Chebyshev's ...

- Motivation, intuition
- Formulation
- Proof
- Example

Markov → Chebyshev

Markov

Probability that non-neg X is α times larger than its mean is $\leq 1/\alpha$

Chebyshev

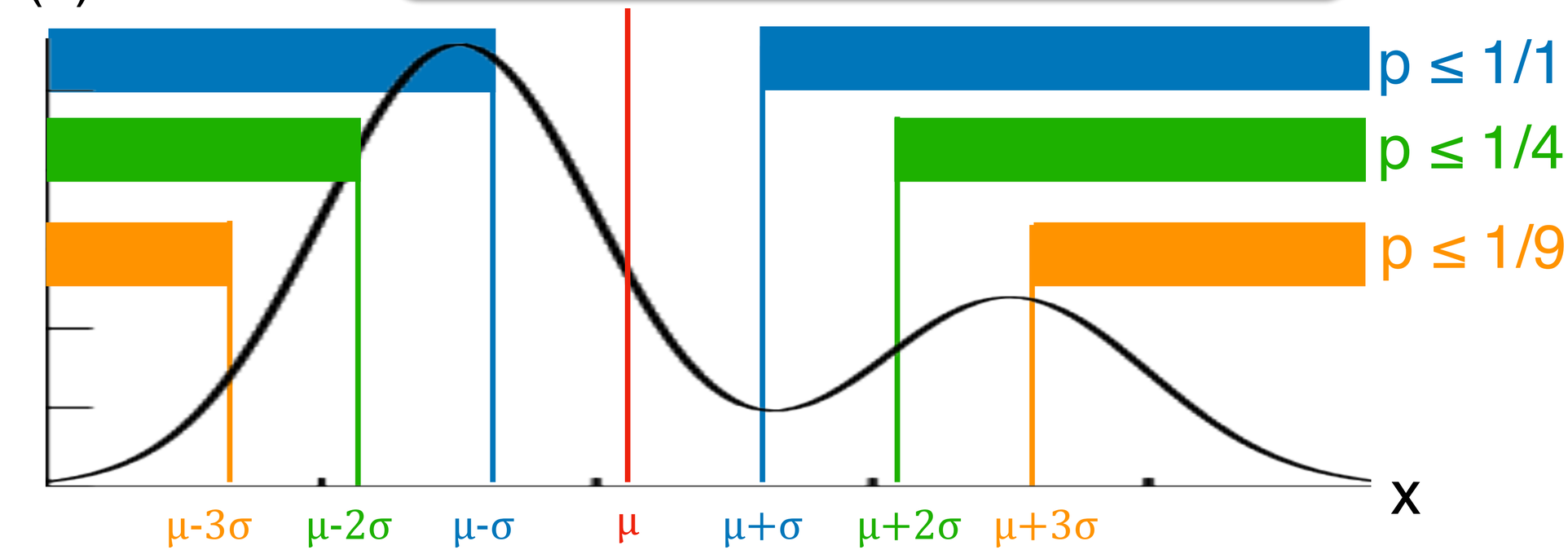
Probability that any X is more than $\alpha\sigma$ times further from μ is $\leq 1/\alpha^2$

Chebyshev's Inequality

Same two versions | 1 | Easier to illustrate, understand, remember

X is any discrete or continuous r.v. with finite mean μ and std σ

$$\forall\ \alpha \geq 1 \qquad P(|X-\mu| \geq \alpha\sigma) \leq \frac{1}{\alpha^2}$$



Two Formulations

X is any discrete or continuous r.v. with finite mean μ and std σ

1 Easier to visualize, understand, remember

$$\forall\ \alpha \geq 1 \qquad P(|X - \mu| \geq \alpha\sigma) \leq \frac{1}{\alpha^2}$$

2 Easier to prove, use $a = \alpha\sigma$

$$\forall\ a \geq \sigma \qquad P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

Towards a Proof

Markov's Inequality

$$\forall\ a \geq \mu \qquad P(X \geq a) \leq \frac{\mu}{a}$$

Chebyshev's Inequality

$$P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

$$P((X - \mu)^2 \geq a^2) \leq \frac{\sigma^2}{a^2}$$

Need: nonnegative r.v. with mean σ^2

Proof

$$P(|X - \mu_X| \geq a) \leq \frac{\sigma_X^2}{a^2}$$

Y Soon

X - any random variable $\mu_X = EX$ $\sigma_X^2 = V(X) = E(X - \mu_X)^2$

$Y = (X - \mu_X)^2$ $Y \geq 0$ $\mu_Y = E(X - \mu_X)^2 = \sigma_X^2$

$$P(|X - \mu_X| \geq a) = P((X - \mu_X)^2 \geq a^2)$$

$$= P(Y \geq a^2)$$

Markov

$$\leq \frac{\mu_Y}{a^2} = \frac{\sigma_X^2}{a^2}$$

Citations

X - # paper citations

$\mu = 8$

Suppose $\sigma = 5$

$P(X \geq 28)$?

Markov

$P(X \geq 28) \leq 8/28 \approx 29\%$

$$P(X \geq a) \leq \frac{\mu}{a}$$

Chebyshev

$P(X \geq 28) = P(X - \mu \geq 20)$

$\leq P(|X - \mu| \geq 20)$

$$P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

Markov: ≤ 0.02%

$\leq (\frac{\sigma}{20})^2 = (\frac{5}{20})^2 = \frac{1}{16} \approx 6.3\%$

$P(X \geq 40{,}000) = P(X - \mu \geq 39{,}992)$

$\leq P(|X - \mu| \geq 39{,}992)$

$\leq (\frac{\sigma}{39{,}992})^2 = (\frac{5}{39{,}992})^2 = 1.6 \times 10^{-6}\ \%$

Survey Responses

Survey expected to result in $\mu = 1M$ responses with $\sigma = 50K$

Bound $P(0.8M < \#\ \text{responses} < 1.2M)$

$0.8M = \mu - 4\sigma$ $\quad$ $1.2M = \mu + 4\sigma$

$P(\mu - 4\sigma < X < \mu + 4\sigma) = P(|X - \mu| < 4\sigma)$

$= 1 - P(|X - \mu| \geq 4\sigma)$

$\geq 1 - 1/16$

$= 15/16$

Mark x Che

	Formula	Applies	Input	Range	Decreases
Markov	$P(X \geq a) \leq \frac{\mu}{a}$	X≥0	μ	a ≥ μ	Linearly
Chebyshev	$P(\|X - \mu\| \geq a) \leq \frac{\sigma^2}{a^2}$	Any X	μ and σ	a ≥ σ	Quadratically

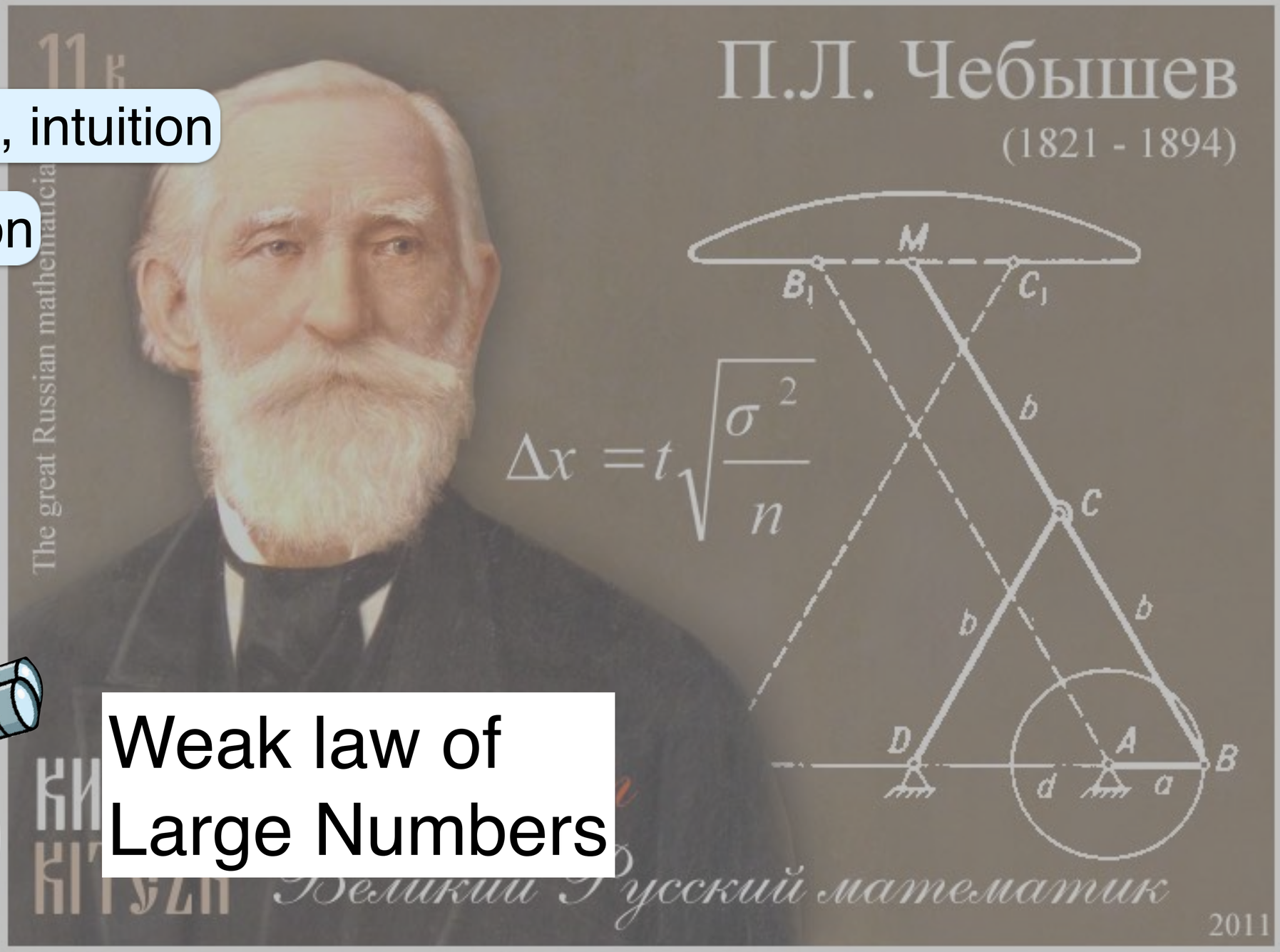
Motivation, intuition

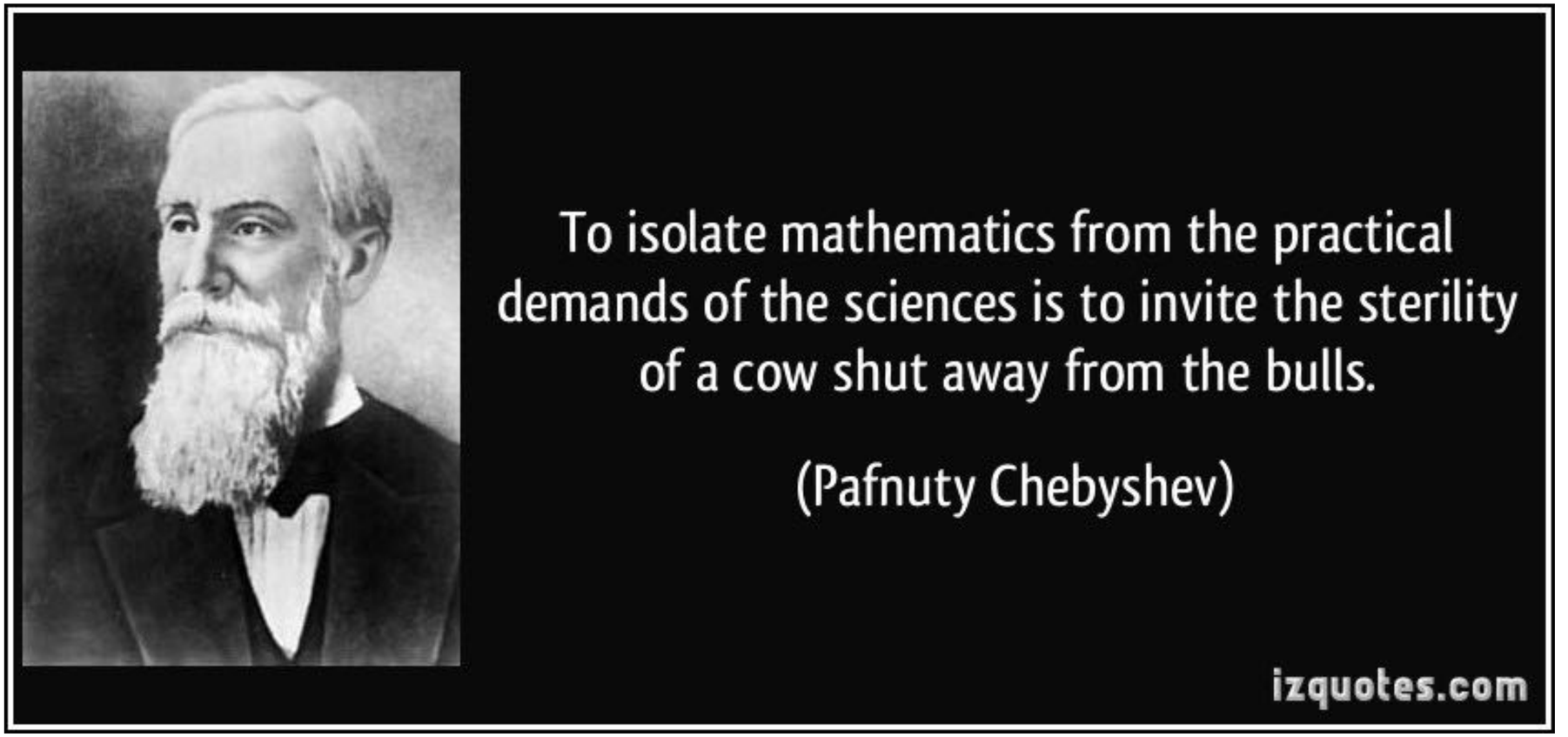
Formulation

Proof

Example

Weak law of Large Numbers





Alternative spellings: *Chebychev*, *Chebysheff*, Chebychov, *Chebyshov*;*Tchebychev*, *Tchebycheff*, *Tschebyschev*, *Tschebyschef*, *Tschebyscheff*