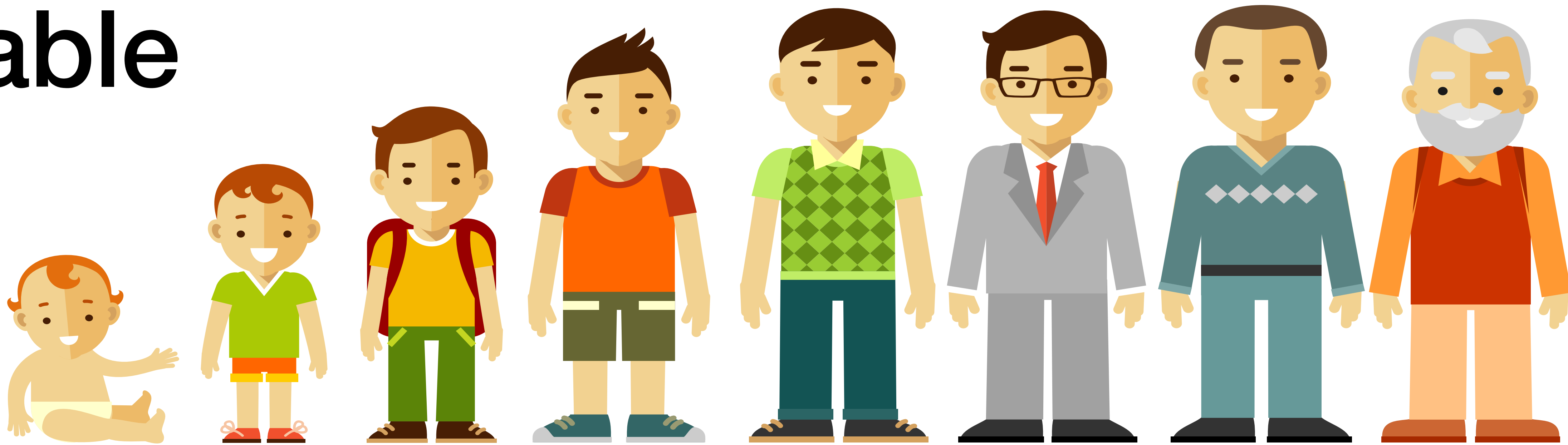


Variable



Modifications (aka functions)



Functions of a Random Variable

Random variables X take values in \mathbb{R}

Often interested in related variable $Y = g(X)$ $g:\mathbb{R} \rightarrow \mathbb{R}$ is a fixed function

X Random salary in \$

\$10 raise

$$Y = X + 10$$

10% raise

$$Y = 1.1X$$

→ CEO

$$Y = X^2$$

Deterministic Functions

$$Y = g(X)$$

g is a **deterministic** function over \mathbb{R} (or Ω)

$$Y = X + 3$$

All randomness in Y derives from X

Deterministically modified by g

$$X = 5$$

$$Y = 8$$

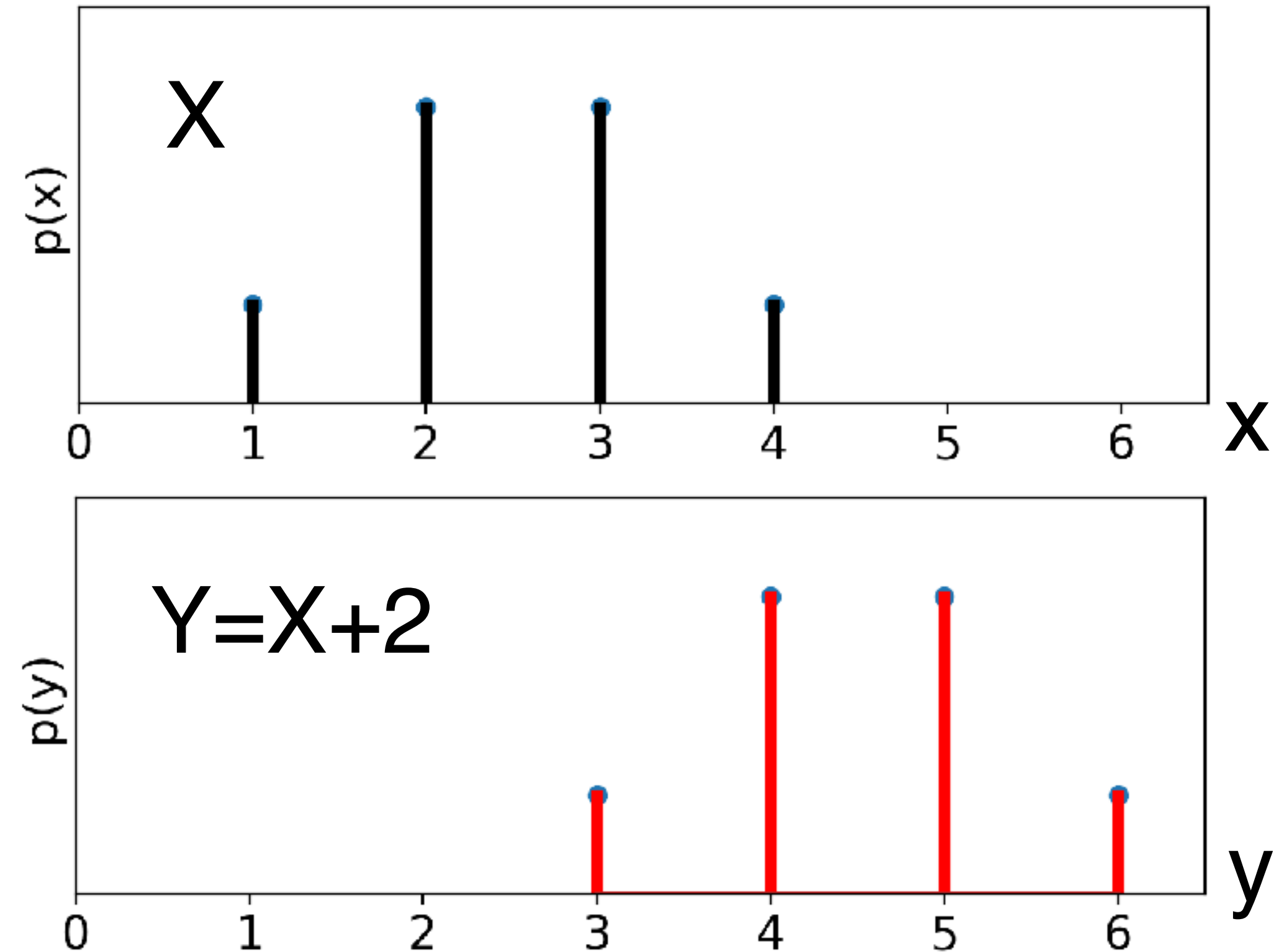
Translation

Add constant b to X

Translate X by b

$$Y = X + b$$

$$P(Y=y) = P(X+b=y) = P(X=y-b)$$



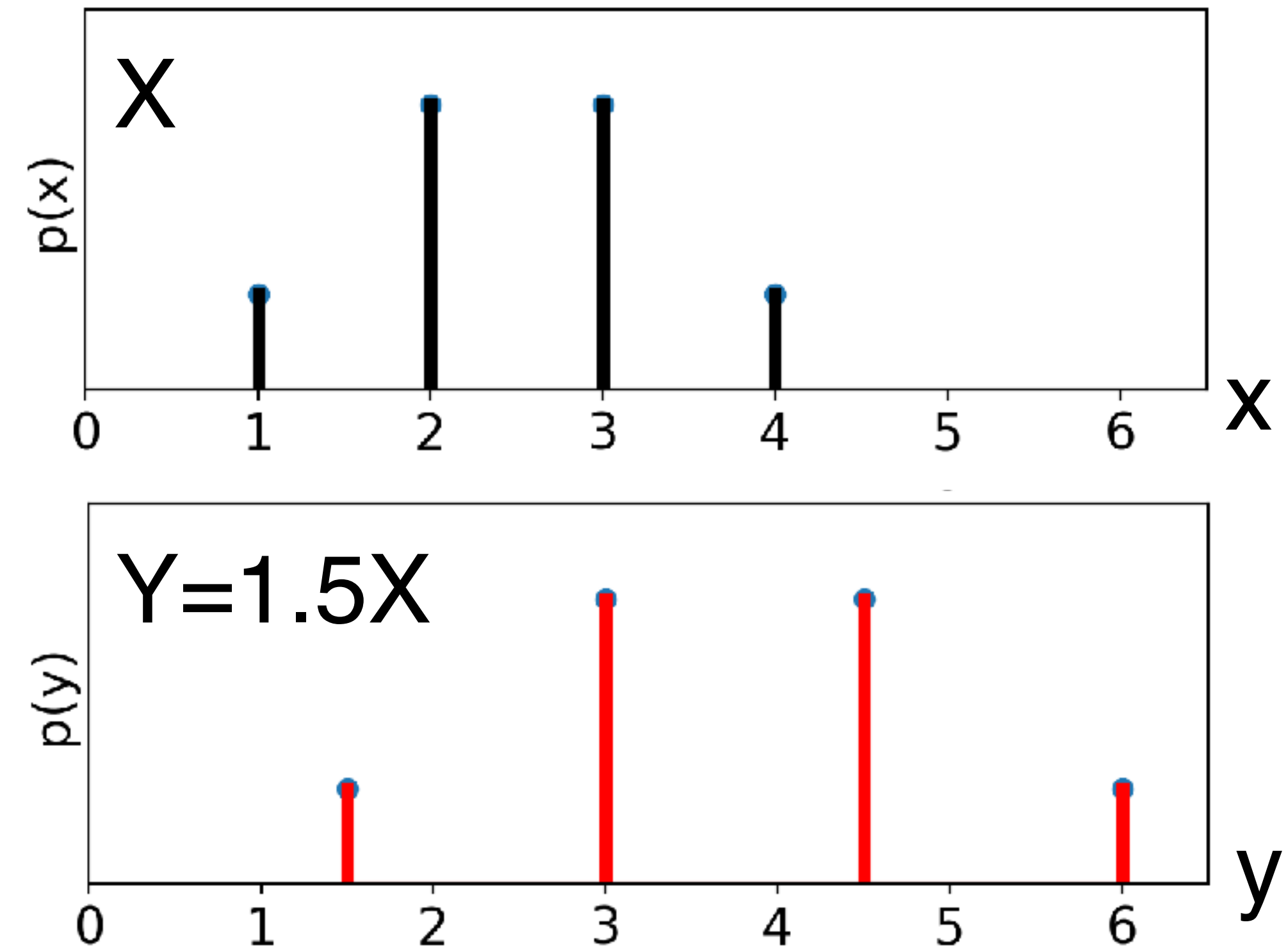
Scaling

Multiply X by a constant b

Scale X by a factor b

$$Y = b \cdot X$$

$$P(Y=y) = P(bX=y) = P(X=y/b)$$



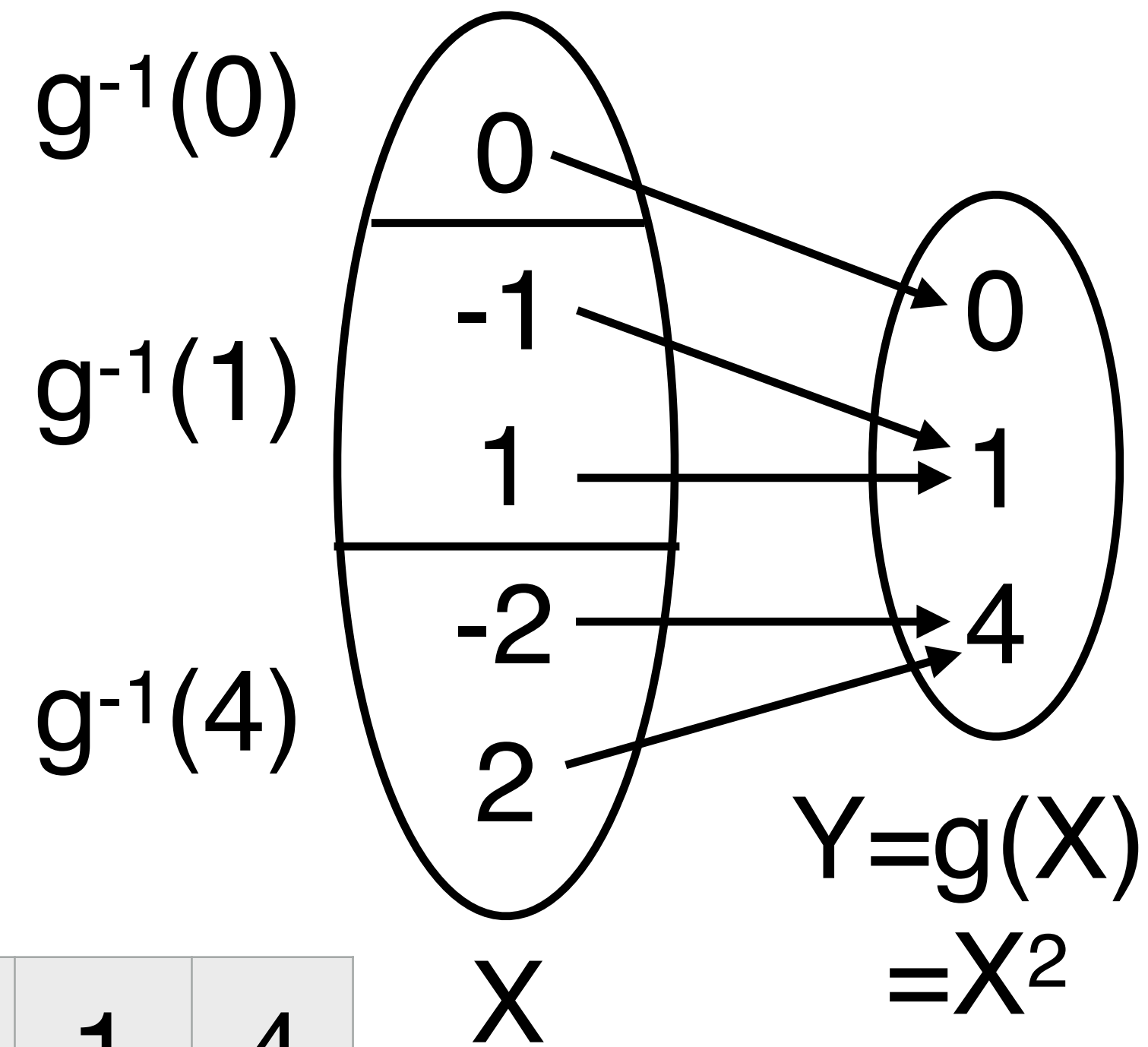
Two Square Examples

Square is 1-1

X	x	0	1	2
	$p(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$$Y = X^2$$

y	0	1	4
$p(Y = y)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$



Square is many to 1

X	x	-2	-1	0	1	2
	$p(X = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$$Y = X^2$$

y	0	1	4
$p(Y = y)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

$$P(Y=y) = P(g(X)=y) = P(X \in g^{-1}(y)) = \sum_{x \in g^{-1}(y)} P(X=x)$$

Variable



Modifications (aka functions)



Expectation of
modification

