

Distribution Families

Bernoulli



Binomial



ChiSq

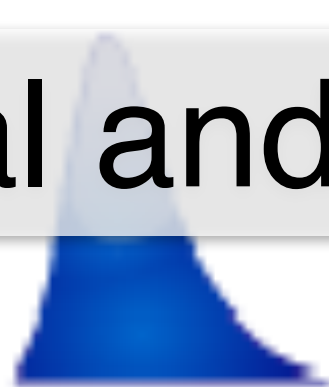


Many distribution families

Cumul



CumulD



Discrete



DoubleTriang



DUniform



Erf



Erlang



Most natural and important

ExponAlt



ExtValue



ExtValueAlt



ExtValueMin



ExtValueMinAlt



F



Gamma



Theoretical and Practical significance

General



Geomet



Histogramm



HyperGeo



IntUniform



InvGauss



InvGaussAlt



JohnsonSB



JohnsonSU



Laplace



LaplaceAlt



Levy



LevyAlt



Logistic



Distributions

Discrete

Bernoulli

Binomial

Poisson

Geometric

Continuous

Uniform

Exponential

Normal

Discuss

Motivation

Applications

Formulate

Visualize

Examples

Properties

μ

V

σ

Python

Plot and experiment

Show Distribution

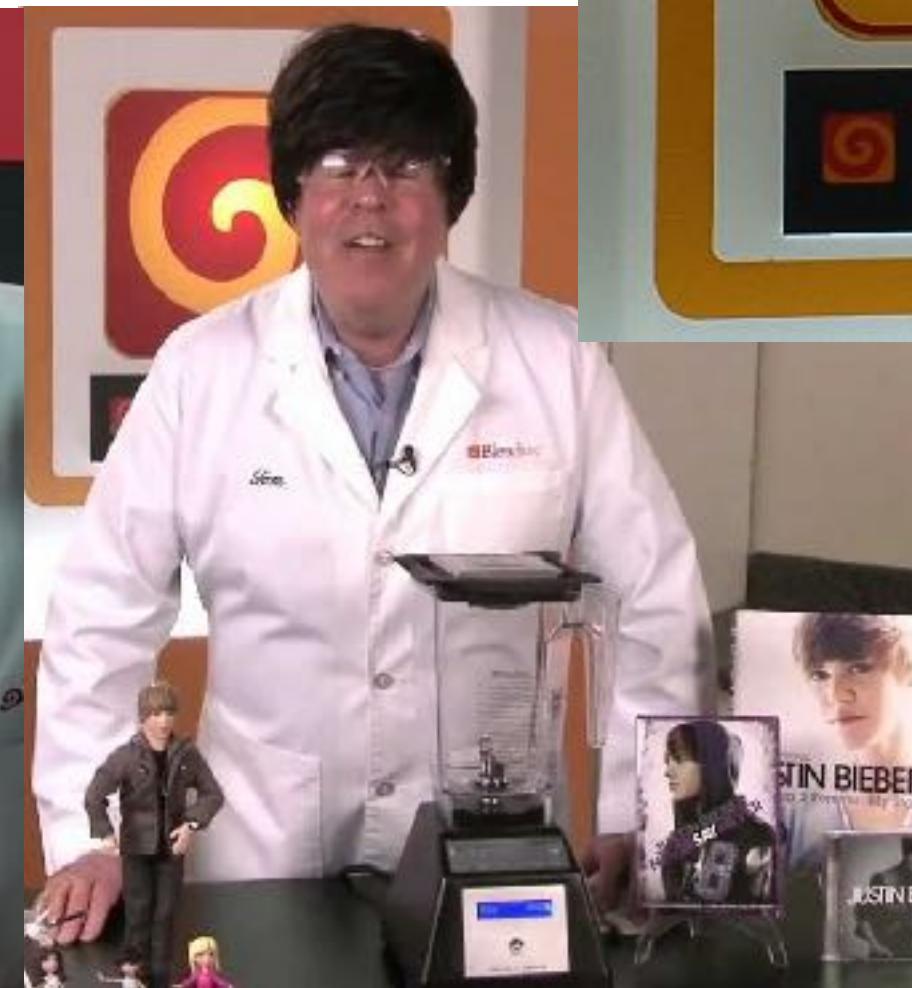
Nonnegative

Sum to 1

Blendtec

Tom Dickson

The logo for the show "Will It Blend?". It features a red and white swirl icon to the left of the text "Will It" in a bold, yellow, sans-serif font. Below "Will It" is the word "BLEND?" in a larger, bold, yellow, sans-serif font. The entire logo has a white outline and a slight drop shadow.



The logo for the show "Sigma Will It Add?". It features a large red Greek letter sigma (Σ) to the left of the text "Will It" in a bold, yellow, sans-serif font. Below "Will It" is the word "ADD?" in a larger, bold, yellow, sans-serif font. The entire logo has a white outline and a slight drop shadow.

Bernoulli Distribution

Simplest non-constant distribution

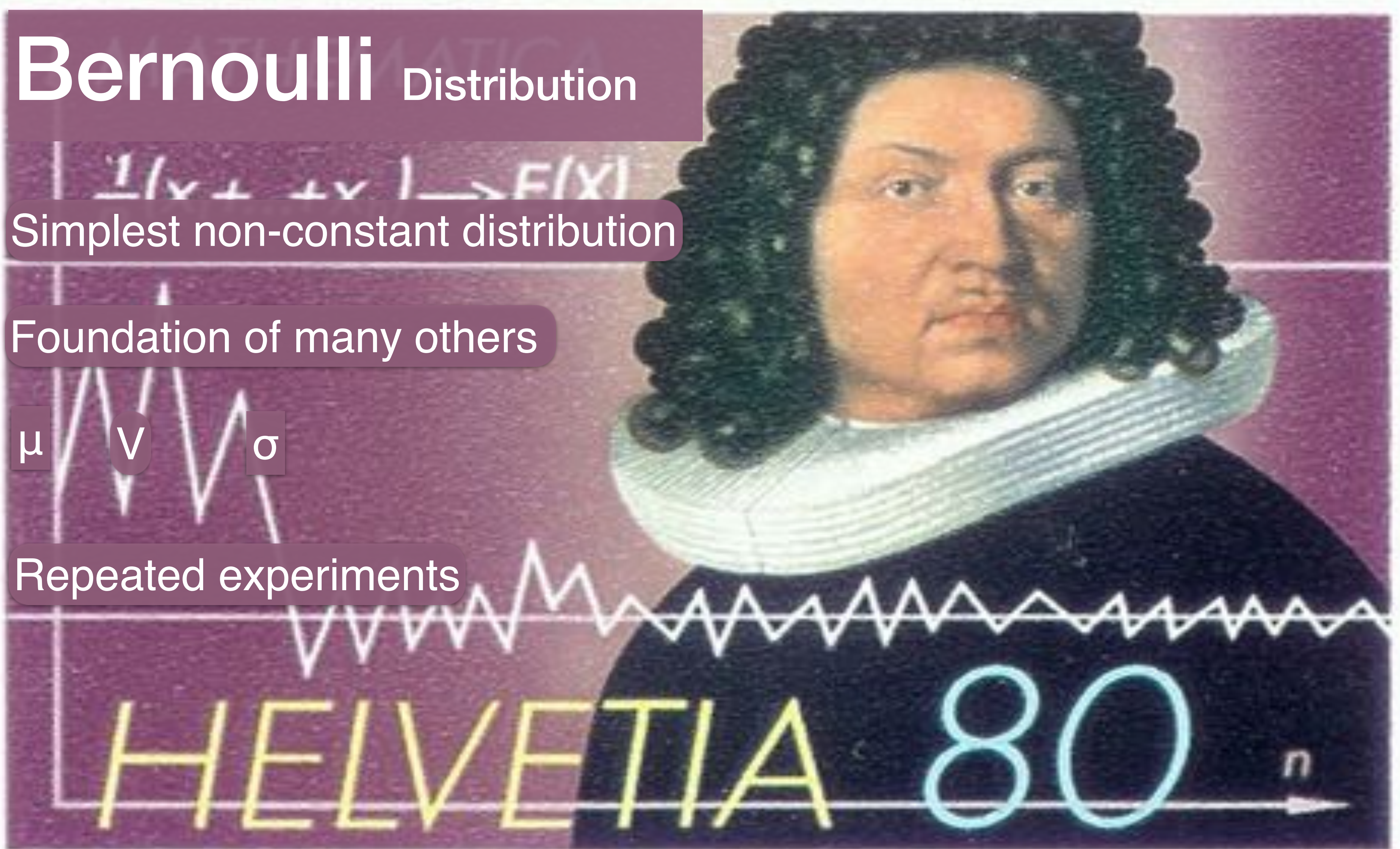
Foundation of many others

μ

V

σ

Repeated experiments



Jacob Bernoulli, 1655-1705

Theology → mathematics

Calculus Integrals

“Euler” number

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$e \rightarrow b$

Ars Conjectandi

First law of large numbers

Mentored brother Johann Medicine → Math Dynasty

The simplest Distribution

Simplest

One value

5

Constant

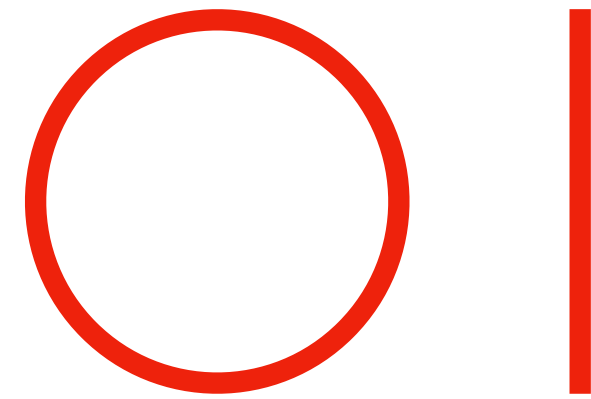
Always same

Trivial

Simplest non-trivial

Two values

Simplest values



0 and 1



Bernoulli Coin!

Bernoulli Distribution

B_p

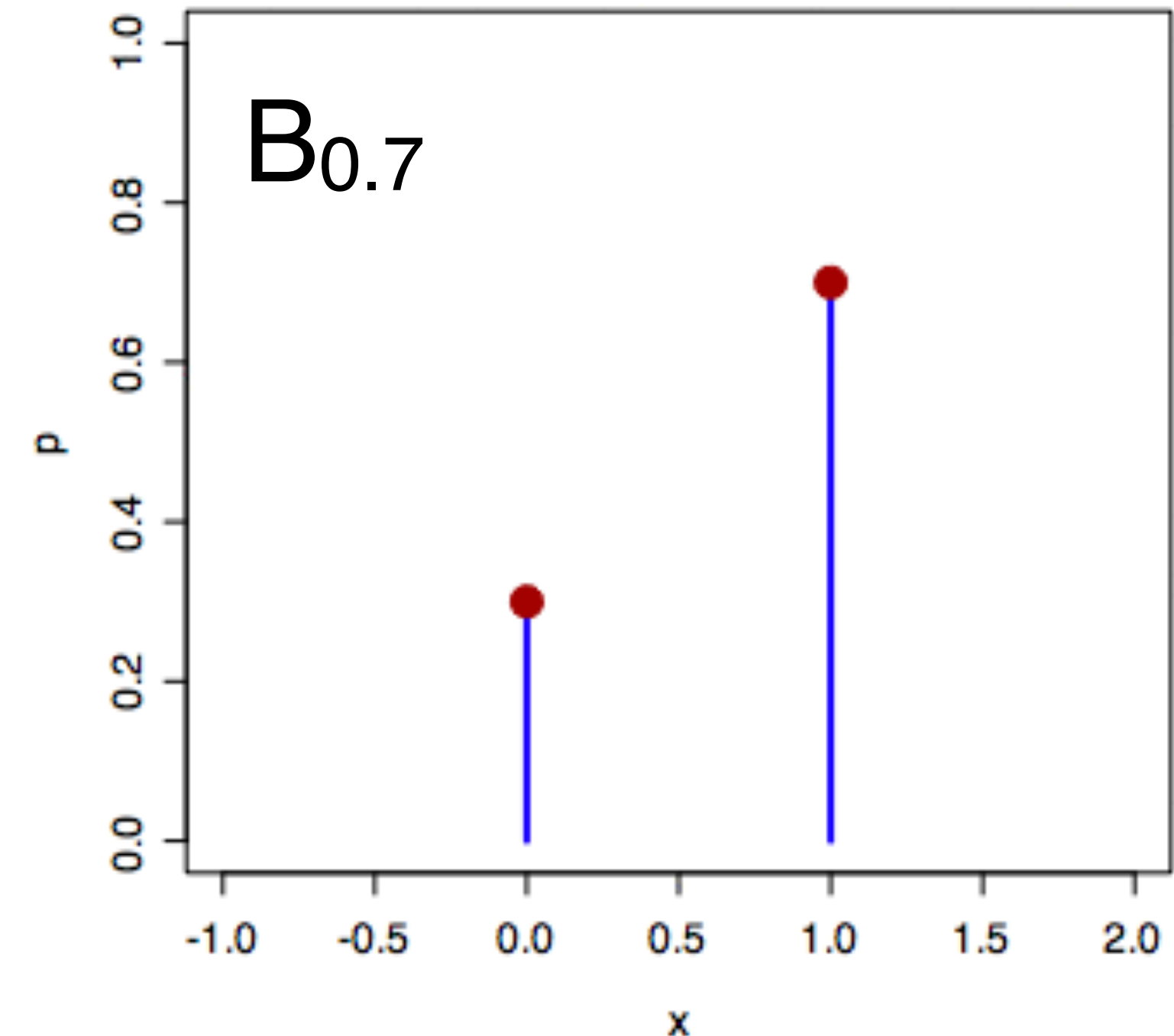
$$0 \leq p \leq 1$$

Two values	0	1
Probability	$1-p$	p

failure

success

\bar{p} q



Σ Will it add?

$$p(0) + p(1) = (1-p) + p = 1$$

YES IT ADDS!

$X \sim B_p$

Bernoulli

random variable, coin, experiment, trial

Who Cares About Two Values?

Everyone!

Binary version of complex events

Products: 80 good, 20 defective

Select one, good or not

$\sim B_{.8}$

Next child will be a boy

$\sim B_{.5}$

Generalizes to more complex variables

Patient has one of three diseases

Repeated trials yield # successes

Many important distributions

Binomial, Geometric, Poisson, Normal

Mean

$$X \sim B_p$$

$$p(0) = 1-p$$

$$p(1) = p$$

$$EX = \sum p(x) \cdot x = (1-p) \cdot 0 + p \cdot 1 = p$$

$$X \sim B_{0.8}$$

$$EX = 0.8$$

$$EX = P(X=1)$$

Fraction of times expect to see 1

Variance

$$X \sim B_p$$

$$EX = p$$

Variance

Easy route

$$0^2 = 0$$

$$1^2 = 1$$

$$X^2 = X$$

$$E(X^2) = EX = p$$

$$V(X) = E(X^2) - (EX)^2 = p - p^2 = p(1-p) = pq$$

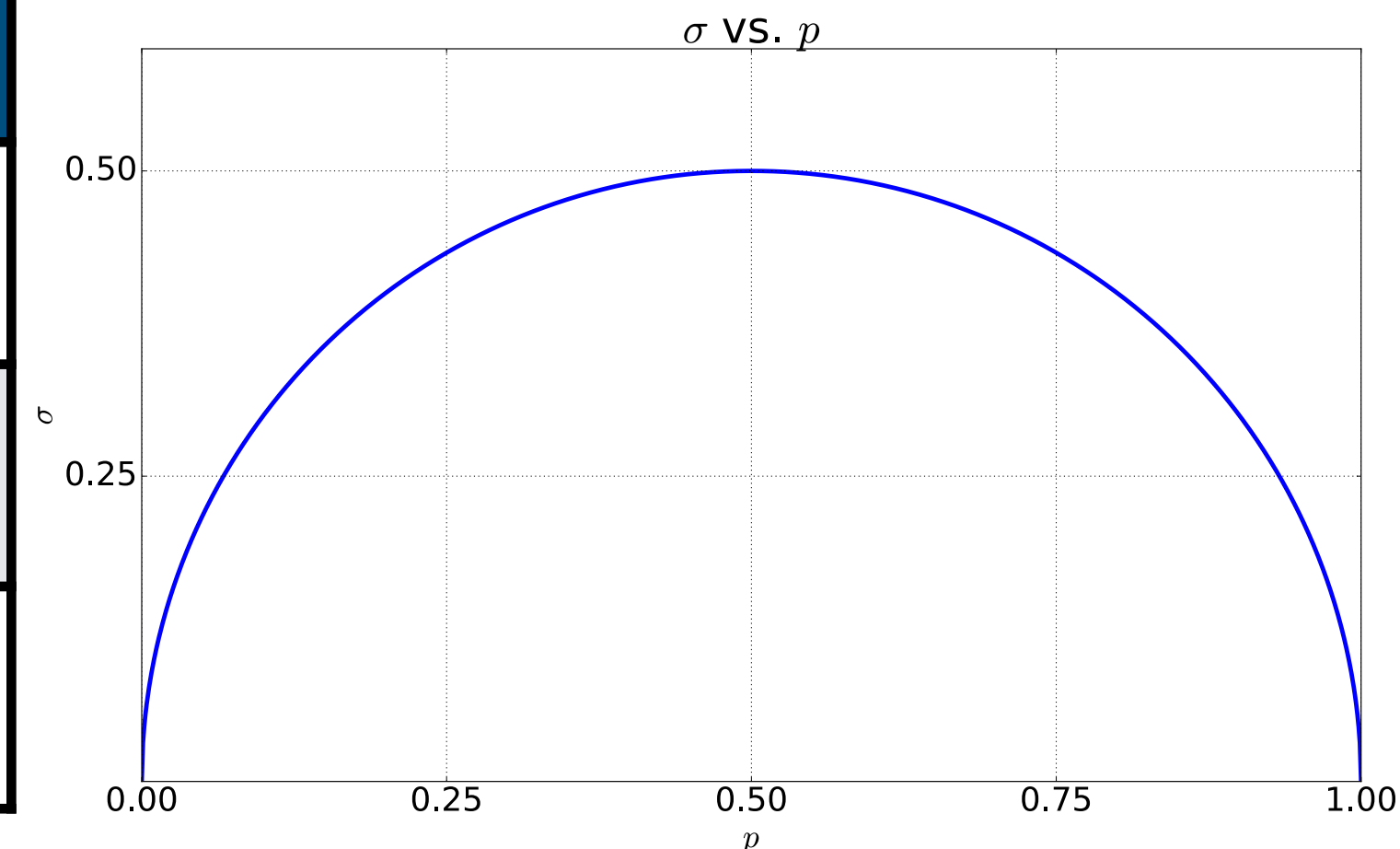
Standard Deviation

$$\sigma = \sqrt{pq}$$

B_p varies most
when $p = \frac{1}{2}$

E, V, σ for
various p

p	EX	V(X)	σ
0	0	0	0
1	1	0	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$



Independent Trials

Much of B_p importance stems from multiple trials

Most common Independent \perp

$$0 \leq p \leq 1 \quad X_1, X_2, X_3 \sim B_p \quad \perp$$

$$q \stackrel{\text{def}}{=} 1-p \quad P(110) = p^2 q = P(101) = P(011)$$

Generally $X_1, X_2, \dots, X_n \sim B_p \quad \perp$

$$\mathbf{x}^n = x_1, x_2, \dots, x_n \in \{0, 1\}^n \quad n_0 \text{ 0's and } n_1 \text{ 1's}$$

$$P(x_1, \dots, x_n) = p^{n_1} q^{n_0} \quad P(10101) = p^{n_1} q^{n_0} = p^3 q^2$$



Typical Samples

Distribution	Typical seq.	Description	Probability
B_0	0000000000	constant 0	$1^{10} = 1$
B_1	1111111111	constant 1	$1^{10} = 1$
$B_{0.8}$	1110111011	80% 1's	$0.8^8 \cdot 0.2^2$
$B_{0.5}$	1011010010	50% 1's	0.5^{10}

Fair coin flip

Not most probable
Most probable: 1...1
Unlikely to be seen

Bernoulli Distribution

Simplest non-constant distribution

$$B_p \quad 0 \leq p \leq 1$$

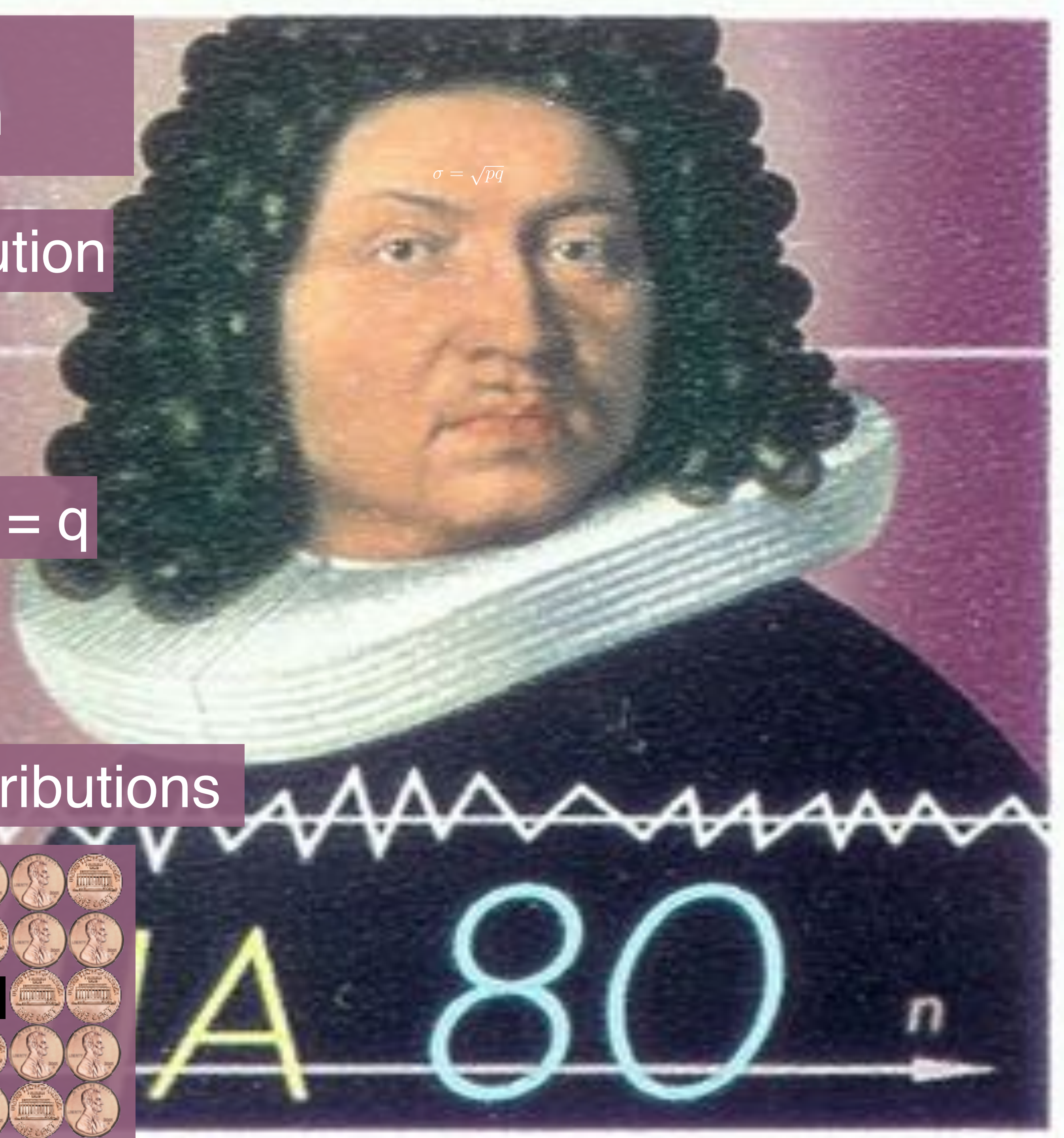
$$0 \text{ and } 1 \quad p(1) = p \quad p(0) = 1 - p = q$$

$$\mu = p \quad V = pq \quad \sigma = \sqrt{pq}$$

Foundation of many other distributions



Binomial



$$\sigma = \sqrt{pq}$$