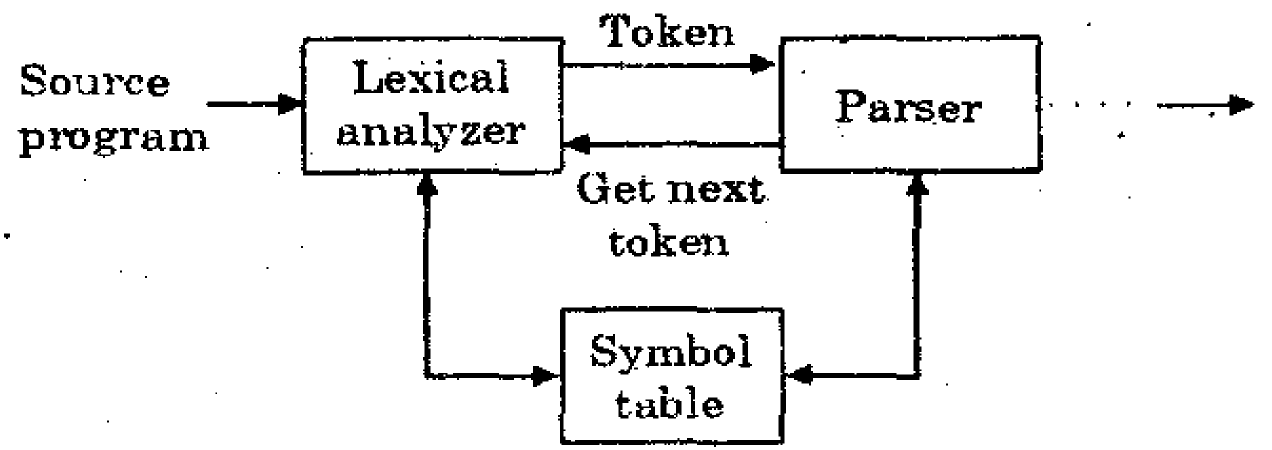
Lexical Analysis and Design of Lexical Analyzer

Lexical Analysis

- Input is scanned completely to identify the tokens
- Tokens (Logical unit)
  - Identifier, Keywords, operators etc.



Specification of Tokens

- Strings and Languages
  - Finite sequence of Symbols is called Strings
  - Set of strings over some alphabet is called Language
- Operation on Languages
  - Concatenation:
    - $L_1L_2 = \{ s_1s_2 \mid s_1 \in L_1 \text{ and } s_2 \in L_2 \}$
  - Union
    - $L_1 \cup L_2 = \{ s \mid s \in L_1 \text{ or } s \in L_2 \}$
  - Kleene Closure
    - $L^* = \bigcup_{i=0}^{\infty} L^i$
  - Positive Closure
    - $L^+ = \bigcup_{i=1}^{\infty} L^i$
- Regular Expressions

Regular Expression

- Notation for representing Tokens
- Ex: Identifiers in Pascal

  letter $\rightarrow$ A | B | ... | Z | a | b | ... | z

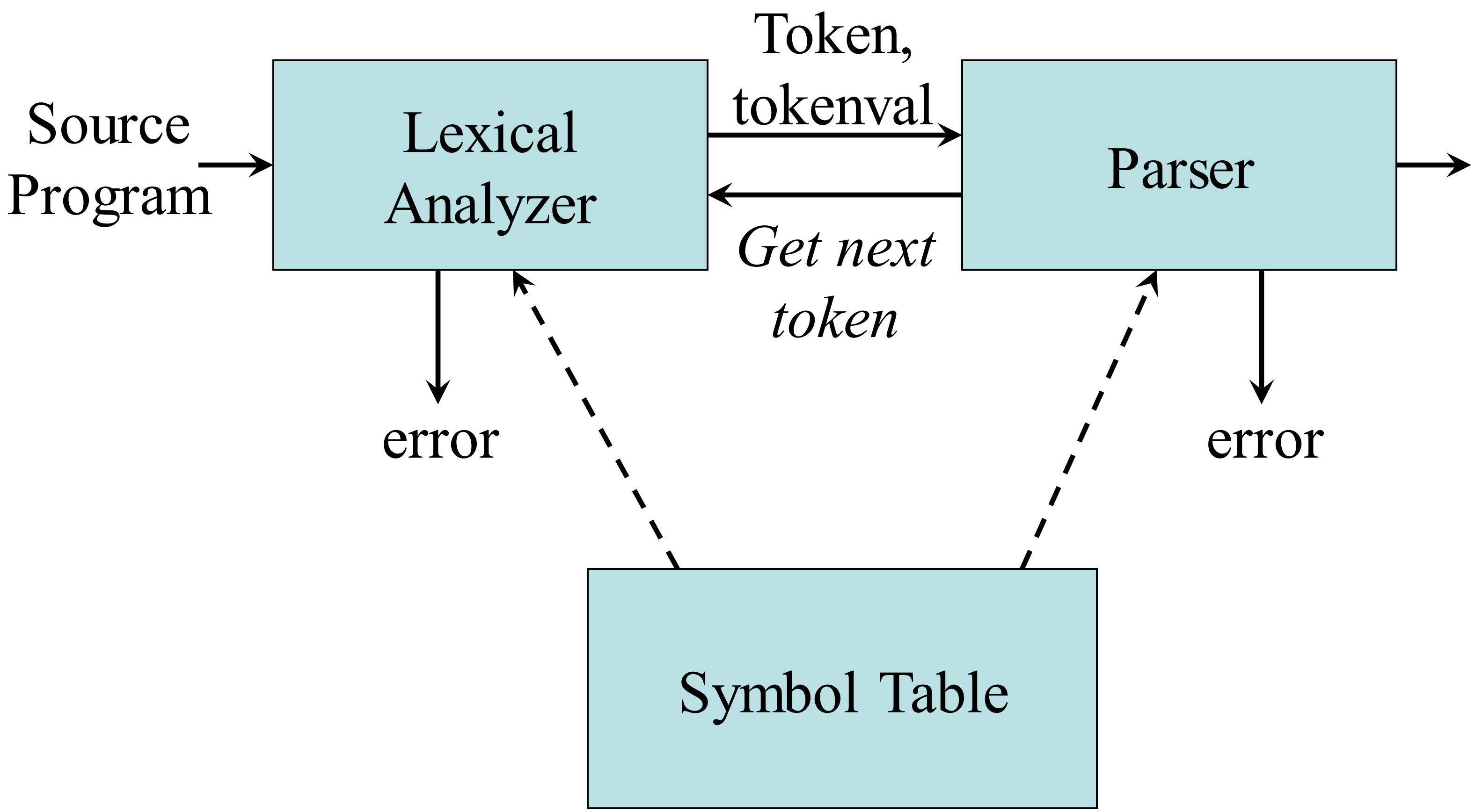
  digit $\rightarrow$ 0 | 1 | ... | 9

  id $\rightarrow$ letter (letter | digit)*

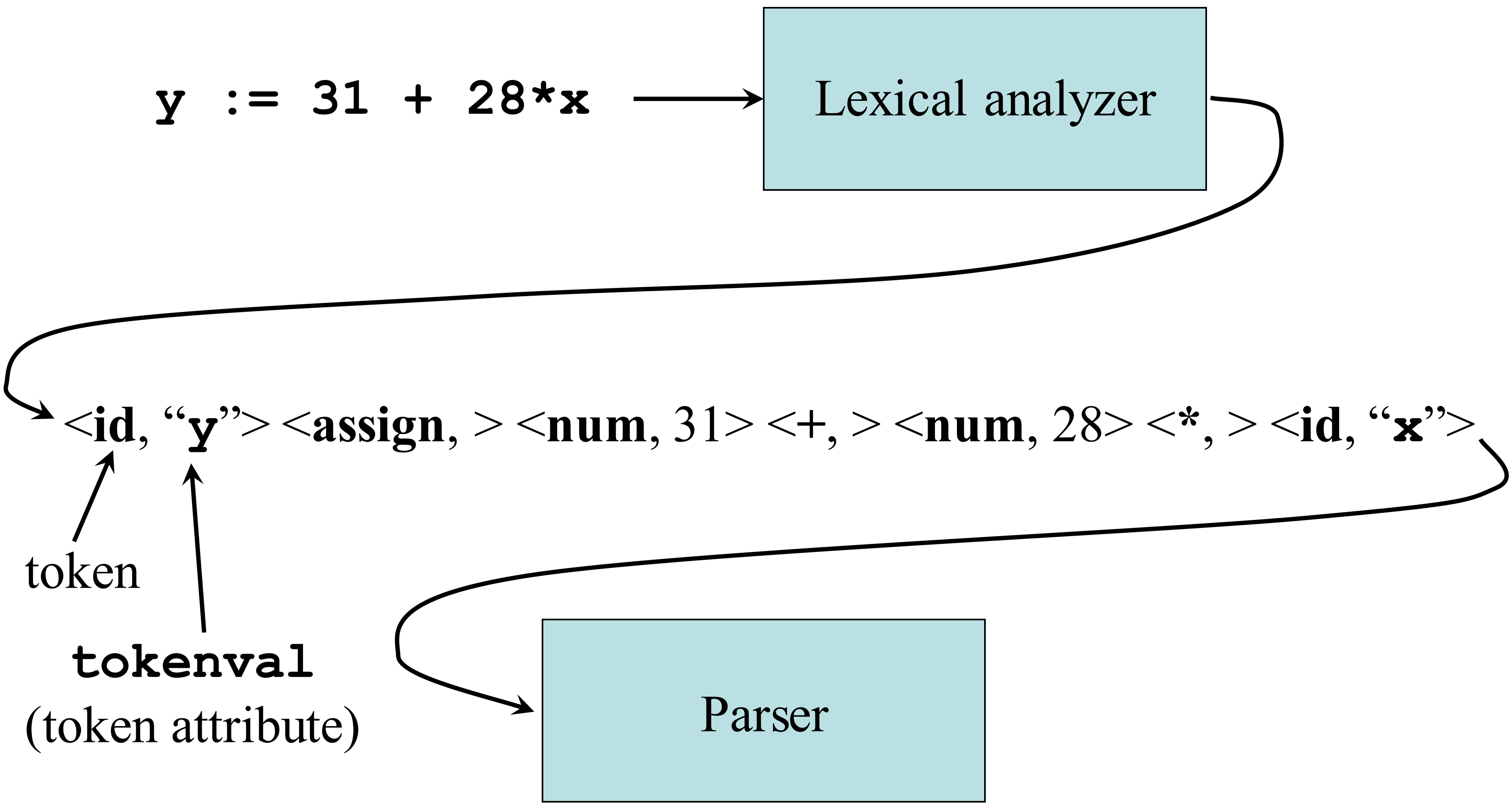
The Reason Why Lexical Analysis is a Separate Phase

- Simplifies the design of the compiler
  - LL(1) or LR(1) parsing with 1 token lookahead would not be possible (multiple characters/tokens to match)
- Provides efficient implementation
  - Systematic techniques to implement lexical analyzers by hand or automatically from specifications
  - Stream buffering methods to scan input
- Improves portability
  - Non-standard symbols and alternate character encodings can be normalized (e.g. trigraphs)

Interaction of the Lexical Analyzer with the Parser



Attributes of Tokens



Tokens, Patterns, and Lexemes

- A *token* is a classification of lexical units
  - For example: **id** and **num**
- *Lexemes* are the specific character strings that make up a token
  - For example: `abc` and `123`
- *Patterns* are rules describing the set of lexemes belonging to a token
  - For example: "*letter followed by letters and digits*" and "*non-empty sequence of digits*"

Specification of Patterns for Tokens: *Definitions*

- An *alphabet* Σ is a finite set of symbols (characters)
- A *string* s is a finite sequence of symbols from Σ
  - $|s|$ denotes the length of string s
  - ε denotes the empty string, thus $|\varepsilon| = 0$
- A *language* is a specific set of strings over some fixed alphabet Σ

Specification of Patterns for Tokens: *String Operations*

- The *concatenation* of two strings x and y is denoted by xy
- The *exponentation* of a string s is defined by

$$s^0 = \varepsilon$$
$$s^i = s^{i-1}s \quad \text{for } i > 0$$

note that $s\varepsilon = \varepsilon s = s$

Specification of Patterns for Tokens: *Language Operations*

- *Union*

  $L \cup M = \{s \mid s \in L \text{ or } s \in M\}$
- *Concatenation*

  $LM = \{xy \mid x \in L \text{ and } y \in M\}$
- *Exponentiation*

  $L^0 = \{\varepsilon\}; \quad L^i = L^{i-1}L$
- *Kleene closure*

  $L^* = \cup_{i=0,\ldots,\infty} L^i$
- *Positive closure*

  $L^+ = \cup_{i=1,\ldots,\infty} L^i$

Specification of Patterns for Tokens: *Regular Expressions*

- Basis symbols:
  - ε is a regular expression denoting language $\{\varepsilon\}$
  - $a \in \Sigma$ is a regular expression denoting $\{a\}$
- If r and s are regular expressions denoting languages $L(r)$ and $M(s)$ respectively, then
  - $r \mid s$ is a regular expression denoting $L(r) \cup M(s)$
  - rs is a regular expression denoting $L(r)M(s)$
  - r^* is a regular expression denoting $L(r)^*$
  - (r) is a regular expression denoting $L(r)$
- A language defined by a regular expression is called a *regular set*

Specification of Patterns for Tokens: *Regular Definitions*

- Regular definitions introduce a naming convention:

  $$d_1 \to r_1$$
  $$d_2 \to r_2$$
  $$\ldots$$
  $$d_n \to r_n$$

  where each r_i is a regular expression over

  $$\Sigma \cup \{d_1, d_2, \ldots, d_{i\text{-}1}\}$$

- Any d_j in r_i can be textually substituted in r_i to obtain an equivalent set of definitions

Specification of Patterns for Tokens: *Regular Definitions*

- Example:

  $$\textbf{letter} \to \texttt{A} \mid \texttt{B} \mid \ldots \mid \texttt{Z} \mid \texttt{a} \mid \texttt{b} \mid \ldots \mid \texttt{z}$$
  $$\textbf{digit} \to \texttt{0} \mid \texttt{1} \mid \ldots \mid \texttt{9}$$
  $$\textbf{id} \to \textbf{letter} \; (\; \textbf{letter} \mid \textbf{digit} \;)^*$$

- Regular definitions are not recursive:

  $$\textbf{digits} \to \textbf{digit} \; \textbf{digits} \mid \textbf{digit}$$ *wrong!*

Specification of Patterns for Tokens: *Notational Shorthand*

- The following shorthands are often used:

$$r^+ = rr^*$$
$$r? = r \mid \varepsilon$$
$$[\mathtt{a\text{-}z}] = \mathtt{a} \mid \mathtt{b} \mid \mathtt{c} \mid \ldots \mid \mathtt{z}$$

- Examples:

digit $\rightarrow$ [`0-9`]

num $\rightarrow$ **digit**$^+$ (. **digit**$^+$)? (**E** (+ | -)? **digit**$^+$)?

Regular Definitions and Grammars

Grammar

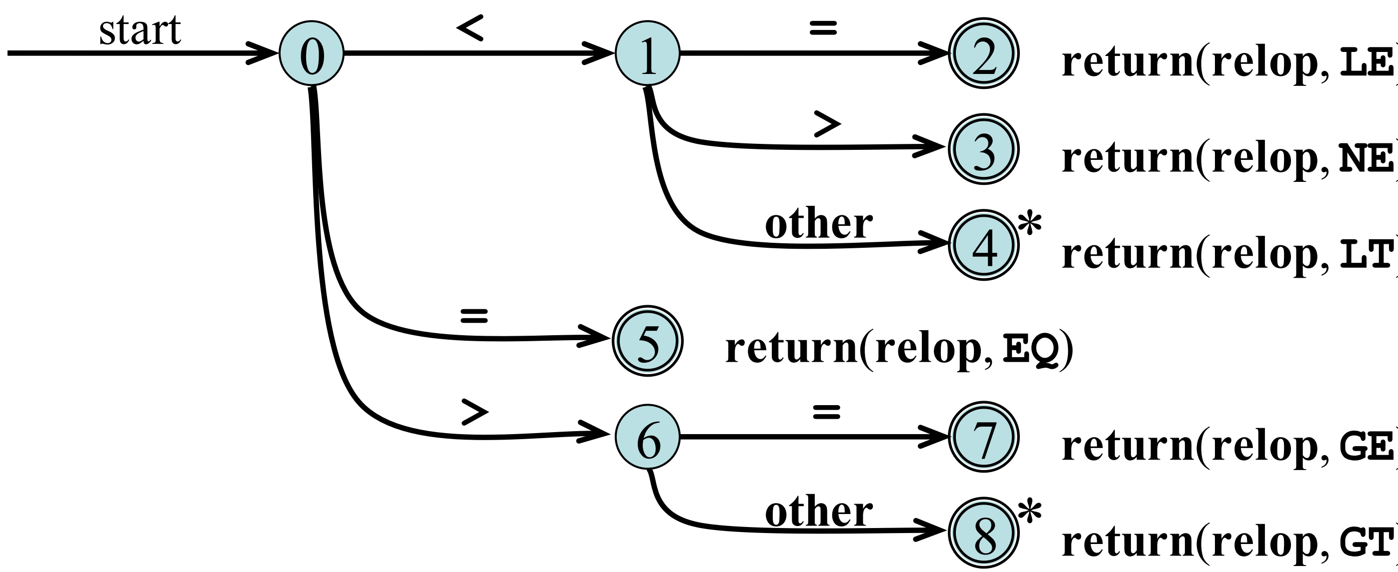
$$
\begin{aligned}
\mathit{stmt} &\rightarrow \textbf{if}\ \mathit{expr}\ \textbf{then}\ \mathit{stmt} \\
&\mid \textbf{if}\ \mathit{expr}\ \textbf{then}\ \mathit{stmt}\ \textbf{else}\ \mathit{stmt} \\
&\mid \varepsilon \\
\mathit{expr} &\rightarrow \mathit{term}\ \textbf{relop}\ \mathit{term} \\
&\mid \mathit{term} \\
\mathit{term} &\rightarrow \textbf{id} \\
&\mid \textbf{num}
\end{aligned}
$$

Regular definitions

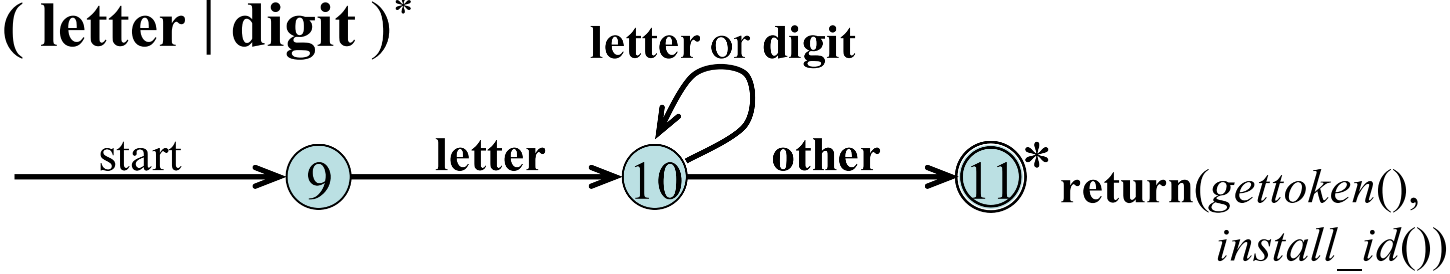
$$
\begin{aligned}
\textbf{if} &\rightarrow \mathtt{if} \\
\textbf{then} &\rightarrow \mathtt{then} \\
\textbf{else} &\rightarrow \mathtt{else} \\
\textbf{relop} &\rightarrow \mathtt{<} \mid \mathtt{<=} \mid \mathtt{<>} \mid \mathtt{>} \mid \mathtt{>=} \mid \mathtt{=} \\
\textbf{id} &\rightarrow \textbf{letter}\ (\ \textbf{letter} \mid \textbf{digit}\ )^{*} \\
\textbf{num} &\rightarrow \textbf{digit}^{+}\ (.\ \textbf{digit}^{+})?\ (\ \textbf{E}\ (+ \mid -)?\ \textbf{digit}^{+}\ )?
\end{aligned}
$$

Coding Regular Definitions in *Transition Diagrams*

$\textbf{relop} \to \texttt{<} \mid \texttt{<=} \mid \texttt{<>} \mid \texttt{>} \mid \texttt{>=} \mid \texttt{=}$



$\textbf{id} \to \textbf{letter}\ (\ \textbf{letter} \mid \textbf{digit}\ )^*$



Coding Regular Definitions in Transition Diagrams: Code


```
token nexttoken()
{ while (1) {
    switch (state) {
    case 0: c = nextchar();
       if (c==blank || c==tab || c==newline) {
         state = 0;
         lexeme_beginning++;
       }
       else if (c=='<') state = 1;
       else if (c=='=') state = 5;
       else if (c=='>') state = 6;
       else state = fail();
       break;
     case 1:
       …
     case 9: c = nextchar();
       if (isletter(c)) state = 10;
       else state = fail();
       break;
     case 10: c = nextchar();
       if (isletter(c)) state = 10;
       else if (isdigit(c)) state = 10;
       else state = 11;
       break;
     …
```

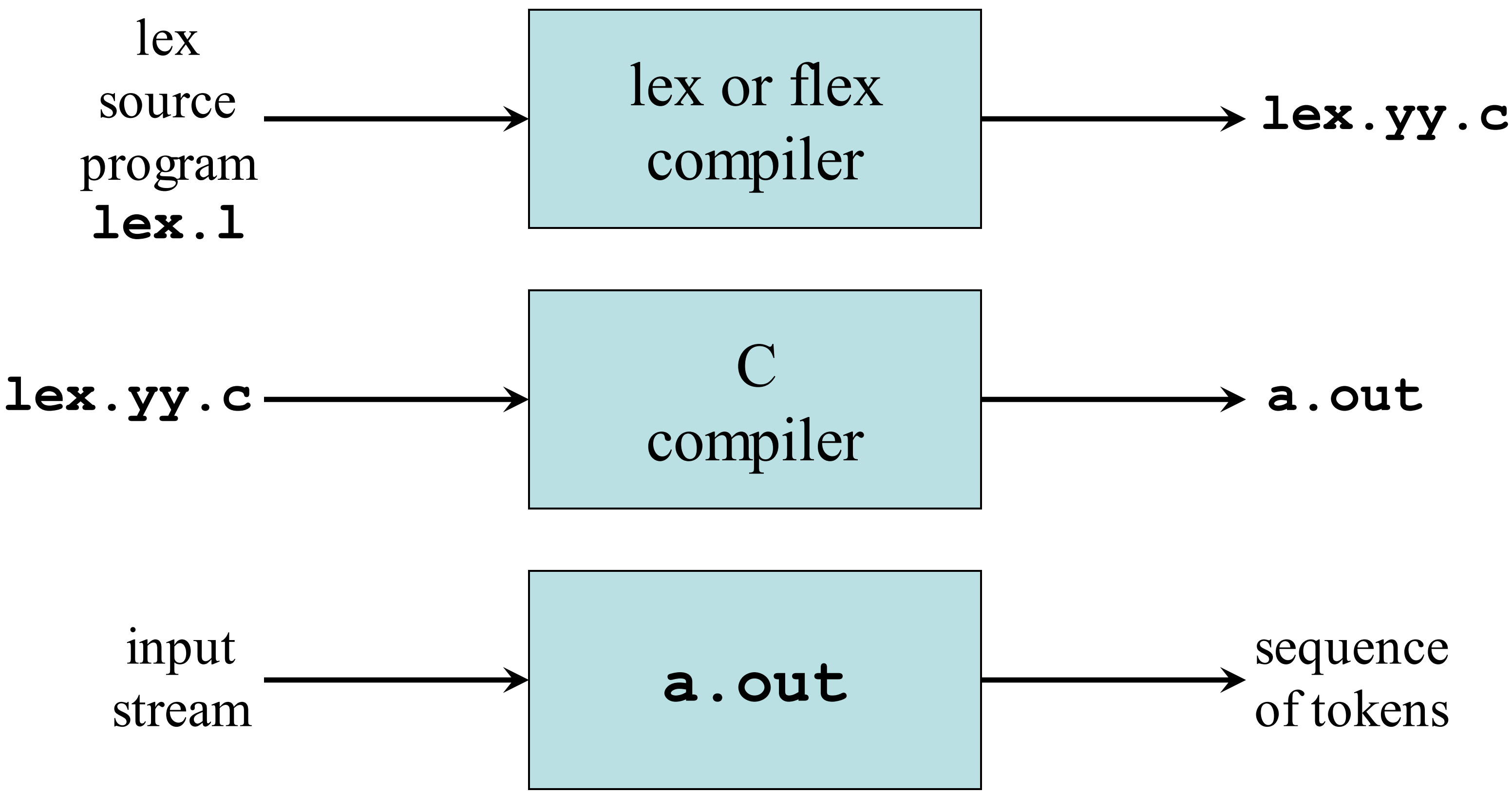
Decides the next start state to check

```
int fail()
{ forward = token_beginning;
  swith (start) {
  case  0: start =  9; break;
  case  9: start = 12; break;
  case 12: start = 20; break;
  case 20: start = 25; break;
  case 25: recover(); break;
  default: /* error */
  }
  return start;
}
```

The Lex and Flex Scanner Generators

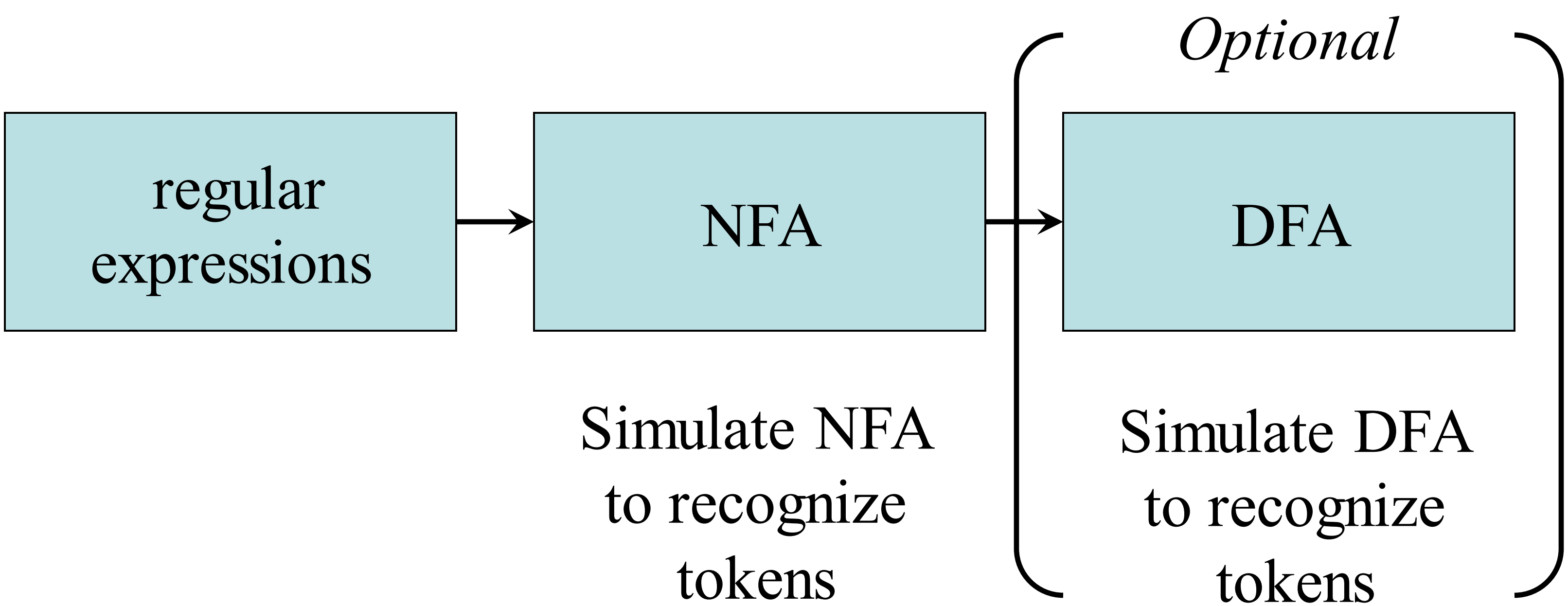
- *Lex* and its newer cousin *flex* are scanner generators
- Systematically translate regular definitions into C source code for efficient scanning
- Generated code is easy to integrate in C applications

Creating a Lexical Analyzer with Lex and Flex



Design of a Lexical Analyzer Generator

- Translate regular expressions to NFA
- Translate NFA to an efficient DFA



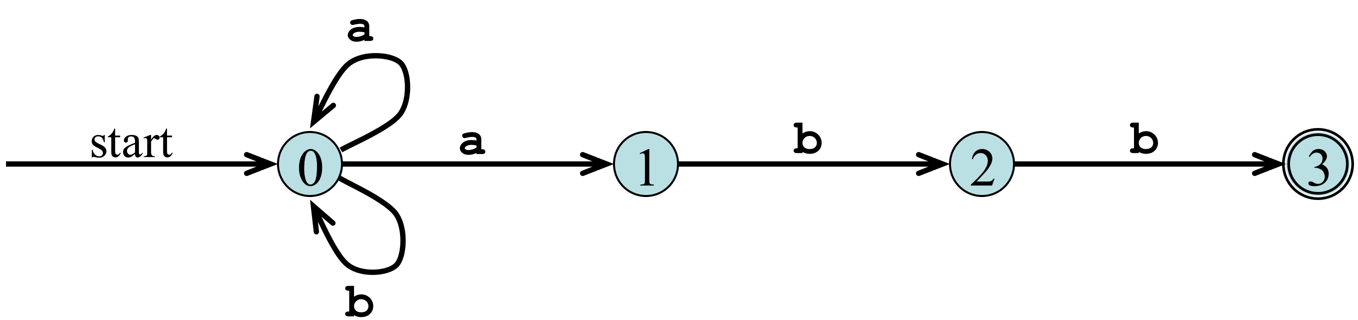
Nondeterministic Finite Automata

- An NFA is a 5-tuple $(S, \Sigma, \delta, s_0, F)$ where

  S is a finite set of *states*
  Σ is a finite set of symbols, the *alphabet*
  δ is a *mapping* from $S \times \Sigma$ to a set of states
  $s_0 \in S$ is the *start state*
  $F \subseteq S$ is the set of *accepting* (or *final) states*

Transition Graph

- An NFA can be diagrammatically represented by a labeled directed graph called a *transition graph*



$S = \{0,1,2,3\}$
$\Sigma = \{\mathbf{a},\mathbf{b}\}$
$s_0 = 0$
$F = \{3\}$

Transition Table

- The mapping δ of an NFA can be represented in a *transition table*

$\delta(0,\mathtt{a}) = \{0,1\}$
$\delta(0,\mathtt{b}) = \{0\}$
$\delta(1,\mathtt{b}) = \{2\}$
$\delta(2,\mathtt{b}) = \{3\}$

$\longrightarrow$

State	*Input* **`a`**	*Input* **`b`**
0	{0, 1}	{0}
1		{2}
2		{3}

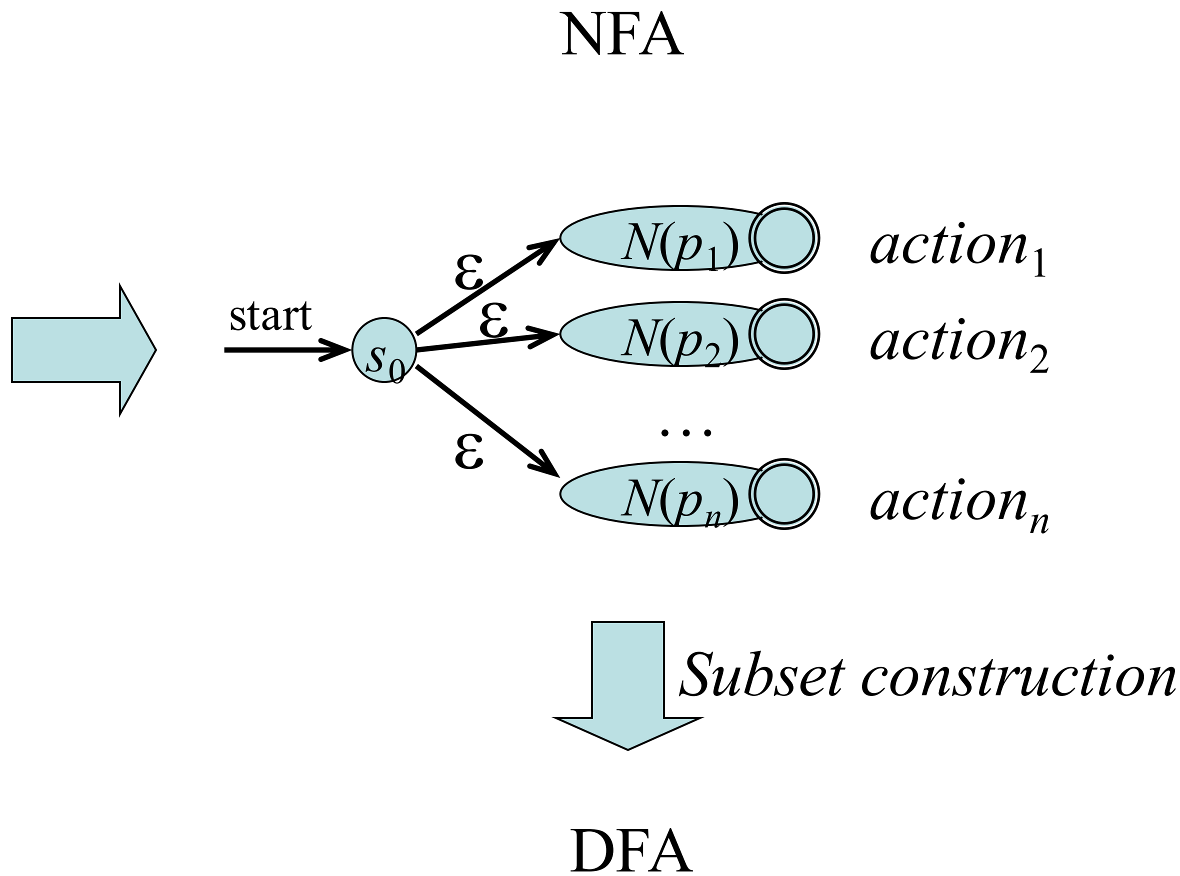
The Language Defined by an NFA

- An NFA *accepts* an input string *x* if and only if there is some path with edges labeled with symbols from *x* in sequence from the start state to some accepting state in the transition graph
- A state transition from one state to another on the path is called a *move*
- The *language defined by* an NFA is the set of input strings it accepts, such as (**a**|**b**)***abb** for the example NFA

Design of a Lexical Analyzer Generator: RE to NFA to DFA

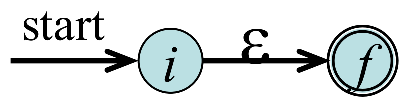
Lex specification with regular expressions

p_1 { $action_1$ }
p_2 { $action_2$ }
…
p_n { $action_n$ }

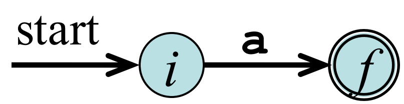


From Regular Expression to NFA (Thompson's Construction)

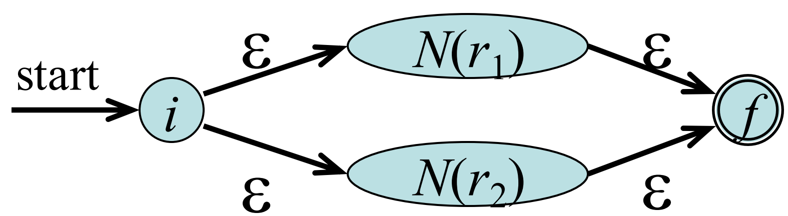
ε



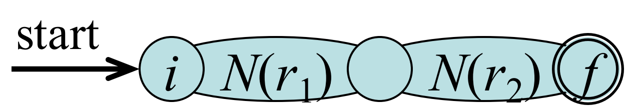
a



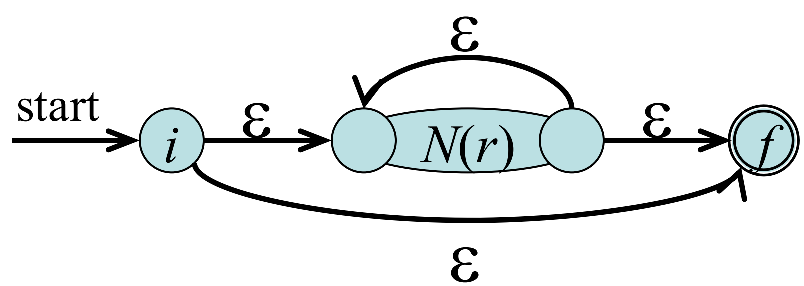
$r_1 \mid r_2$



$r_1 r_2$

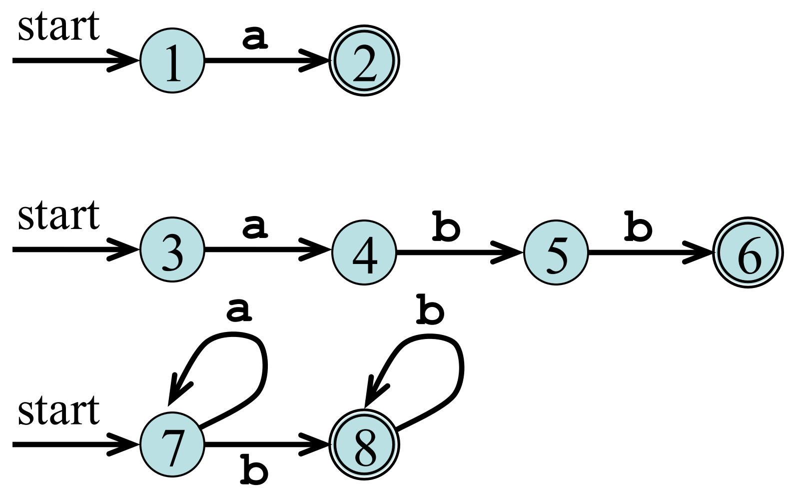


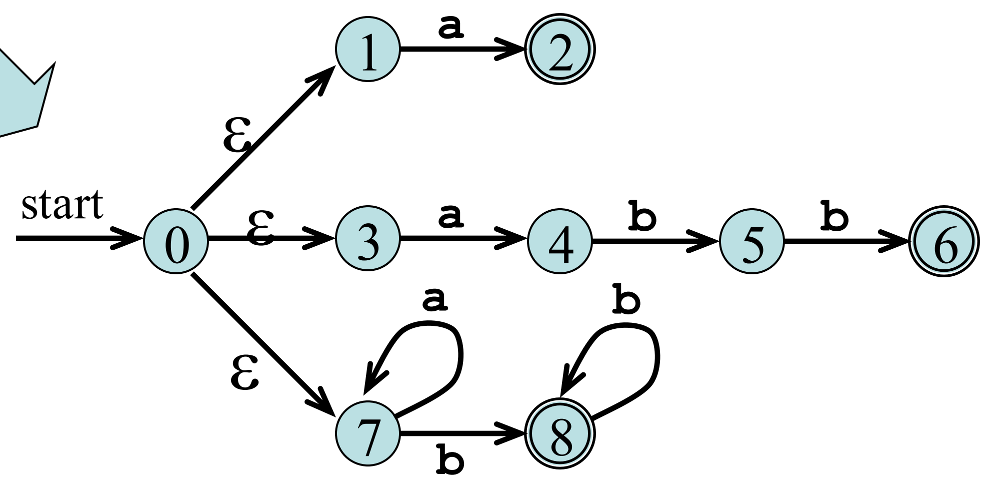
r^*



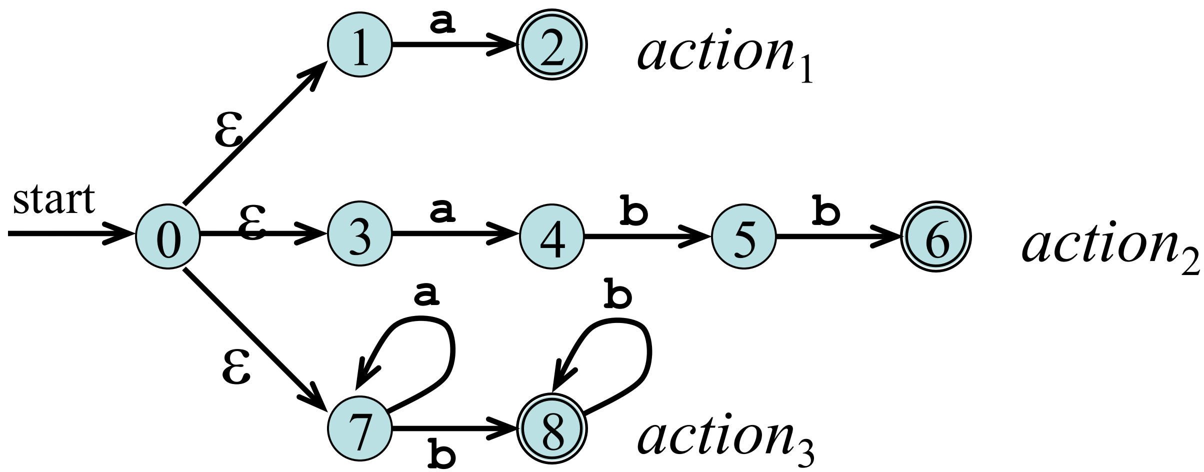
Combining the NFAs of a Set of Regular Expressions

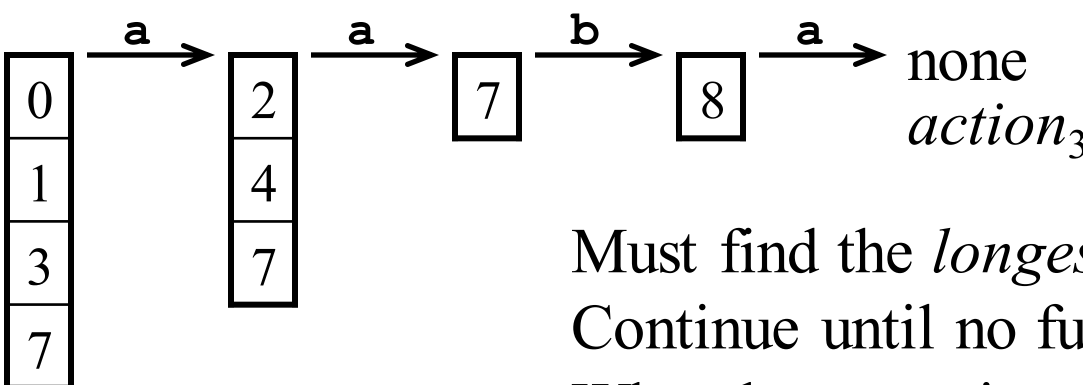
a { $action_1$ }
abb { $action_2$ }
a*b+ { $action_3$ }





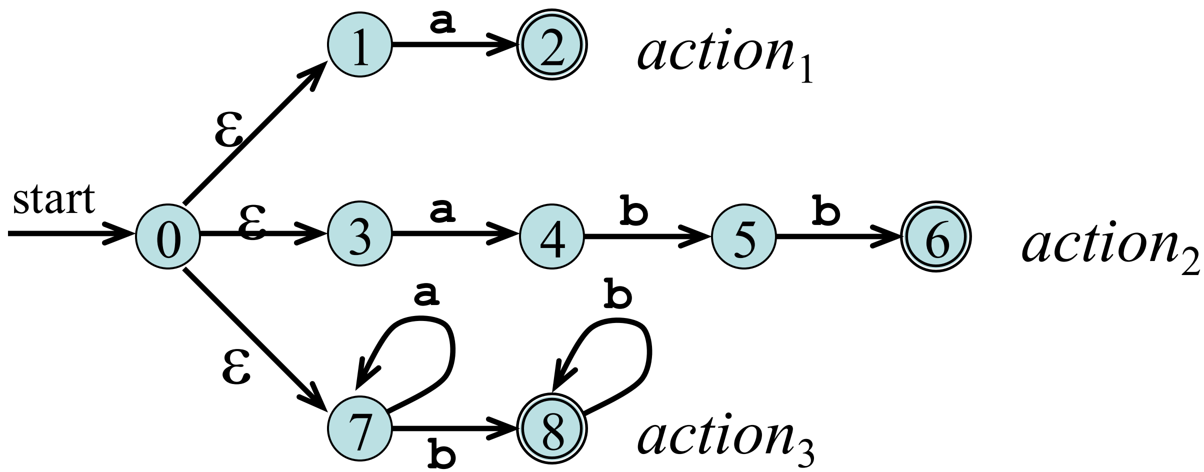
Simulating the Combined NFA Example 1

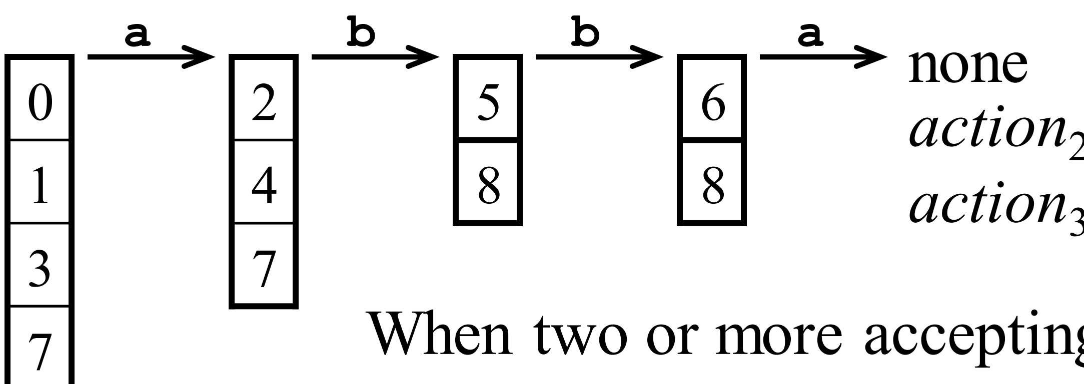




Must find the *longest match*:
Continue until no further moves are possible
When last state is accepting: execute action

Simulating the Combined NFA Example 2





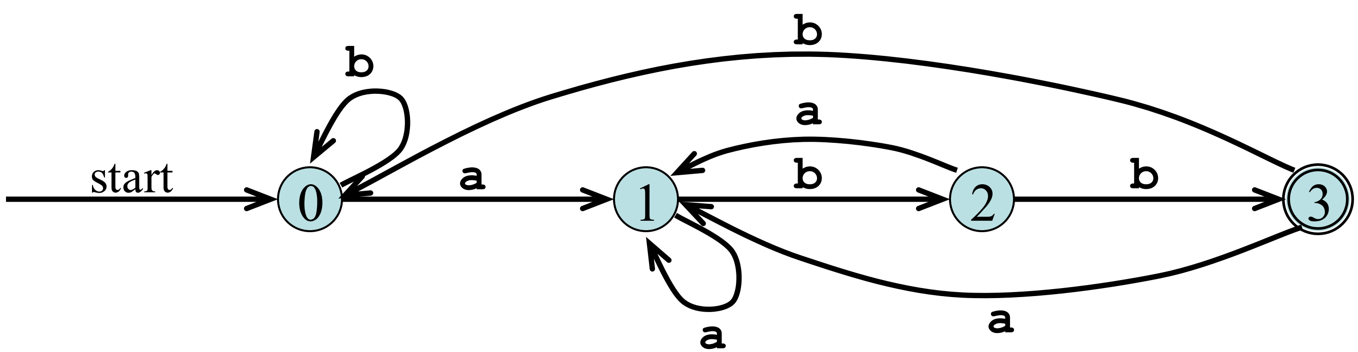
When two or more accepting states are reached, the first action given in the Lex specification is executed

Deterministic Finite Automata

- A *deterministic finite automaton* is a special case of an NFA
  - No state has an ε-transition
  - For each state *s* and input symbol *a* there is at most one edge labeled *a* leaving *s*
- Each entry in the transition table is a single state
  - At most one path exists to accept a string
  - Simulation algorithm is simple

Example DFA

A DFA that accepts (**a** | **b**)***abb**



Conversion of an NFA into a DFA

- The *subset construction algorithm* converts an NFA into a DFA using:

  $\varepsilon\text{-}closure(s) = \{s\} \cup \{t \mid s \rightarrow_\varepsilon \ldots \rightarrow_\varepsilon t\}$

  $\varepsilon\text{-}closure(T) = \cup_{s \in T}\, \varepsilon\text{-}closure(s)$

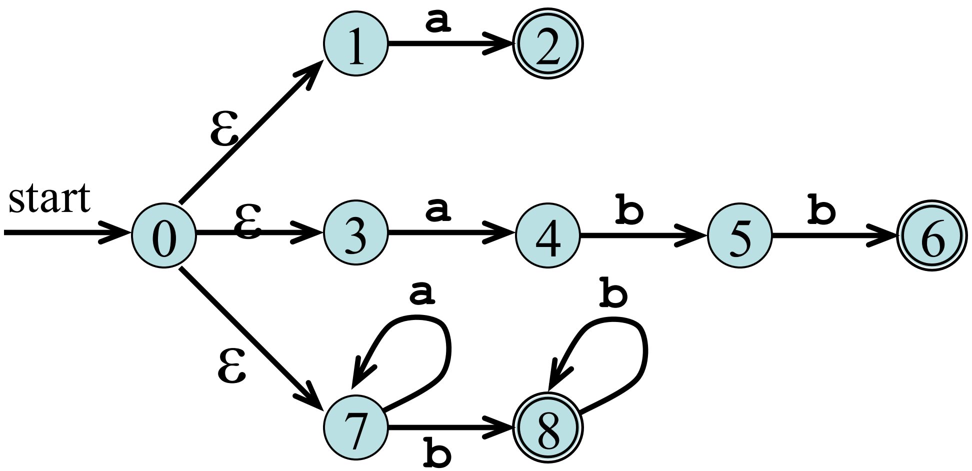
  $move(T,a) = \{t \mid s \rightarrow_a t \text{ and } s \in T\}$

- The algorithm produces:

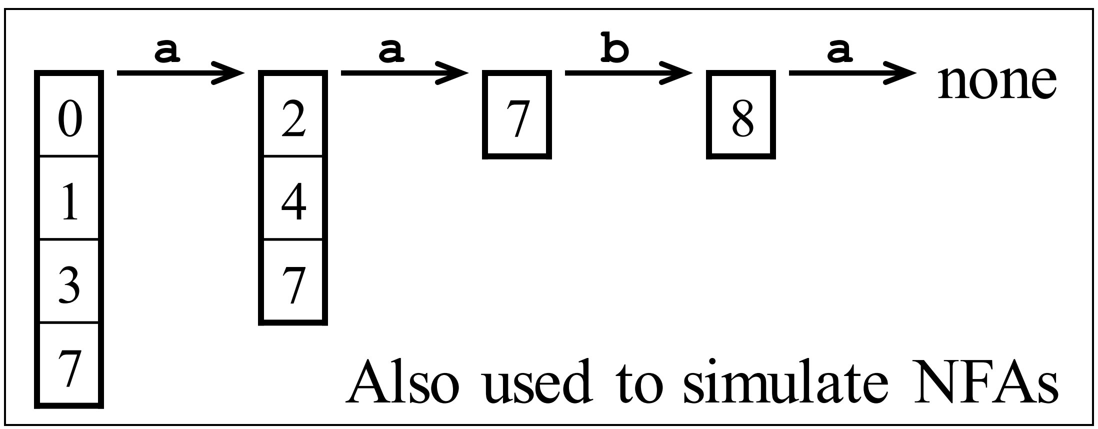
  Dstates is the set of states of the new DFA consisting of sets of states of the NFA

  Dtran is the transition table of the new DFA

ε-*closure* and *move* Examples



ε-*closure*({0}) = {0,1,3,7}
move({0,1,3,7},**a**) = {2,4,7}
ε-*closure*({2,4,7}) = {2,4,7}
move({2,4,7},**a**) = {7}
ε-*closure*({7}) = {7}
move({7},**b**) = {8}
ε-*closure*({8}) = {8}
move({8},**a**) = ∅



Simulating an NFA using ε-*closure* and *move*

S := ε-*closure*($\{s_0\}$)
S_{prev} := $\emptyset$
a := *nextchar*()
while $S \neq \emptyset$ **do**
    S_{prev} := S
    S := ε-*closure*(*move*(S,a))
    a := *nextchar*()
end do
if $S_{prev} \cap F \neq \emptyset$ **then**
    execute *action in* S_{prev}
    return "yes"
else **return** "no"

Minimizing the Number of States of a DFA