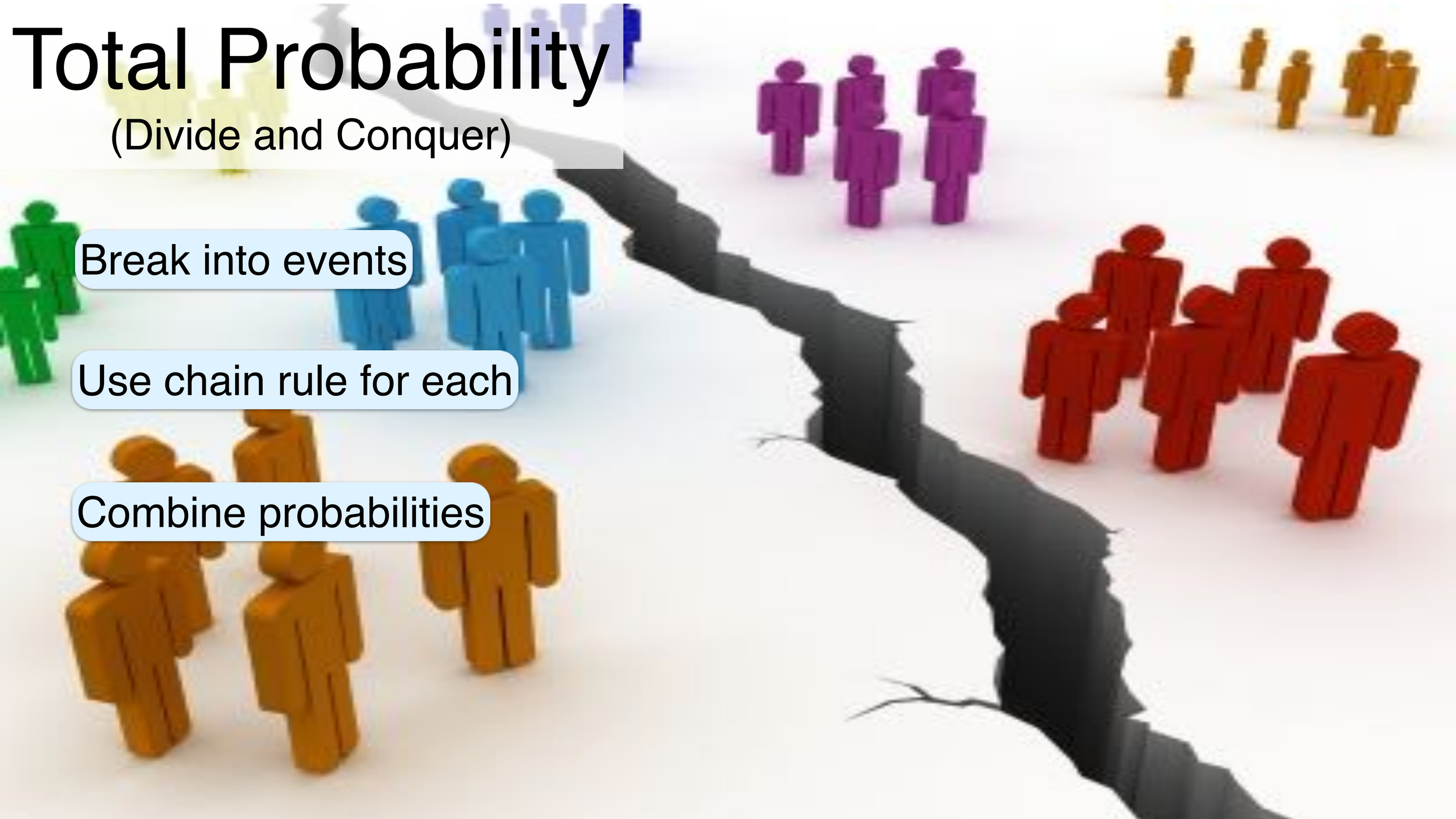
Total Probability

(Divide and Conquer)



Break into events

Use chain rule for each

Combine probabilities

Divide and Conquer

When evaluating probability of an event

Sometimes easier to split event into different parts

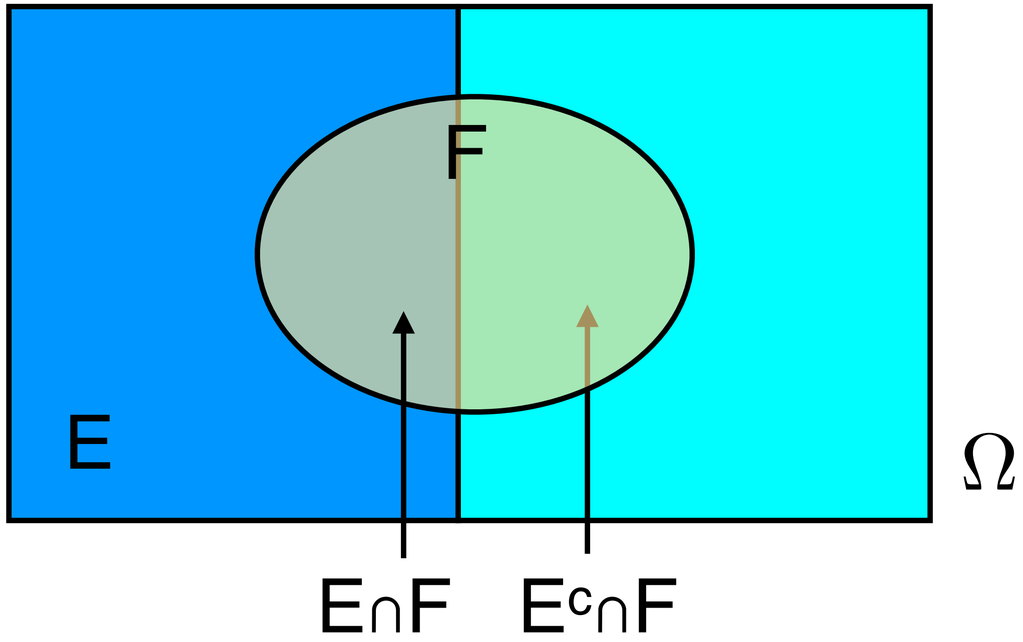
Calculate probability of each part

Add probabilities

Law of Total Probability

E,F events, P(F)=?

$F = E \cap F \cup E^c \cap F$



$P(F) = P(E \cap F) + P(E^c \cap F)$

$= P(E) \cdot P(F \mid E) + P(E^c) \cdot P(F \mid E^c)$

Product rule

2 Fair Coins

H_i - coin i is h

$\exists H$ - at least one h

$P(\exists H)$?

$$P(\exists H) = \frac{|\exists H|}{|\Omega|} = \frac{3}{4}$$

h h
h t
t h
t t

$P(\exists H) = P(H_1 \cap \exists H) + P(H_1^C \cap \exists H)$

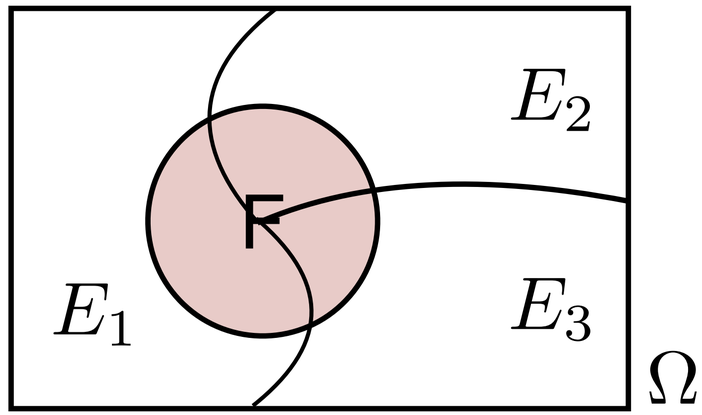
$= P(H_1) \cdot P(\exists H \mid H_1) + P(H_1^C) \cdot P(\exists H \mid H_1^C)$

$= \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

Total Probability - n Conditions

Let $E_1, E_2, \ldots E_n$ partition Ω

$$F = \bigcup_{i=1}^{n} (E_i \bigcap F)$$



$$P(F) = \sum_{i=1}^{n} P(E_i \cap F) \quad = \sum_{i=1}^{n} P(E_i) \cdot P(F|E_i)$$

2 Dice

D_i - outcome of die i

$S = D_1 + D_2$ sum of 2 dice

P(S = 5) = ?

$$P(S = 5) = \sum_{i=1}^{4} P(D_1 = i) \cdot P(D_2 = 5 - i \mid D_1 = i)$$

$$= \sum_{i=1}^{4} P(D_1 = i) \cdot P(D_2 = 5 - i)$$

$$= 4 \cdot \frac{1}{36} = 1/9$$

iPhone X

Three factories produce 50%, 30%, and 20% of iPhones

Their defective rates are 4%, 10%, and 5% respectively

What is the overall fraction of defective iPhones?

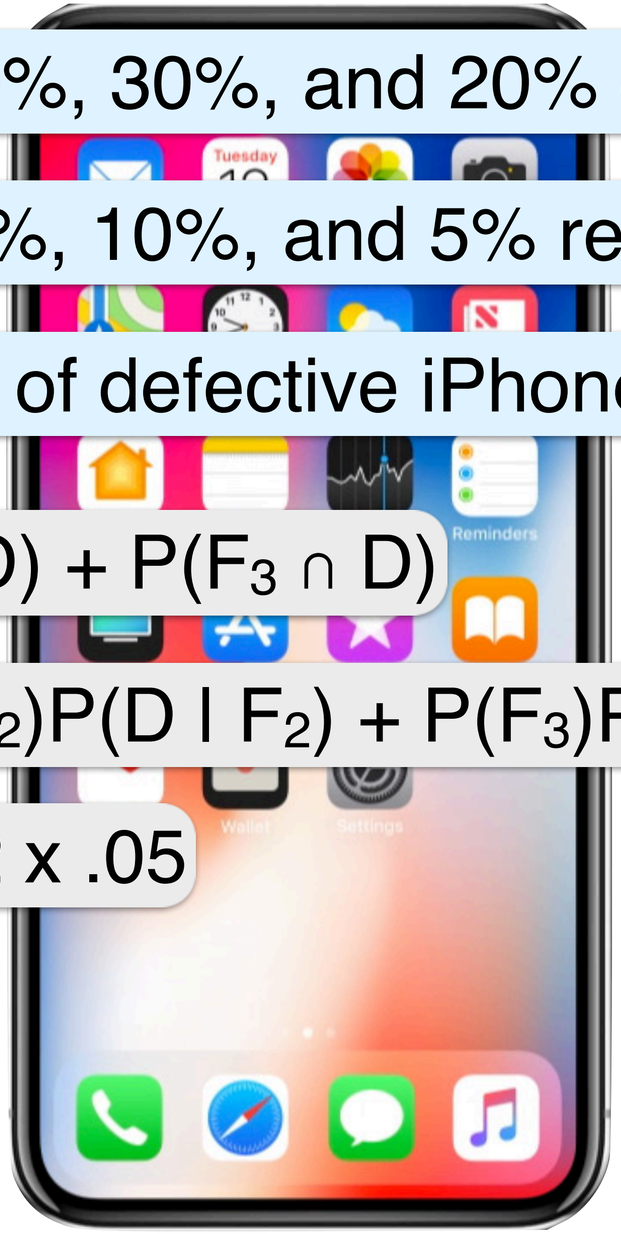
$$P(D) = P(F_1 \cap D) + P(F_2 \cap D) + P(F_3 \cap D)$$

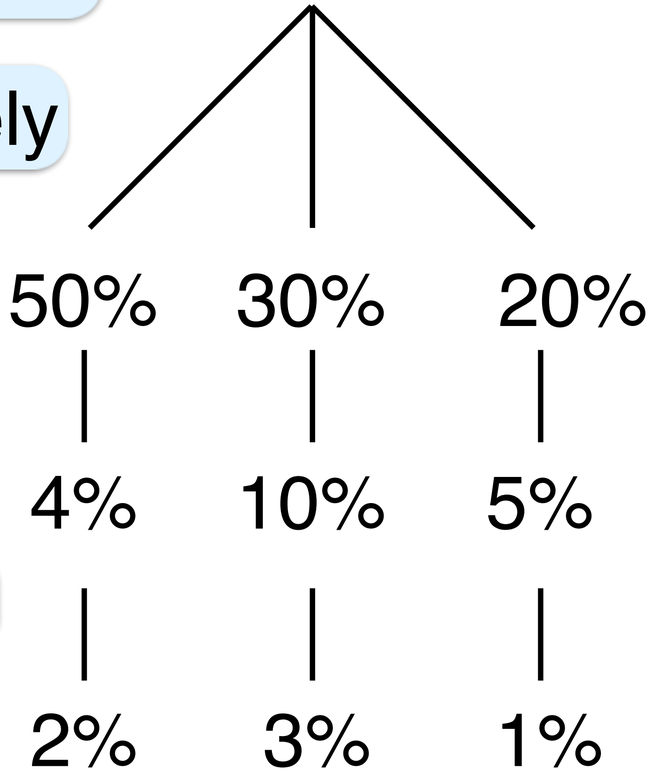
$$= P(F_1)P(D \mid F_1) + P(F_2)P(D \mid F_2) + P(F_3)P(D \mid F_3)$$

$$= .5 \times .04 + .3 \times .1 + .2 \times .05$$

$$= .02 + .03 + .01$$

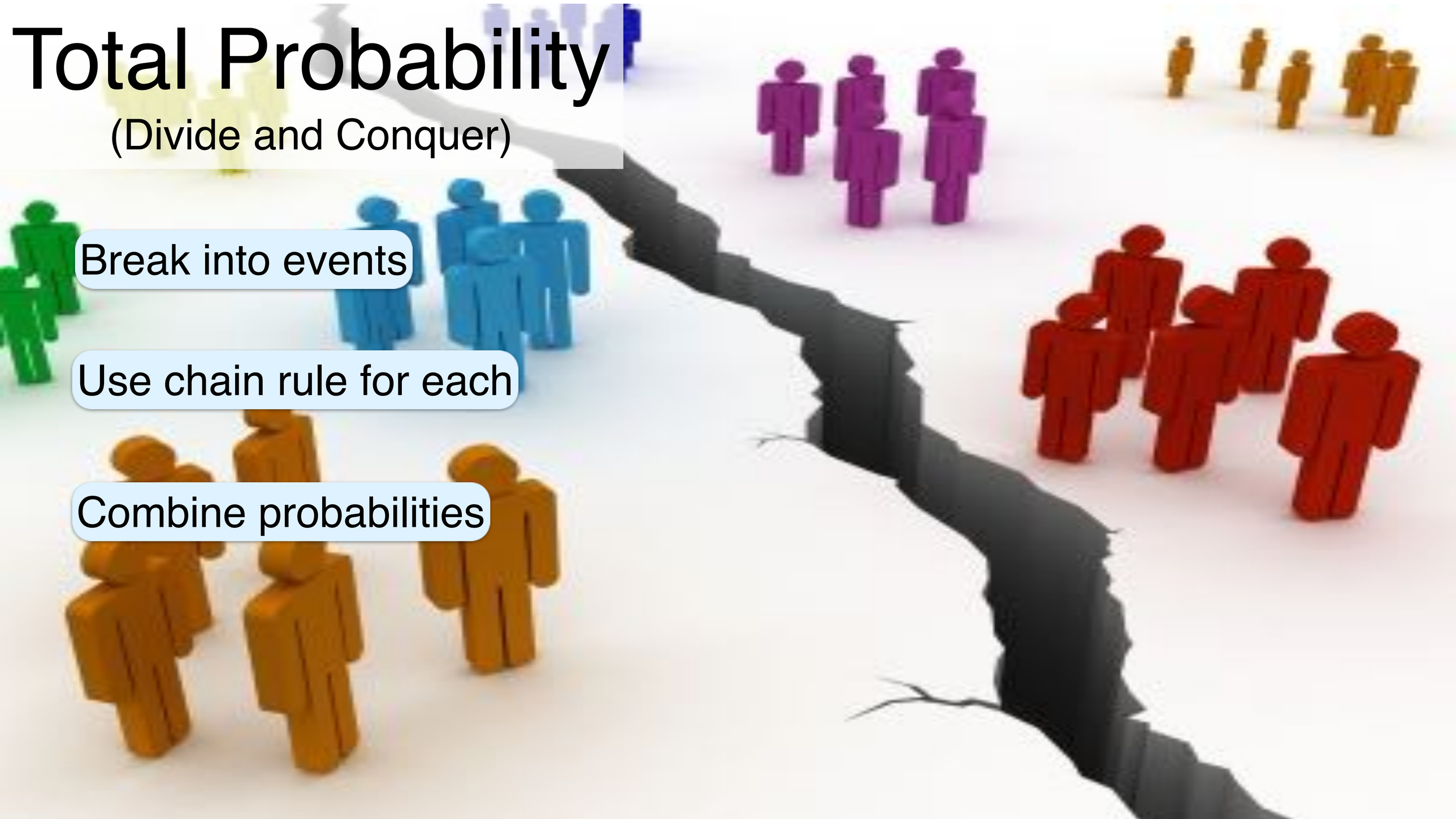
$$= .06$$





Total Probability

(Divide and Conquer)



Break into events

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Combine probabilities

This lecture: Total Probability

Next: Bayes' Rule