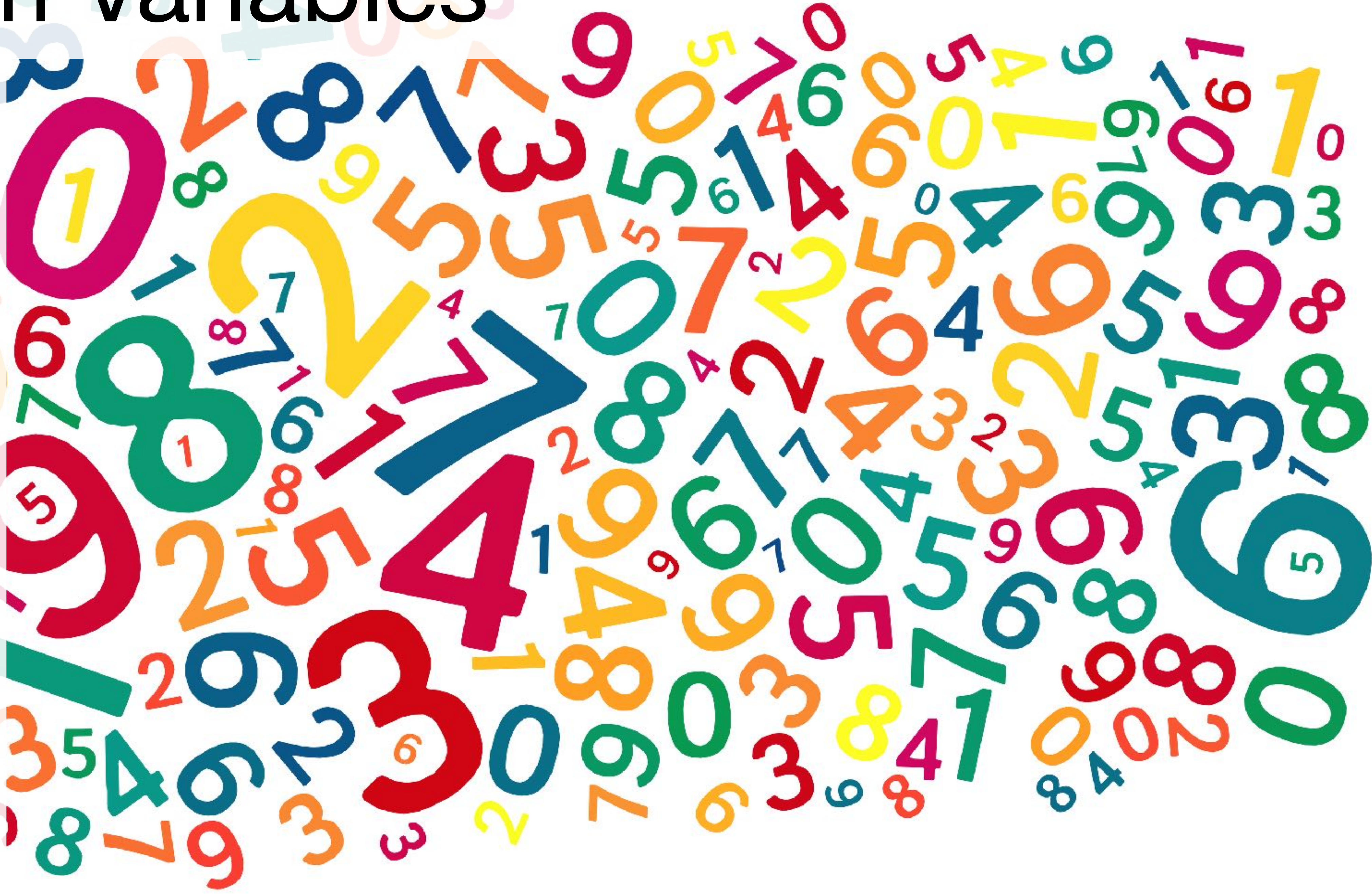# Random Variables

Motivation

Examples

Definitions

Visualization

What Matters

So far coins, dice, cards, dominoes, marbles, ….

Often subscribers clicks viewers yield weight sales

time congestion delay age temperature heart rate

GPA tuition assignment income cost
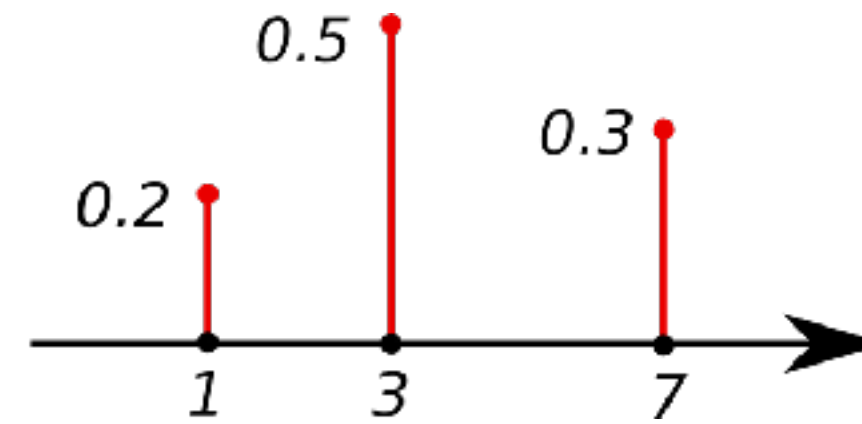
Numbers!

Random variable Number-valued random outcome

Xtra with Numbers

Distri-bution

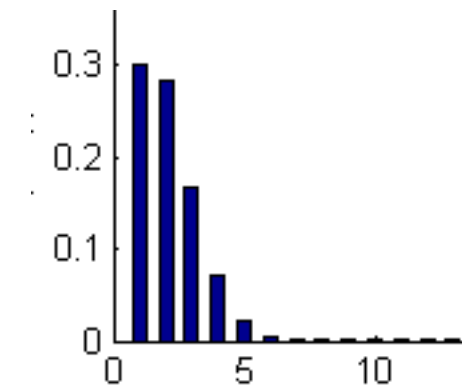**p(x)**

- View on a line

- Express as function

$p(x) = 1/x^2$

- Consider properties

Decreasing

concentrated

Random Variable

X

- Perform operations

$X+1$ X^2

- Combine variables

$X+Y$

- Consider properties

average value of X

Two Types

Size of sample space Ω

Ω is finite   {1, 2, 3}   {e, π}   or countably infinite $\mathbb{N}$ $\mathbb{Z}$ Discrete

Ω is uncountably infinite   [0,2]   (-1,3) $\cup$ [4,5] $\mathbb{R}$ Continuous

Combination   [0,2] $\cup$ {e, π}   Mixed

Begin with discrete

Been There

Several past examples had number outcomes

Outcome of a die roll   {1,…,6}

Number of heads in 3 coin tosses   {0,…,3}

Values of a domino tile   {0,…,6}

Did not use numerical features   →   Use extensively

Familiar and new examples   General formulation

Heads

3 fair coins

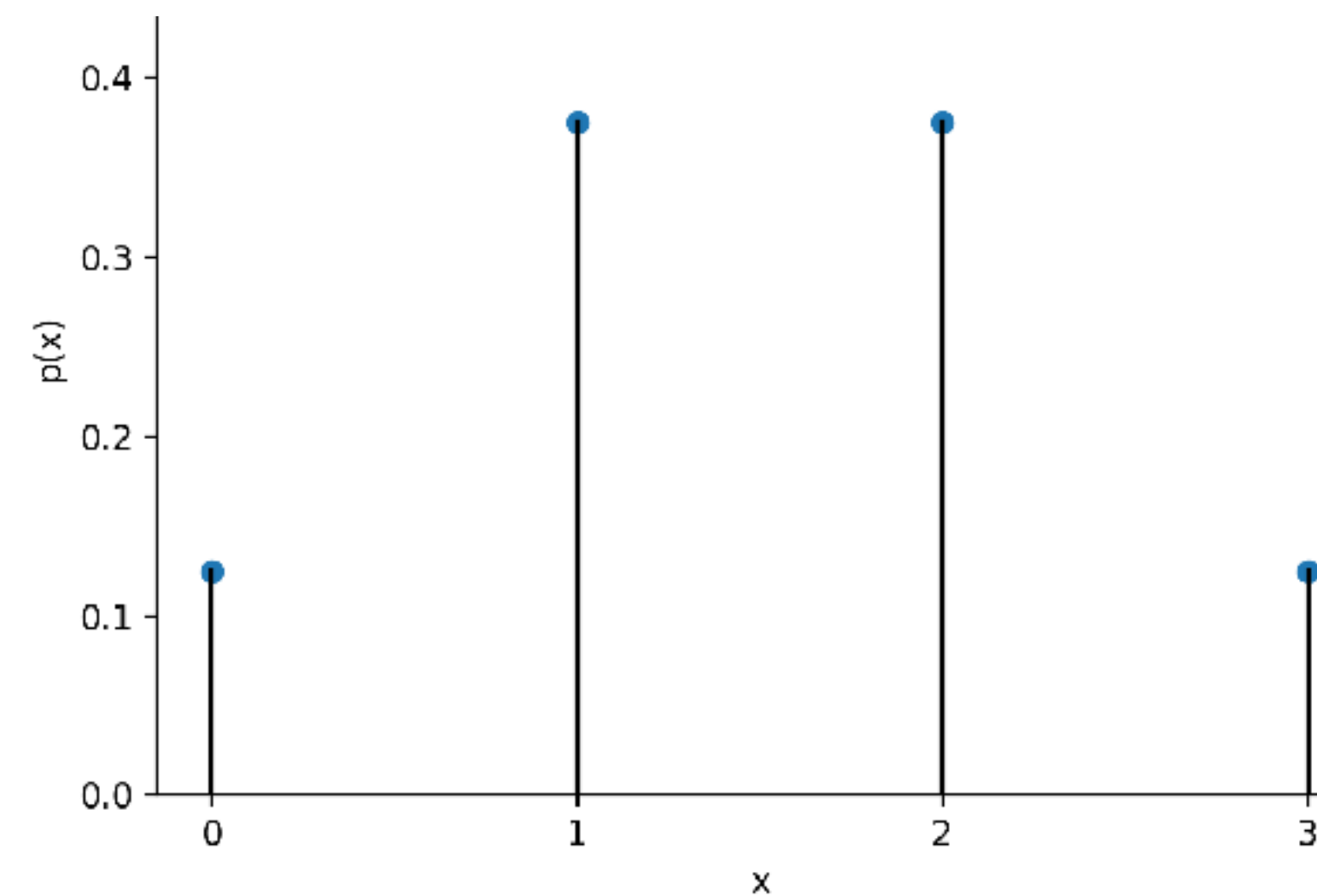Ω = { ttt, tth, tht, thh, htt, hth, hht, hhh } |Ω| = 8

Equiprobable   p = 1/8

X   # heads

x	Outcomes	p(x)
0	ttt	⅛
1	tth, tht, htt	⅜
2	thh, hth, hht	⅜
3	hhh	⅛

Specification

As before   Explicit   p(1)=.1   p(2)=.2   p(3)=.3   p(4)=.4

Table

x	1	2	3	4
p(x)	.1	.2	.3	.4

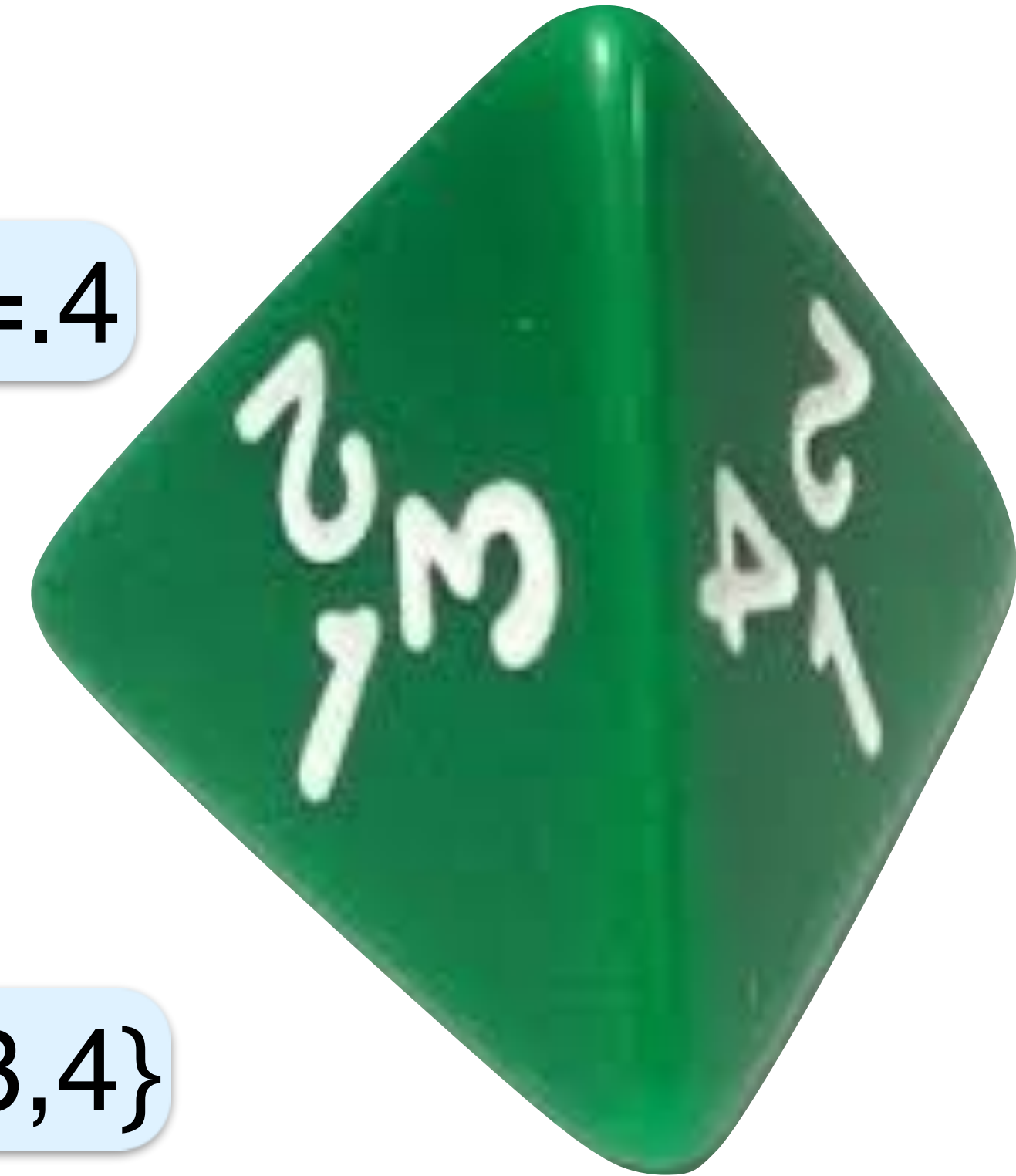With numbers   Function $p(x) = x / 10$ $x \in \{1,2,3,4\}$
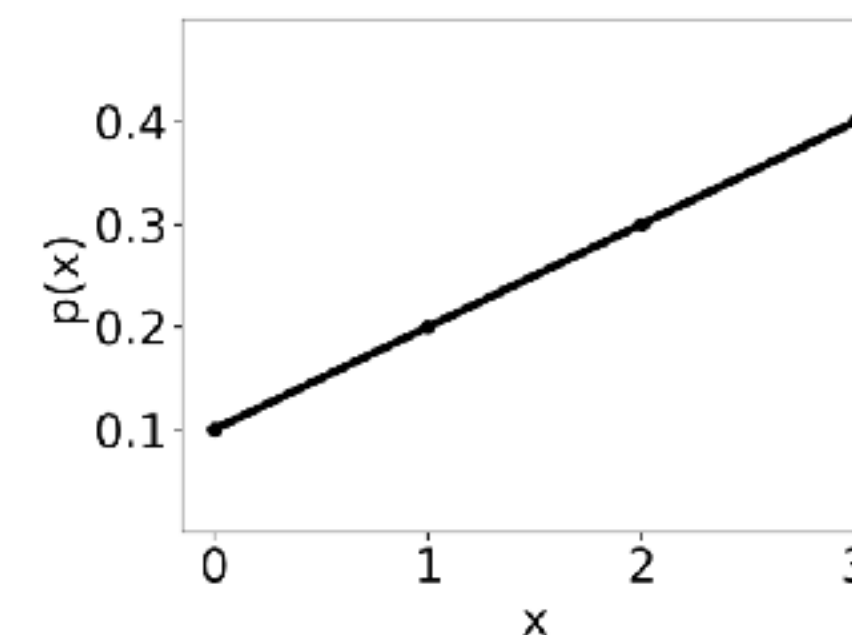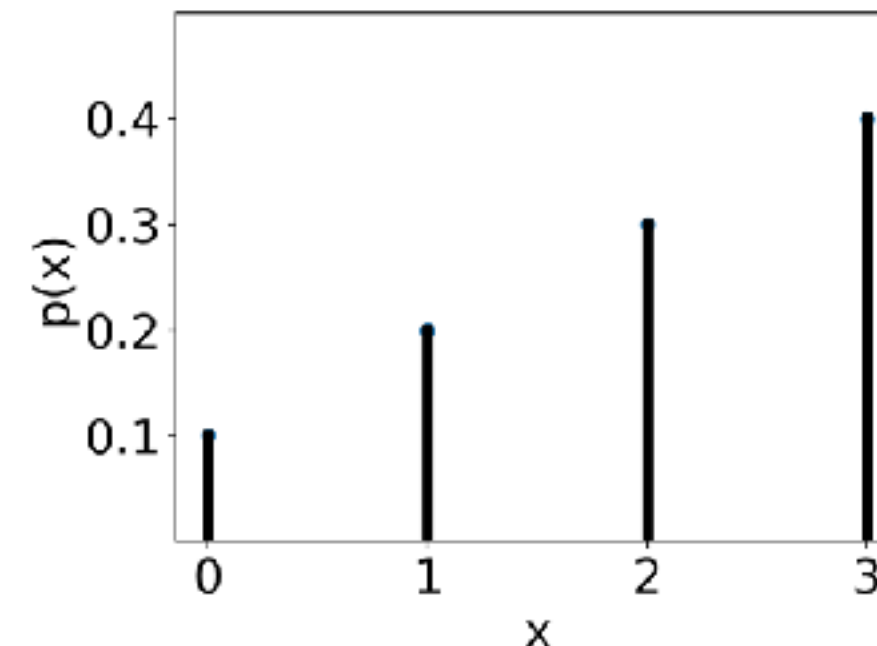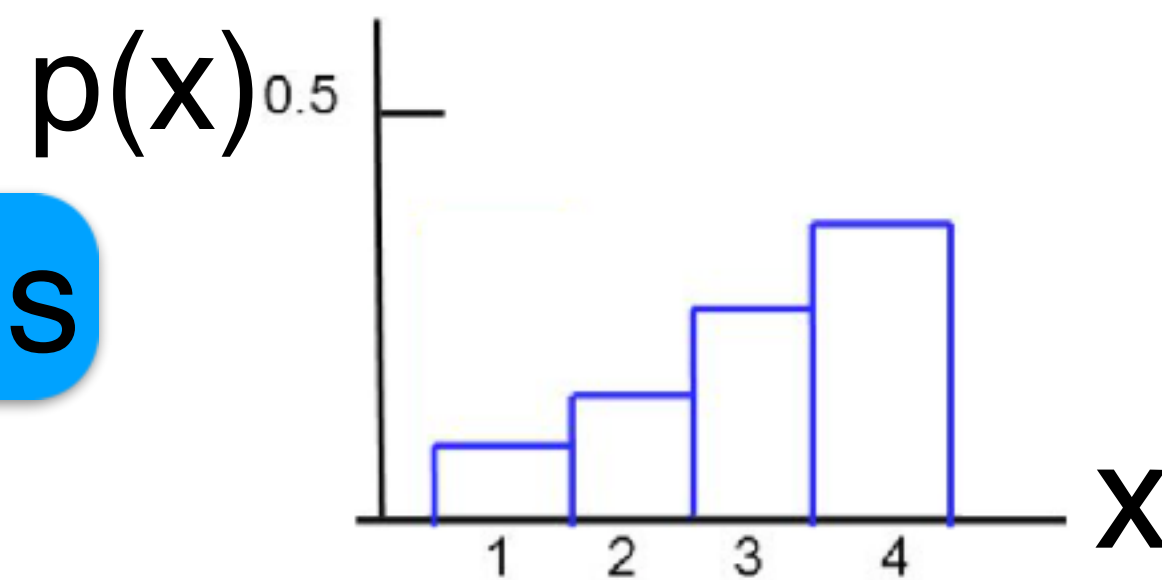
Histogram   Stem plot   Plot

Graphs

Probability Mass Function

As before   pmf $p : \Omega \rightarrow \mathbb{R}$ Specify Ω and p

Ω Random variable $\rightarrow$ $\subseteq \mathbb{R}$

Discrete $\rightarrow$ finite or countably infinite

p $p(x) \geq 0 \quad \forall x \in \Omega$ $\displaystyle\sum_{x \in \Omega} p(x) = 1$

If X is distributed according to p, we write X ~ p

Alternative Notation

Discrete

$\Omega \subseteq \mathbb{R}$ Often $\mathbb{Z}$ $\mathbb{N}$ $\mathbb{P}$ $\{1,...n\}$

$p(x)$ $\rightarrow$ p_x p_i

$p_i \geq 0$ $\sum_i p_i = 1$

Types of Discrete Distributions

Finite $|\Omega| = n \in \mathbb{P}$

Infinite $|\Omega| = \infty = \aleph_0$

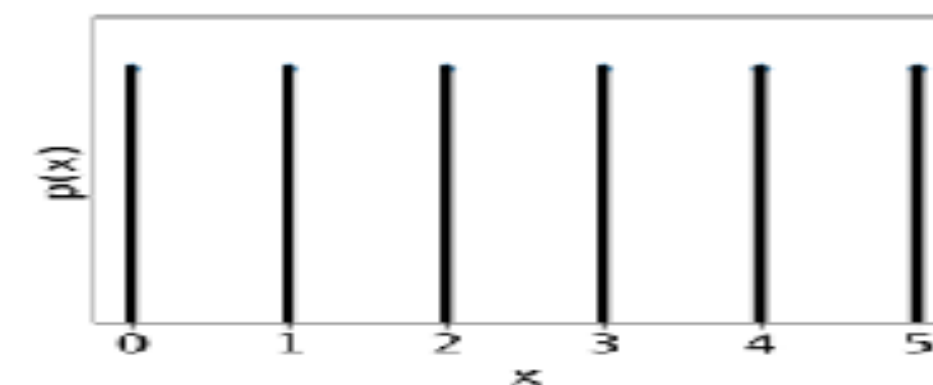# Finite Distributions

$|\Omega| = n$ Specify pmf

$p_1, p_2, \ldots, p_n$ $\forall_{1 \leq i \leq n} \quad p_i \geq 0 \qquad \sum_{i=1}^{n} p_i = 1$
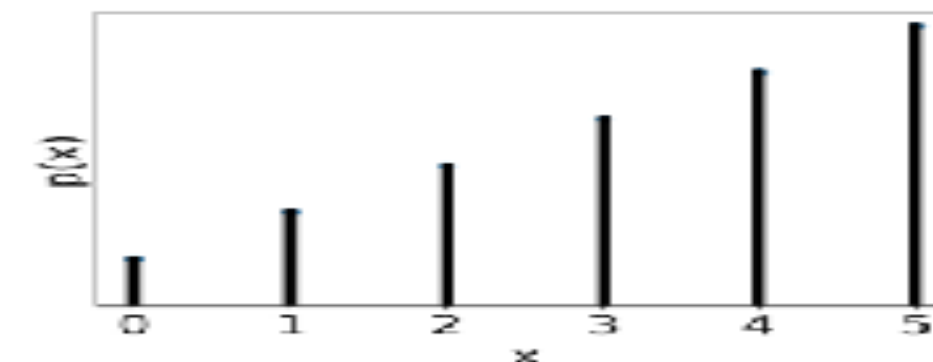

Uniform $p_1 = p_2 = \ldots = p_n$ $= 1/n$
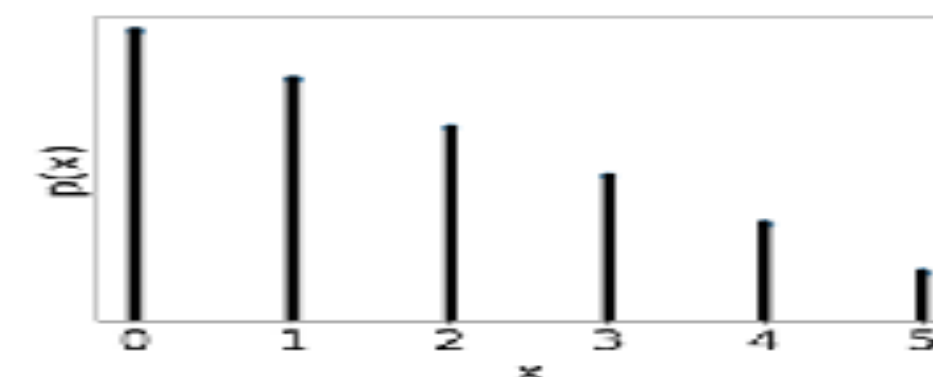
Increasing $p_1 \leq p_2 \leq \ldots \leq p_n$

Decreasing $p_1 \geq p_2 \geq \ldots \geq p_n$

Infinite Distributions

$|\Omega| = \infty$

One-sided infinite $p_1, p_2, p_3, \ldots$
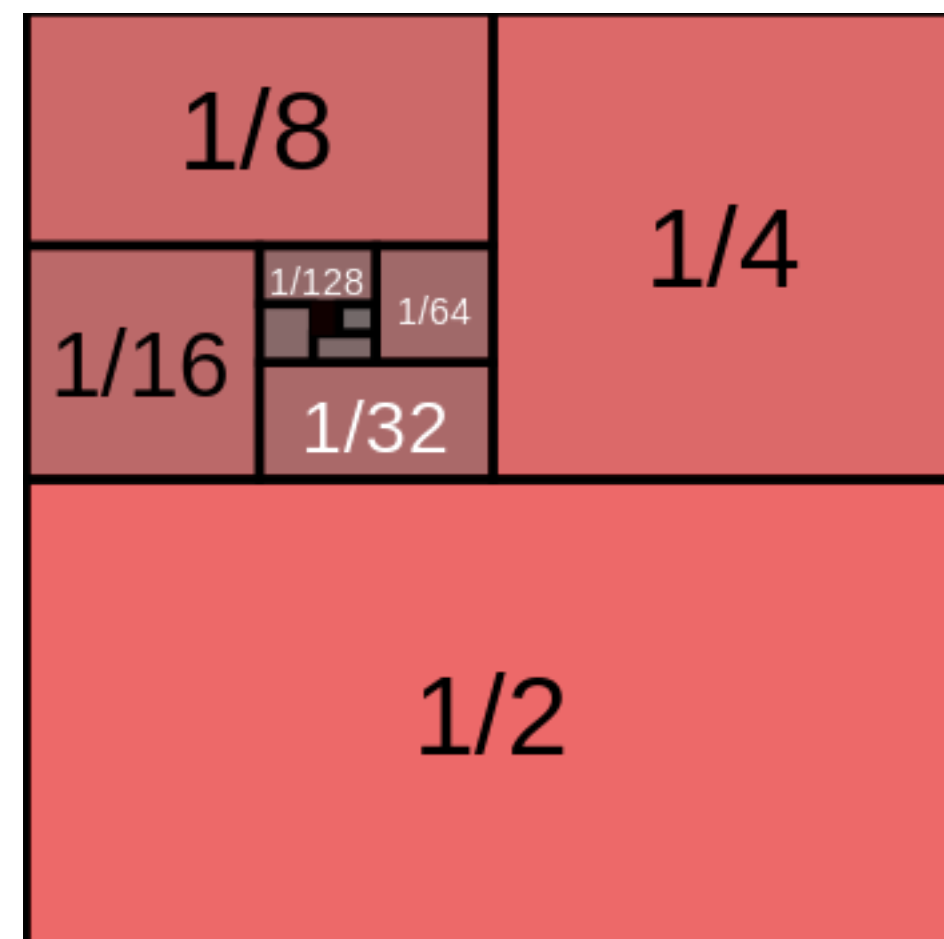
Cannot be uniform $p = 0 \rightarrow \sum = 0$ $p > 0 \rightarrow \sum = \infty$

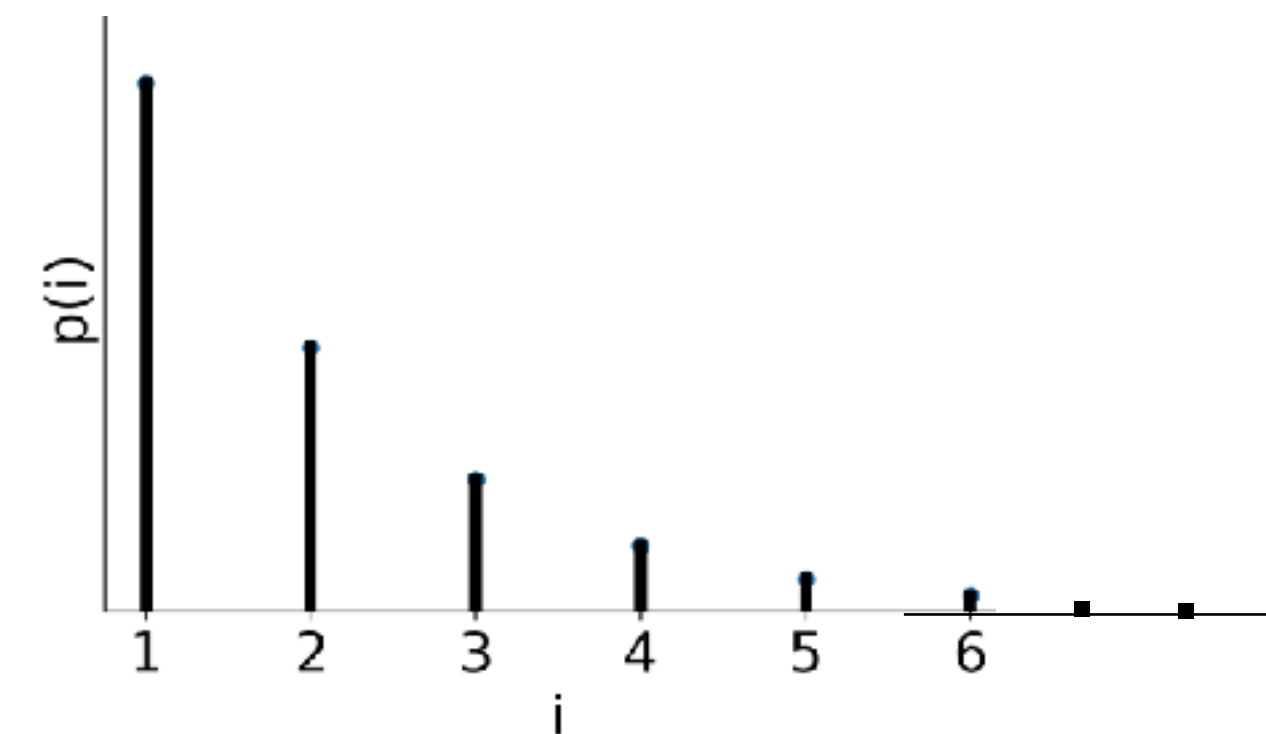Cannot increase $p_i > 0 \rightarrow p_{i+1}, p_{i+2}, \ldots > 0 \rightarrow \sum = \infty$

Can decrease $\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \ldots$

$\dfrac{1}{8}$ $\dfrac{1}{4}$ $\dfrac{1}{16}$ $\dfrac{1}{128}$ $\dfrac{1}{64}$ $\dfrac{1}{32}$ $\dfrac{1}{2}$

$$\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$$

$$\sum_{i=1}^{\infty} \frac{1}{2^i} = 1$$

Doubly infinite $\ldots, p_{-2}, p_{-1}, p_0, p_1, p_2, \ldots$

$\ldots, \tfrac{1}{8}, \tfrac{1}{4}, 0, \tfrac{1}{4}, \tfrac{1}{8}, \ldots$

Formal Definition

Random variable is a mapping $f : \Omega \rightarrow \mathbb{R}$

Simplify terminology, focus on math

Number-valued random experiment

Random Variables

Motivation Numbers Operations

Examples

Definitions pmf

Visualization Histogram Plot Stem

Cumulative Distribution Functions