Bayes’ Rule

Asymmetry

Bought Google at IPO → Wealthy ← ?

Alive today → Born after 1800 ← ?

Forward - Backward

At times	$P(F \mid E)$ - easy	$P(E \mid F)$ - hard
2 coins	H_i - coin i is h	$\exists H$ - at least one h
	$P(\exists H \mid H_1) = 1$	$P(H_1 \mid \exists H)$?
2 dice	D_i - face of die i	$S = D_1+D_2$ sum of 2 faces
	$P(S=5 \mid D_1=2) = P(D_2=3) = \frac{1}{6}$	$P(D_1=2 \mid S=5)$?

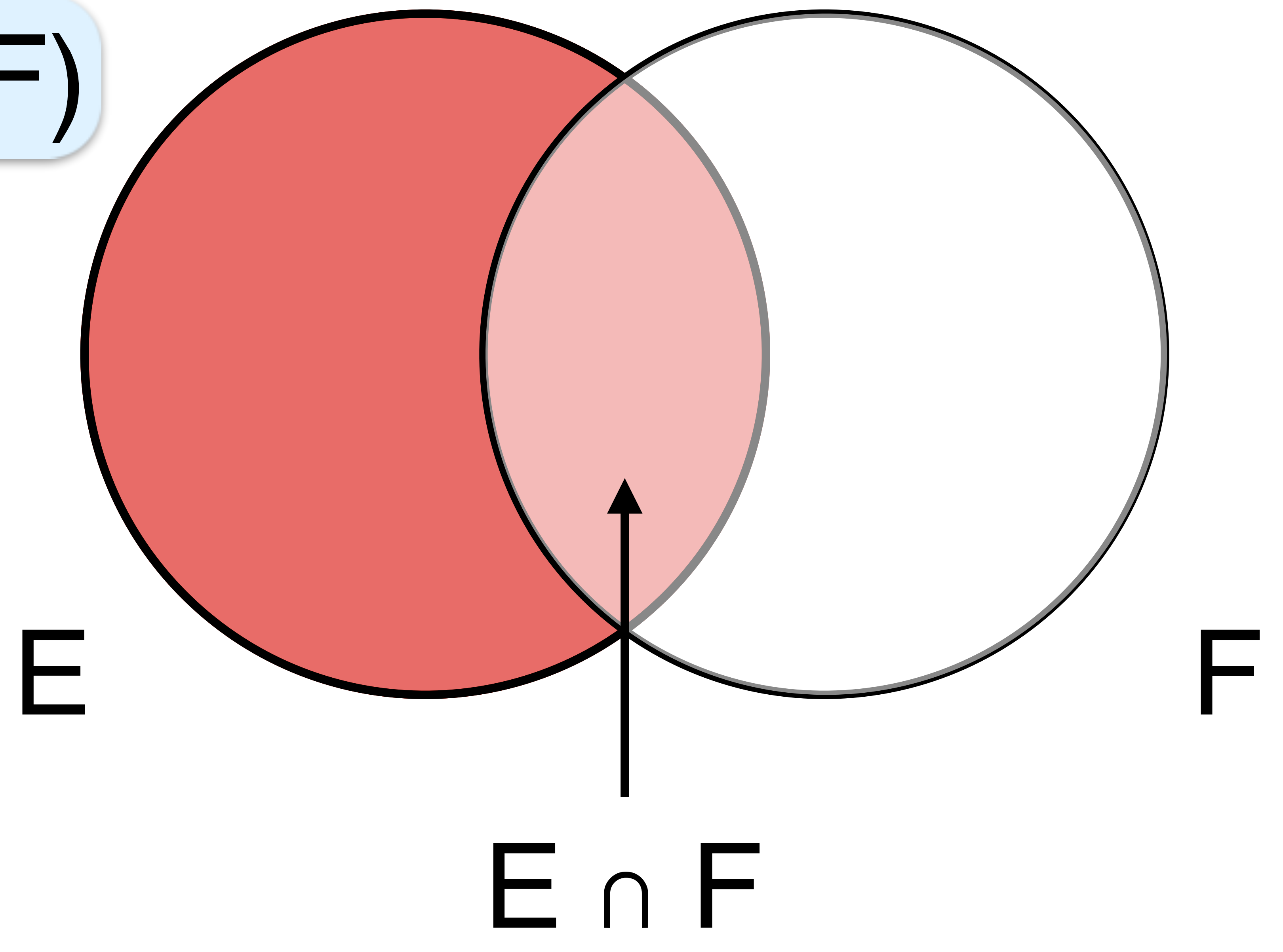
Bayes' Rule: Method for converting $P(F \mid E)$ to $P(E \mid F)$

Bayes' Rule

P(E), P(F)

Given P(F | E) (and a bit more) determine P(E | F)

$$\boxed{P(E \mid F) = \frac{P(E) \cdot P(F \mid E)}{P(F)}}$$



μ-proof

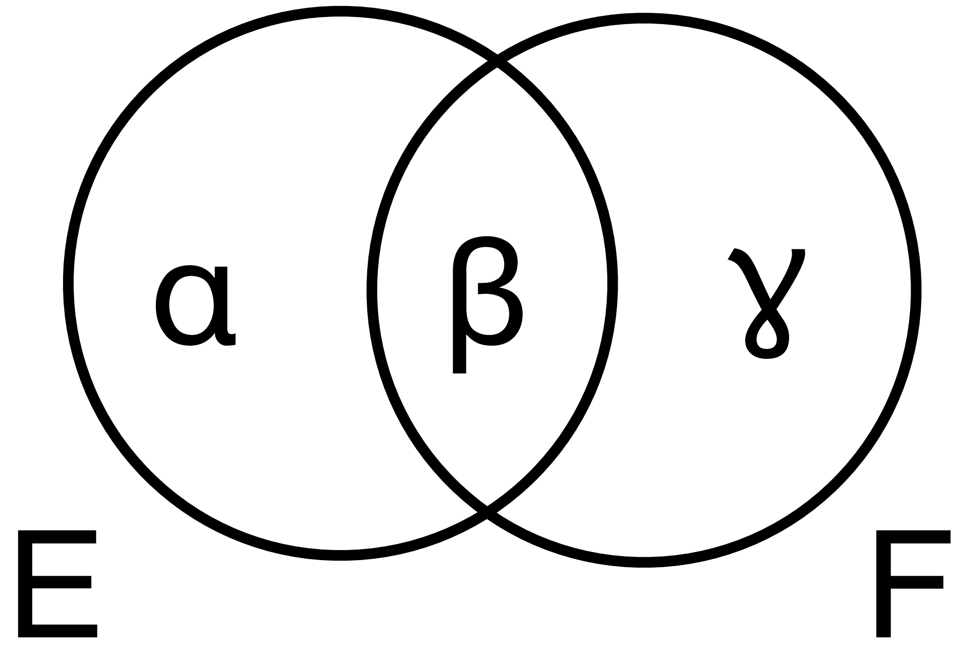
$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E) \cdot P(F \mid E)}{P(F)}$$

Two More Views

$$\boxed{P(E \mid F) = \frac{P(E) \cdot P(F \mid E)}{P(F)}}$$

$$P(F) \cdot P(E \mid F) = P(E \cap F) = P(E) \cdot P(F \mid E)$$

$$P(F \mid E) = \frac{\beta}{\alpha + \beta}$$



$$P(E \mid F) = \frac{\beta}{\beta + \gamma}$$

$$P(E \mid F) = \frac{\beta}{\beta + \gamma} = \frac{\beta}{\alpha + \beta} \cdot \frac{\alpha + \beta}{\beta + \gamma} = \frac{P(F \mid E) \cdot P(E)}{P(F)}$$

Two Fair Coins

H_i - coin i is h $\qquad\qquad$ $\exists H$ - at least one h

$$P(H_1 \mid \exists H) = \frac{|H_1 \cap \exists H|}{|\exists H|}$$

$$\exists H \left\{ \begin{array}{l} \left.\begin{array}{c} h\ h \\ h\ t \end{array}\right\} H_1 \\ \ \ t\ h \end{array} \right.$$

$$t\ t$$

$P(H_1 \mid \exists H) = ?$

$$P(H_1 \mid \exists H) = P(\exists H \mid H_1) \cdot \frac{P(H_1)}{P(\exists H)} = 1 \cdot \frac{1/2}{3/4} = 2/3$$

$P(\exists H \mid H_1) = 1$

$P(H_1) = 1/2$

$P(\exists H) = 3/4$ $\qquad$ Last video

Two Fair Dice

D_i - outcome of die i $S = D_1 + D_2$ Sum of 2 dice

$P(D_1 = 2 \mid S = 5)$?

$$P(D_1 = 2 \mid S = 5) = \frac{P(S = 5 \mid D_1 = 2) \cdot P(D_1 = 2)}{P(S = 5)} = \frac{\frac{1}{6} \cdot \frac{1}{6}}{\frac{1}{9}} = \frac{1}{4}$$

$$P(S = 5 \mid D_1 = 2) = P(D_2 = 3 \mid D_1 = 2) = P(D_2 = 3) = \frac{1}{6}$$

$$P(D_1 = 2) = \frac{1}{6}$$

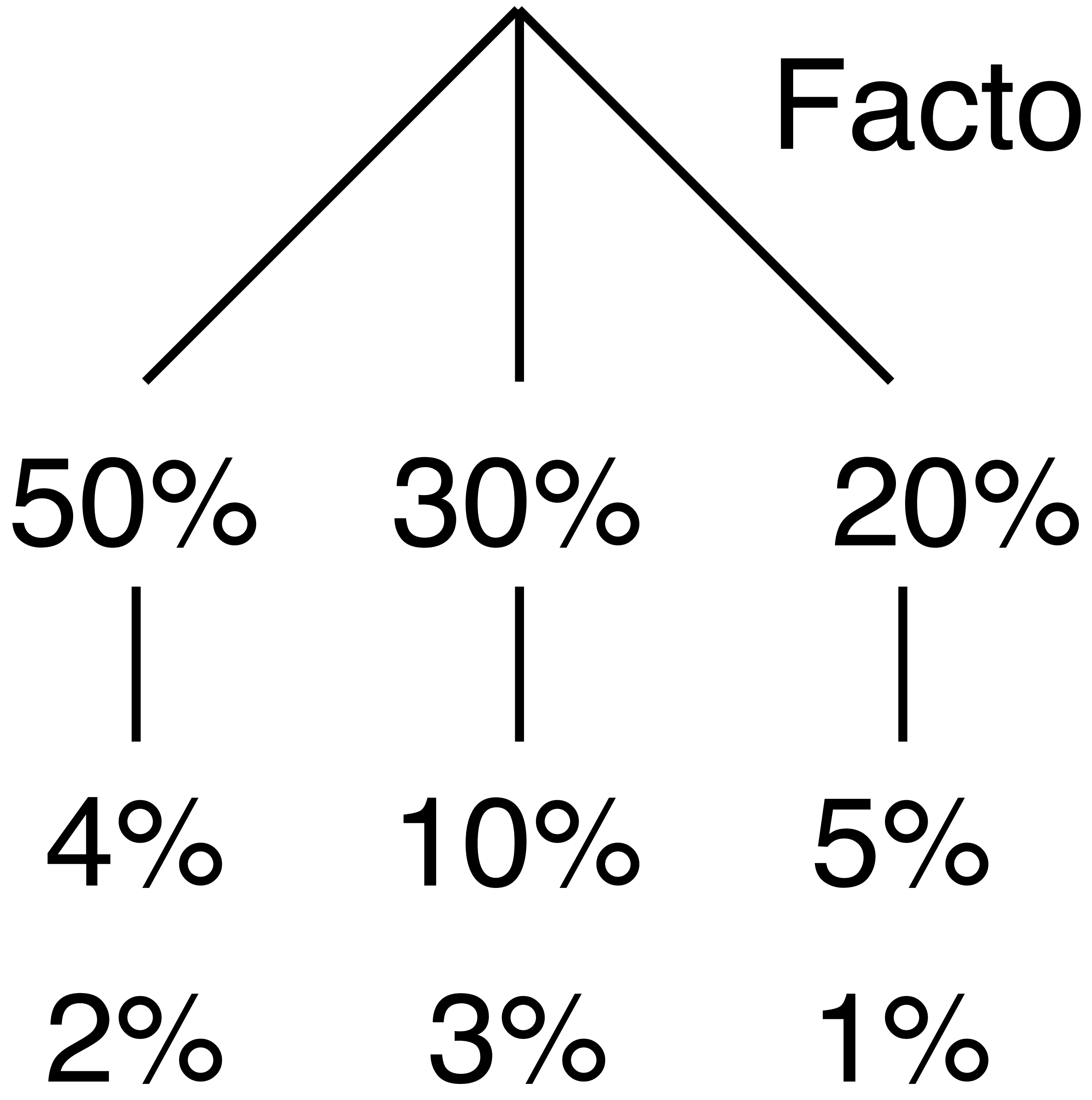
$P(S = 5) = \frac{1}{9}$ Last video

$$S = 5 \begin{cases} 1\ 4 \\ 2\ 3 \} \ D_1 = 2 \\ 3\ 2 \\ 4\ 1 \end{cases}$$

$$P(D_1 = 2 \mid S = 5) = \frac{|D_1 = 2 \cap S = 5|}{|S = 5|}$$

Foxconn

Foxconn has 3 factories producing 50%, 30%, and 20% of its iPhones



Factory defective fractions 4%, 10%, and 5% respectively

Overall fraction of defective phones?

$$
\begin{aligned}
P(D) &= P(D \cap F_1) + P(D \cap F_2) + P(D \cap F_3) \\
&= P(F_1)P(D \mid F_1) + P(F_2)P(D \mid F_2) + P(F_3)P(D \mid F_3) \\
&= .5 \times .04 + .3 \times .1 + .2 \times .05 \\
&= .02 + .03 + .01 \\
&= .06
\end{aligned}
$$

Culprit?

$$P(F_1 \mid D) = \frac{P(D \mid F_1) \cdot P(F_1)}{P(D)} = \frac{.04 \cdot .5}{.06} = \frac{.02}{.06} = ⅓$$

$P(D \mid F_1) = .04$ $\quad$ $P(F_1) = .5$ $\quad$ $P(D) = .06$

$$P(F_2 \mid D) = \frac{.1 \cdot .3}{.06} = \frac{.03}{.06} = ½$$

$$P(F_3 \mid D) = \frac{.05 \cdot .2}{.06} = \frac{.01}{.06} = ⅙$$

Conditional probabilities add to 1

Conditional order determined by both $P(F_i)$ and $P(D \mid F_i)$

Bayes' Rule

From P(F|E), find P(E|F)

$$P(E \mid F) = \frac{P(E) \cdot P(F \mid E)}{P(F)}$$

Taxi

This Lecture: Bayes' Rule

Next: Random Variables