ENAC	Lab on inertial navigation 2D land-vehicle navigation	ENSEIRB-MATMECA January 2026

1. Introduction

The purpose of this project is to give hands-on experience with inertial navigation.

You will implement a simplified IRS mechanisation for a land-vehicle in a local-tangent plane (LTP). Then, you will integrate this mechanization into a GPS/IRS hybridization process based on linearized Kalman filtering.

The work to do is organized in 13 questions. The written report to deliver shall be organised around your responses to these questions.

The question paper is organized as follows:

- Part 2 (page 2) describes the data at your disposal.
- Part 3 (page 6) describes the theoretical analysis to conduct for both the IRS 2D simplified mechanisation and the hybridization filter implementations.
- Part 4 (page 9) describes the implementation tasks to do and the results analysis to conduct.

2. Data description

GPS and IMU measurements are real data.

A GPS receiver and an IMU were installed in a car: the IMU was located inside the car; the GPS receiver antenna was located on the top of the car. For sake of simplicity, we neglect the lever arm between the IMU and the GPS receiver antenna locations, as well as the lever arm between the vehicle gravity centre and the platform.

The car travelled 3 rounds around ENAC Ziegler building. The travelled path is illustrated in figure 1; its duration is about 4.5 minutes.

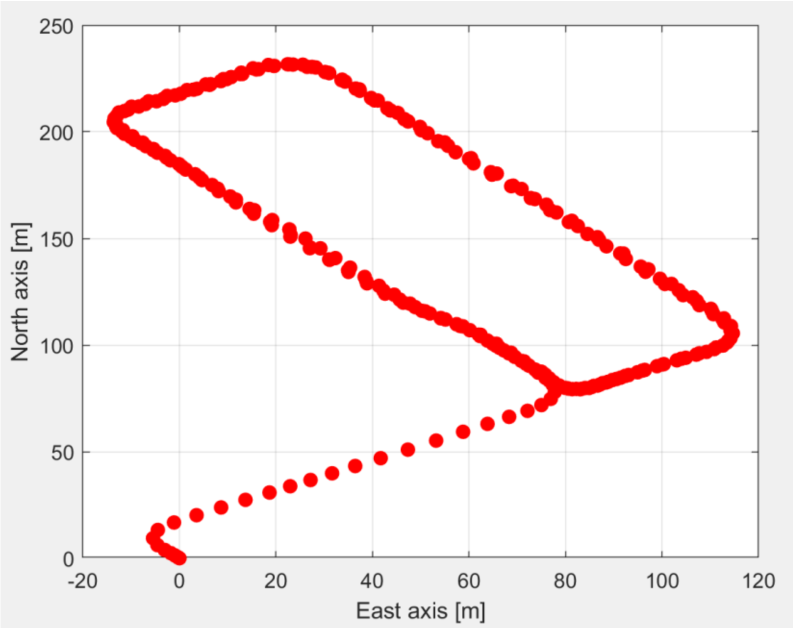


Figure 1 – Reference trajectory (recorded using a trajectography equipment)

In the frame of this lab work, **we interest to the 2D motion of the car in local-tangent plane (t-frame)**.

2.1.Frames definition

2.1.1. Vehicle frame

The vehicle frame, b-frame, is the one defined in the course.

2.1.2. Navigation frame

The navigation frame, t-frame, is a local tangent plane (LTP) with centre, 0, is at the trajectory start point. x-axis extends through the North direction, y-axis extends through the East direction and z-axis is along the downward vertical, as illustrated in figure 2.

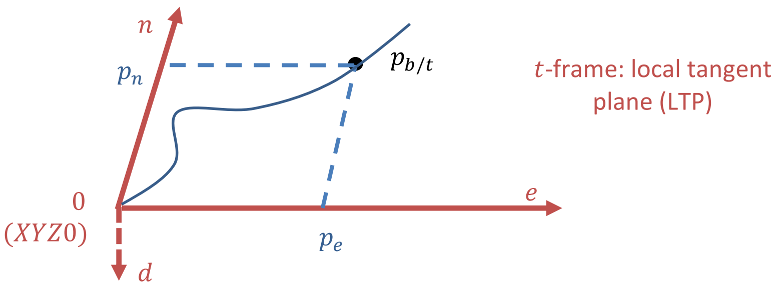
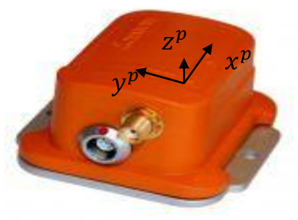


Figure 2 – Navigation frame definition: local tangent plane

2.1.3. Platform frame

The IMU platform frame, p-frame, is illustrated in figure 3.



(x^p, y^p, z^p) defines the platform coordinate frame, p.

This is a NWU coordinate frame

Figure 3 – Platform frame definition

2.2. GPS measurements

GPS measurements were collected using a commercial off-the-shell *u-blox* GPS receiver, which provides the user position and velocity estimates. The collected data rate is 1 Hz.

GPS data are stored in file *GPS.mat*. They are described in table 1.

Data	Size	Description
t_GPS	nPts_GPSx1	Time [s]
nPts_GPS	1x1	Number of GPS samples collected
ned_GPS	nPts_GPSx3	GPS position in LTP [m], $p_{b/t}$ $ned\_GPS(i,:) = [p_n(i), p_e(i), p_d(i)]$
v_ned_GPS	nPts_GPSx3	GPS velocity in LTP [m/s], ${v_{b/e}}^t$

Table 1 – GPS.mat file data

2.3. Inertial measurements

Accelerometer and gyrometer measurements were collected using a *MTi Xsens* IMU. The collected data rate is 100 Hz.

The IMU is composed of a triad of accelerometers, a triad of gyrometers (and a triad of magnetometers that are not used in this project). They are strapped-down mounted. Accelerometer and gyrometer sensors are based on MEMS technologies.

Because of the sensors' quality, we assume the sensors measurement errors are mainly driven by noise and bias such as the measurement mathematical models we assume are:

$$\tilde{f}^p(t) = f^p(t) - b^f(t) - n^f(t)$$

$$\tilde{\omega}^p(t) = \omega^p(t) - b^\omega(t) - n^\omega(t)$$

where,

$\tilde{\omega}^p$ and $\tilde{f}^p$ are the accelerometer and gyrometer measurements at the IMU outputs parametrized in p-frame; by definition, $f^p(t) = f^p_{b/i}(t)$ and $\omega^p(t) = \omega^p_{b/i}(t)$.

ω^p and f^p are the sensed specific force and angular rate values

b^ω and b^f are biases that affect the measurements; they are assumed to be 1st-order Markov process:

$\dot{b}^\omega(t) = -\frac{1}{\tau_\omega} b^\omega(t) + n^{b^\omega}(t)$ with $n^{b^\omega}(t) \sim N(0, \sigma_{b^\omega})$

$\dot{b}^f(t) = -\frac{1}{\tau_f} b^f(t) + n^{b^f}(t)$ with $n^{b^f}(t) \sim N(0, \sigma_{b^f})$

n^ω and n^f are the measurements noise modelled as WCGN:

$n^\omega(t) \sim N(0, \sigma_\omega)$

$n^f(t) \sim N(0, \sigma_f)$

A preliminary analysis has been conducted to determine the IMU errors characteristics; the assumed values are summarized in Table 2.

Error / Characteristic	Value
Accelerometer noise standard deviation ($\boldsymbol{\sigma_f}$)	$0.1\ m/s^2$
Accelerometer bias time constant ($\boldsymbol{\tau_\omega}$)	$5\ s$
Accelerometer bias temporal standard deviation ($\boldsymbol{\sigma_{b^f}}$)	$0.05\ m/s^2$
Gyrometer noise standard deviation ($\boldsymbol{\sigma_\omega}$)	$0.01\ rad/s$
Gyrometer bias time constant ($\boldsymbol{\tau_\omega}$)	$4\ s$
Gyrometer bias temporal standard deviation ($\boldsymbol{\sigma_{b^\omega}}$)	$0.001\ rad/s$

Table 2 – Error characteristics of IMU assumed

The IMU was installed such as x^p axis is aligned with the car along track displacement axis (ie. x^b and $x^p$are perfectly aligned) and z^p is in the upward direction (ie. z^b and $z^p$are in opposite directions). IMU position in the car in illustrated in figure 4.

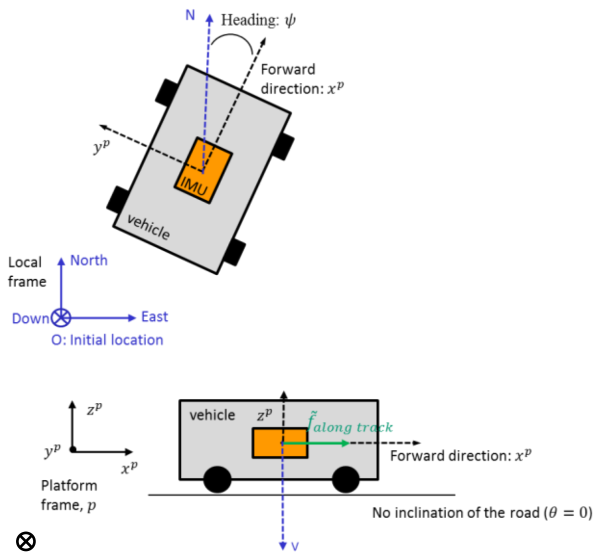


Figure 4 – IMU position in the vehicle

IMU data are stored in file *IMU.mat*. They are described in table 3.

Data	Size	Description
t_IMU	nPts_IMUx1	Time [s]
nPts_IMU	1x1	Number of IMU samples collected
Fs	1x1	IMU data collection rate [Hz]
Ts	1x1	IMU data collection sampling period [s]
f_bi_p	nPts_IMUx3	Specific force measured by the triad of accelerometers [m/s²], ${f_{b/i}}^p$
w_bi_p	nPts_IMUx3	Angular rate measured by the triad of gyrometers [rad/s], ${\omega_{b/i}}^p$

Table 3 – IMU.mat file data

2.4. Reference solution

For each travelled path, a reference solution was computed using a Novatel SPAN trajectography equipment (tactical grade IRU + differential GPS).

The reference solution data are stored in file *Reference.mat*. They are described in table 4.

Data	Size	Description
t_Ref	nPts_Refx1	Time [s]
nPts_Ref	1x1	Number of Reference solution samples
llh_Ref	nPts_Refx3	Reference position in e-frame [m], $p_{b/e}$ $llh\_Ref(i,:) = [\lambda\ [rad], \varphi\ [rad], h[m]]$
ned_Ref	nPts_Refx3	Reference position in LTP [m], $p_{b/t}$ $ned\_Ref(i,:) = [p_n, p_e, p_d]$
v_ned_Ref	nPts_Refx3	Reference velocity in LTP [m/s], ${v_{b/e}}^t$
heading_Ref	nPts_Refx1	Reference heading [rad], ψ
theta_Ref	nPts_Refx1	Reference pitch [rad], θ
XYZ0	1x3	Initial point location, $p_{b/e}(0) = [x, y, z]^e$

Table 4 – Reference.mat file data

3. Theoretical analysis

3.1. 2D IRS mechanisation for land-vehicle

You will implement a simplified 2D IRS mechanisation appropriate for low-cost inertial sensors. It is based on the use of a limited number of sensors: 1 accelerometer (we neglect the road inclination) and 1 gyrometer, only.

3.1.1. Assumptions

For the IMU measurements, we only consider:

- the accelerometer measurement in the car along track direction: this is the specific force along x^p, $\tilde{f}^p_{\_IMU}(x)$.
- the gyrometer measurement in vertical direction, which is along z^p: this is $\widetilde{\omega}^p_{\_IMU}(z)$.

We do not consider the information provided by the 4 other sensors.

3.1.2. Theoretical analysis of the simplified mechanisation in LTP

Question 1

Run ***IRS_Simplified_Mechanization_2D.m*** (line 58 must be commented).

Present the figures displayed (Figures 1, 2 and 3) and give a short analysis about the IMU measurements (bias, noise).

Find out the main characteristics of the path travelled in the measurements.

Question 2

Calculate the rotation matrix R_{p2b}, which allows aligning the platform frame with the vehicle frame.

Give the expression of the components of $\tilde{f}^b_{\_IMU}$ and $\widetilde{\omega}^b_{\_IMU}$ (specific force and gyrometer measurements parametrized in b-frame) as a function of $\tilde{f}^p_{\_IMU}$ and $\widetilde{\omega}^p_{\_IMU}$ components.

Question 3

Assuming,

- Hypothesis 1 : the mechanisation is developed in the local tangent plane defined in 2.1.2.
- Hypothesis 2 : the Earth rotation may be neglected for the trajectory duration (ie. $\omega_{e/i} \approx 0$)
- Hypothesis 3 : the local gravity acceleration is constant.

Calculate the motion equations of:

- the along track velocity parametrized in b-frame, $v^b_{AT}(t)$
- the North-position parametrized in t-frame, $p^t_n(t)$
- the East-position parametrized in t-frame, $p^t_e(t)$
- the heading angle, $\psi(t)$ (based on Euler derivatives)

as a function of ${f_{b/i}}^b(x)(t)$ and ${\omega_{b/i}}^b(z)(t)$.

where, ${f_{b/i}}^b(x)(t)$ is the x-component of ${f_{b/i}}^b(t)$,

${\omega_{b/i}}^b(z)(t)$ is the z-component of ${\omega_{b/i}}^b(t)$.

Deduce the corresponding mechanization equations.

Question 5

Assuming Δt - the step time between 2 consecutive IMU measurements - is constant, use trapezoidal integration to give the expression of:

- The IRS heading at time k, $\hat{\psi}_{\_IRS}(k)$
- The IRS along-track velocity parametrized in b-frame, at time k, $\hat{v}^b_{AT\_IRS}(k)$
- The IRS North-position parametrized in t-frame, at time k, $\hat{p}^t_{n\_IRS}(k)$
- The IRS East-position parametrized in t-frame, at time k, $\hat{p}^t_{e\_IRS}(k)$

from the IMU measurements parametrized in b-frame at times k and $k-1$ ($\tilde{f}^b_{\_IMU}(x)(k)$, $\tilde{\omega}^b_{\_IMU}(z)(k)$, $\tilde{f}^b_{\_IMU}(x)(k-1)$, $\tilde{\omega}^b_{\_IMU}(z)(k-1)$), and the estimates at time $k-1$.

3.2. GPS/IRS loose coupling for land-vehicle

GPS and IRS data are hybridized using a linearized Kalman filter implementation that aims at estimating the IRS errors. Those estimates are used to correct the IRS navigation solution to provide the hybridized (GPS/IRS) navigation solution.

We assume the IRS solution results from the former 2D simplified mechanisation.

The integration process is done using a linearized Kalman filter.

Let $\delta x(t)$ be the linearized Kalman filter error state vector. It is composed of:

- IRS North position error parametrized in t-frame, $\delta p_n(t)$
- IRS East position error parametrized in t-frame, $\delta p_e(t)$
- IRS Along track velocity error parametrized in b-frame, $\delta v^b_{AT}(t)$
- IMU Accelerometer bias (1 states), $b^f(t)$
- IRS Heading error, $\delta\psi(t)$
- IMU Gyrometer bias (1 state), $b^\omega(t)$

where, for any error term, $\delta s = s - \hat{s}_{IRS}$.

The underlying system to model the integration process is:

$$\delta\dot{x}(t) = F(t).\delta x(t) + u(t)$$
$$y(t) = H(t).\delta x(t) + w(t)$$

where,

$\delta x(t)$ is the error state vector
$F(t)$ is the state transition matrix
$u(t)$ is the process noise
$y(t)$ is the observation vector

$H(t)$ is the observation matrix
$w(t)$ is the observation model noise

3.2.1. Theoretical study of the Kalman filter model

3.2.1.1. State transition model

Question 6

With help of the methodology detailed in Annex 1 and the assumed IMU measurements mathematical models (section 2.3), calculate the Kalman filter state transition model in continuous time (give the details of the calculation).

Identify the state transition matrix, $F(t)$, and the process noise, $u(t)$.

Assuming all the components of the process noise are independent and not time-correlated, give the expression of the covariance matrix, $Q(t)$.

What is the value of the Kalman filter initial state vector, $\hat{x}_{0|0}$?

What is the value of the initial state covariance matrix, $S_{0|0} = cov(\hat{x}_{0|0})$?

3.2.1.2. Measurement model calculation

GPS receiver feeds the hybridized filter (Kalman filter) with the GPS estimated position available every 1 second.

At the discrete time k, we assume the GPS observations are,

$$\tilde{p}^t_{n\_GPS}(k) = p^t_n(k) + n_{p_{n\_GPS}}(k)$$
$$\tilde{p}^t_{e\_GPS}(k) = p^t_e(k) + n_{p_{e\_GPS}}(k)$$

where

$\tilde{p}^t_{n\_GPS}$ and $\tilde{p}^t_{e\_GPS}$ are the GPS position North and East-coordinates parametrized in t-frame, respectively
n_{n_GPS} and n_{e_GPS} stand for the GPS position solution error along the North and East axes, respectively. We assume $n_{p_{n\_GPS}}(k) \sim N(0, \sigma_{p_{n\_GPS}})$ and $n_{p_{e\_GPS}}(k) \sim N(0, \sigma_{p_{e\_GPS}})$.

At first, we assume $\sigma_{p_{n\_GPS}} = \sigma_{p_{e\_GPS}} = 10m$.

Question 7

Calculate the Kalman filter measurement model (give the details of the calculation).

Identify the measurement matrix, $H(k)$, and the measurement noise, $w(k)$.

Assuming the components of the measurement noise are independent and not time-correlated, give the expression of the covariance matrix, $R(k)$.

4. Implementation and results analysis

4.1. 2D IRS simplified mechanisation

The templates for implementing the simplified 2D mechanization are in *IRS_Navigation.m* and in *IRS_Simplified_Mechanization_2D.m*.

A process for correcting the IMU measurements by their initial bias is implemented in *IRS_Simplified_Mechanization_2D.m* (lines 66 to 75):

- aAT is the along tracked acceleration measurement, $\tilde{f}^{b}_{\_IMU}(x)$, where the initial bias has been removed
- $HeadingRate$ is the gyrometer measurement in vertical direction, $\widetilde{\omega}^{b}_{\_IMU}(z)$, where the initial bias has been removed

Using the results of Question 5, complete the Matlab® function, *IRS_Navigation.m*. This function implements the IRS 2D navigation solution for:

- the vehicle heading angle estimate
- the vehicle along-track velocity in LTP estimate
- the vehicle velocity along the North- and East-axis of the LTP estimate
- the vehicle location along the North- and East-axis of the LTP estimate

Complete the Matlab® program, *IRS_Simplified_Mechanization_2D.m*:

- Rotation matrix from platform frame to body frame (line 62)
- Matrices containing the IMU measurements parametrized in the body frame (lines 63 and 64)
- IRS platform initialization (line 105 to 110), assuming at start, the vehicle is static, and the car heading is equal to the heading angle estimated by the Reference solution.
- IRS 2D navigation (call to the function *IRS_Navigation.m*)

Uncomment the lines you have completed.

Question 8

Comment line 58 ("return" command) and **run *IRS_Simplified_Mechanization_2D.m***.

Present the figures displayed.

Using these figures, analyse the solution provided by this 2D IRS simplified mechanisation.

Question 9 – Initial bias correction analysis

Now, assume the initial biases affecting the IMU measurements are not removed. Modify *IRS_Simplified_Mechanization_2D.m* to set $bgyro$ to $0\ rad/s$, and $bacc$ to $0\ m/s^2$.

Analyse and compare the new solution obtained to the one of Question 6.

4.2. GPS/IRS loose coupling for land-vehicle

Initially, the hybridization process is implemented in feedforward configuration (open loop coupling) as illustrated in figure 5.

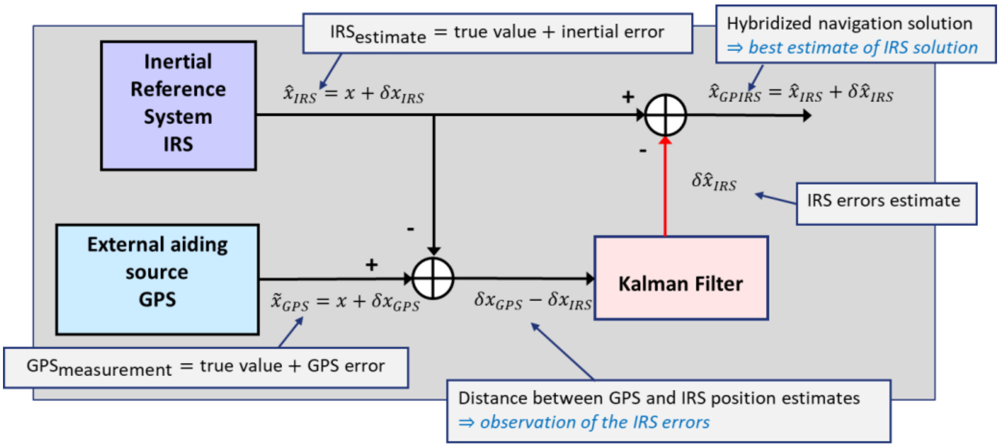


Figure 5 – Open loop GPS/IRS hybridization

The template of the GPS/IRS hybridization is in *Hybridization_Simplified_Mechanization_2D.m*.

Using the elements you have already completed in *IRS_Simplified_Mechanization_2D.m*, **complete the IRS platform part** with:

- The rotation matrix from platform frame to body frame (line 78)
- The matrices of the IMU measurements parametrized in the body frame (lines 79 and 80)
- IRS platform initialization (line 161 to 166).
- The IRS 2D navigation (line 169)

(the notations are similar; the initial conditions are the same).

Complete the Kalman filter initialization part:

- The Kalman filter initial state vector, $\hat{x}_{0|0}$, noted $X0$ (line 99)
- What is the value of the initial state covariance matrix, $S_{0|0}$, noted $S0$ (line 101)

Complete the Kalman filter state transition model,

- The continuous-time state transition matrix, Ft (line 179)
- The continuous-time process noise matrix, Qt (line 182)

Complete the Kalman filter measurement model,

- The measurement vector, $meas$ (line 205)
- The measurement matrix, H (line 208)
- The measurement noise covariance matrix, R (line 211)

Complete the GPS/IRS navigation solution computation (line 241 to 246).

Question 10

IRS and GPS data are not available at the same rate: how is this considered in the Kalman filter implementation?

Question 11

A GPS signals masking may be simulated after 100s by defining a masking interval duration in seconds through the parameter, *MaskingInterval* (line144).

Run M-file with *MaskingInterval = 0s.* Present the figures displayed.

Analyse the IRS/GPS navigation solution obtained and compare it to the IRS solution of Question 8.

Explain the saw teeth shape you observe in the GPS/IRS position solution.

Question 12

Now run M-file with *MaskingInterval = 60s*. Compare these results with the ones of Question 11.

4.3. Close loop GPS/IRS coupling for land vehicle

Because of the IMU sensors low-quality, the Kalman filter estimates may quickly drift with time.

A solution is to implement the hybridization process in close loop as illustrated in figure 6.

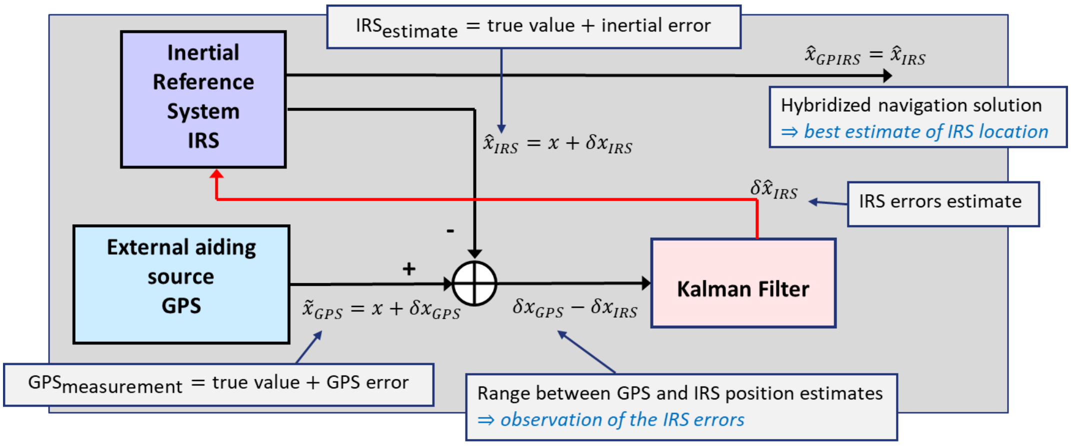


Figure 6 – Close loop GPS/IRS hybridization

Modify the functions *Hybridization_Simplified_Mechanization_2D.m* and *IRS_Navigation.m* to implement a close loop GPS/IRS hybridization.

I advise to save the modified versions of *Hybridization_Simplified_Mechanization_2D.m* and *IRS_Navigation.m* with new names.

Question 13

Run the close loop implementation and compare the results with the ones of the open loop implementation.

5. Annex 1 – Position North-component equations

In question 3, you will show the **motion equation of the position North-component** is

$$\dot{p}_n^t(t) = v_{AT}^b(t) \cdot \cos\big(\psi(t)\big)$$

This means $\dot{p}_n^t(t)$ is a function of $v_{AT}^b(t)$ and $\psi(t)$. Let us name g this function. We may write:

$$\dot{p}_n^t(t) = g\big(v_{AT}^b(t), \psi(t)\,\big)$$

Thus, the **corresponding mechanization equation** is given by

$$\dot{\hat{p}}_{n\_IRS}^t(t) = g\big(\hat{v}_{AT\_IRS}^b(t), \hat{\psi}_{\_IRS}(t)\,\big) = \hat{v}_{AT\_IRS}^b(t) \cdot \cos\Big(\hat{\psi}_{\_IRS}(t)\Big)$$

The **transition equation of the IRS North position error parametrized in *t*-frame** is defined as:

$$\delta\dot{p}_n(t) = \dot{p}_n^t(t) - \dot{\hat{p}}_{n\_IRS}^t(t)$$

Or,

$$\delta\dot{p}_n(t) = g\big(v_{AT}^b(t), \psi(t)\,\big) - g\big(\hat{v}_{AT\_IRS}^b(t), \hat{\psi}_{\_IRS}(t)\,\big)$$

Let us consider the 1st order Taylor series expansion of $g\big(v_{AT}^b(t), \psi(t)\,\big)$ around the IRS solution.

Assuming $\delta v_{AT}^b(t)$ and $\delta\psi(t)$ are small enough, it is calculated as

$$\begin{aligned} g\big(v_{AT}^b(t), \psi(t)\,\big) \\ &= g\big(\hat{v}_{AT_{IRS}}^b(t), \hat{\psi}_{\_IRS}(t)\,\big) + \left.\frac{\partial g\big(v_{AT}^b(t), \psi(t)\,\big)}{\partial v_{AT}^b(t)}\right|_{\substack{v_{AT}^b(t)=\hat{v}_{AT_{IRS}}^b(t) \\ \psi(t)=\hat{\psi}_{\_IRS}(t)}} \cdot \delta v_{AT}^b(t) \\ &+ \left.\frac{\partial g\big(v_{AT}^b(t), \psi(t)\,\big)}{\partial \psi(t)}\right|_{\substack{v_{AT}^b(t)=\hat{v}_{AT_{IRS}}^b(t) \\ \psi(t)=\hat{\psi}_{\_IRS}(t)}} \cdot \delta\psi(t) \end{aligned}$$

Thus,

$$\delta\dot{p}_n(t) = \left.\frac{\partial g\big(v_{AT}^b(t), \psi(t)\,\big)}{\partial v_{AT}^b(t)}\right|_{\substack{v_{AT}^b(t)=\hat{v}_{AT_{IRS}}^b(t) \\ \psi(t)=\hat{\psi}_{\_IRS}(t)}} \cdot \delta v_{AT}^b(t) + \left.\frac{\partial g\big(v_{AT}^b(t), \psi(t)\,\big)}{\partial \psi(t)}\right|_{\substack{v_{AT}^b(t)=\hat{v}_{AT_{IRS}}^b(t) \\ \psi(t)=\hat{\psi}_{\_IRS}(t)}} \cdot \delta\psi(t)$$

So finally,

$$\delta\dot{p}_n(t) = \cos\Big(\hat{\psi}_{\_IRS}(t)\Big) \cdot \delta v_{AT}^b(t) - \sin\Big(\hat{\psi}_{\_IRS}(t)\Big) \cdot \hat{v}_{AT\_IRS}^b(t) \cdot\ \delta\psi(t)$$