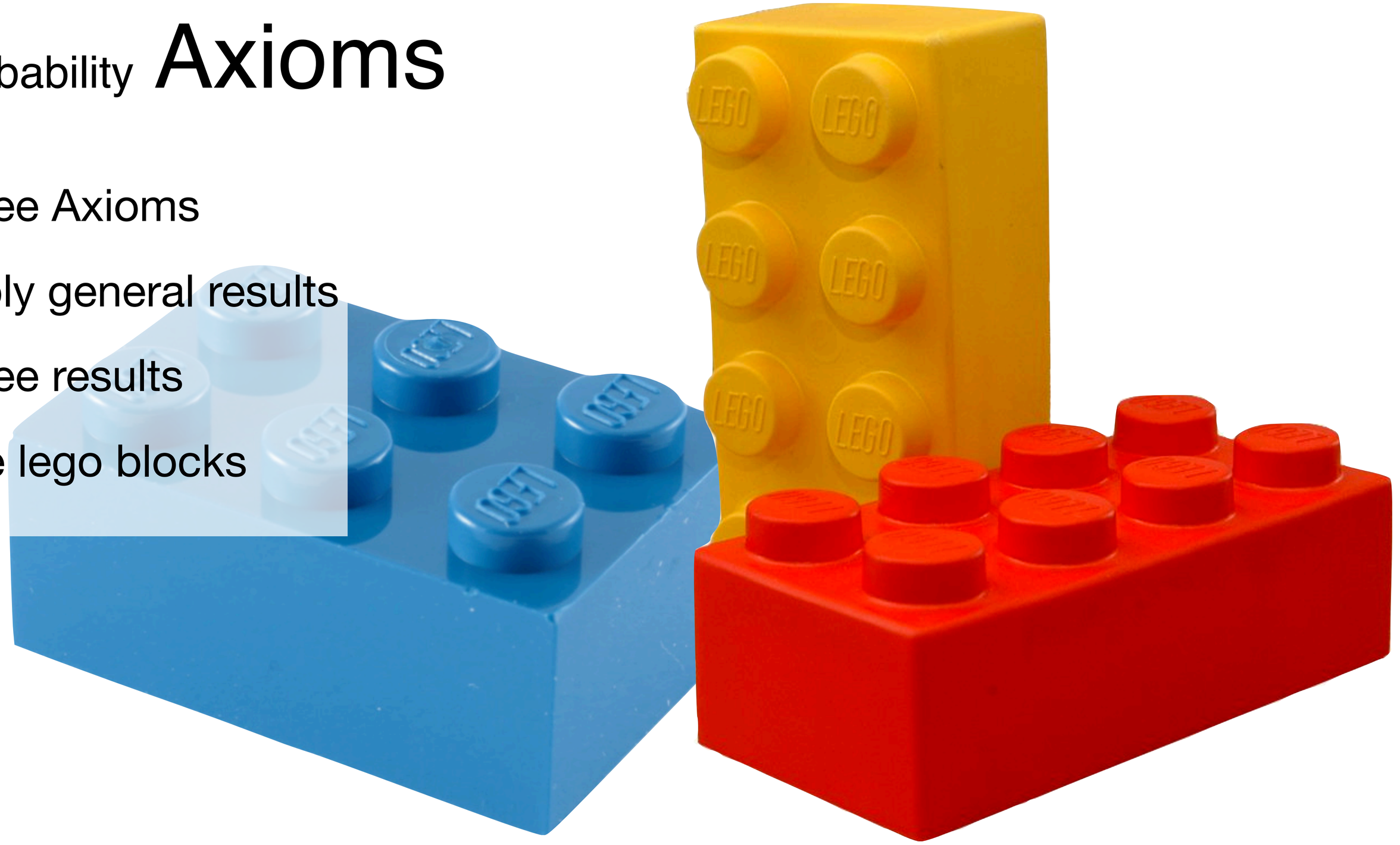
Probability Axioms

Three Axioms

Imply general results

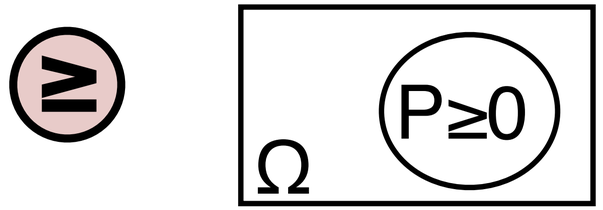
Three results

Use lego blocks



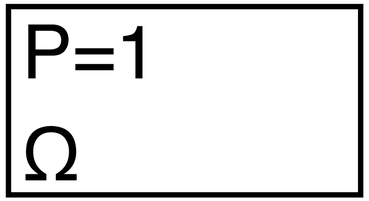
Three Axioms

Non-negativity $P(A) \geq 0$

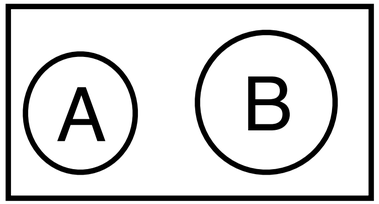


Short for $\forall A \;\; P(A) \geq 0$

Unitarity $P(\Omega) = 1$



Addition rule A, B disjoint → $P(A \cup B) = P(A) + P(B)$



$A_1, A_2, \ldots$ disjoint → $P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$

Countable unions only

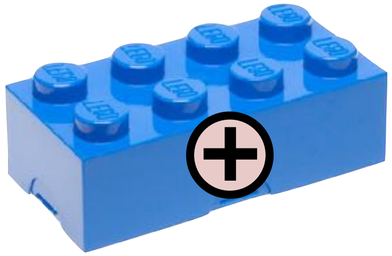
Uncountable later

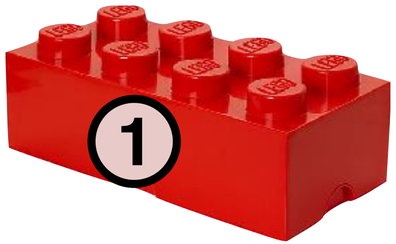
“Building” Results

Use lego blocks

construct three simple results

Complement rule

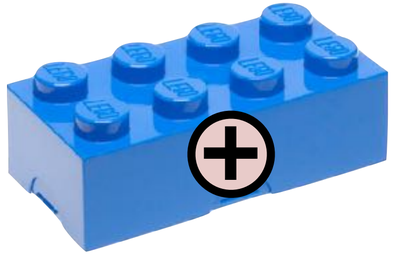




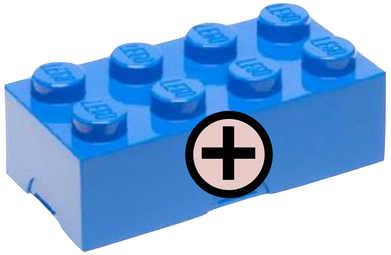
Subtraction

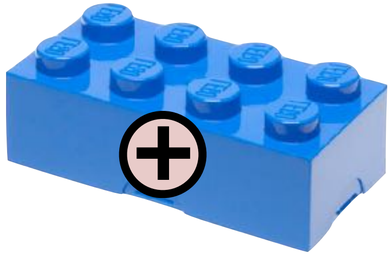
nested sets

general sets



Inclusion-exclusion





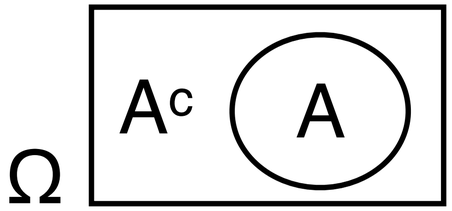
Complement Rule

Complement rule for counting

$|A^c| = |\Omega| - |A|$

Same for 

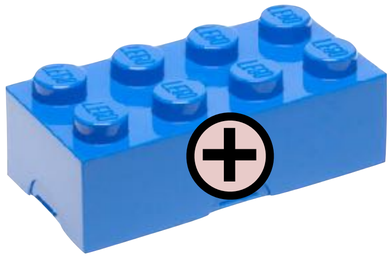
$A \cup A^c = \Omega$

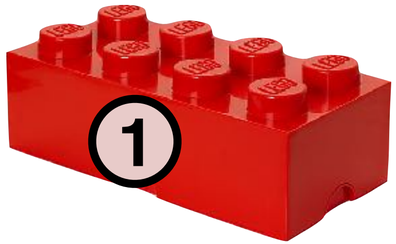


$P(A) + P(A^c) = P(A \cup A^c) = P(\Omega) = 1$

$P(A^c) = 1 - P(A)$

Complement rule for probability



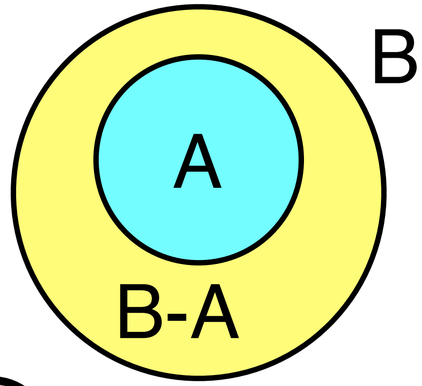


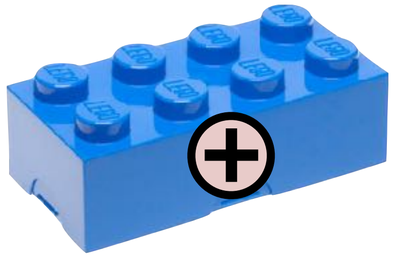
Subtraction Rule - Nested Sets

Complement rule $A \subseteq \Omega$ $P(A^c) = 1 - P(A)$ $P(\Omega - A) = P(\Omega) - P(A)$

Generalize $A \subseteq B$ → $P(B-A) = P(B) - P(A)$

$A \subseteq B$ → $B = A \cup (B-A)$





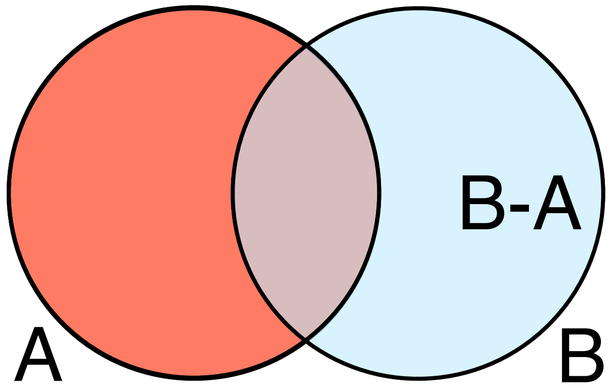
$$P(B) = P(A \cup (B-A)) = P(A) + P(B-A)$$

$P(B-A) = P(B) - P(A)$ **Subtraction rule for nested sets**

Subtraction Rule - General Sets

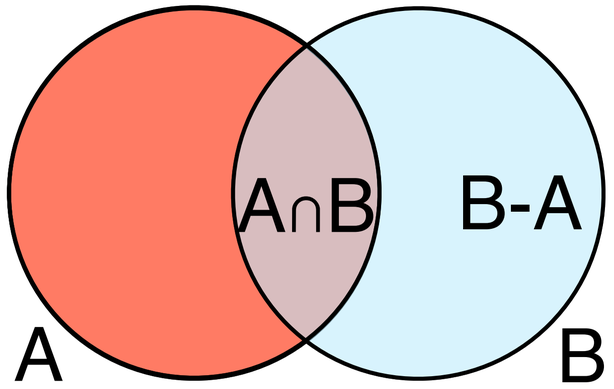
Nested $A \subseteq B$ $P(B-A) = P(B) - P(A)$

General $\forall A, B$ $P(B-A) = P(B) - P(A \cap B)$

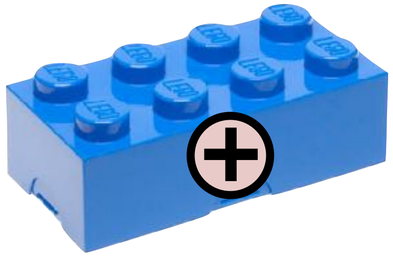


$B - A = B - (A \cap B)$

$A \cap B \subseteq B$

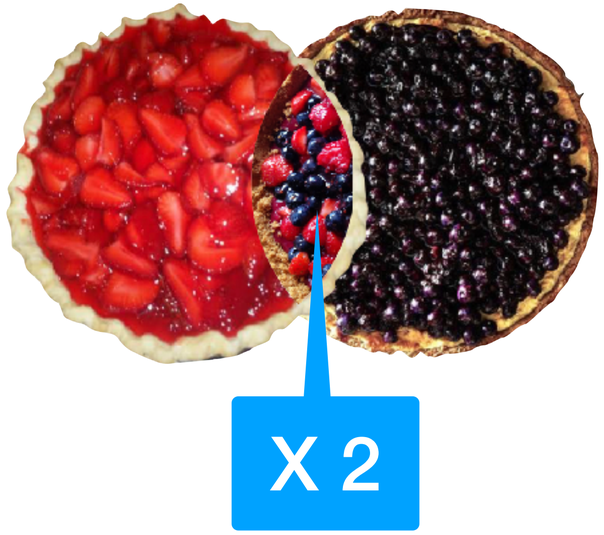


$P(B - A) = P(B - (A \cap B)) = P(B) - P(A \cap B)$

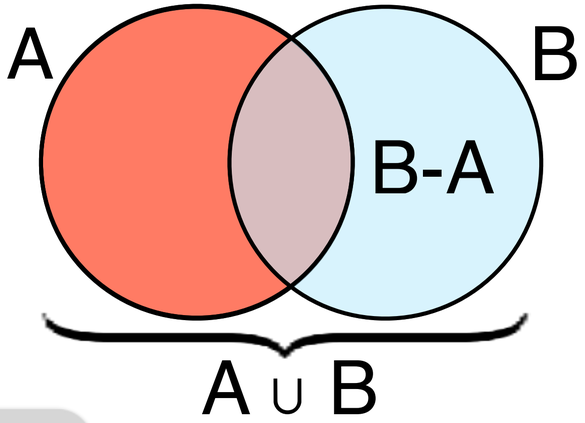


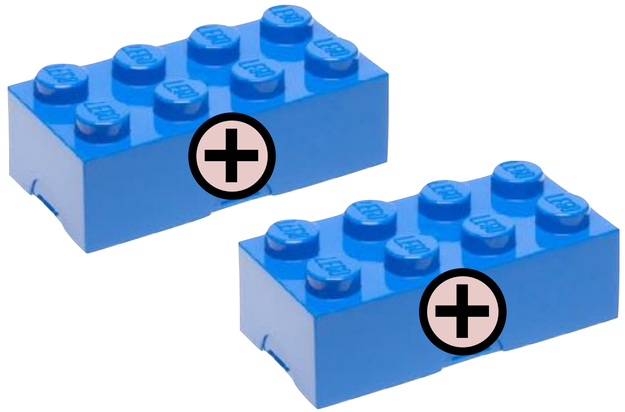
Inclusion Exclusion

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$A \cup B = A \cup B\text{-}A$





$P(A \cup B) = P(A \cup B\text{-}A)$

$\oplus$

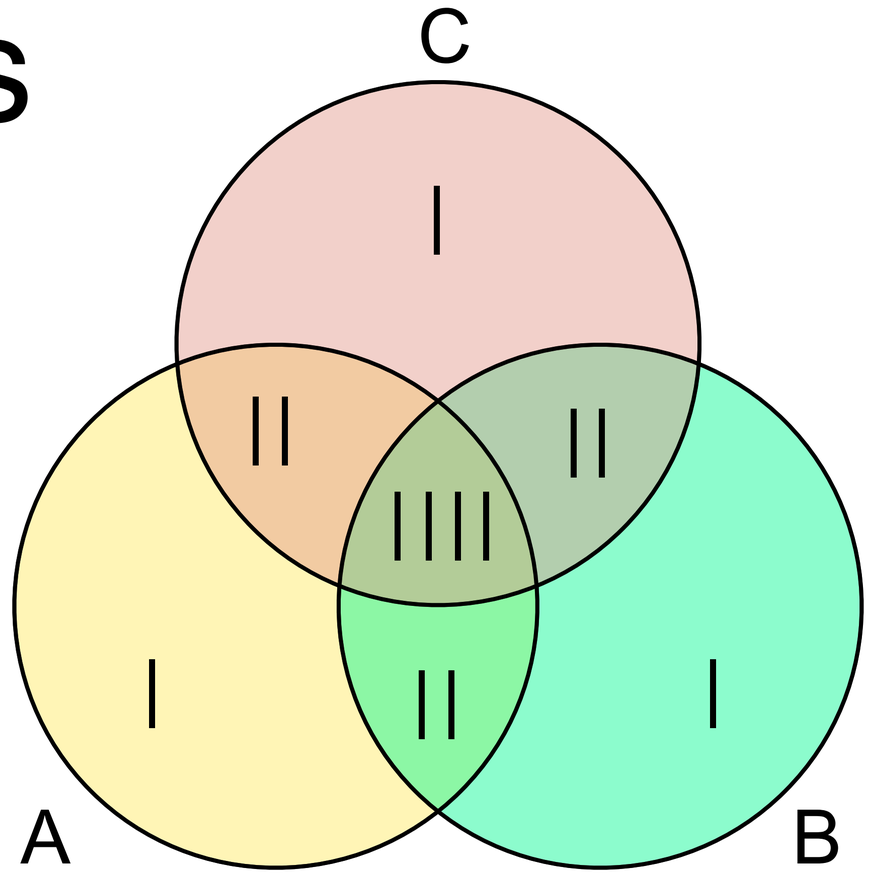
$= P(A) + P(B\text{-}A)$

General subtraction

$P(B\text{-}A) = P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A \cap B)$

More Sets



Two sets

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Three sets

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

n sets

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1 \le i \le n} P(A_i) - \sum_{1 \le i < j \le n} P(A_i \cap A_j) + \ldots + (-1)^{n-1} P\left(\bigcap_{i=1}^{n} A_i\right)$$

Probability Axioms

Three Axioms

Imply general results

Three results

Use lego blocks

Axioms

Three Axioms

Some implications

$1.4M question