

GOAL PROGRAMMING

Linear optimization problems or linear programming only focuses on a single linear objective function with linear constraints. Goal programming is much more general. It is allowed to have multiple objective function which might be conflicting.

APPROACH TO SOLVE A GOAL PROGRAMMING PROBLEM:

1. Establish a specific numeric goal for each of the objectives
2. Formulate an objective function for each objective, and then
3. Find a solution that minimizes the (weighted) sum of deviations of these objective functions from their respective goals.

THERE ARE THREE POSSIBLE TYPES OF GOALS:

1. A lower, one-sided goal sets a lower limit that we do not want to fall under (but exceeding the limit is fine).
2. An upper, one-sided goal sets an upper limit that we do not want to exceed (but falling under the limit is fine).
3. A two-sided goal sets a specific target that we do not want to miss on either side.

DIFFERENT METHODS OF GOAL PROGRAMMING:

Goal programming problems can be categorized according to how the goals compare in importance.

In one case, called non-preemptive goal programming, all the goals are of roughly comparable importance

In another case, called preemptive goal programming, there is a hierarchy of priority levels for the goals, so that the goals of primary importance receive first priority attention, those of secondary importance receive second-priority attention, and so forth (if there are more than two priority levels).

There are two approaches:

- Sequential procedure
- Streamlined procedure.

Here we will be seeing an example of preemptive goal Programming with streamlined approach.

PROBLEM:

Original Problem:

A company is considering three new products to replace current models that are being discontinued. Management wants primary consideration given to three factors: long-run profit, stability in the workforce, and the level of capital investment. In particular, management has established the goals of (1) achieving a long-run profit (net present value) of at least \$125 million, (2) maintaining the current employment level of 4,000 employees, and (3) holding the capital investment to less than \$55 million. However, management realizes that it probably will not be possible to attain all these goals simultaneously, so it has discussed priorities with the OR department. This discussion has led to setting penalty weights of 5 for missing the profit goal (per \$1 million under), 2 for going over the employment goal (per 100 employees), 4 for going under this same goal, and 3 for exceeding the capital investment goal (per \$1 million over). Each new product's contribution to profit, employment level, and capital investment level is proportional to the rate of production.

■ **TABLE 1** Data for the Dewright Co. nonpreemptive goal programming problem

Factor	Unit Contribution			Goal (Units)	Penalty Weight
	Product:				
	1	2	3		
Long-run profit	12	9	15	≥ 125 (millions of dollars)	5
Employment level	5	3	4	$= 40$ (hundreds of employees)	2(+), 4(-)
Capital investment	5	7	8	≤ 55 (millions of dollars)	3

Revised Problem for preemptive approach:

Faced with the unpleasant recommendation to increase the company's work- force by more than 20 percent, the management has reconsidered the original formulation of the problem that was summarized in the earlier Table. This increase in workforce probably would be a rather temporary one, so the very high cost of training 833 new employees would be largely wasted, and the large (undoubtedly well- publicized) layoffs would make it more difficult for the company to attract high-quality employees in the future. Consequently, management has concluded that a very high priority should be placed on avoiding an increase in the workforce. Furthermore, management has learned that raising more than \$55 million for capital investment for the new products would be extremely difficult, so a very high priority also should be placed on avoiding capital investment above this level.

■ **TABLE 2** Revised formulation for the Dewright Co. preemptive goal programming problem

Priority Level	Factor	Goal	Penalty Weight
First priority	Employment level	≤ 40	2M
	Capital investment	≤ 55	3M
Second priority	Long-run profit	≥ 125	5
	Employment level	≥ 40	4

Streamlined Procedure:

For the Dewright Co. preemptive goal programming problem summarized in Table 2, note that (1) different penalty weights are assigned within each of the two priority levels and (2) the individual penalty weights (2 and 3) for the first-priority goals have been multiplied by M. These penalty weights yield the following single linear programming model that incorporates all the goals.

$$\begin{aligned}
& \text{Minimize } Z = 5y_1^- + 2My_2^+ + 4y_2^- + 3My_3^+ \\
& 12x_1 + 9x_2 + 15x_3 - (y_1^+ - y_1^-) = 125 \\
& 5x_1 + 3x_2 + 4x_3 - (y_2^+ - y_2^-) = 40 \\
& 5x_1 + 7x_2 + 8x_3 - (y_3^+ - y_3^-) = 55 \\
& x_j \geq 0; y_i^+ \geq 0; y_i^- \geq 0
\end{aligned}$$

The lp file and solution is available in `pgp.lp` and `gp_mpadhye.rmd`.