Conditional Probability

- Probability with additional information
- Definition
- Examples

Why Condition

Often have partial information about the world

Modifies event probabilities

- Unemployment numbers - stock prices
- LeBron James injured - Cavaliers game result
- Sunny weekend - beach traffic

Can help

- Improve estimates
- Determine original unconditional probabilities

Back to Basics

Empirical frequency interpretation of probability

The probability P(E) of an event E is the fraction of experiments where E occurs when the number of experiments grows to infinity

To estimate P(E) repeat the experiment many times, calculate the fraction of experiments where E occurs

Fair Die

$$P(2) = \frac{2}{12} = \frac{1}{6}$$

Estimate

$$\underbrace{\boxed{2}\ 1\ 3\ 6\ 4\ \boxed{2}\ 5\ 4\ 3\ 6\ 5\ 1}_{12}$$

Conditional Probability

Let E and F be events. The conditional probability P(F | E) of F given E is the fraction of times F occurs in experiments where E occurs

To estimate P(F | E) take many samples, consider only experiments where E occurs, and calculate the fraction therein where F occurs too

$$P(2 \mid \text{Even}) = \frac{2}{6} = \frac{1}{3}$$

$\text{Even} = \{2, 4, 6\}$

$\boxed{2}$ 1 3 6 4 $\boxed{2}$ 5 4 3 6 5 1

Die

$P(\{2\}) = P(2) = ⅙$

P(2) = ?

$P(2 \mid \text{Odd}) = P(2 \mid \{1,3,5\}) = \frac{0}{6} = 0$

2 1 3 6 4 2 5 4 3 6 5 1

$P(\leq 2) = P(\{1,2\}) = ⅓$

P(≤ 2) = ?

$P(\leq 2 \mid \geq 2) = P(\{1,2\} \mid \{2,3,4,5,6\}) = \frac{2}{10} = \frac{1}{5}$

$\boxed{2}$ 1 3 6 4 $\boxed{2}$ 5 4 3 6 5 1

General Events - Uniform Spaces

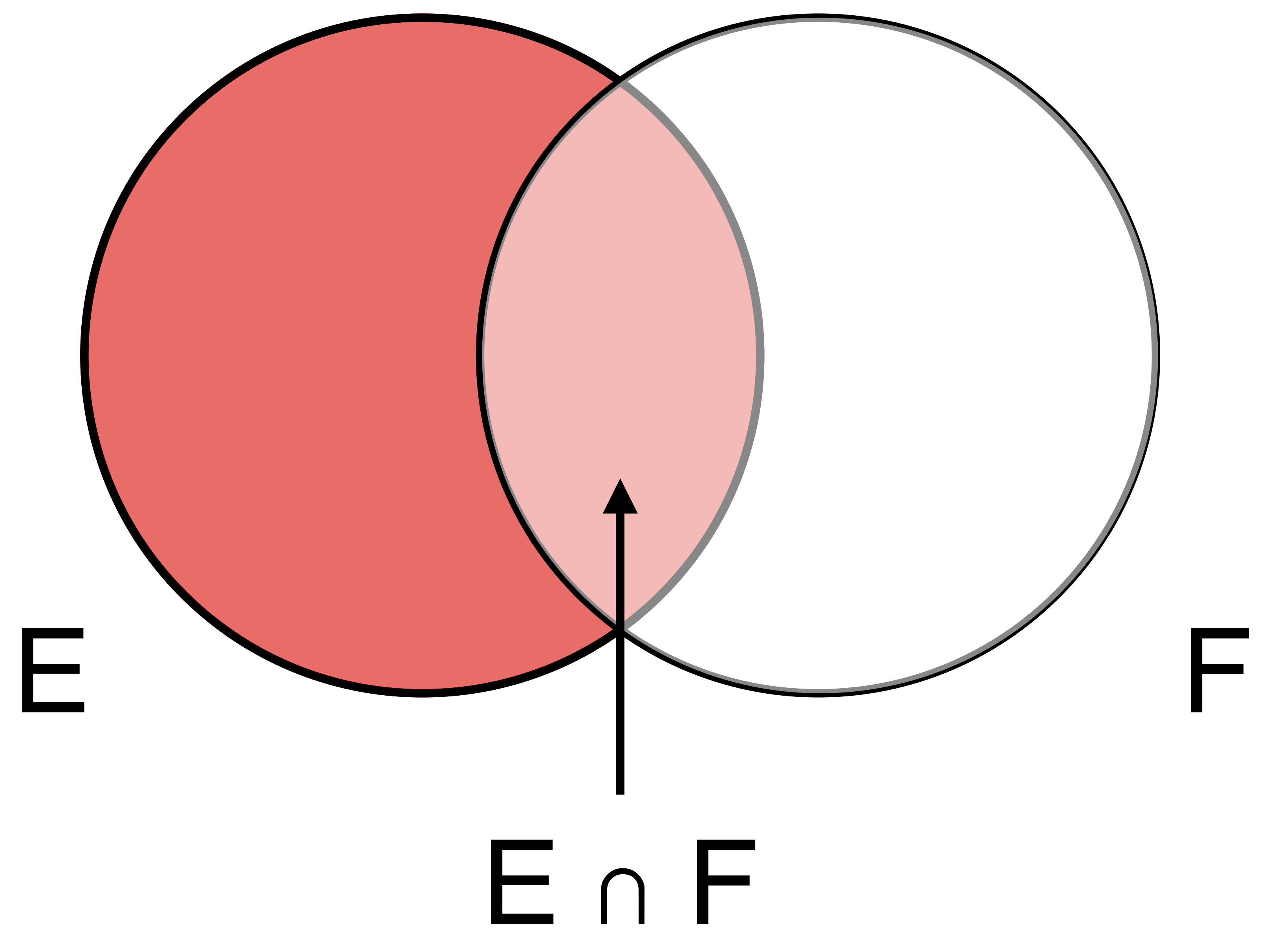
$P(F \mid E) = P(\, X \in F \mid X \in E \,)$

$= P(\, X \in E \text{ and } X \in F \mid X \in E \,)$

$= P(\, X \in E \cap F \mid X \in E \,)$

$= P(\, E \cap F \mid E \,)$

$$= \frac{|E \cap F|}{|E|}$$



Fair Die Again

$$P(\text{Prime} \mid \text{Odd}) = P(\{2,3,5\} \mid \{1,3,5\})$$

$$= \frac{|\{2,3,5\} \cap \{1,3,5\}|}{|\{1,3,5\}|} = \frac{|\{3,5\}|}{|\{1,3,5\}|} = \frac{2}{3}$$

$$P(\{4\} \mid \text{Prime}) = P(\{4\} \mid \{2,3,5\})$$

$$= \frac{|\{4\} \cap \{2,3,5\}|}{|\{2,3,5\}|} = \frac{|\emptyset|}{|\{2,3,5\}|} = 0$$

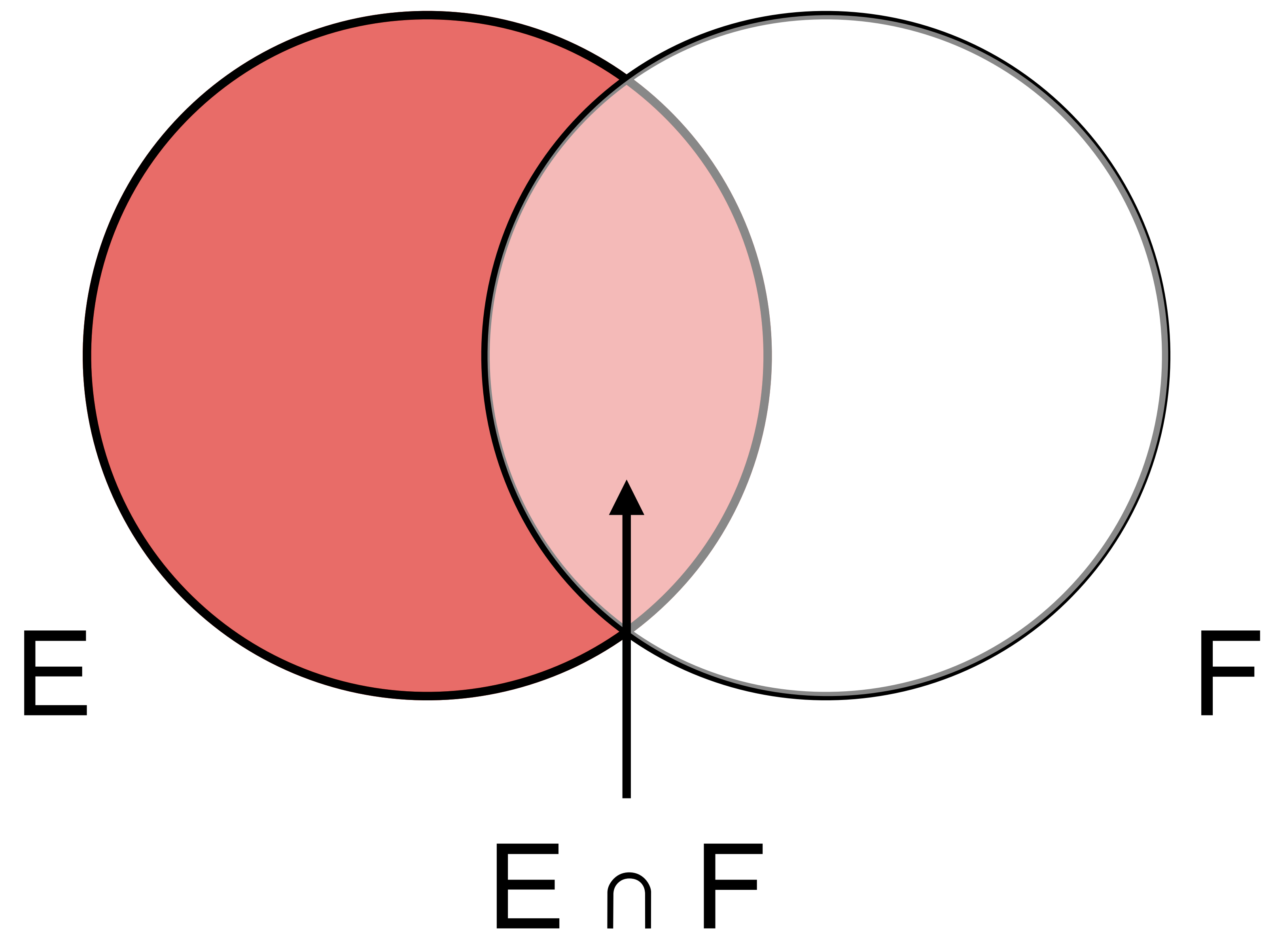
General Spaces

$$P(\,F \mid E\,) = P(\,X{\in}F \mid X{\in}E\,)$$

$$= P\,[\,X{\in}E \cap X{\in}F \;\mid\; X{\in}E\,]$$

$$= P\,[\,X \in E{\cap}F \;\mid\; X{\in}E\,]$$

$$= \frac{P(E \cap F)}{P(E)}$$



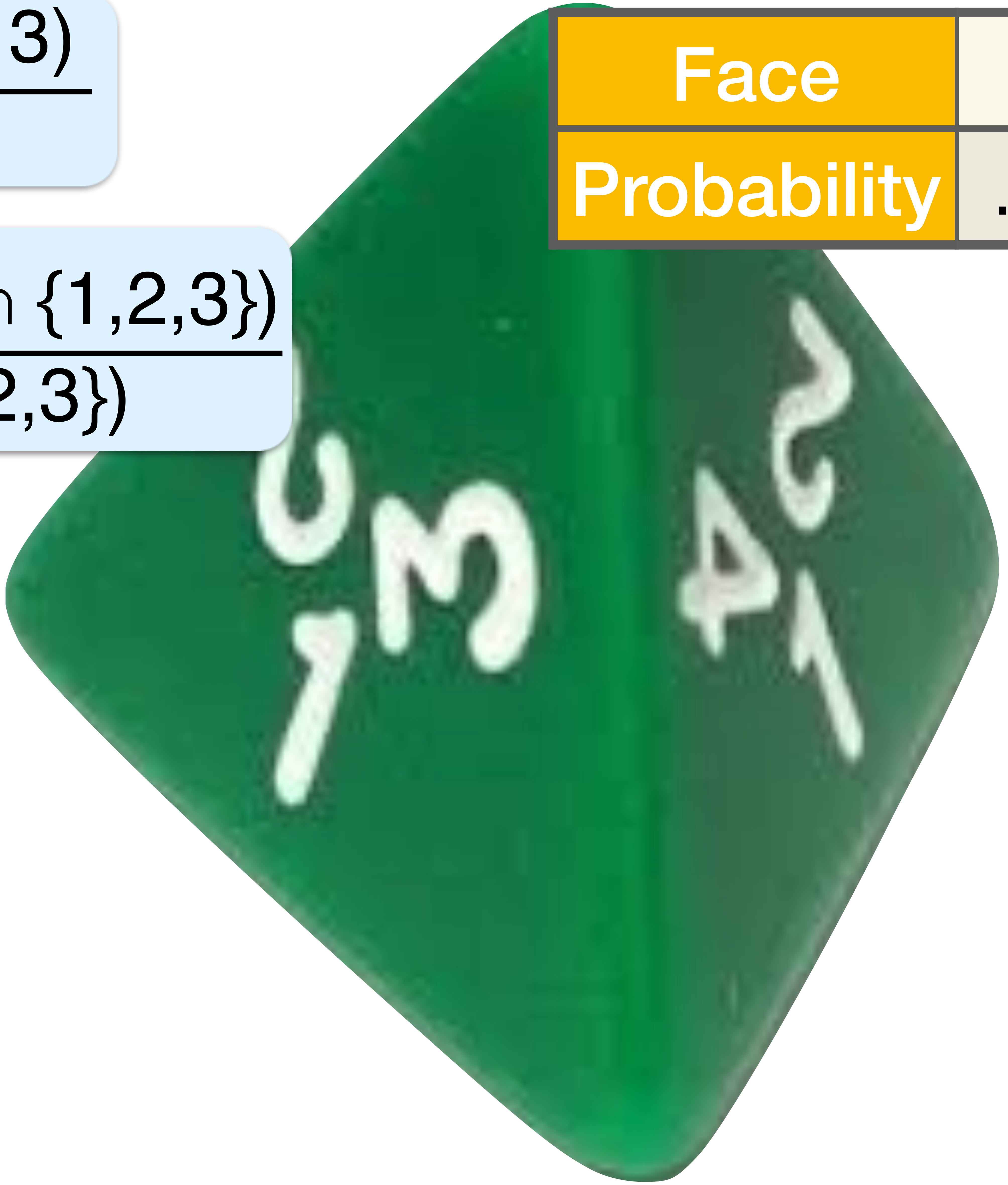
4-Sided Die

$$P(\geq 2 \mid \leq 3) = \frac{P(\geq 2 \cap \leq 3)}{P(\leq 3)}$$

$$= \frac{P(\{2,3,4\} \cap \{1,2,3\})}{P(\{1,2,3\})}$$

$$= \frac{P(\{2,3\})}{P(\{1,2,3\})}$$

$$= \frac{.5}{.6} = \frac{5}{6}$$



Face	1	2	3	4
Probability	.1	.2	.3	.4


KIDS
ARE PEOPLE TOO

Conditionals are Probabilities Too

$P(x \mid a) \geq 0$

$\sum_{x \in \Omega} P(x \mid a) = 1$

$P(x \mid a) \geq 0$

Conditional Probability

Probability with additional information

Definition

Examples

Independence

Independence