum_{x \in \Omega} P(x) = 1$

Probability distribution function (PDF)

distribution

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