Image processing II
feature extraction

Feb. 4, 2025

Today's Class

- Notes from the first project
- Review: image processing and edge detection
- Feature extraction

Project 1 observations

- Add details to support your design decisions
  - Be precise, concise, and to the point.
- Go beyond what is needed
- Rubric review

	10-9	9-8	8-7	7-6	6 or less
Solution	Outstanding. Solution considers all aspects of the problem and provides an innovative design that shows deep understanding of the tools and methods used in machine vision	Excellent. Considered all aspects of the problem and provided a solid design that shows very good understanding of the tools and methods used in machine vision. One mistake	Very Good. Solution considers most aspects of the problem and provides a good design that shows good understanding of the tools and methods of machine vision. Multiple mistakes were made	Good. Several mistakes are made. Solution doesn't consider important aspects of the problem but generally correct. Provided design shows a basic understanding of the tools and methods used in machine vision.	Poor or wrong solution that ignores the problem defenition. Superficial understanding of machine vision tools and methods
Question 1 (70%)		9			
Question 2&3 (20%) DOF and Cell camera				6	
Question 4 (10%)				6	
Total for solution (90%)			7.29		
Presentation and write up (10%)	Professional and clear		Good but suffers from lack of solution details		Poor presentation. Hard to understand the solution
presentation mark	9				
Total			8.2		

Project 2 rubric

	10-9	9-8	8-7	7-6	6 or less	Grade
Tasks completed (40%)	Completed all required tasks in the assignment.	Completed at least 75% of required tasks	Completed at least 50% of required tasks	Completed at least 30% of required tasks	Completed less than 30% of required tasks	10
Quality of Solution (30%)	Outstanding. Solution considers all aspects of the problem and provides an innovative robust design that is likely to work under different sources of noise and environmental conditions.	Excellent. Solution considers most aspects of the problem and provides a robust design that is likely to work under different sources of noise and environmental conditions.	Very Good. Solution considers most aspects of the problem and provides a good design that is based on heuristics. It requires specific assumptions to work.	Good. Solution doesn't consider important aspects of the problem but generally correct. Provided design shows a basic understanding of the tools and methods used in machine vision. Solution is very dependent on specific conditions	Poor or wrong solution that ignores the problem definition. Superficial understanding of machine vision tools and methods. Solution is ad hoc and arbitrary.	9.5
Discussion of solution (30%)	Provide an excellent discussion of the results that explores different solutions and design options. Point to limitation(s) of the proposed solution and possible errors. Find and use external sources beyond the ones provided in class. Provide a brief but excellent review of existing related literature.				No discussion of results is provided. Doesn't provide any details on the rational behind different design steps or components. Use external machine vision tools without understanding limitations and appropriateness.	5
Total for solution (90%)						8.35
Presentation and write up (10%)	Professional and clear		Good but suffers from many writing and organizational mistakes		Poor presentation. Hard to understand the solution	7
Total						8.2

IMAGE OPERATIONS AND FILTERING

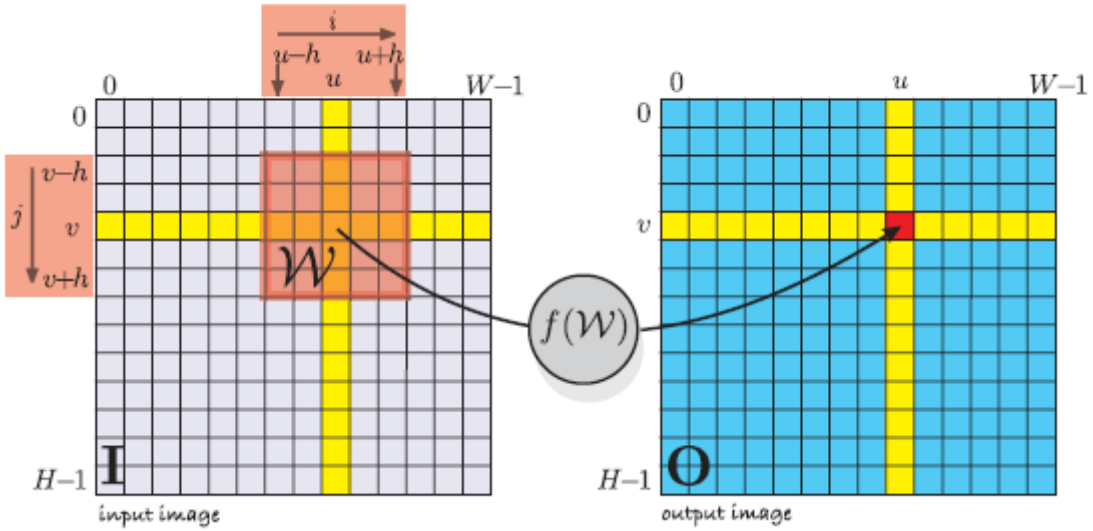
Review: Types of image processing operations

- Monadic
  - $O[u,v] = f(I[u,v]), \forall (u,v) \in I$
  - Example: color to grey
- Diadic
  - $O[u,v] = f(I_1[u,v], I_2[u,v]), \forall (u,v) \in I_i$
  - Example: extract background
- Image filters in spatial domain
  - $O[u,v] = f(I[u+i, v+j]), \forall (i,j) \in W, \forall (u,v) \in I$
- Templates and Image Pyramids

Review: Spatial operations (filters)

- $O[u,v] = f(I[u+i, v+j]), \forall (i,j) \in W, \forall (u,v) \in I$

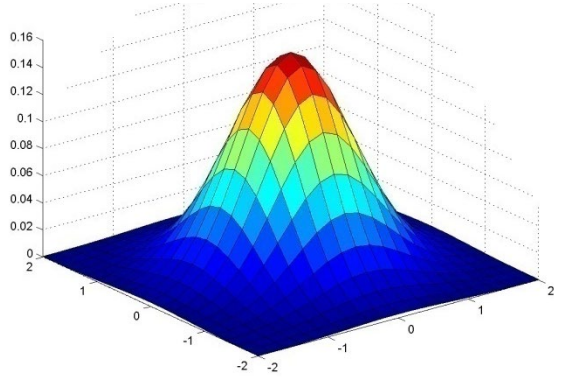
  Where *W* is the window, *w* x *w* square region

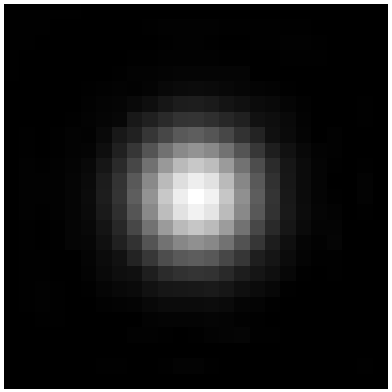


f(.) can be linear and non-linear functions

Important filter: Gaussian

- Weight contributions of neighboring pixels by nearness





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

5 x 5, $\sigma = 1$

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Slide credit: Christopher Rasmussen

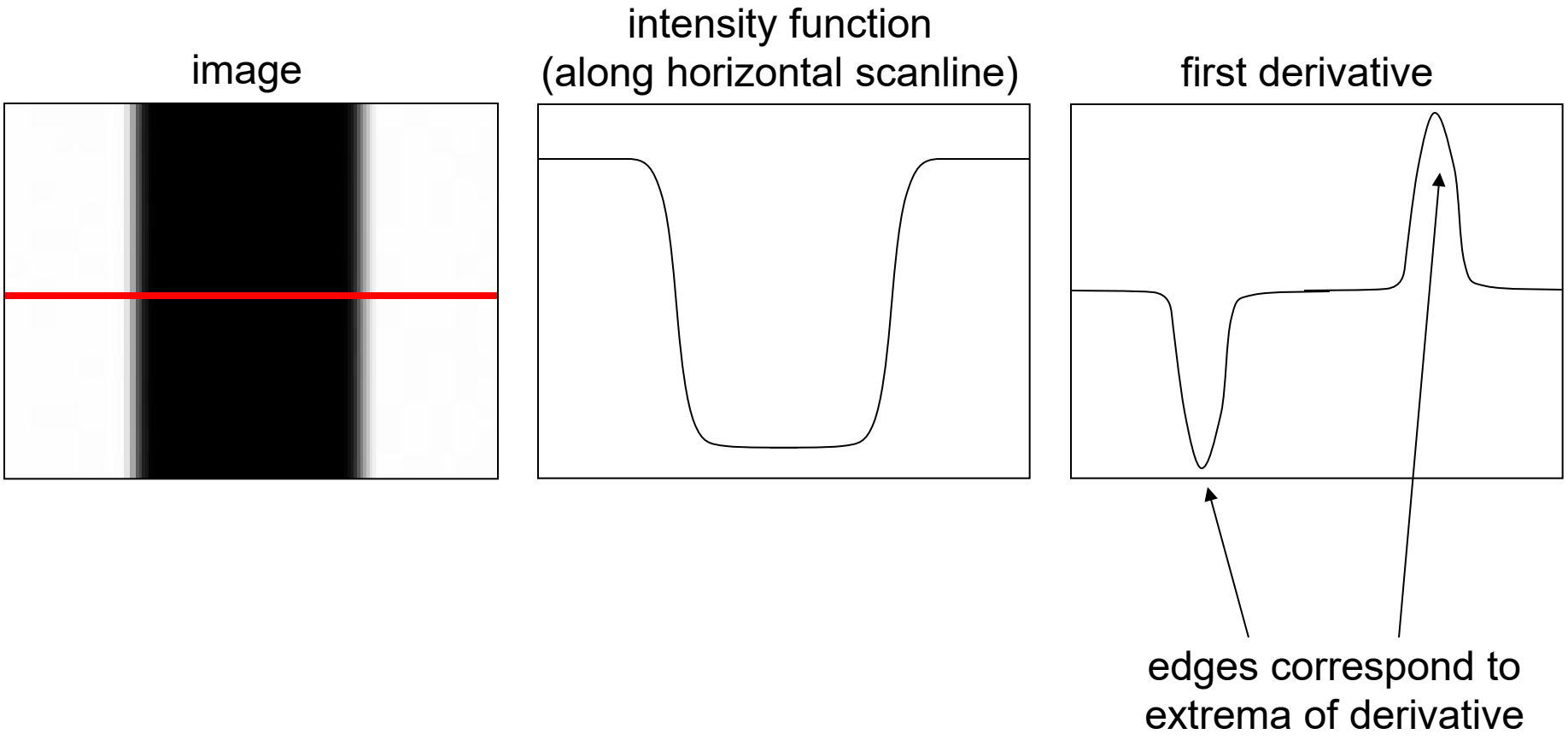
Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

Source: D. Lowe

Characterizing edges

- An edge is a place of rapid change in the image intensity function



Canny edge detector

- This is probably the most widely used edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of *signal-to-noise ratio* and localization

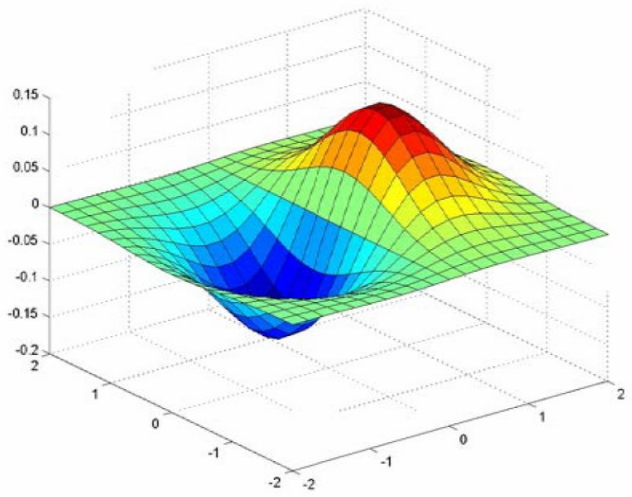
J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Source: L. Fei-Fei

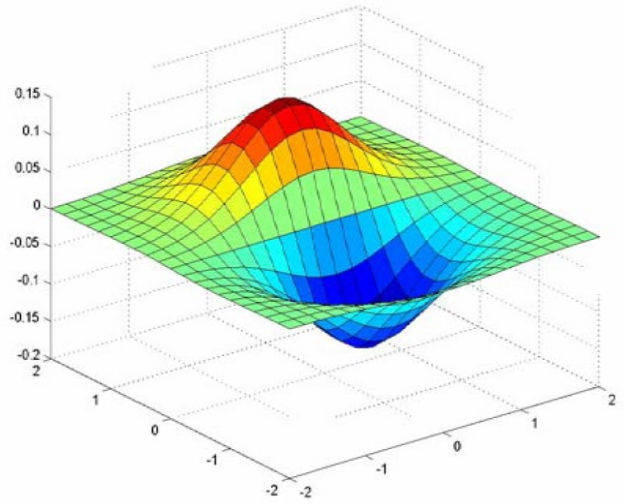
Example

original image (Lena)

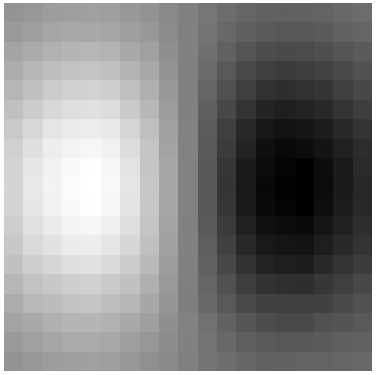
Derivative of Gaussian filter

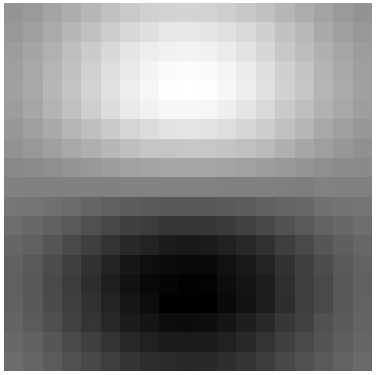


x-direction

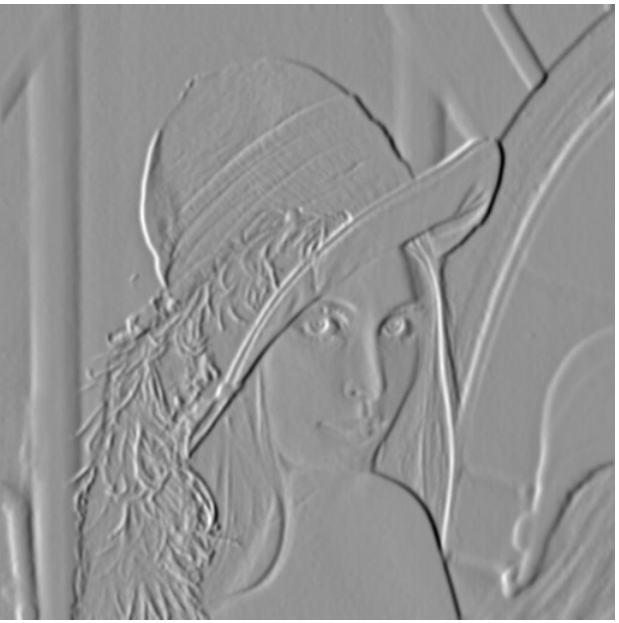


y-direction





Compute Gradients (DoG)



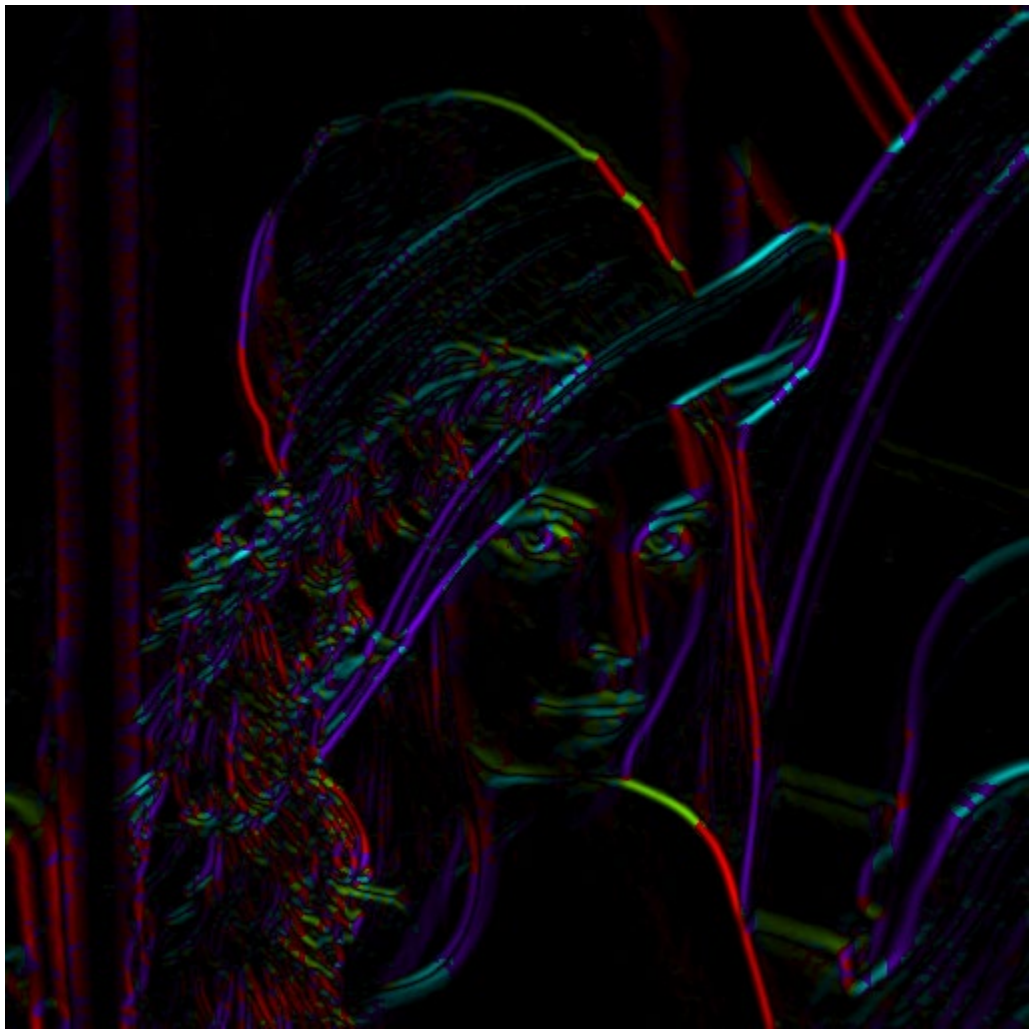
X-Derivative of Gaussian

Y-Derivative of Gaussian

Gradient Magnitude

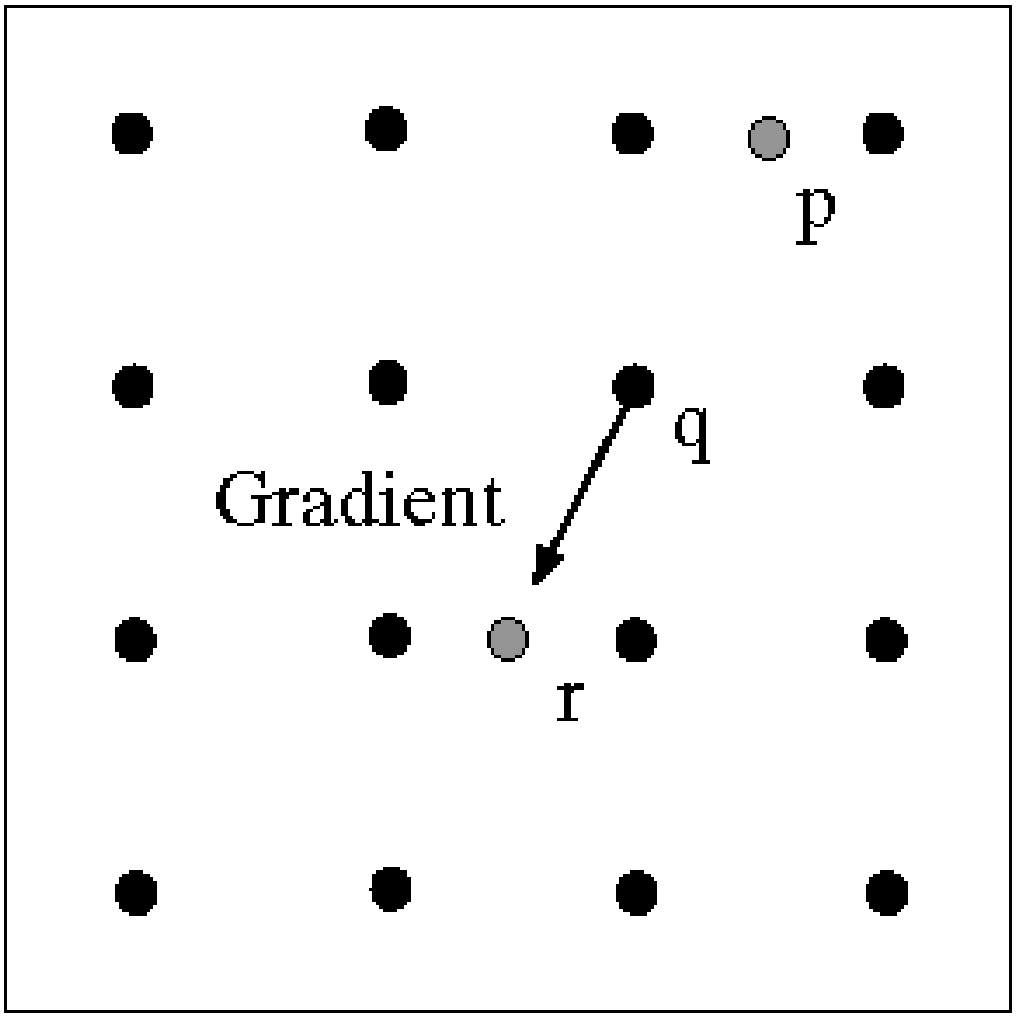
Get Orientation at Each Pixel

- Threshold at minimum level
- Get orientation

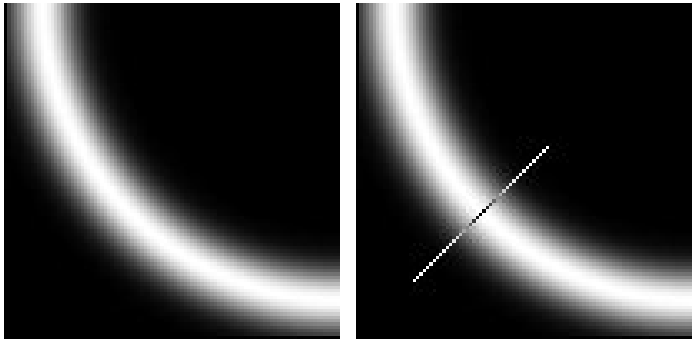


theta = atan2(gy, gx)

Non-maximum suppression for each orientation

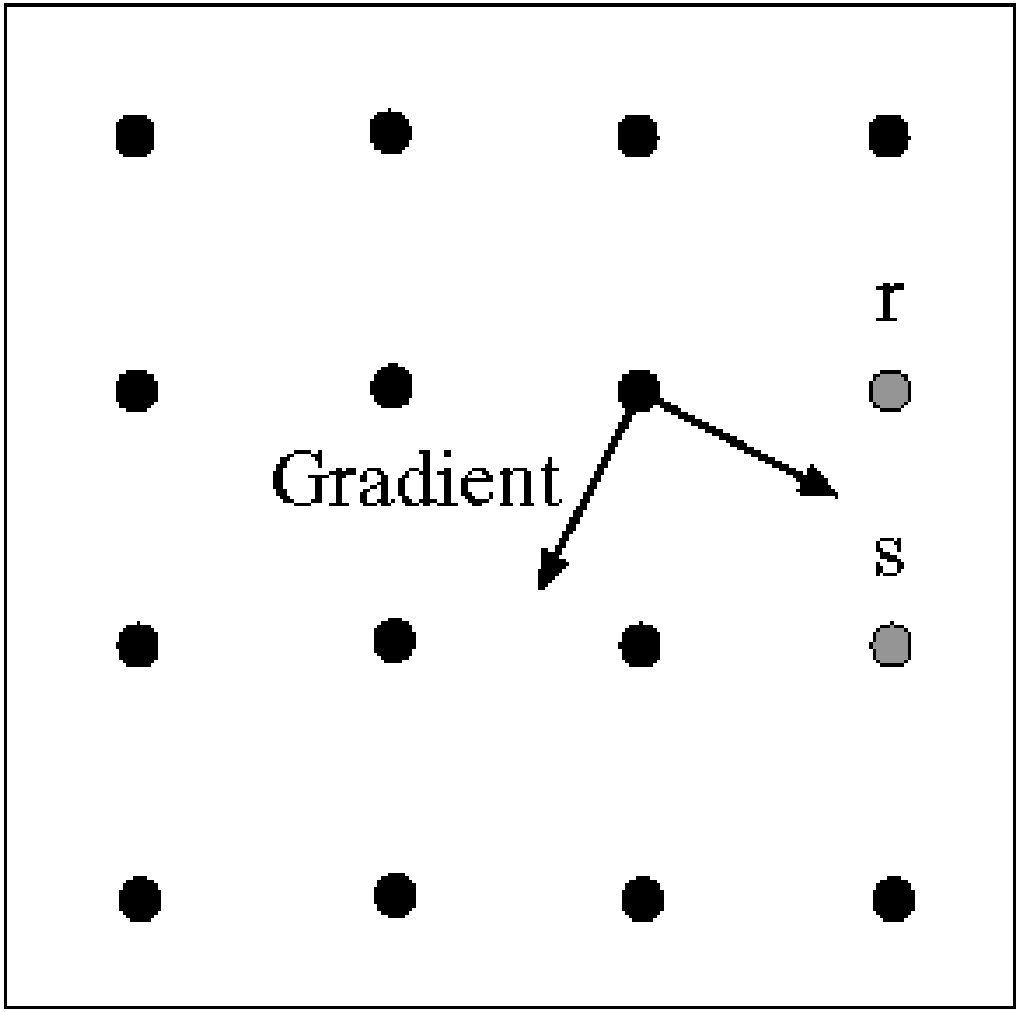


At q, we have a maximum if the value is larger than those at both p and at r. Interpolate to get these values.

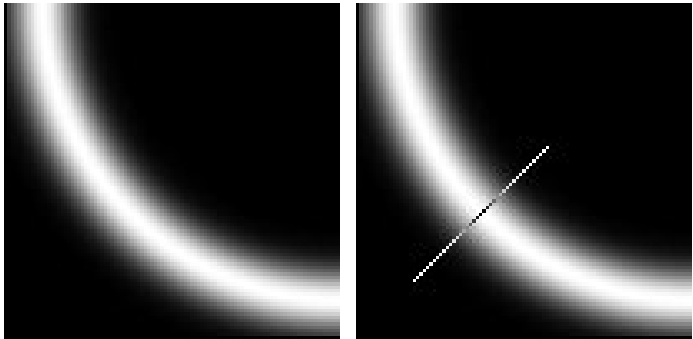


Source: D. Forsyth

Edge linking



Assume the marked point is an edge point. Then we construct the tangent to the edge curve (which is normal to the gradient at that point) and use this to predict the next points (here either r or s).



Source: D. Forsyth

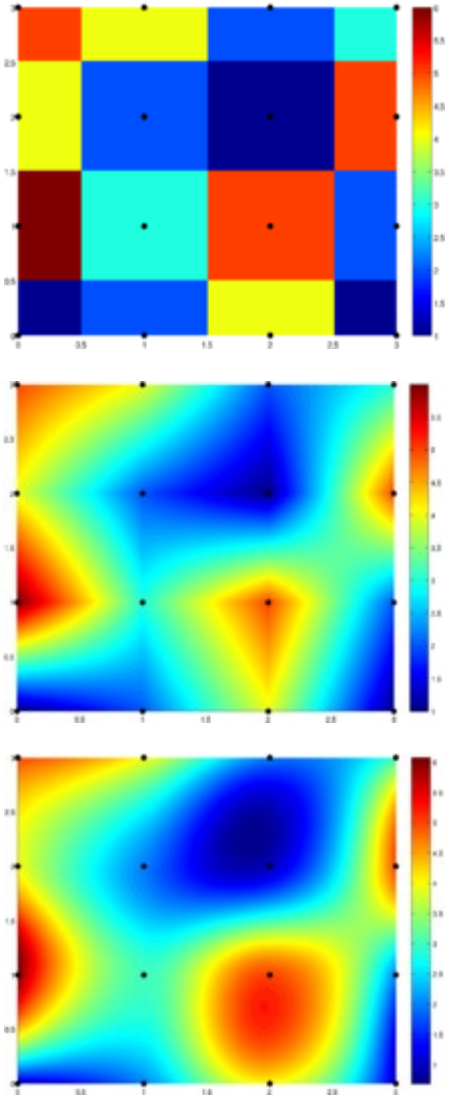
Sidebar: Interpolation options

- imx2 = imresize(im, 2, interpolation_type)

- ‘nearest’
  - Copy value from nearest known
  - Very fast but creates blocky edges

- ‘bilinear’
  - Weighted average from four nearest known pixels
  - Fast and reasonable results

- ‘bicubic’ (default)
  - Non-linear smoothing over larger area (4x4)
  - Slower, visually appealing, may create negative pixel values



Examples from http://en.wikipedia.org/wiki/Bicubic_interpolation

Before Non-max Suppression

After non-max suppression

Before Non-max Suppression

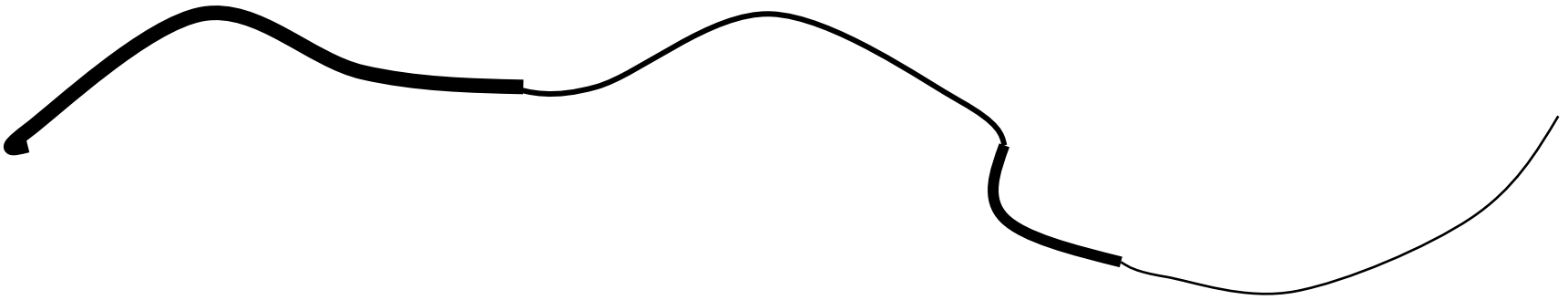
After non-max suppression

Hysteresis thresholding

- Threshold at low/high levels to get weak/strong edge pixels
- Do connected components, starting from strong edge pixels

Hysteresis thresholding

- Check that maximum value of gradient value is sufficiently large
  - drop-outs? use **hysteresis**
    - use a high threshold to start edge curves and a low threshold to continue them.



Source: S. Seitz

Final Canny Edges

Canny edge detector

1. Filter image with x, y derivatives of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin multi-pixel wide “ridges” down to single pixel width
4. Thresholding and linking (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: edge(image, ‘canny’)
- Vision toolbox: icanny(image, sigma)

Source: D. Lowe, L. Fei-Fei

Effect of σ (Gaussian kernel spread/size)

original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

Source: S. Seitz

The Laplacian operator

- Another alternative to calculate the second derivative
- Klaplace()

FEATURE EXTRACTION

Review

- Image processing: image -> image
- Feature extraction (segmentation): image -> image features or objects
- Information concentration or reduction step
  - Regions
  - Lines
  - Interest or keypoint

Region features

- Segment or extract regions that represent objects
- Three steps
  - Classify: assign pixels to classes (binary or more)
  - Represent: connect regions to form spatial labeled sets
  - Describe: size, position, shape, etc.

Binary classification

$$c[u,v] = \begin{cases} 0 & I[u,v] < t \\ 1 & I[u,v] \geq t \end{cases} \quad \forall (u,v) \in I$$

- Matlab example
  - Basic histogram thresholding
  - More advanced thresholding
    - N. Otsu, A Threshold Selection Method from Gray-Level Histograms, IEEE Trans. Systems, Man and Cybernetics, Vol SMC-9(1), Jan 1979, pp 62-66
    - W. Niblack, An Introduction to Digital Image Processing, Prentice-Hall, 1986.
    - J. Matas, O. Chum, M. Urban, and T. Pajdla, Robust wide-baseline stereo from maximally stable extremal regions", Image and Vision Computing, vol. 22, pp. 761-767, Sept. 2004.

LINE FEATURES

Hough Transform: Outline

1. Create a grid of parameter values

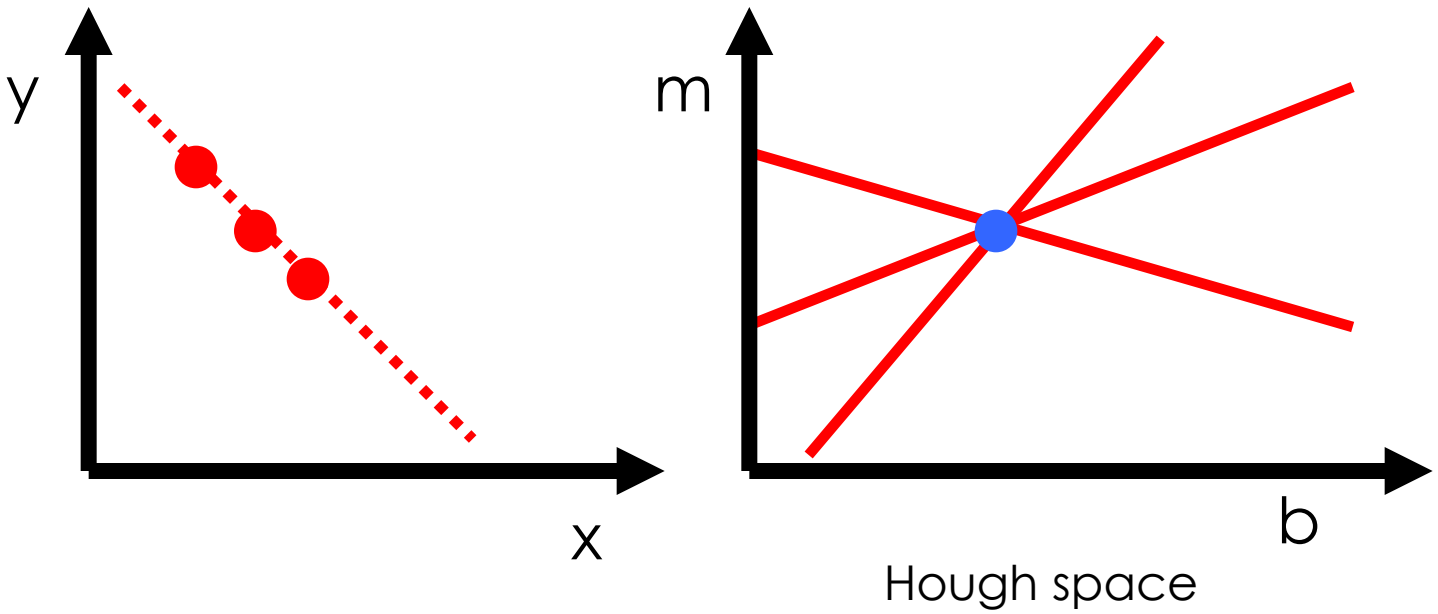
2. Each point votes for a set of parameters, incrementing those values in grid

3. Find maximum or local maxima in grid

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

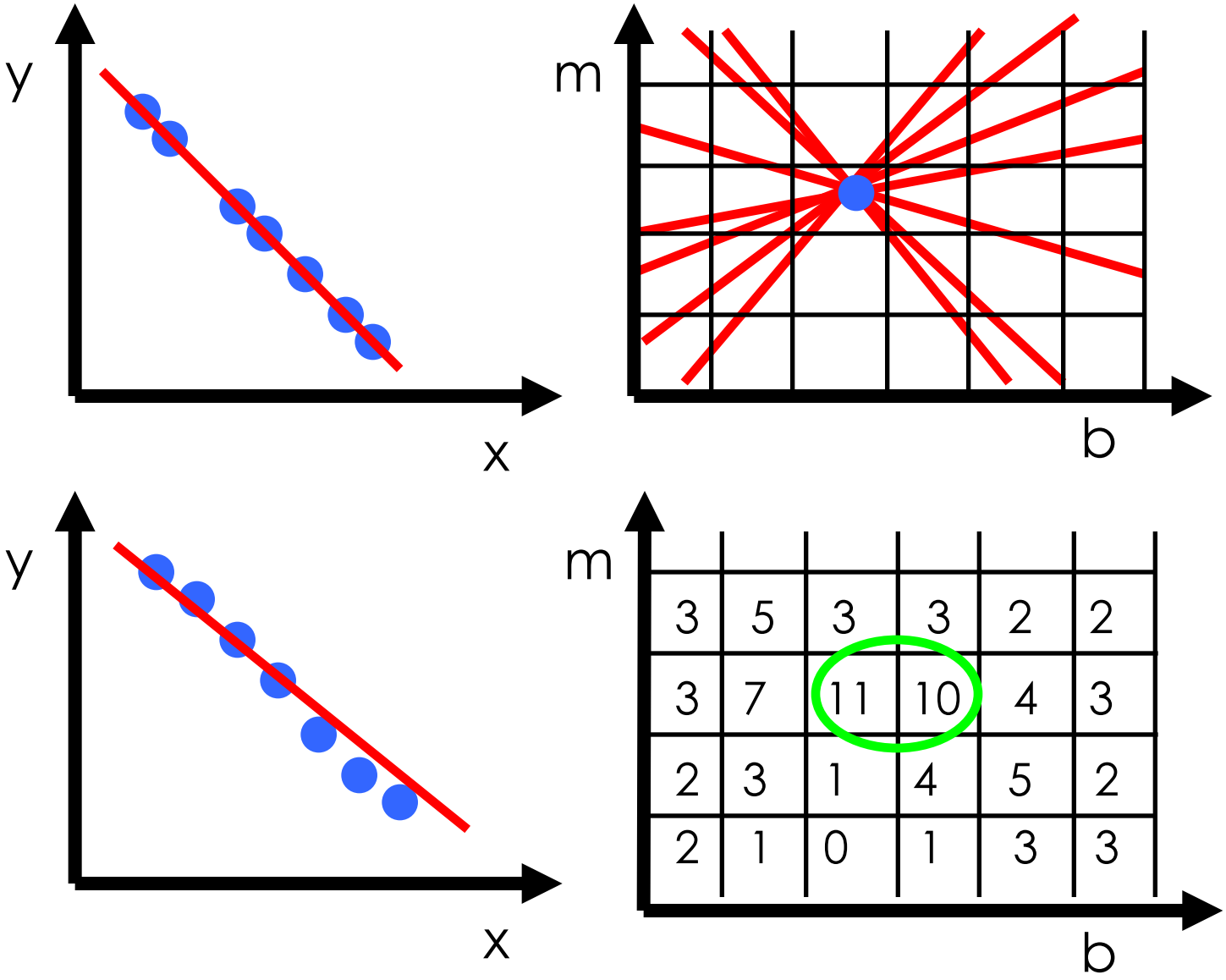
Given a set of points, find the curve or line that explains the data points best



$$y = m\,x + b$$

Slide from S. Savarese

Hough transform



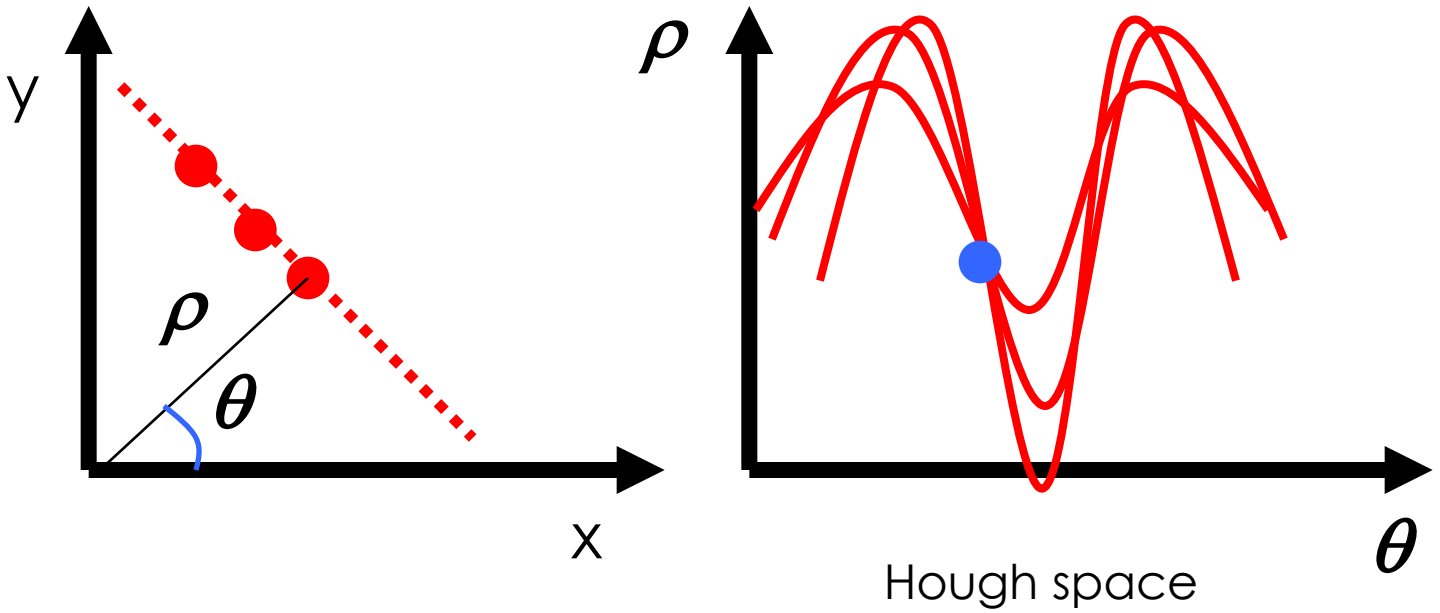
Slide from S. Savarese

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Issue : parameter space [m,b] is unbounded…

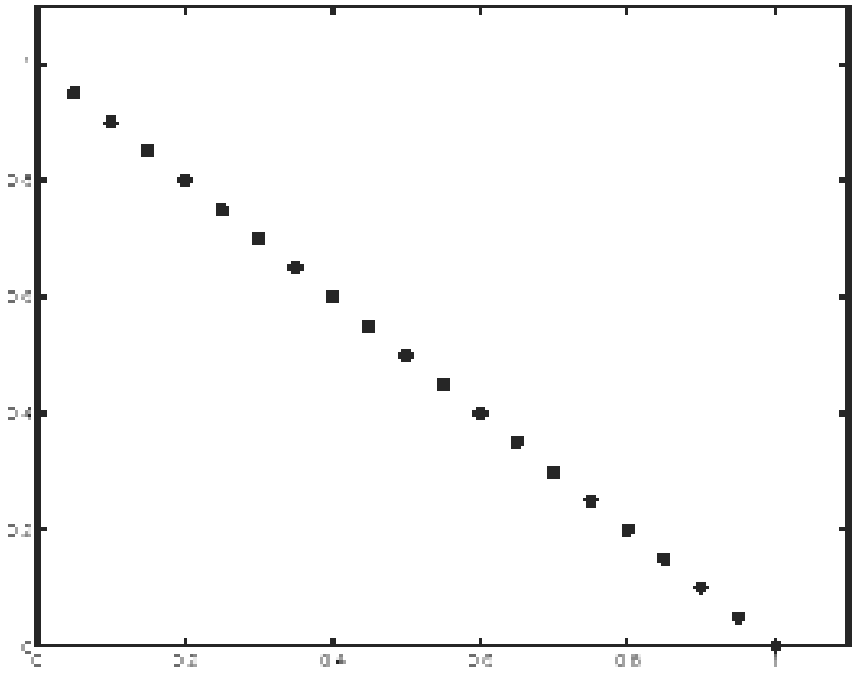
Use a polar representation for the parameter space



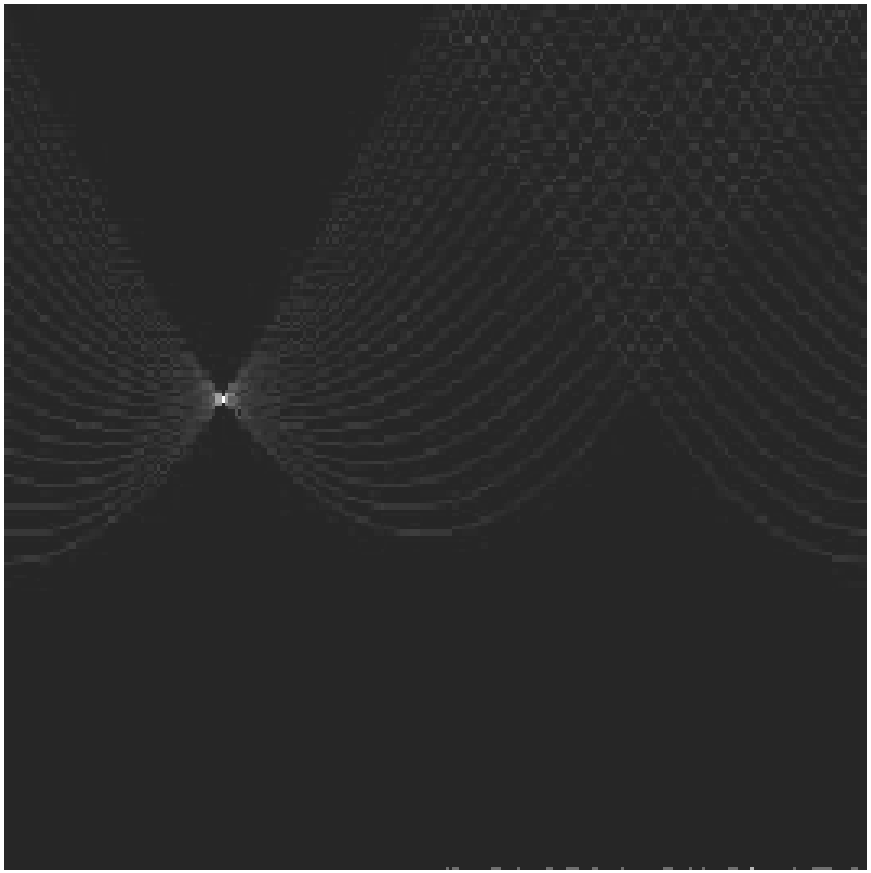
$$x\cos\theta + y\sin\theta = \rho$$

Slide from S. Savarese

Hough transform - experiments



features

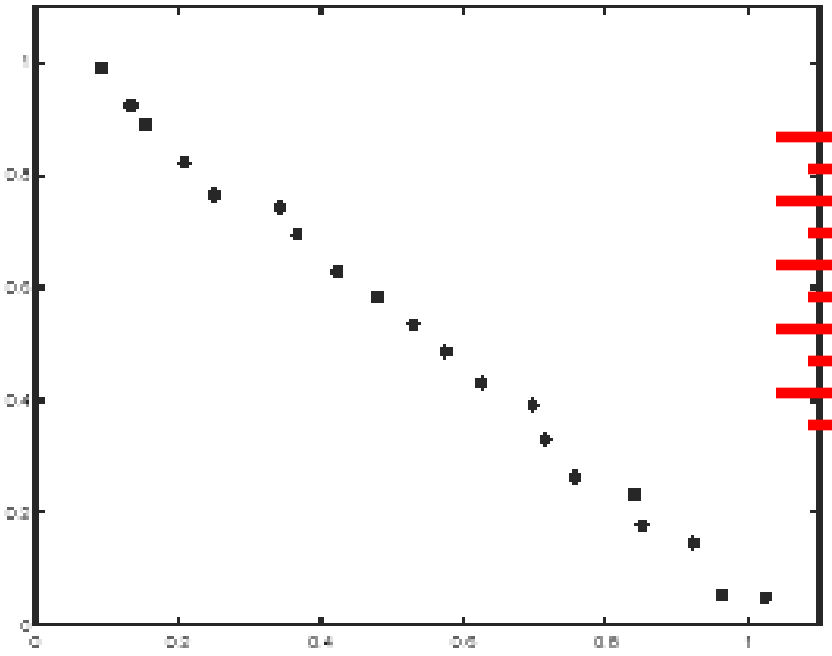


votes

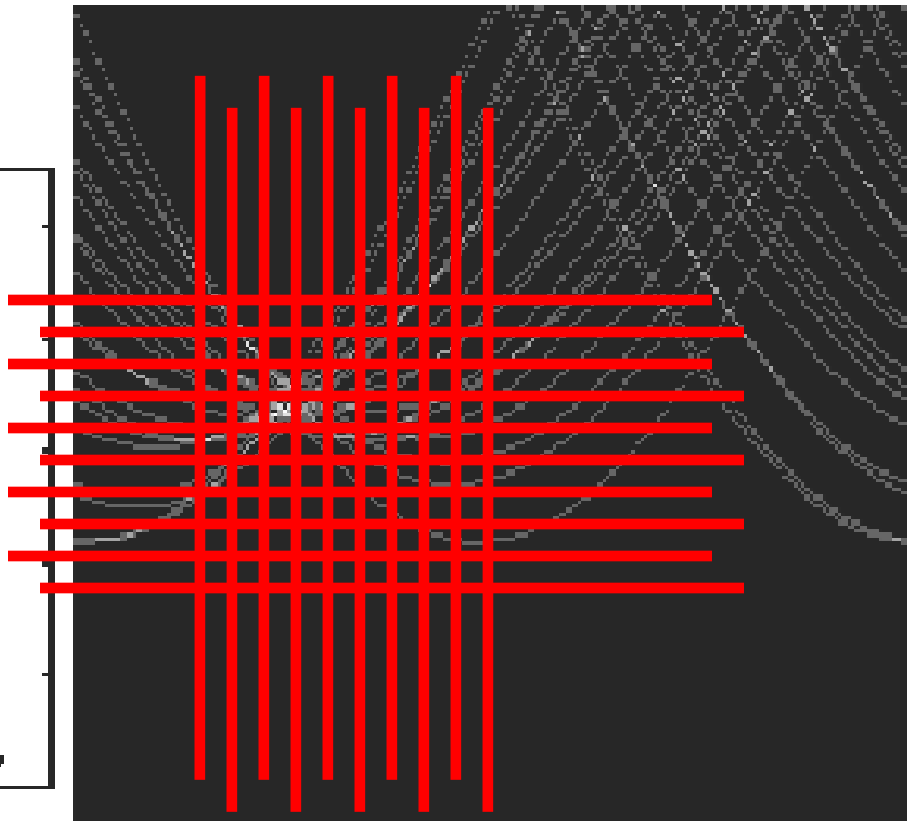
Slide from S. Savarese

Hough transform - experiments

Noisy data



features

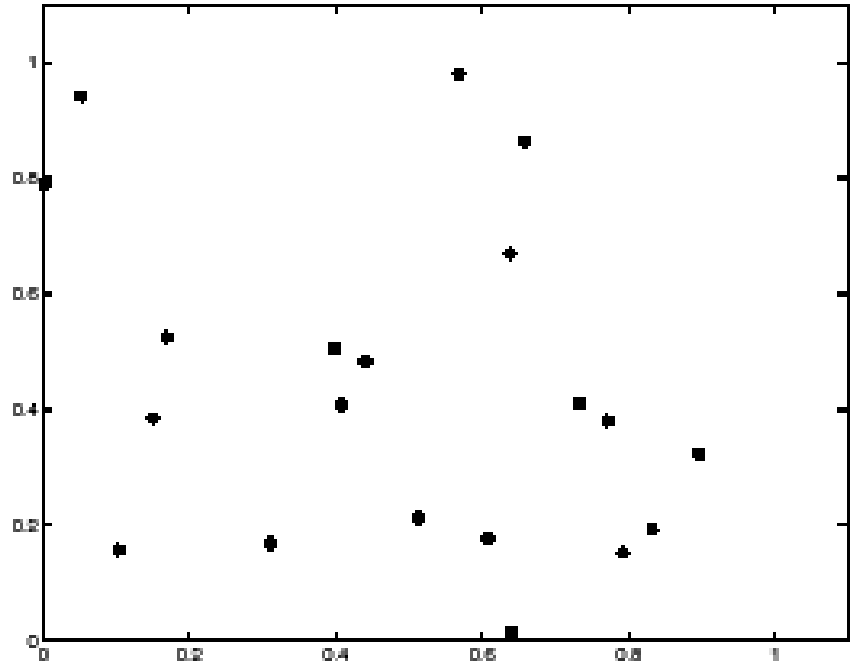


votes

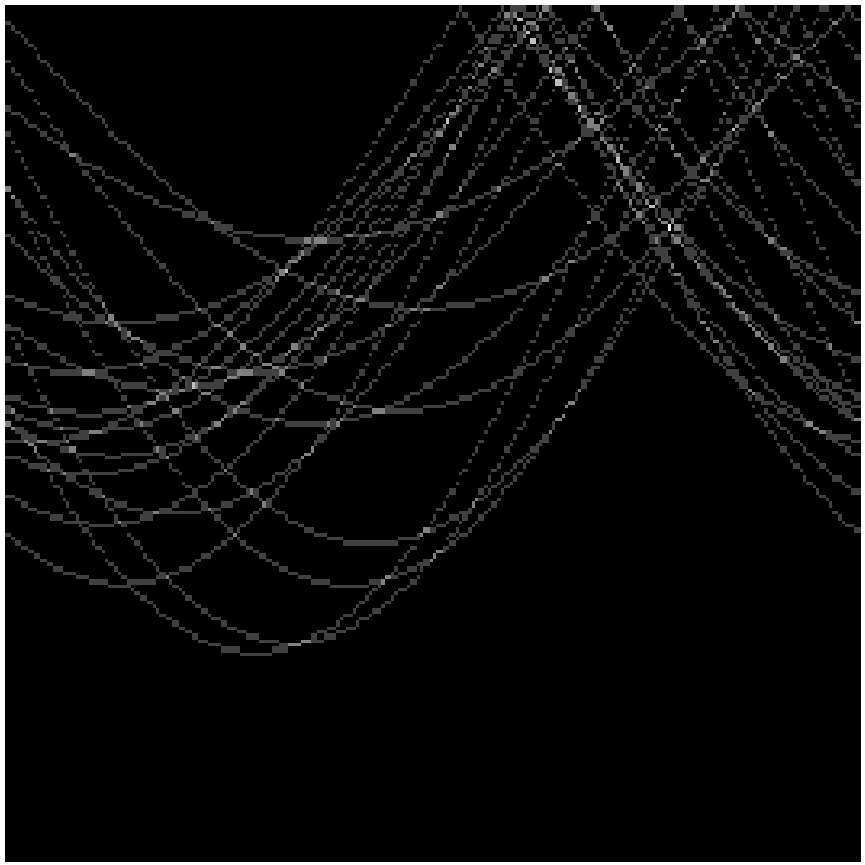
Need to adjust grid size or smooth

Slide from S. Savarese

Hough transform - experiments



features



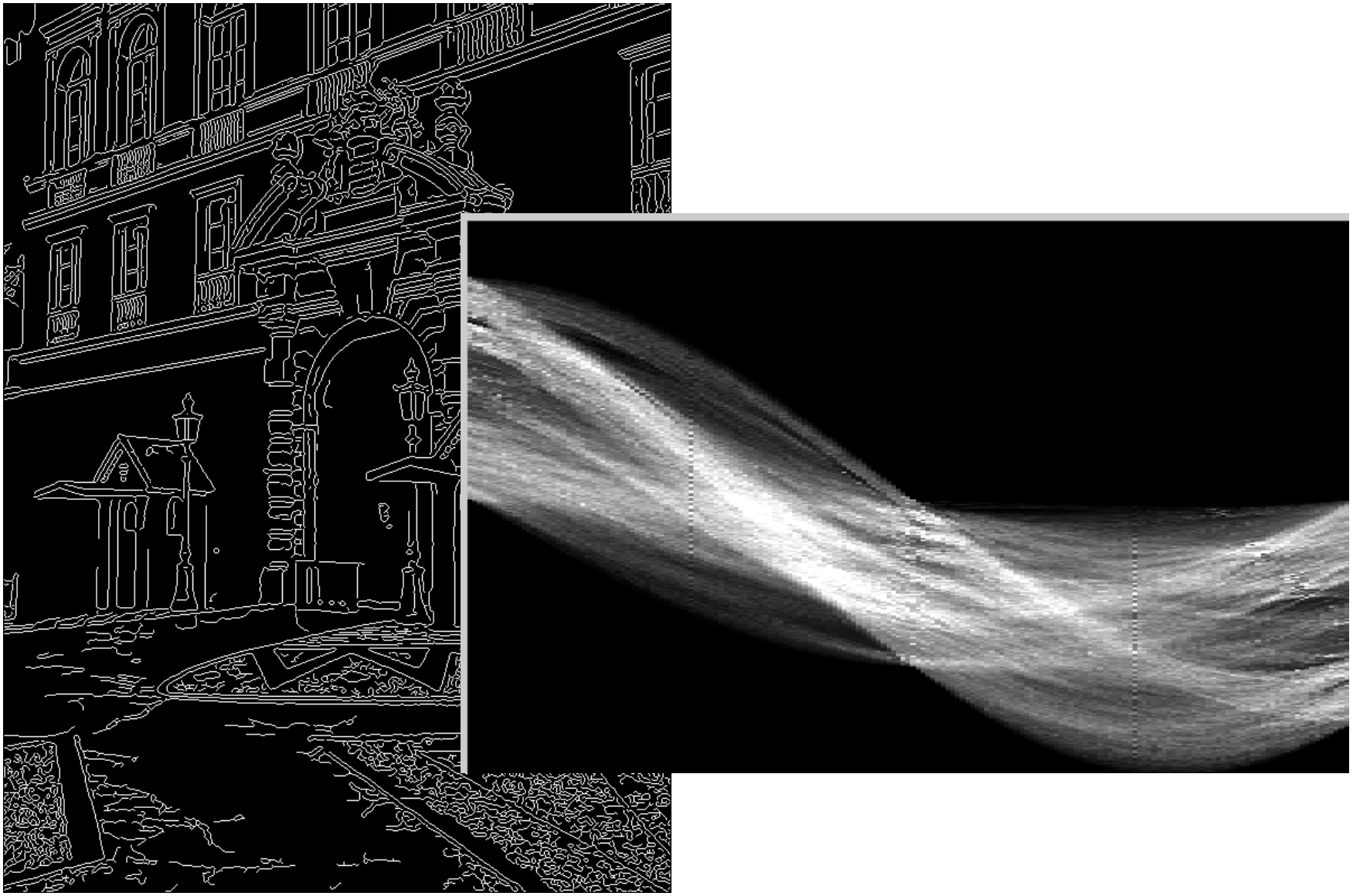
votes

Issue: spurious peaks due to uniform noise

Slide from S. Savarese

1. Image → Canny

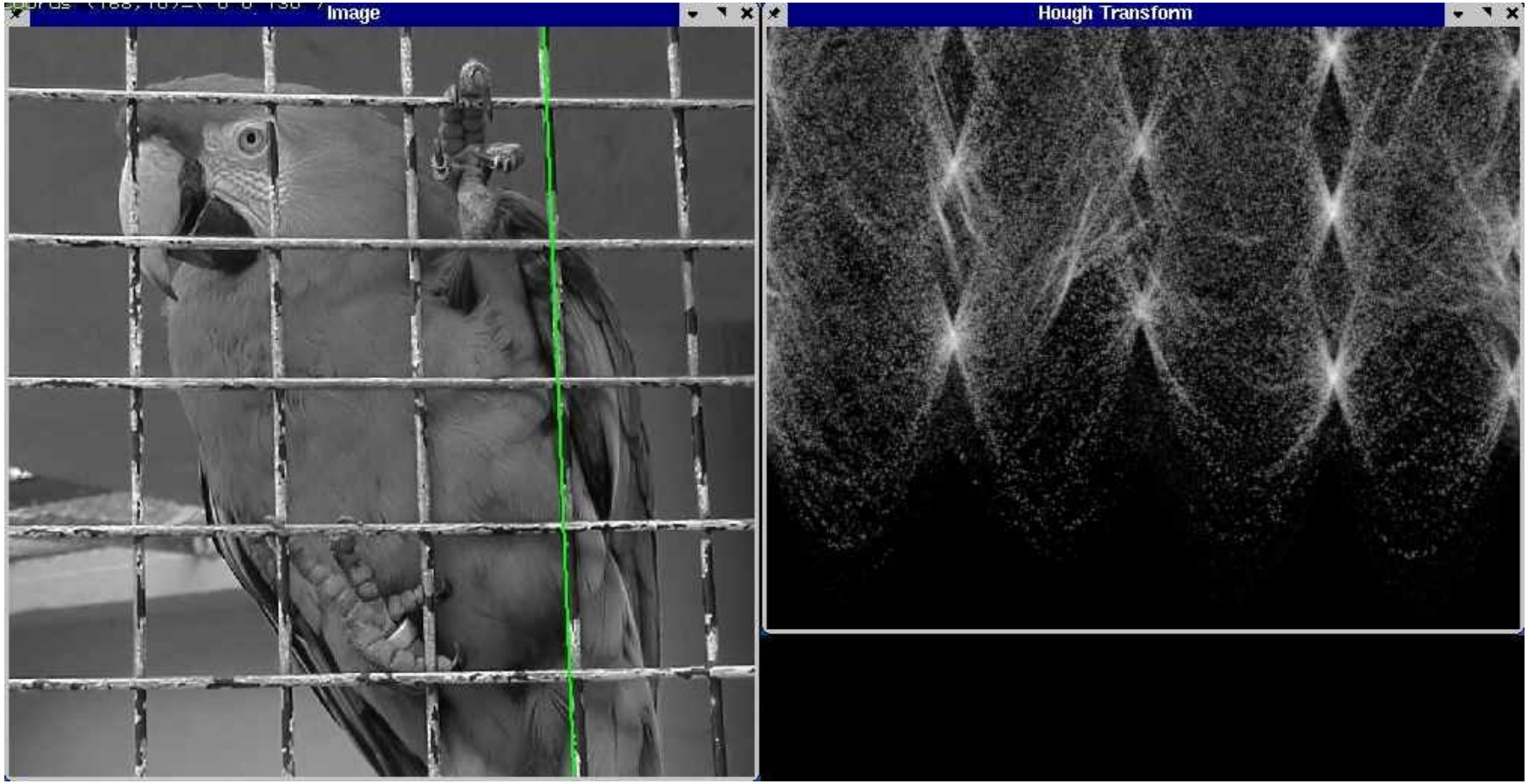
2. Canny → Hough votes



3. Hough votes → Edges

Find peaks and post-process

Hough transform example



http://ostatic.com/files/images/ss_hough.jpg

Finding lines using Hough transform

- Using m,b parameterization
- Using r, theta parameterization
  - Using oriented gradients
- Practical considerations
  - Bin size
  - Smoothing
  - Finding multiple lines
  - Finding line segments

INTEREST POINTS

Read Szeliski 4.1, Corke 13.3

Slides from Rick Szeliski, Svetlana Lazebnik, Derek Hoiem, Grauman&Leibe 2008 AAAI Tutorial, and James Hayes

Image matching

by Diva Sian

by swashford

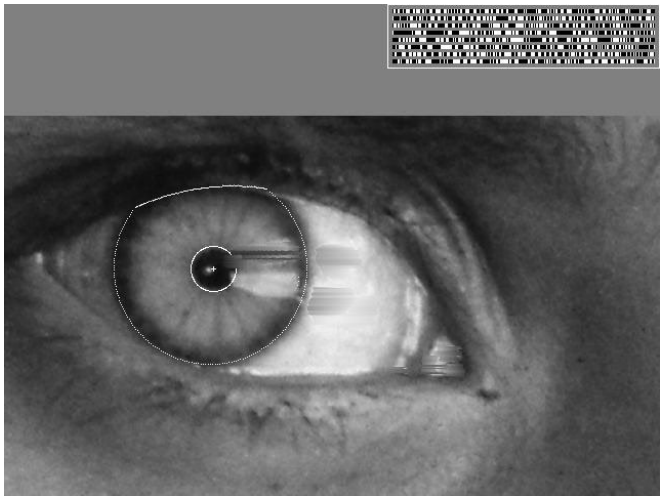
Harder case

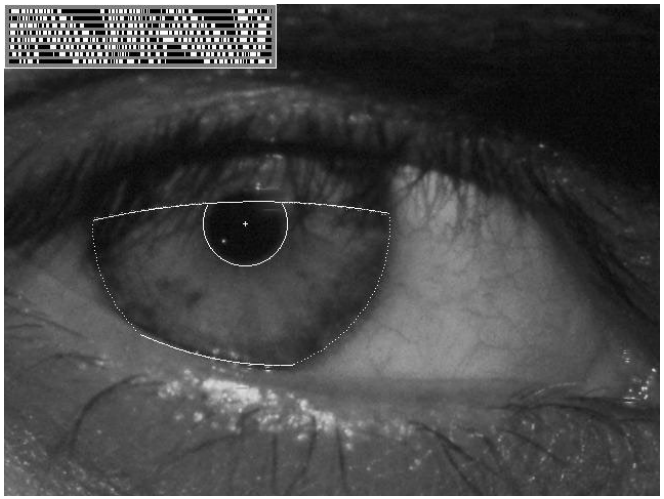
by Diva Sian

by scgbt

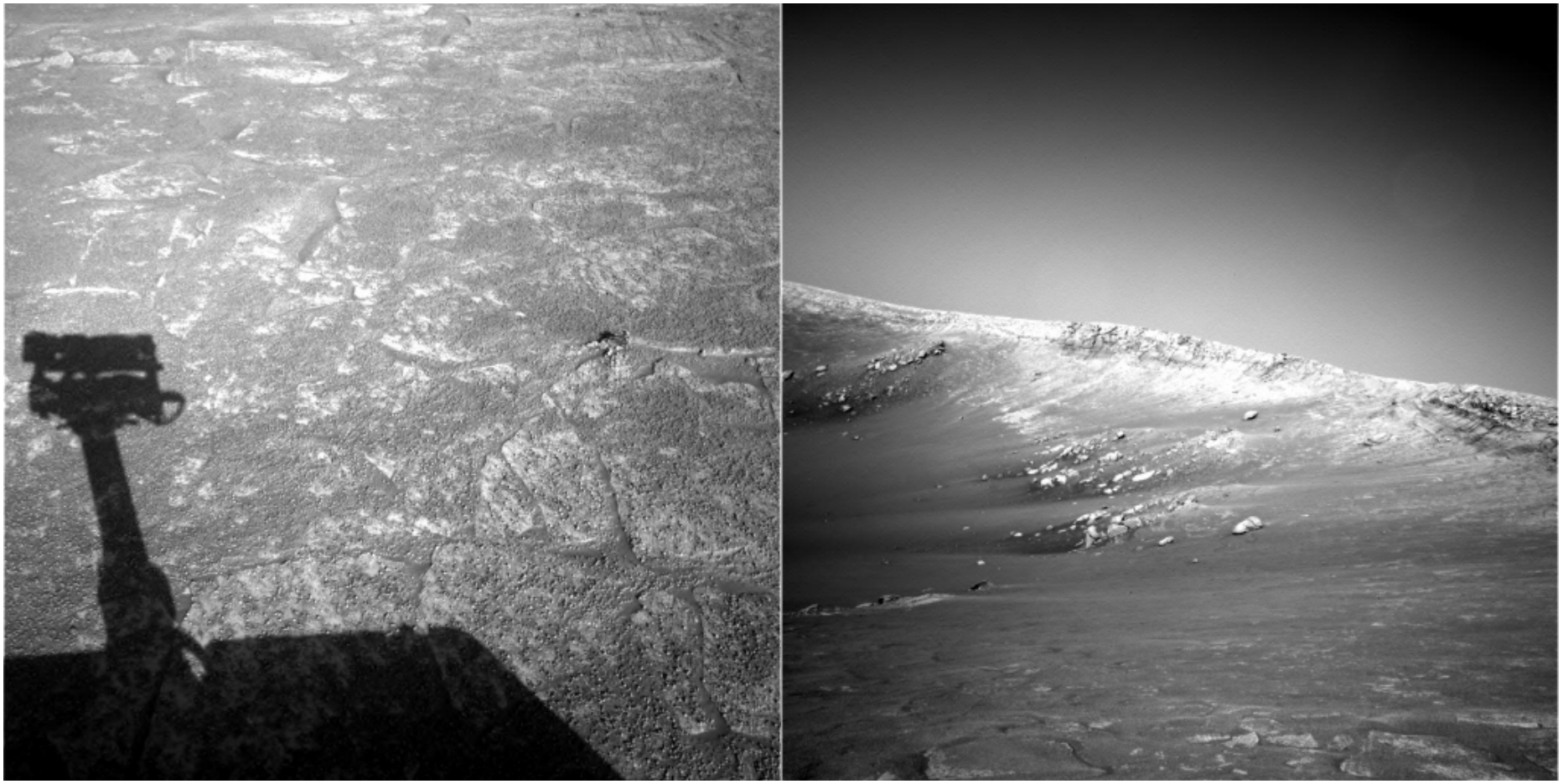
Even harder case

"***How the Afghan Girl was Identified by Her Iris Patterns***" **Read the story**



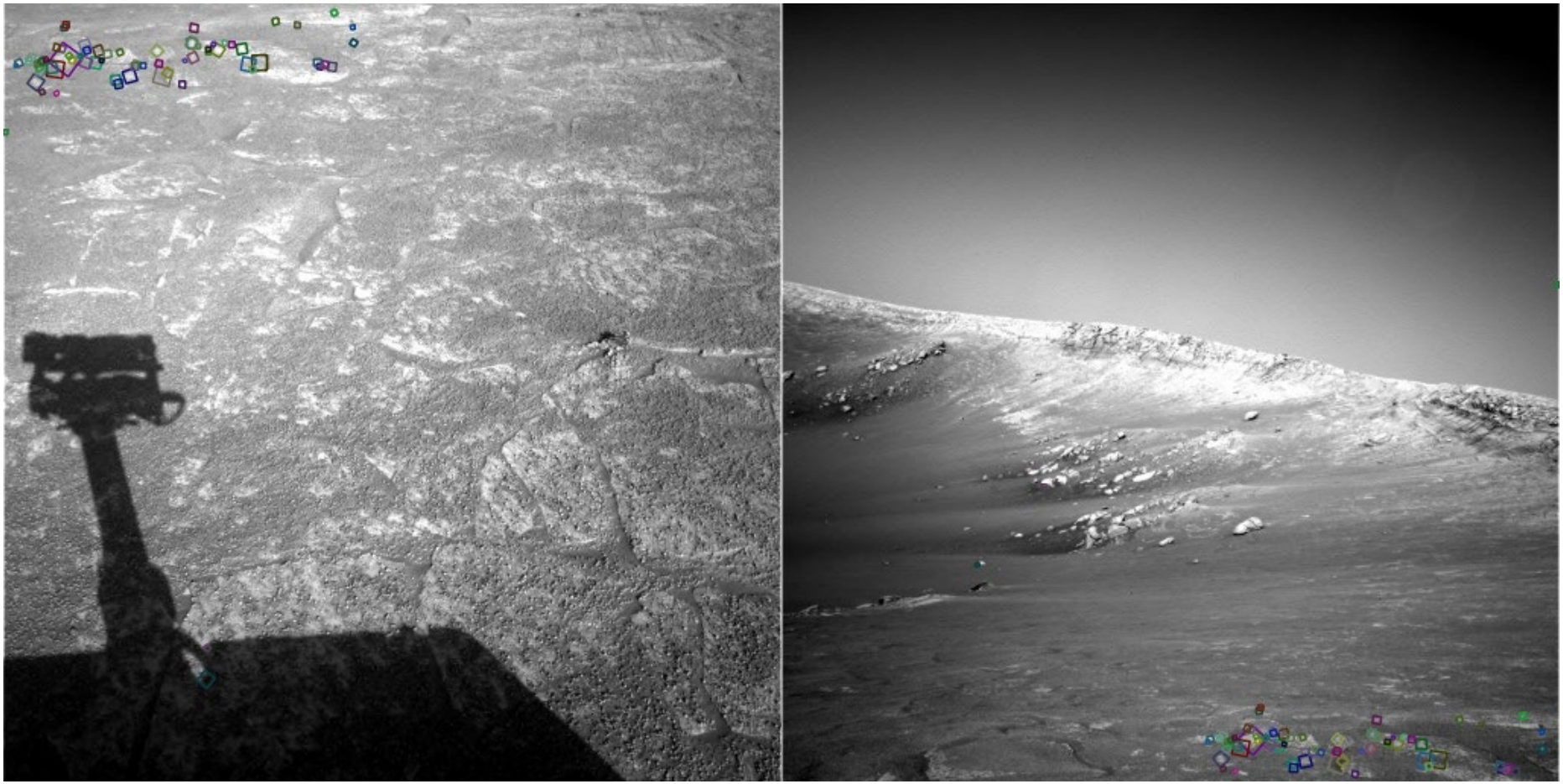


Harder still?



NASA Mars Rover images

Answer below (look for tiny colored squares…)



NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Image Matching

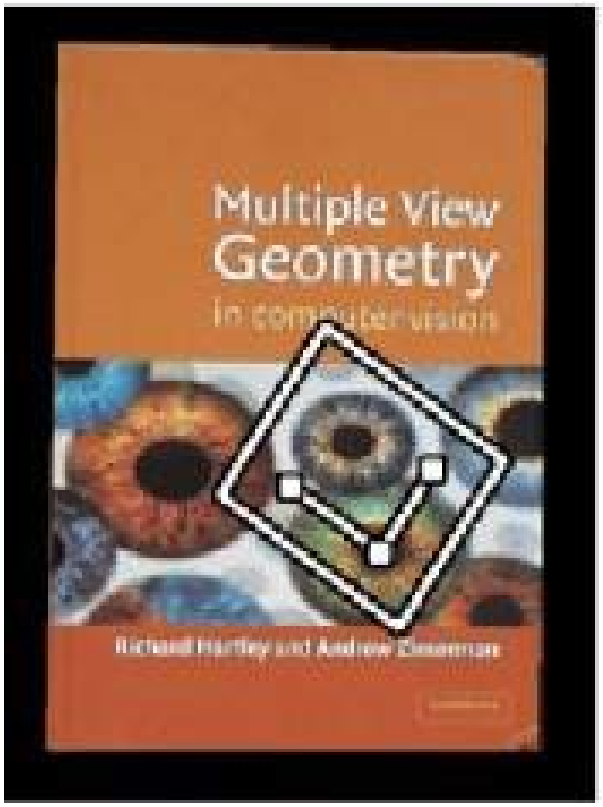
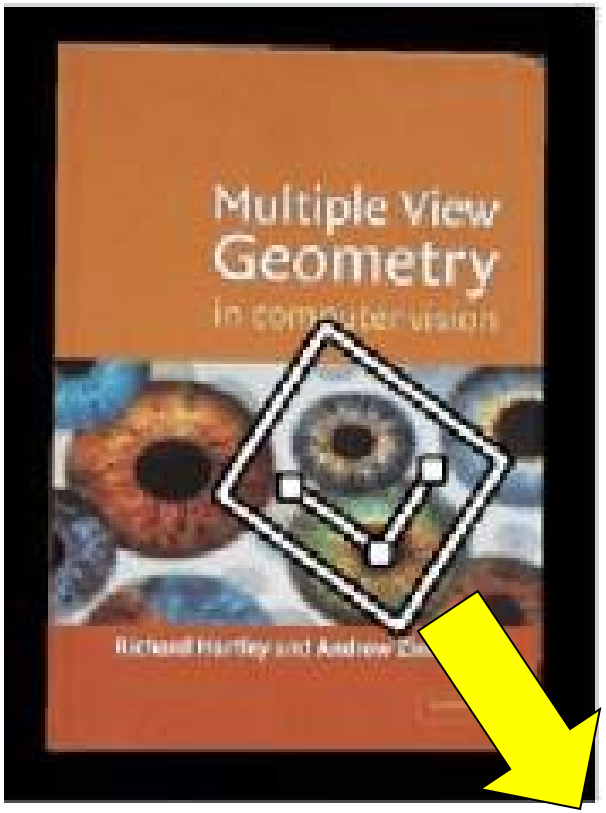
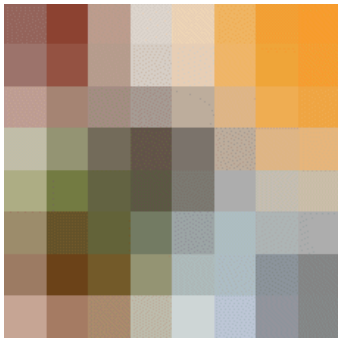
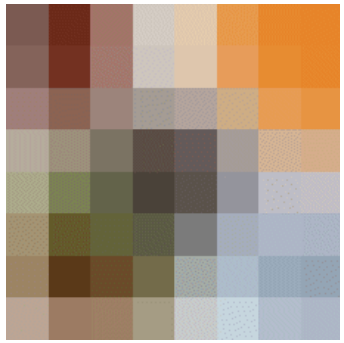


Image Matching



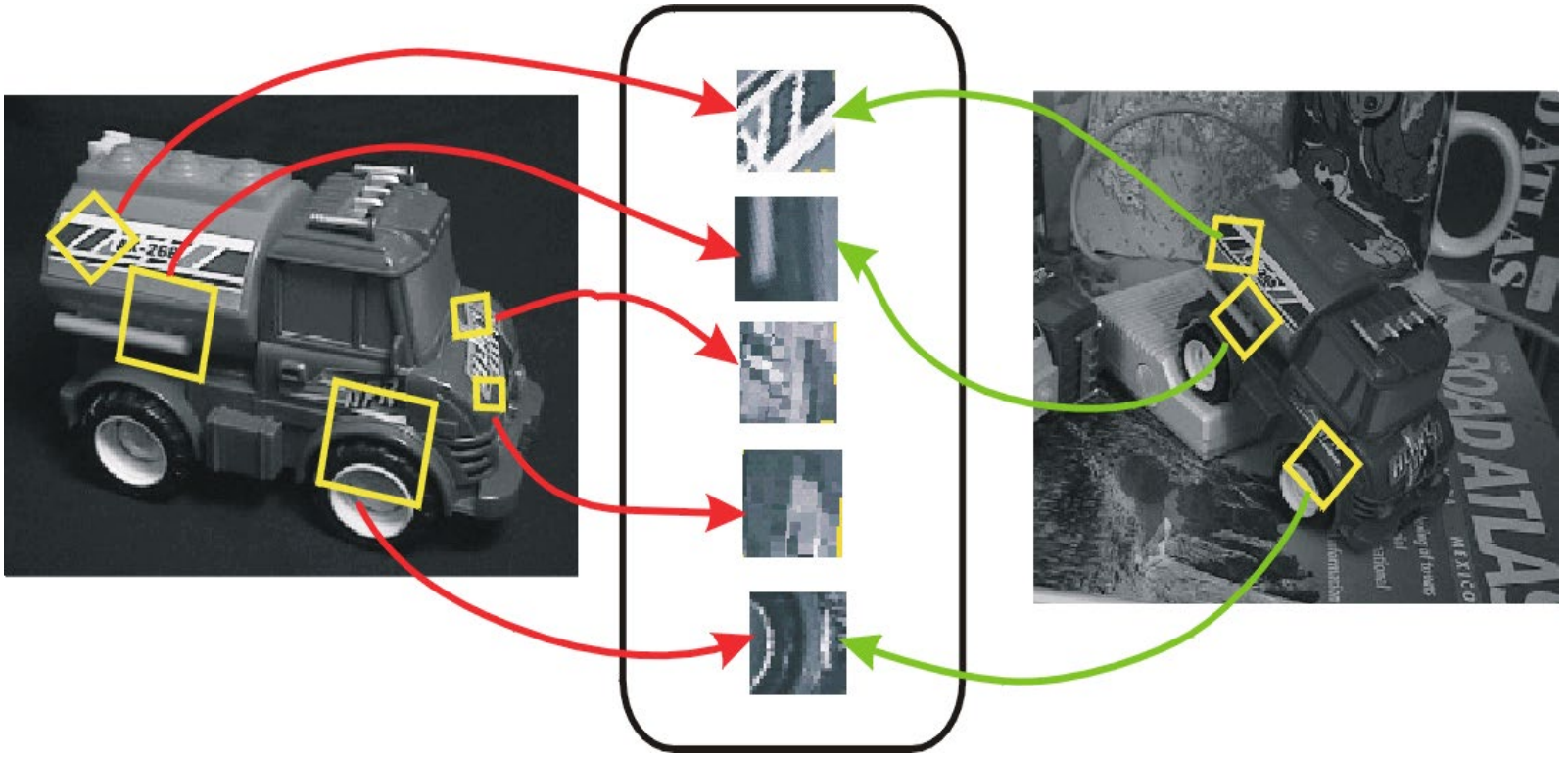




Invariant local features

Find features that are invariant to transformations

- geometric invariance: translation, rotation, scale
- photometric invariance: brightness, exposure, …



Feature Descriptors

Advantages of local features

Locality

- features are local, so robust to occlusion and clutter

Distinctiveness:

- can differentiate a large database of objects

Quantity

- hundreds or thousands in a single image

Efficiency

- real-time performance achievable

Generality

- exploit different types of features in different situations

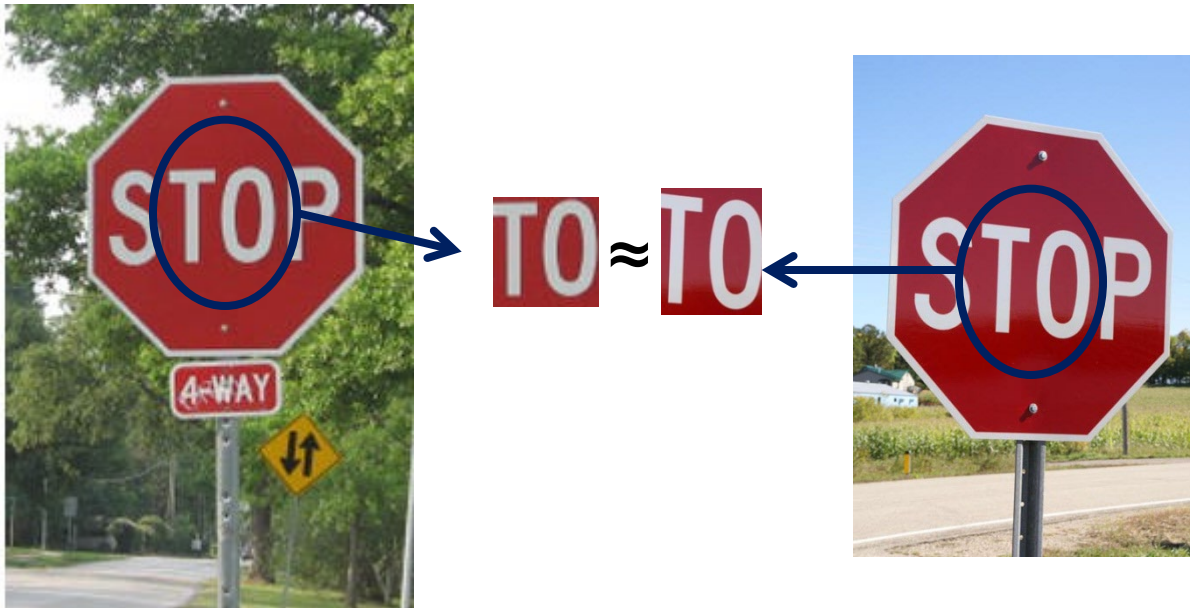
More motivation…

Feature points (aka keypoints are used for:

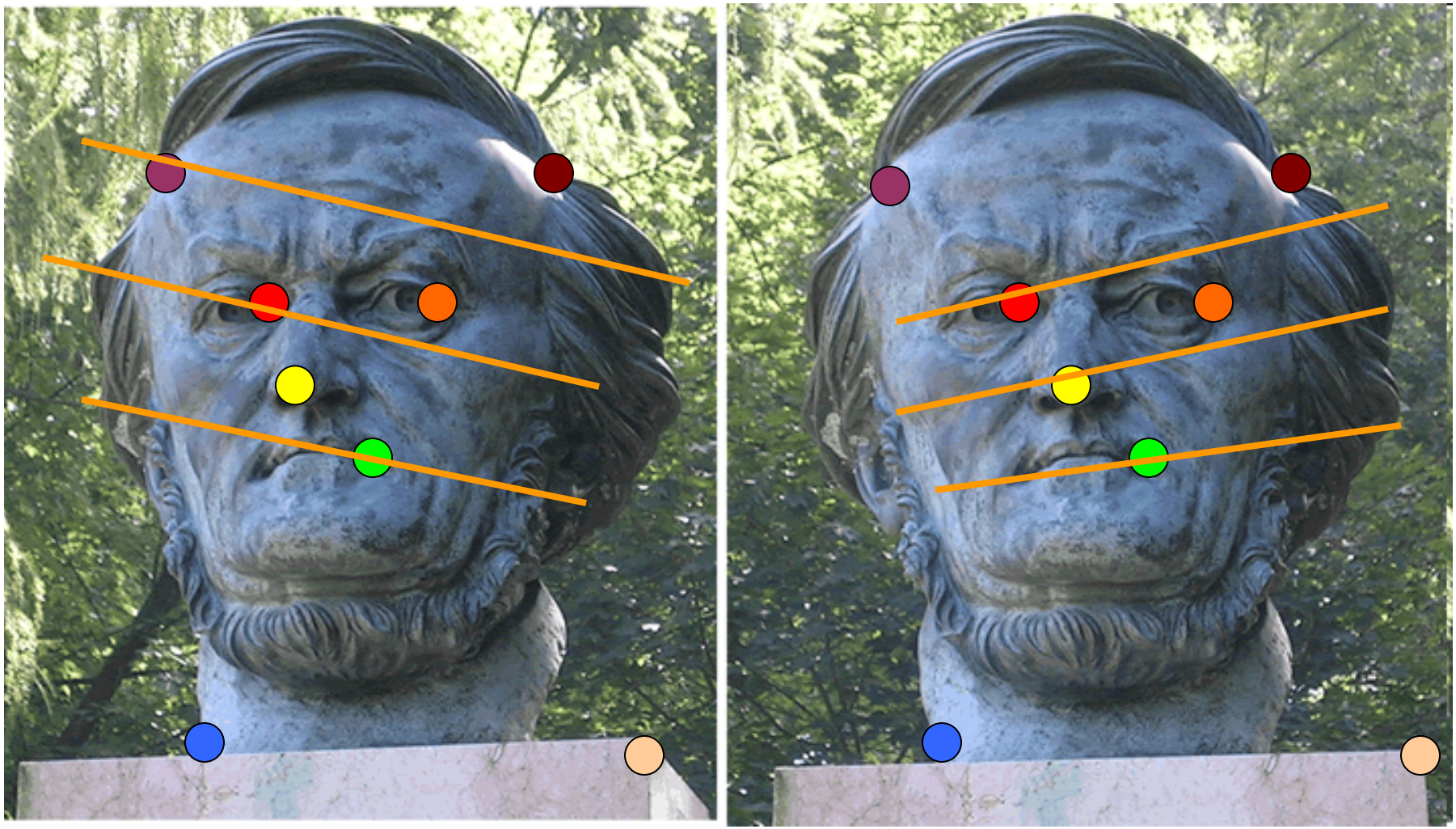
- Image alignment (e.g., mosaics)
- 3D reconstruction
- Motion tracking
- Object recognition
- Indexing and database retrieval

Correspondence across views

- Correspondence: matching points, patches, edges, or regions across images



Example: estimating “fundamental matrix” that corresponds two views



Slide from Silvio Savarese

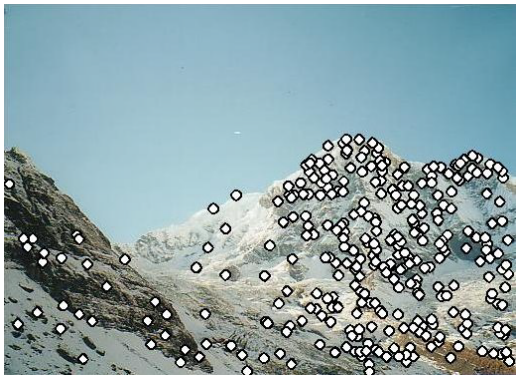
Example: structure from motion

Example: panorama stitching

- We have two images – how do we combine them?

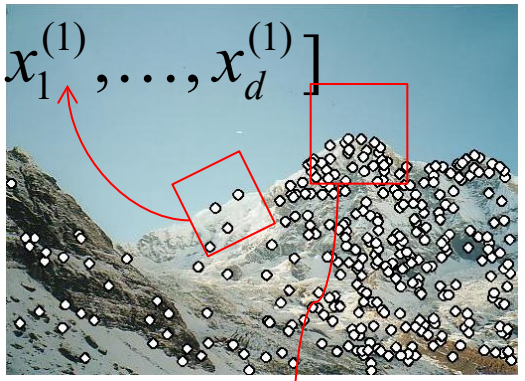
Local features: main components

1) **Detection:** Identify the interest points



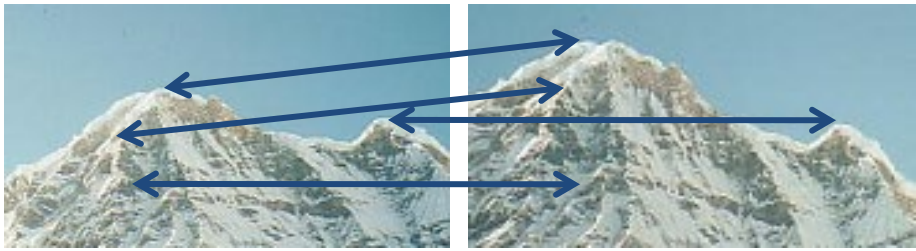
2) **Description**: Extract vector feature descriptor surrounding each interest point.

$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$

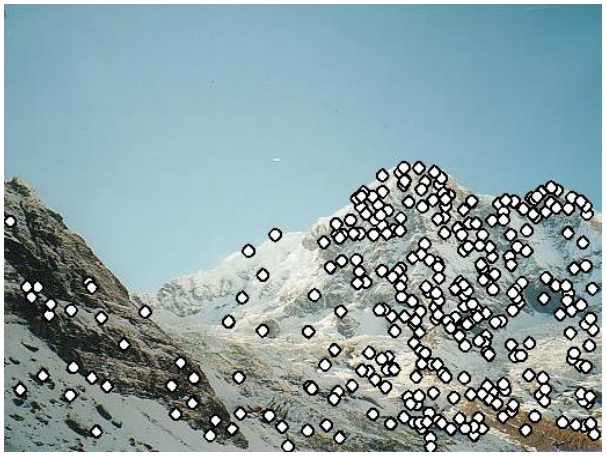
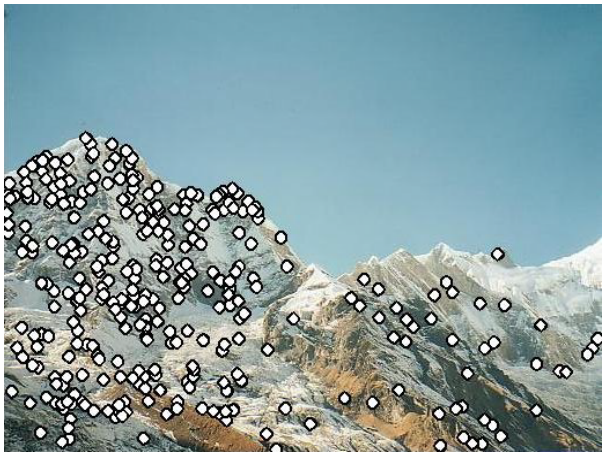


3) **Matching:** Determine correspondence between descriptors in two views

$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$



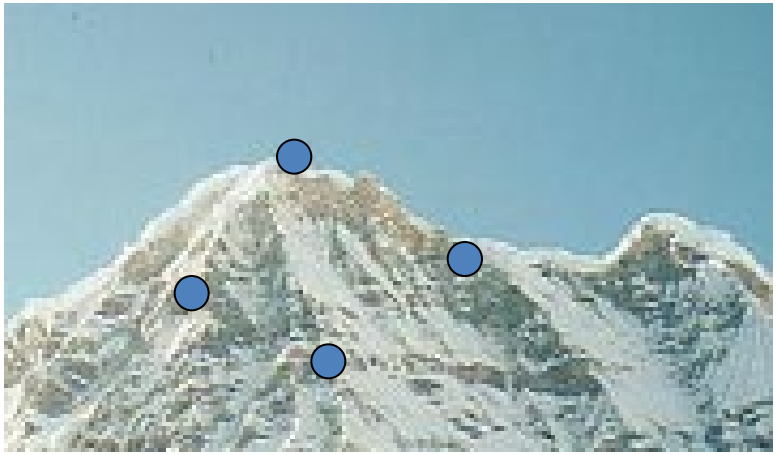
Characteristics of good features

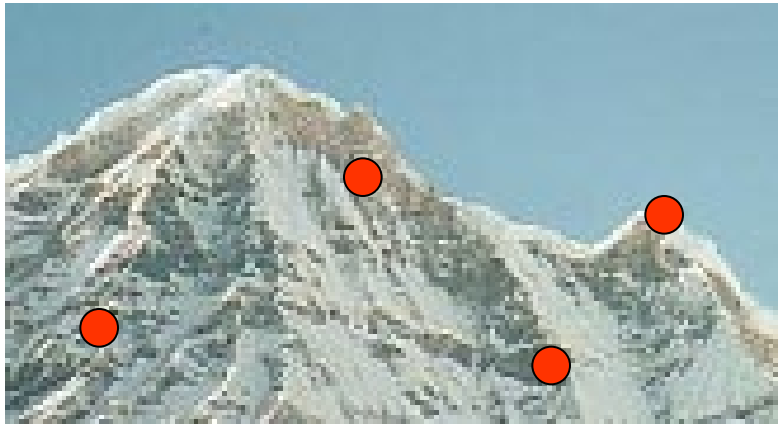



- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Goal: interest operator repeatability

- We want to detect (at least some of) the same points in both images.

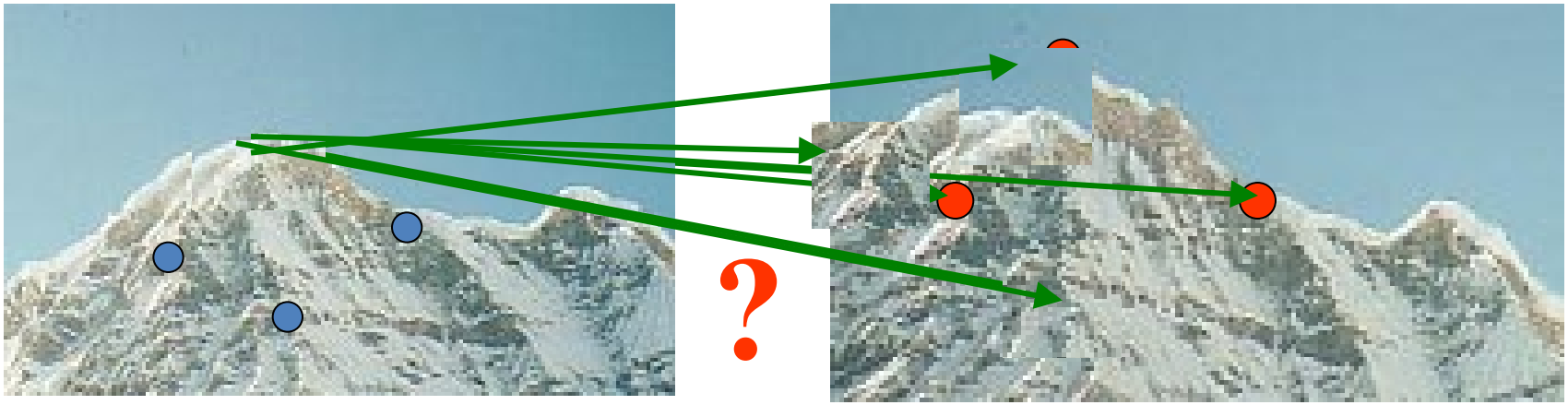




- Yet we have to be able to run the detection procedure *independently* per image.

Goal: descriptor distinctiveness

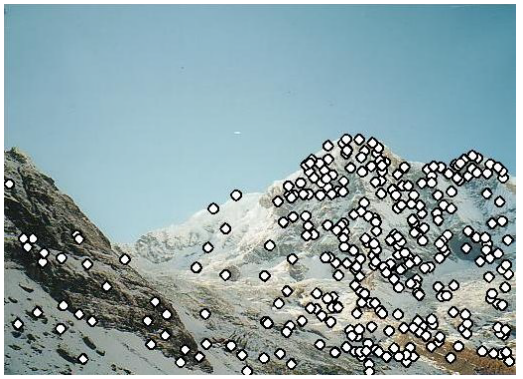
- We want to be able to reliably determine which point goes with which.



- Must provide some invariance to geometric and photometric differences between the two views.

Local features: main components

1) Detection: Identify the interest points (aka. Keypoints)



2) Description:Extract vector feature descriptor surrounding each interest point.

3) Matching: Determine correspondence between descriptors in two views

Choosing distinctive interest points

- If you wanted to meet a friend would you say
  a) “Let’s meet on campus.”
  b) “Let’s meet on Green street.”
  c) “Let’s meet at Green and Wright.”
  – Corner detection

- Or if you were in a secluded area:
  a) “Let’s meet in the Plains of Akbar.”
  b) “Let’s meet on the side of Mt. Doom.”
  c) “Let’s meet on top of Mt. Doom.”
  – Blob (valley/peak) detection

Choosing interest points

Where would you tell your friend to meet you?

Choosing interest points

Where would you tell your friend to meet you?

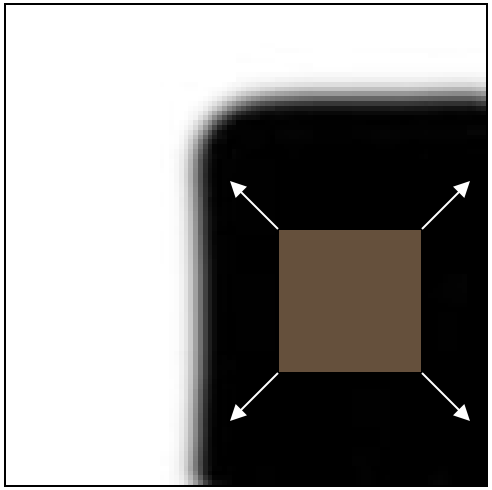
Many Existing Detectors Available

Hessian & Harris	[Beaudet ‘78], [Harris ‘88]
Laplacian, DoG	[Lindeberg ‘98], [Lowe 1999]
Harris-/Hessian-Laplace	[Mikolajczyk & Schmid ‘01]
Harris-/Hessian-Affine	[Mikolajczyk & Schmid ‘04]
EBR and IBR	[Tuytelaars & Van Gool ‘04]
MSER	[Matas ‘02]
Salient Regions	[Kadir & Brady ‘01]
Others…	

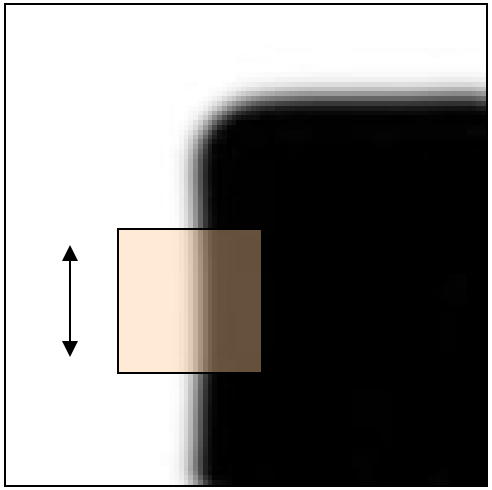
CORNERS

Corner Detection: Basic Idea

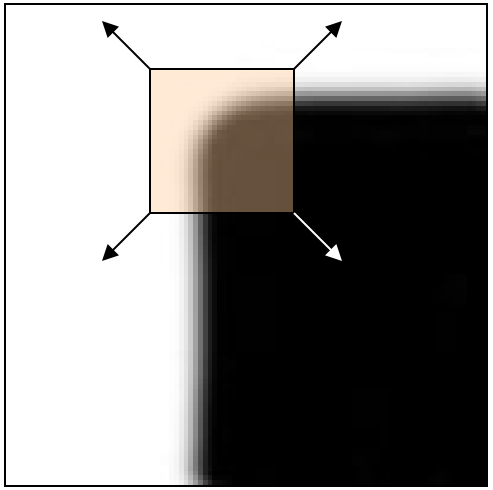
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions



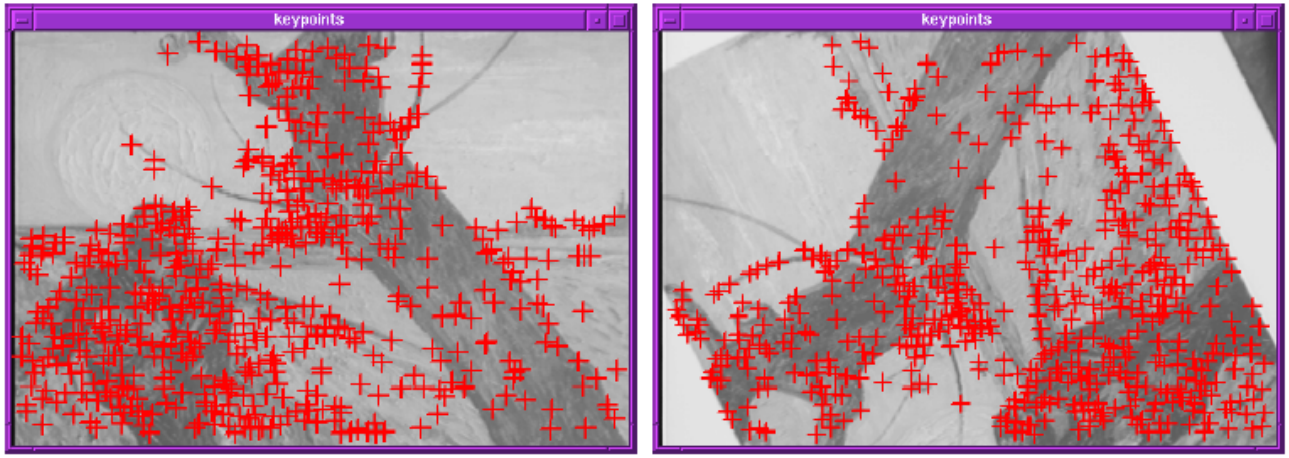
"edge":
no change along the edge direction



"corner":
significant change in all directions

Source: A. Efros

Finding Corners



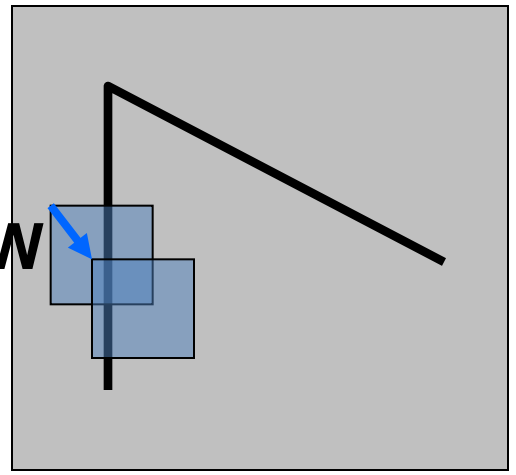
- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“
Proceedings of the 4th Alvey Vision Conference: pages 147--151.

Corner detection: the math

Consider shifting the window W by (u,v)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD "error" of *E(u,v)*:

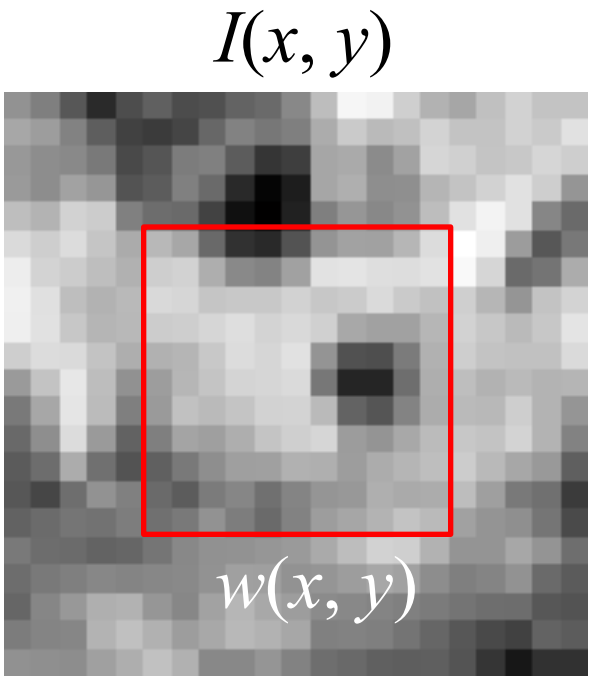


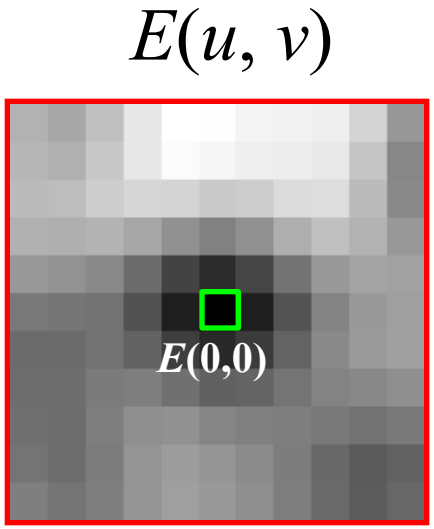
$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

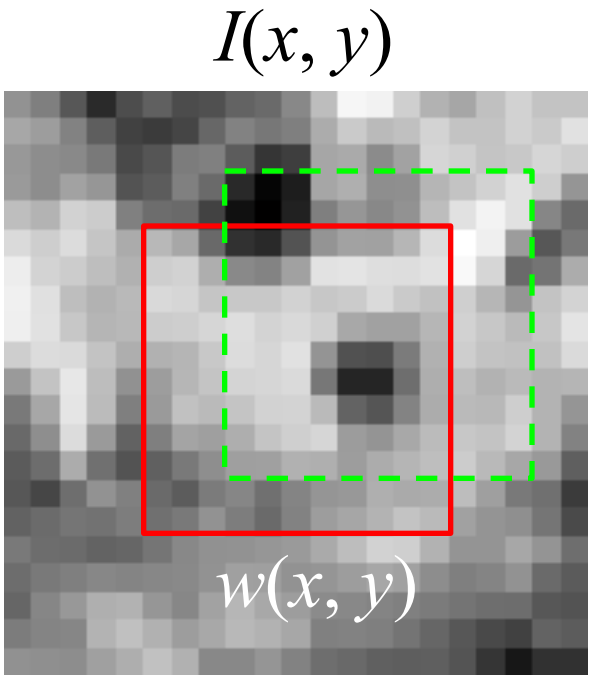


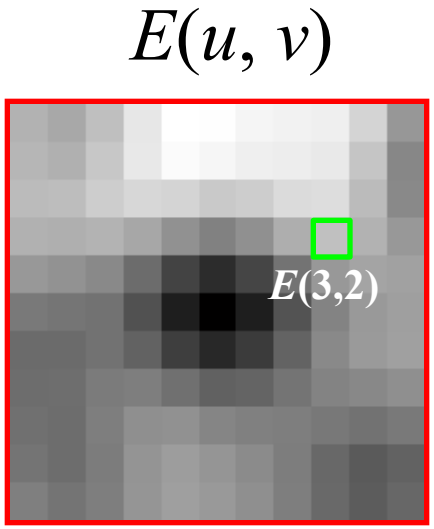


Corner Detection: Mathematics

Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$





Corner Detection: Mathematics

Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

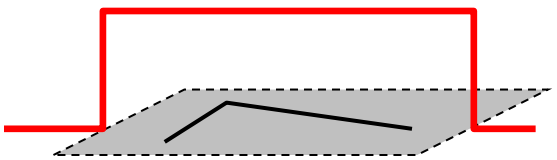
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

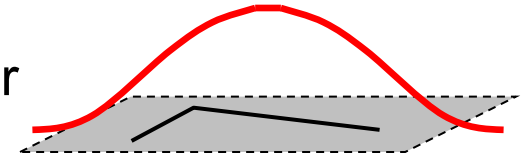
Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Source: R. Szeliski

Small motion assumption

Taylor Series expansion of I:

$$I(x+u, y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u,v) is small, then first order approx is good

$$I(x+u, y+v) \approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x,y) + [I_x \; I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

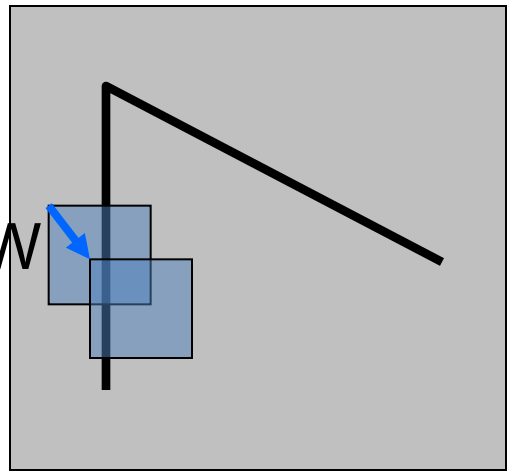
$$\text{shorthand: } I_x = \frac{\partial I}{\partial x}$$

Plugging this into the formula on the previous slide…

Corner detection: the math

Consider shifting the window W by (u,v)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences
- this defines an "error" of E(u,v):



$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

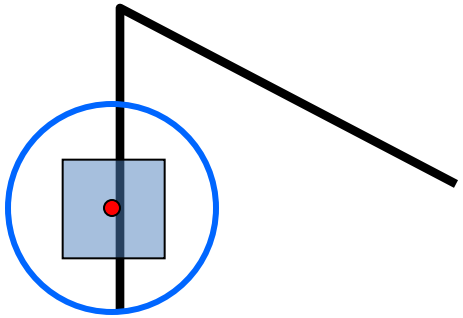
$$\approx \sum_{(x,y)\in W} [I(x,y) + [I_x \; I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x,y)]^2$$

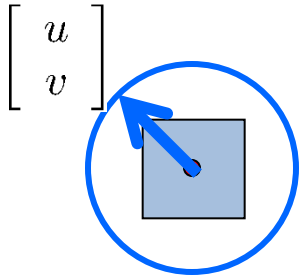
$$\approx \sum_{(x,y)\in W} \left[[I_x \; I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right]^2$$

Corner detection: the math

This can be rewritten:

$$E(u,v) = \sum_{(x,y)\in W} [u\ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$





For the example above

- You can move the center of the green window to anywhere on the blue unit circle
- Which directions will result in the largest and smallest E values?
- We can find these directions by looking at the eigenvectors of *H*

Quick eigenvalue/eigenvector review

The **eigenvectors** of a matrix **A** are the vectors **x** that satisfy:

$$Ax = \lambda x$$

The scalar λ is the **eigenvalue** corresponding to **x**

- The eigenvalues are found by solving:

$$det(A - \lambda I) = 0$$

- In our case, ***A*** = ***H*** is a 2x2 matrix, so we have

$$det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$$

- The solution:

$$\lambda_{\pm} = \frac{1}{2} \left[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2} \right]$$

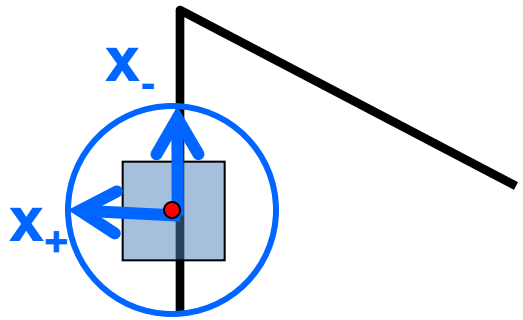
Once you know λ, you find **x** by solving

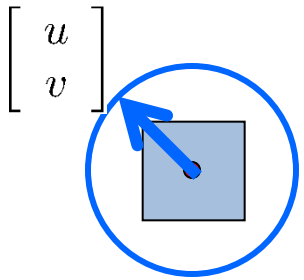
$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Corner detection: the math

This can be rewritten:

$$E(u,v) = \sum_{(x,y)\in W} [u\ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$





Eigenvalues and eigenvectors of H

- Define shifts with the smallest and largest change (E value)
- x_+ = direction of largest increase in E.
- λ_+ = amount of increase in direction x_+
- x_- = direction of smallest increase in E.
- λ_- = amount of increase in direction x_+

$$Hx_+ = \lambda_+ x_+$$
$$Hx_- = \lambda_- x_-$$

Corner Detection: Mathematics

The quadratic approximation simplifies to

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a *second moment matrix* computed from image derivatives:

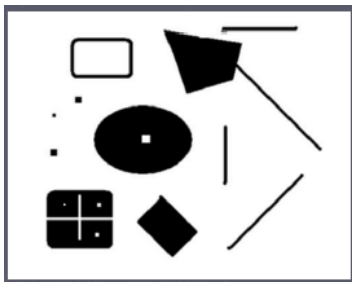
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

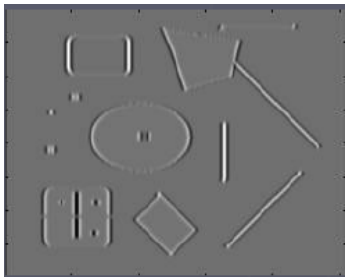
$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y] = \sum \nabla I (\nabla I)^T$$

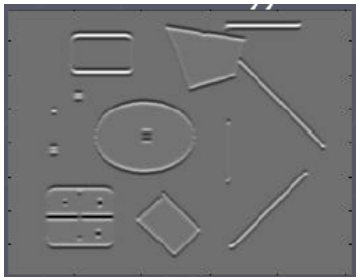
Corners as distinctive interest points

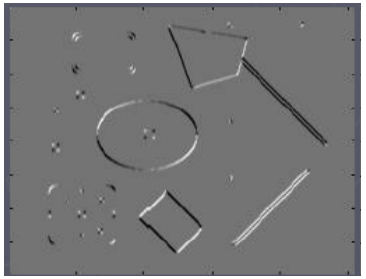
$$M = \sum w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives (averaged in neighborhood of a point).









Notation:

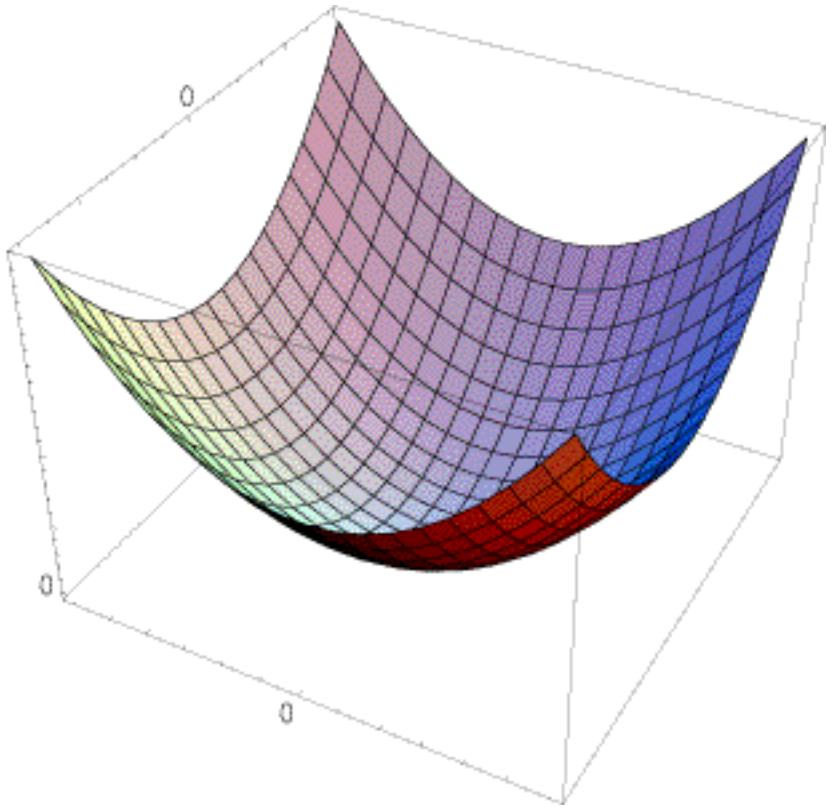
$$I_x \Leftrightarrow \frac{\partial I}{\partial x} \qquad I_y \Leftrightarrow \frac{\partial I}{\partial y} \qquad I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$$

Interpreting the second moment matrix

The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

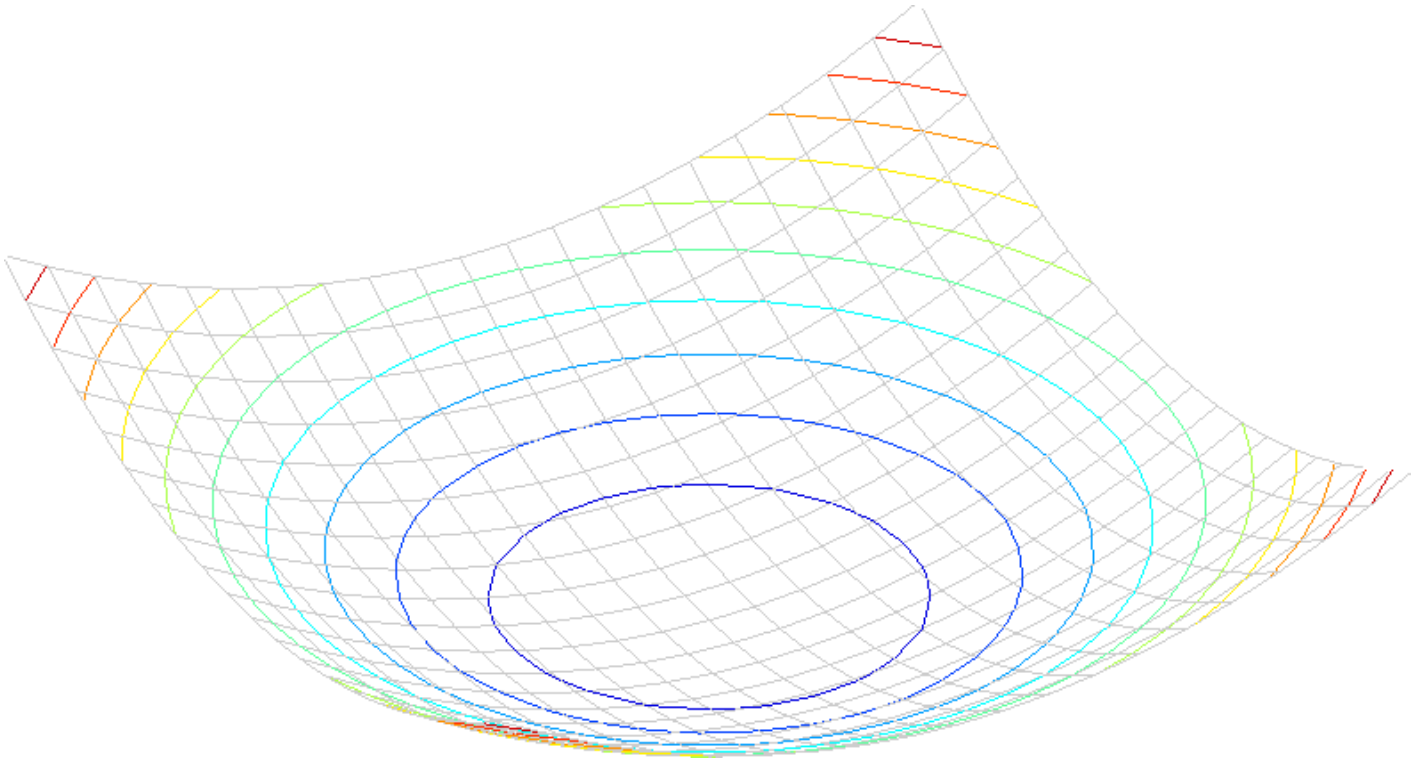
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.



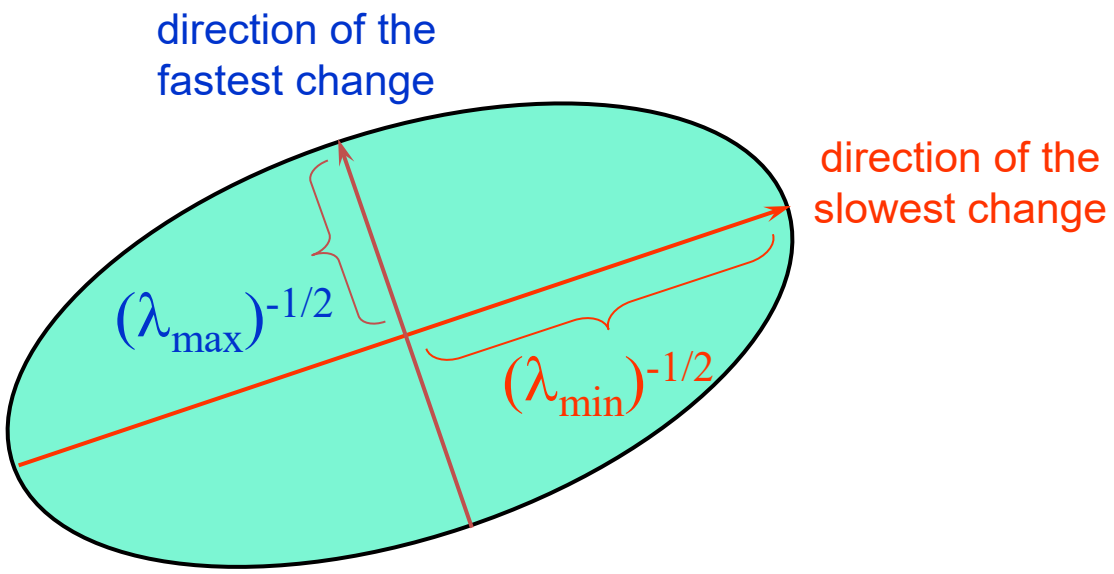
Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.

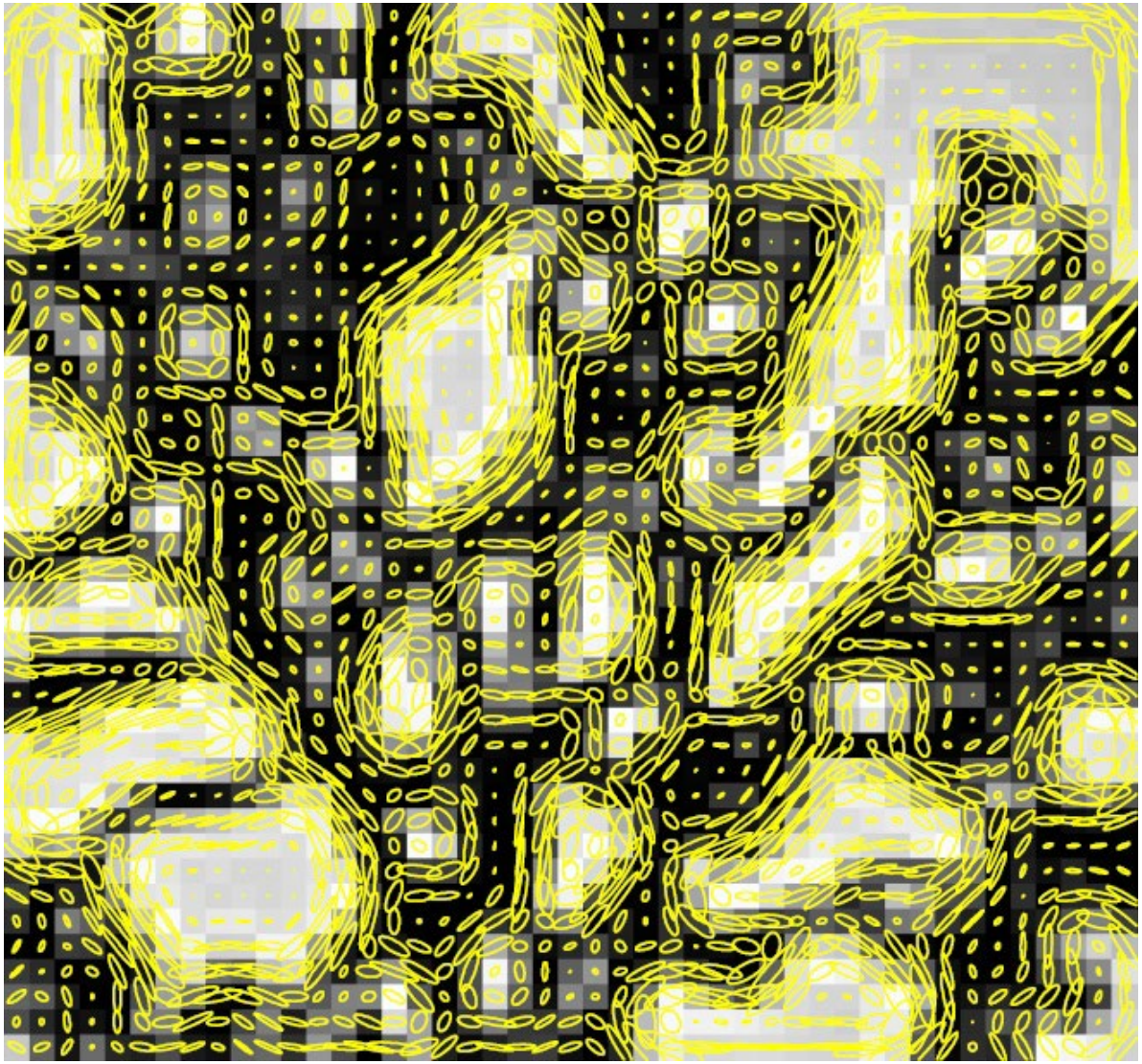
Diagonalization of M: $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by R



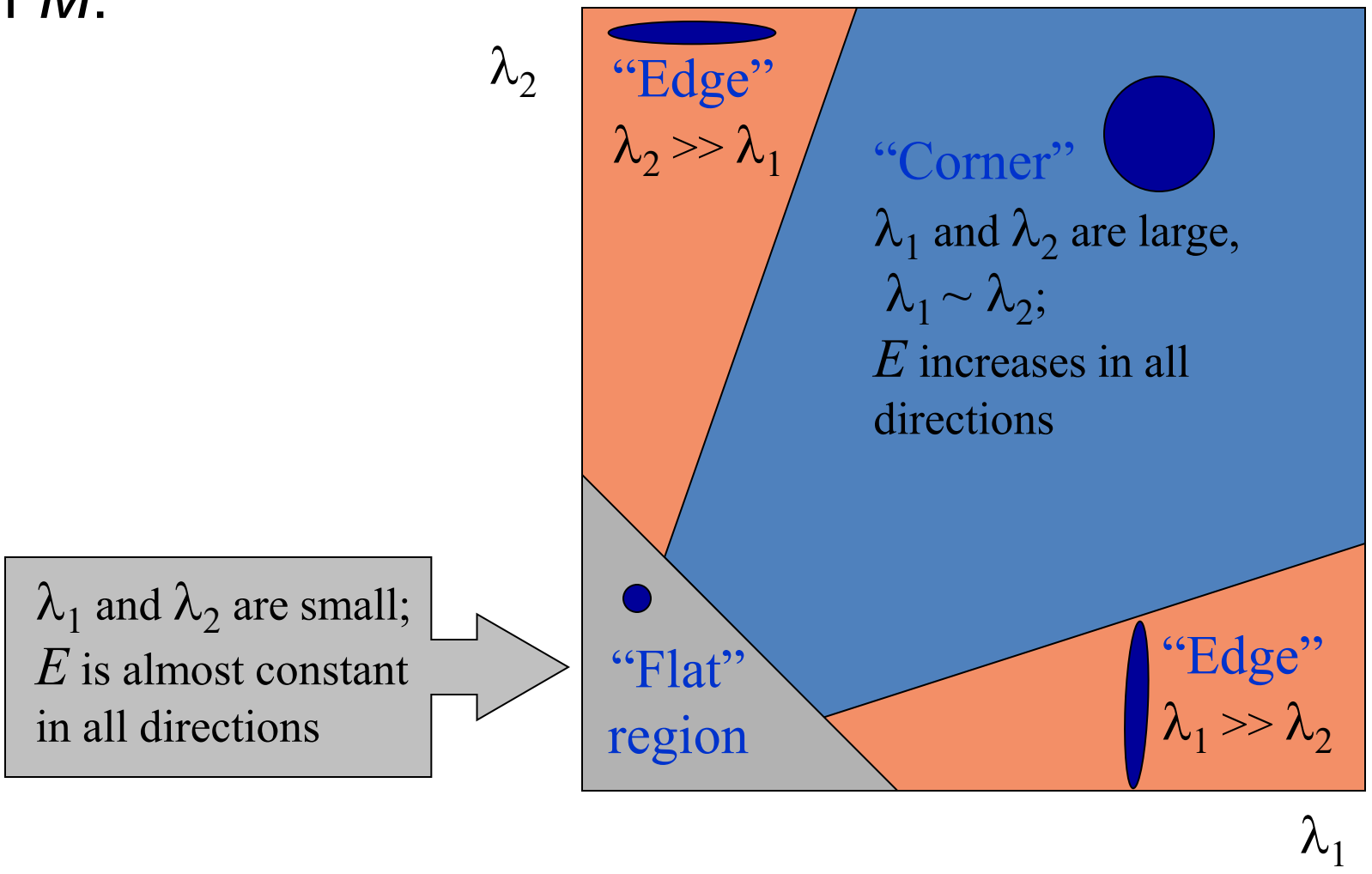
Visualization of second moment matrices

Visualization of second moment matrices



Interpreting the eigenvalues

Classification of image points using eigenvalues of *M*:



The Harris operator

λ_- is a variant of the "Harris operator" for feature detection

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

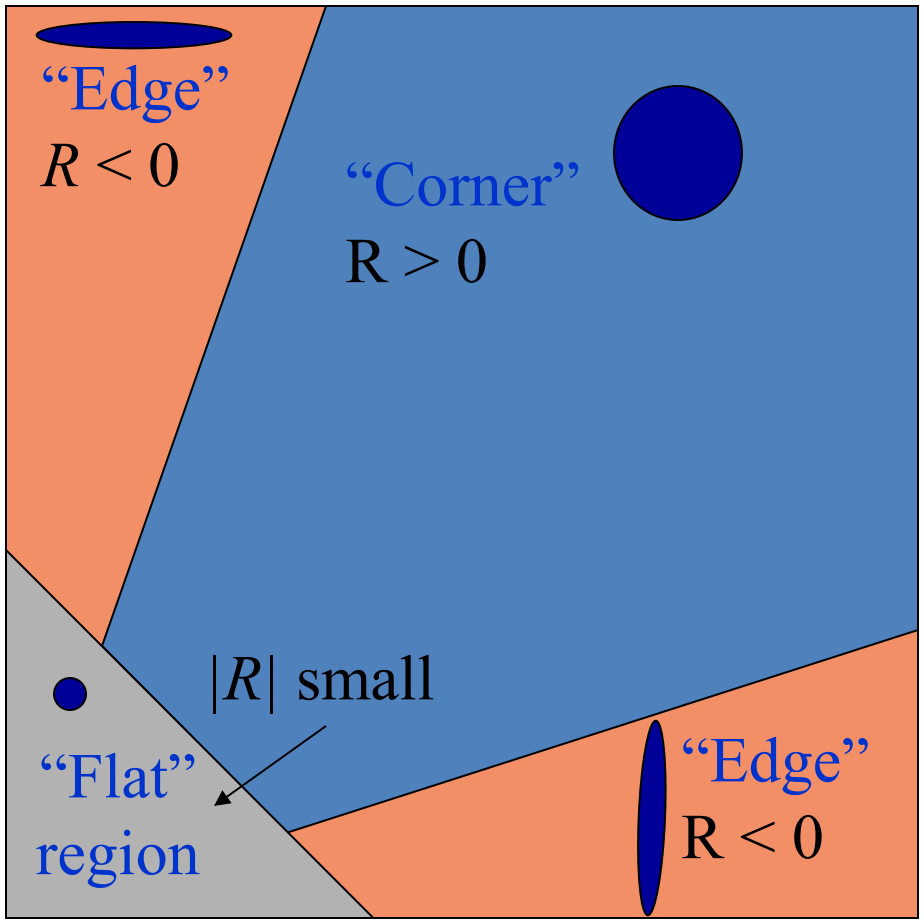
$$= \frac{determinant(H)}{trace(H)}$$

- The *trace* is the sum of the diagonals, i.e., *trace(H)* = $h_{11} + h_{22}$
- Very similar to λ_- but less expensive (no square root)
- Called the "Harris Corner Detector" or "Harris Operator"
- Lots of other detectors, this is one of the most popular

Corner response function

$$R = \det(M) - \alpha \,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



Harris corner detector

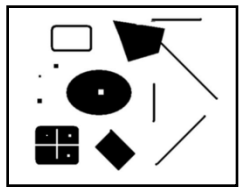
1) Compute *M* matrix for each image window to get their *cornerness* scores.
2) Find points whose surrounding window gave large corner response (*f*> threshold)
3) Take the points of local maxima, i.e., perform non-maximum suppression

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

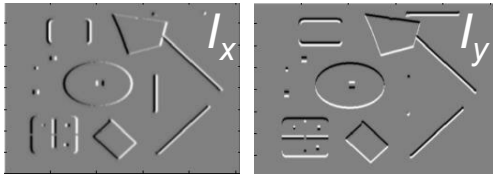
Harris Detector [Harris88]

- Second moment matrix

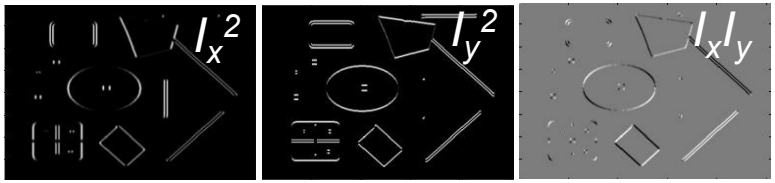
$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$



1. Image derivatives (optionally, blur first)



2. Square of derivatives

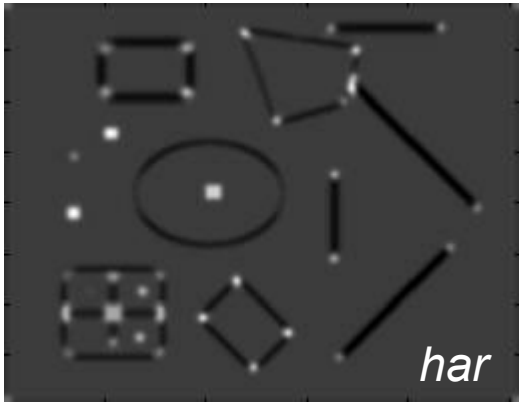


$$\det M = \lambda_1 \lambda_2$$
$$\text{trace}\, M = \lambda_1 + \lambda_2$$

3. Gaussian filter $g(\sigma_I)$

4. Cornerness function – both eigenvalues are strong

$$har = \det[\mu(\sigma_I, \sigma_D)] - \alpha[\text{trace}(\mu(\sigma_I, \sigma_D))^2] =$$
$$g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

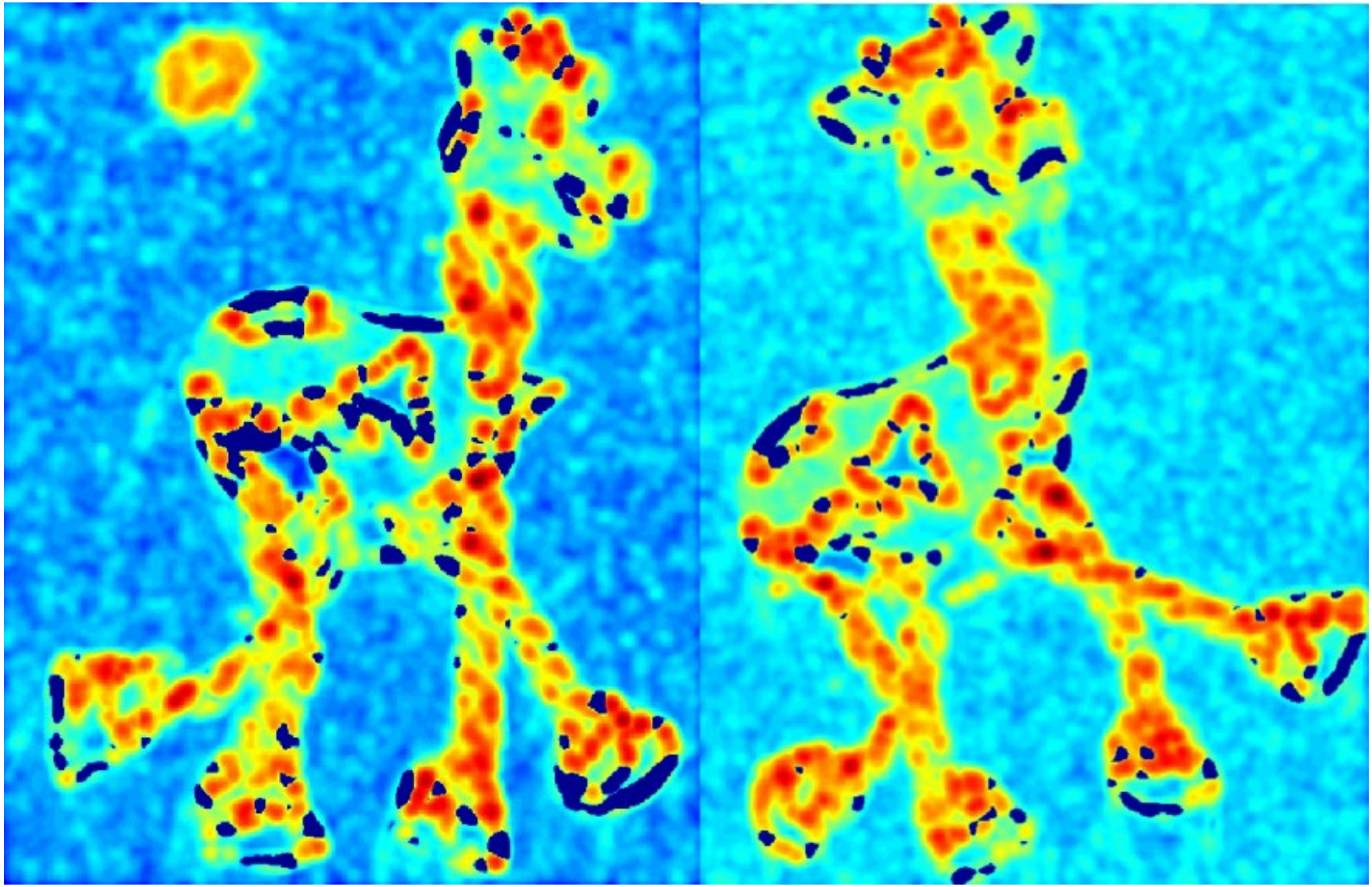


5. Non-maxima suppression

Harris Detector: Steps

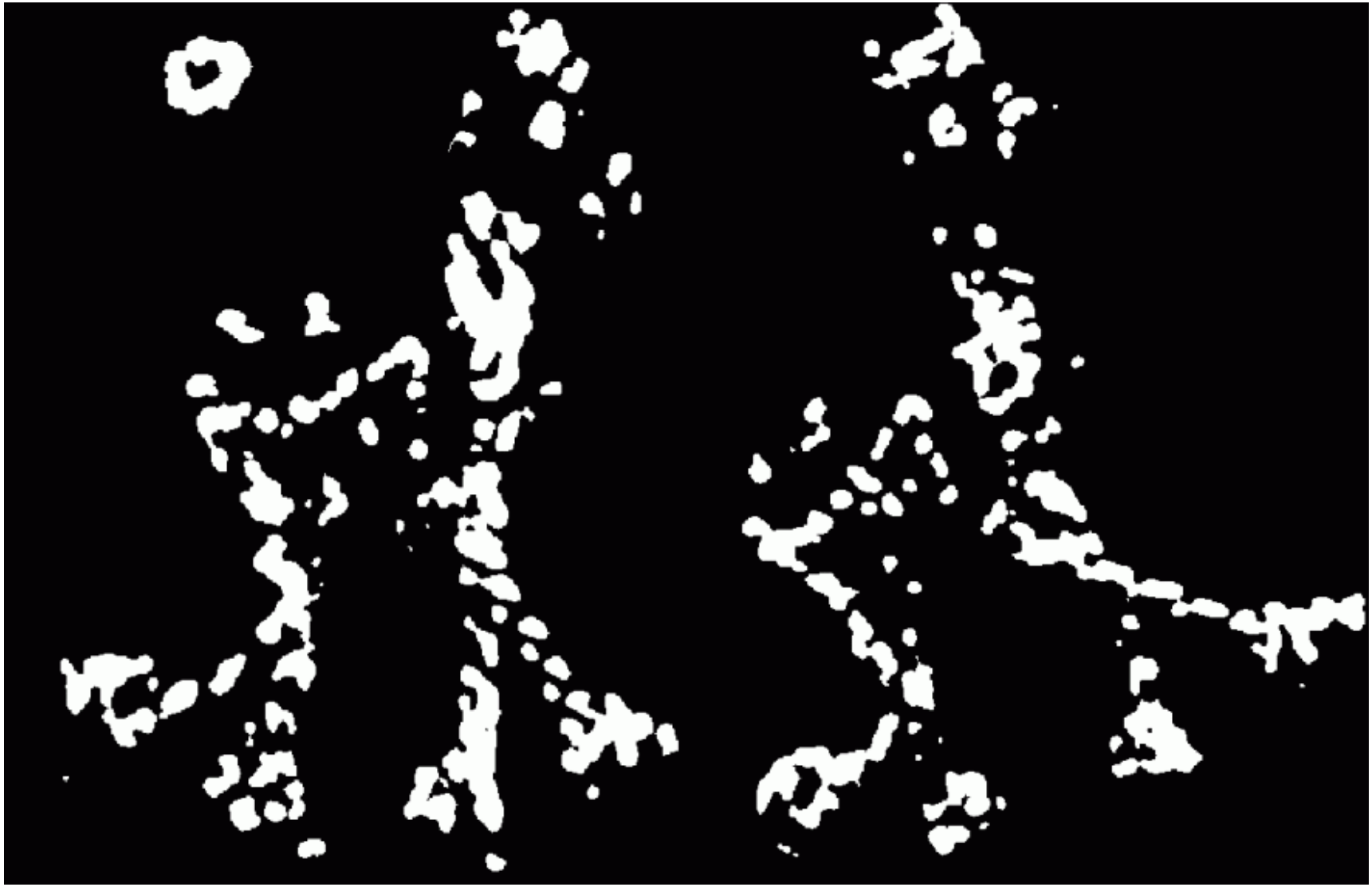
Harris Detector: Steps

Compute corner response R



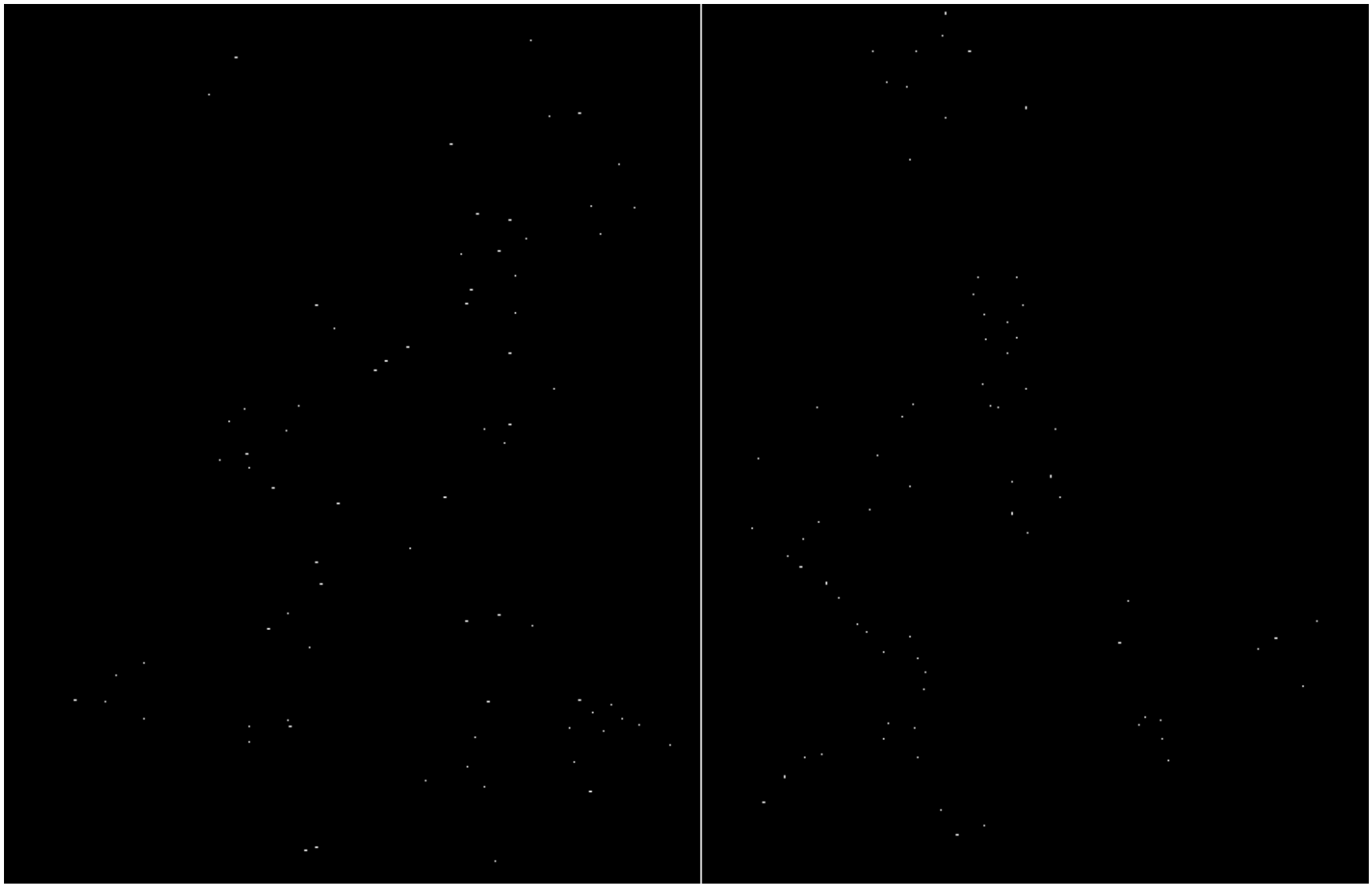
Harris Detector: Steps

Find points with large corner response: R>threshold



Harris Detector: Steps

Take only the points of local maxima of R

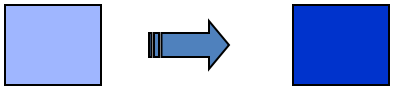


Harris Detector: Steps

Invariance and covariance

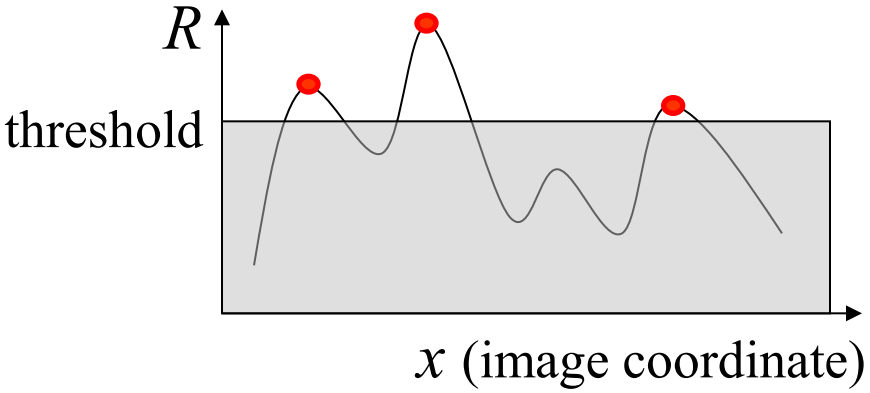
- We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
  - **Invariance:** image is transformed and corner locations do not change
  - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

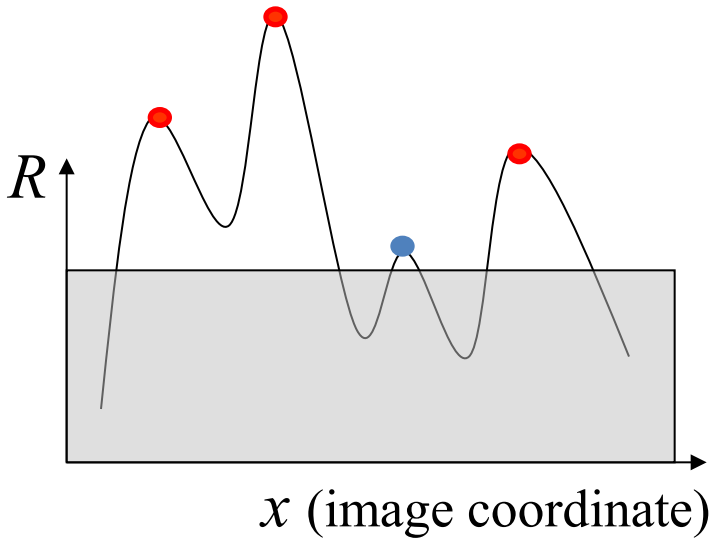
Affine intensity change



$$I \rightarrow a\,I + b$$

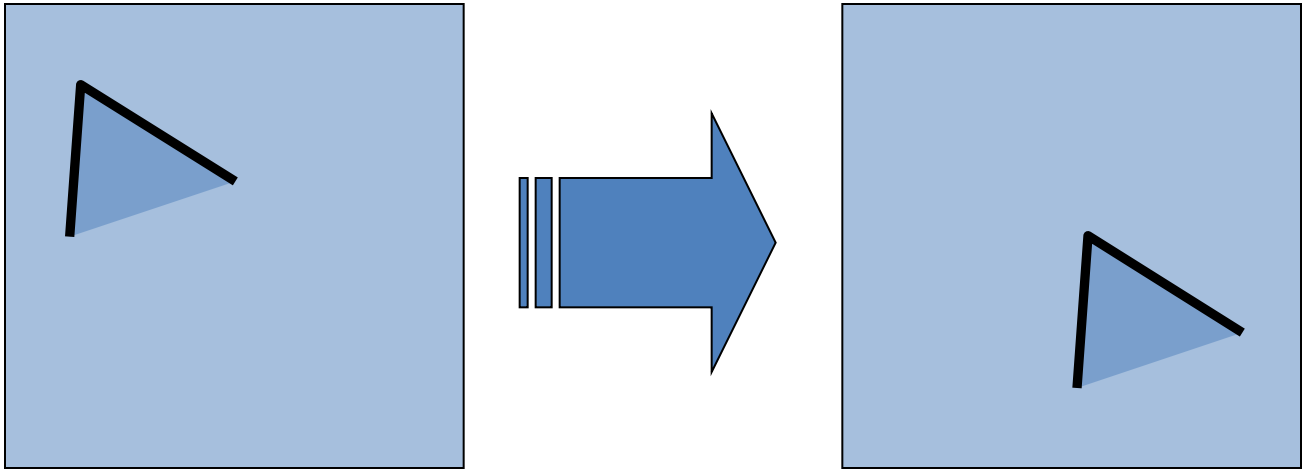
- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a\,I$





Partially invariant to affine intensity change

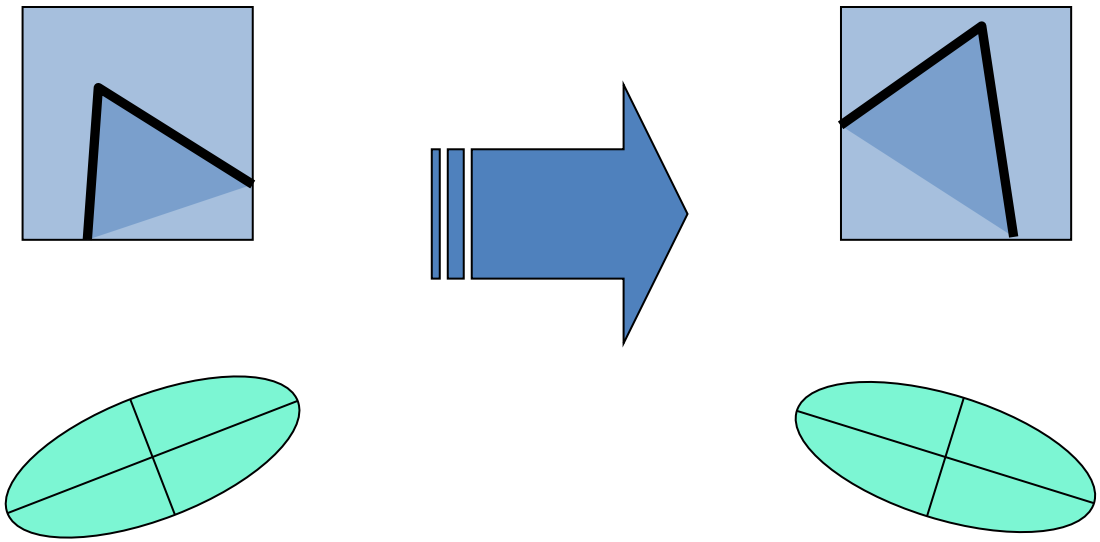
Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

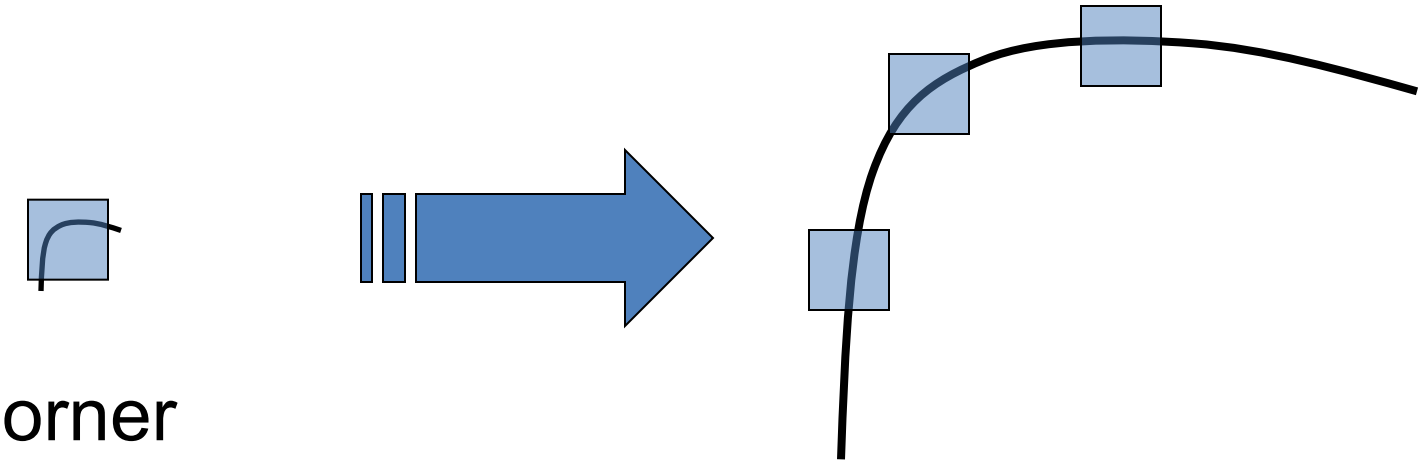
Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Scaling



Corner

All points will be classified as edges

Corner location is not covariant to scaling!