Control Engineering

Experiment 7: Controller design on MATLAB platform by digital root loci.

GROUP 1:

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OBJECTIVE:

- To design a cascade feedback controller for a given digital transfer function to stabilise its open loop poles.
- To best stabilise the given digital system.
- Perform sensitivity analysis for variation in key parameters.

TRANSFER FUNCTION:

The given equation is:

$$z^3 + Kz^2 + 1.75Kz - (K+1) = 0$$

The original equation is of the form K*N(s) + D(s) = 0, where N(s) and D(s) are respectively the numerator and denominator polynomial of the open loop system. Rearranging the terms will yield the transfer function:

$$G_{OLTF} = \frac{z^2 + 1.75z - 1}{z^3 - 1}$$

The poles (which are clearly the cube roots of unity and thus lie on the unit circle) and zeros of of the transfer function are as follows: -

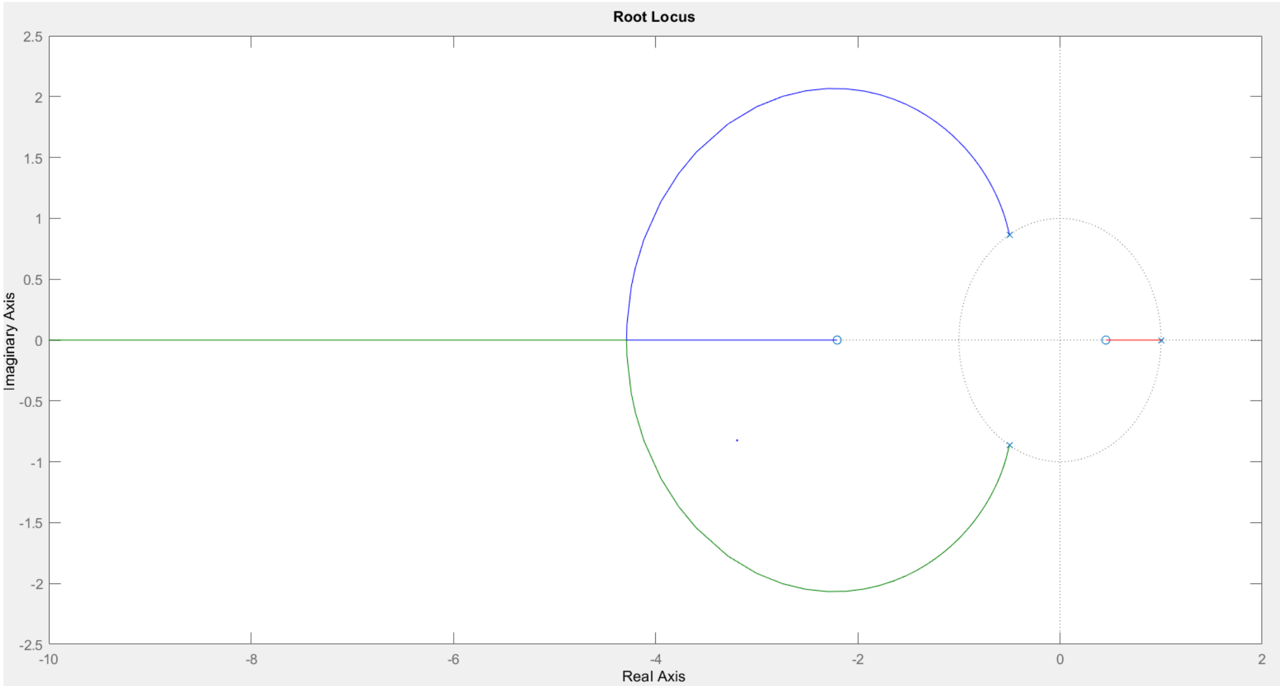
```
poles =

  -0.5000 + 0.8660i
  -0.5000 - 0.8660i
   1.0000 + 0.0000i


zeroes =

   -2.2038
    0.4538
```

Through the root locus plot we can clearly see that when we close the OLTF with a gain of K and unity negative feedback, the system is unstable for all values of K.

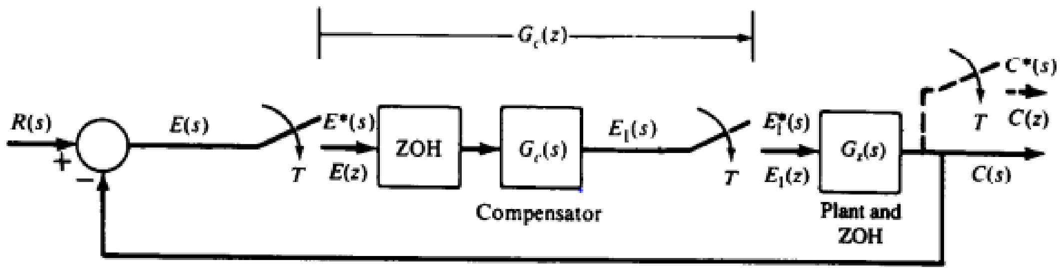


THEORY:

Generally, just a simple gain adjustment is not enough to achieve the desired specifications. Thus, a cascade and/or feedback compensator (controller) can be used to achieve the design objectives.

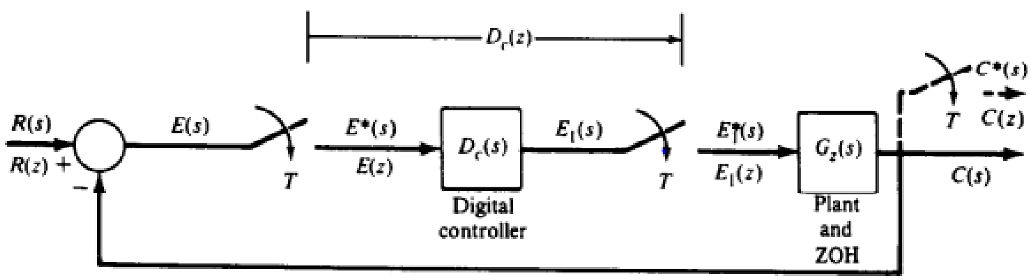
We are attempting to design a cascade controller. For real world digital systems this can be done in two ways: -

a) By using an analog unit Gc as shown below -



(Reference from Digital Control Systems Houpis-Lamont)

b) By using a digital controller directly as shown below



(Reference from Digital Control Systems Houpis-Lamont)

We will be using method b) for the purpose of this report, i.e., we will assume values of gain, poles and zeros for a digital controller.

SENSITIVITY ANALYSIS:

Sensitivity of the system is the ratio of change in the CLTF to change in the OLTF. Mathematically,

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{\partial T}{\partial G} \cdot \frac{G}{T}$$

The parameter sensitivity is defined as the sensitivity of the transfer function, to a percentage change in a system parameter.

$$S_\alpha^T = S_G^T S_\alpha^G$$

In this project, we will vary the gain of the closed loop controller, to see how it affects the stability of the system. For every controller designed, we will study the effect of increasing the gain on the system, and will also try to find the limit at which the system becomes unstable.

CASCADE COMPENSATION :

We use the control system designer app to analyse the effect of addition of poles and zeros in the compensator on the CLTF. Two cases are studies in detail:

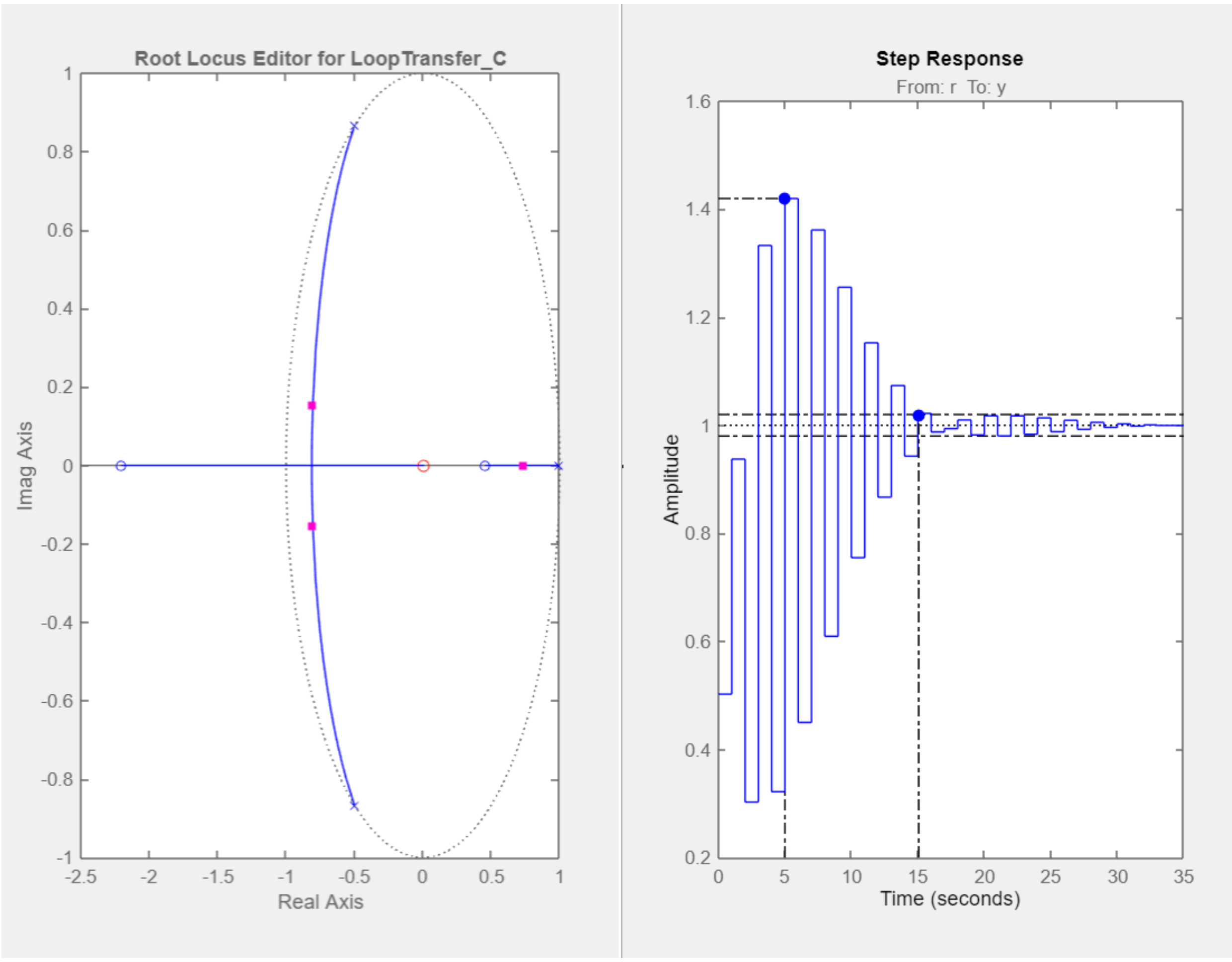
- Adding a single zero to the compensator. The intuition behind this stems from observing the root locus plot; note that both open loop poles end up *outside* the unit circle. We need a zero to “pull” the poles inside the unit circle so that it does not end up at infinity.
- Adding two zeros and one pole to the compensator. We will do this to see how the addition of poles and zeros affect the system characteristics like peak overshoot and settling time.

In this project, we have taken the sampling time to be 1 second.

Single Zero

Our primary observation was that for a gain of **K** = **1**, the system was stable for the compensator **zero** lying between the values of **-0.34 and 0.06**.

Placing a zero at 0.01:



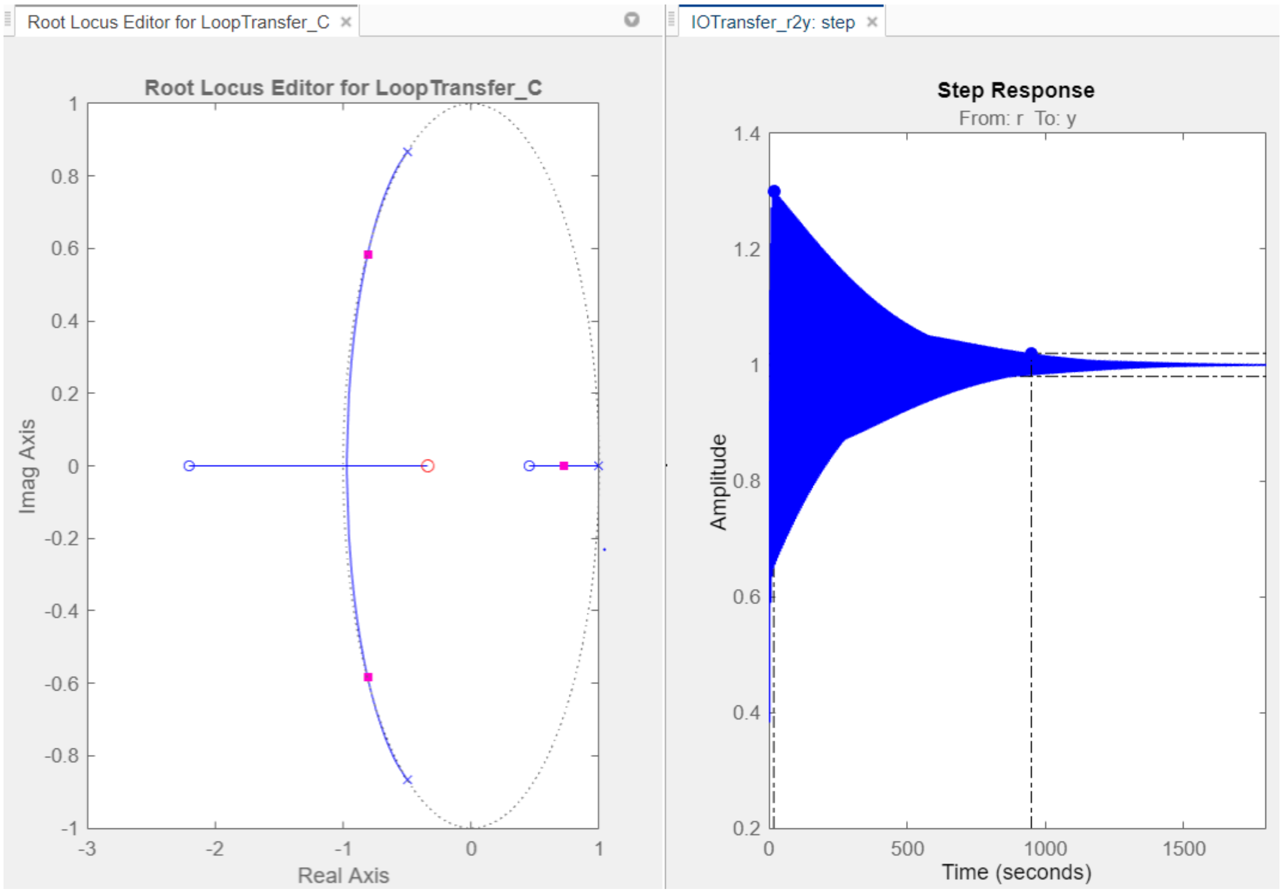
This is the optimal position for the zero, as far as the settling time is concerned.

Settling Time = 15 s

Peak Overshoot = 0.42

However this zero is susceptible to gain variations. **It is observed that even a marginal increase in gain (K = 1.2) makes the system unstable.**

Placing a zero at -0.34:



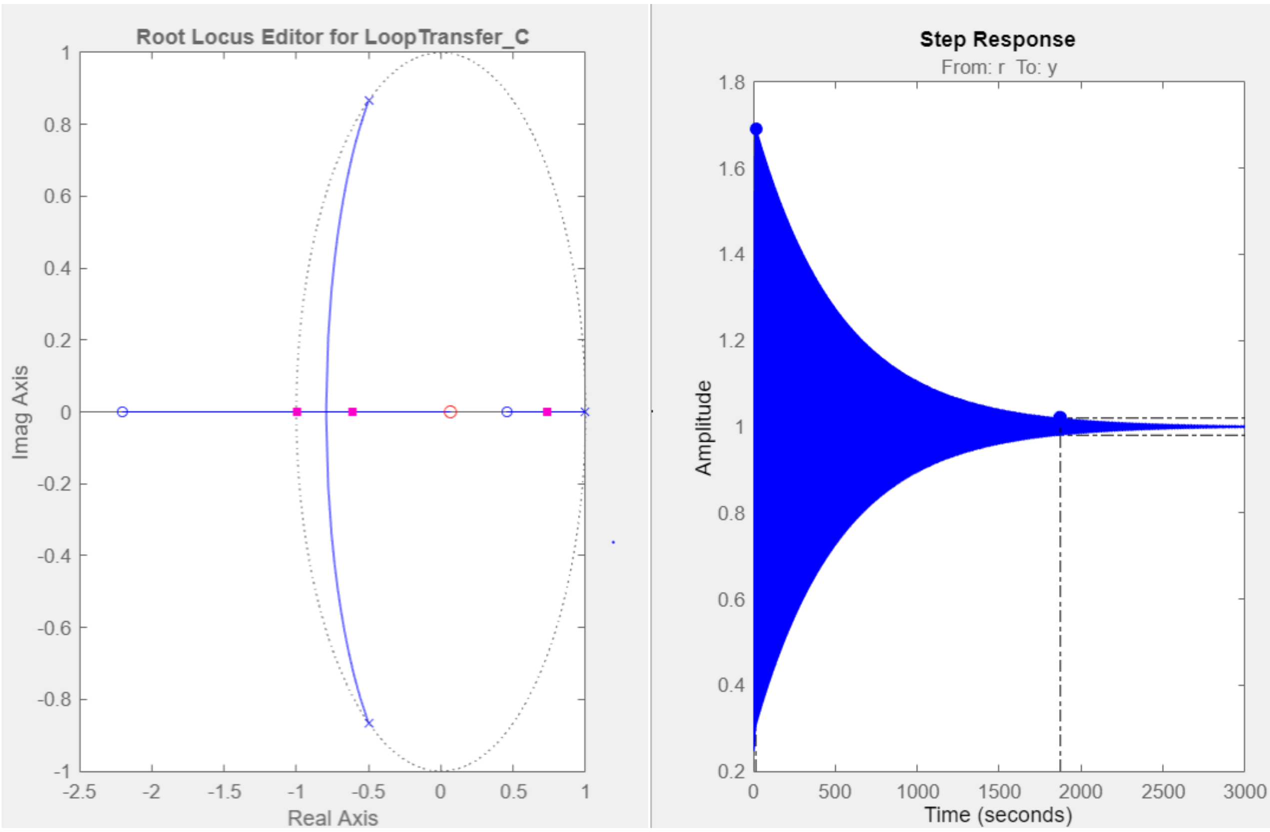
Beyond this point, the system becomes unstable, with the output to the step response blowing up to infinity. This is the negative edge case for the introduced controller zero.

Settling Time = 950 s

Peak Overshoot = 0.3

Although this zero performs better than the previous case for peak overshoot, it is significantly worse at settling time. **One interesting observation deals with the gain variation; even at K = 2 the system remains stable, although with a worse peak overshoot (0.76) but a better settling time (150 s).**

Placing a zero at 0.066:



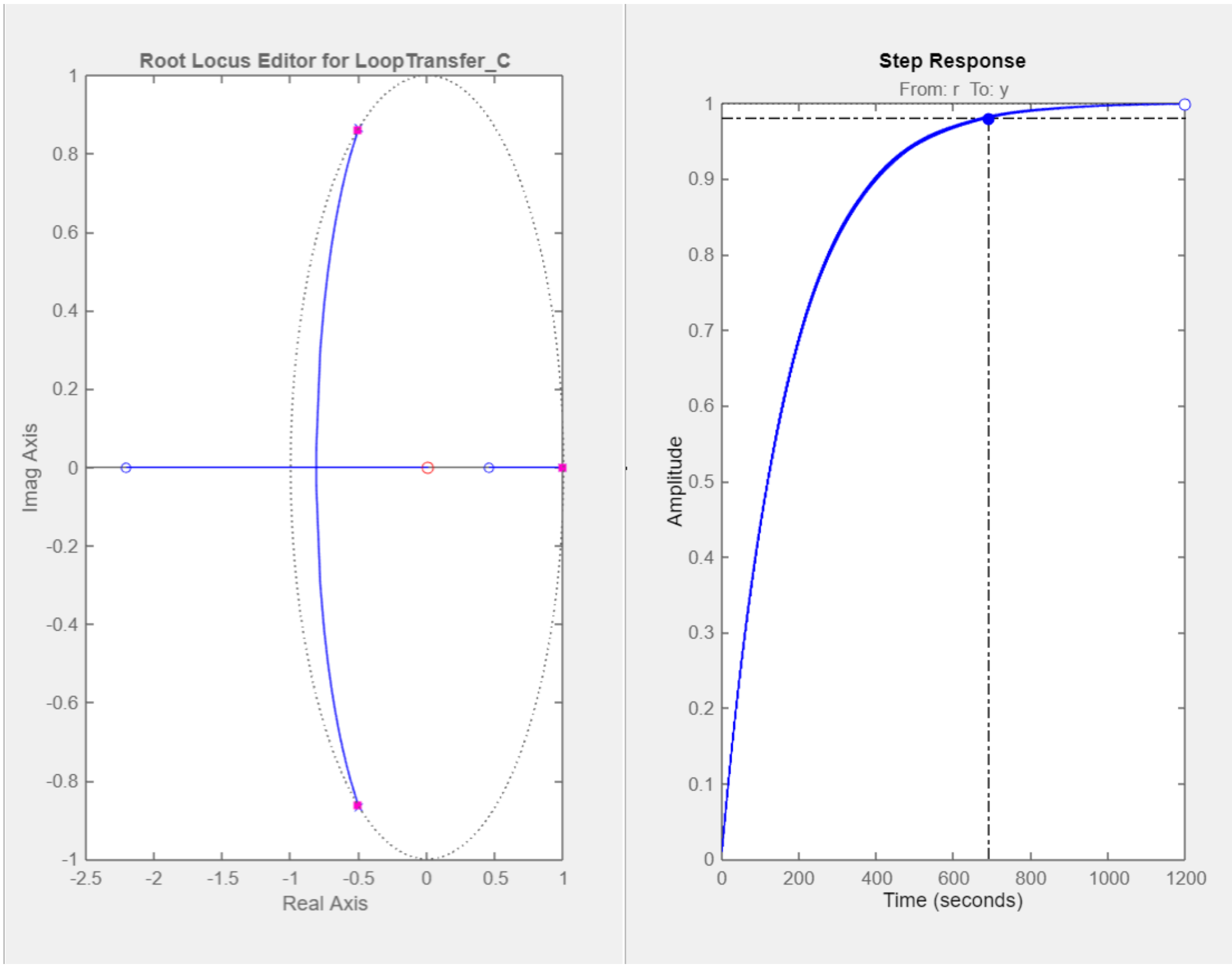
Beyond this point, the system becomes unstable again, with the output to the step response blowing up to infinity. This is the positive edge case for the introduced controller zero.

Settling Time = 1870 s

Peak Overshoot = 0.69

This zero fares worse than the two previously discussed, for both the parameters. **The susceptibility to closed-loop gain variation is even higher, with the system getting unstable for even small perturbations (K = 1.1).**

Placing a zero at 0.01 with a small closed-loop gain:



At low values of gain (**K = 0.01**), the step response of the system becomes lag type. This comes at a cost of a significantly higher settling time.

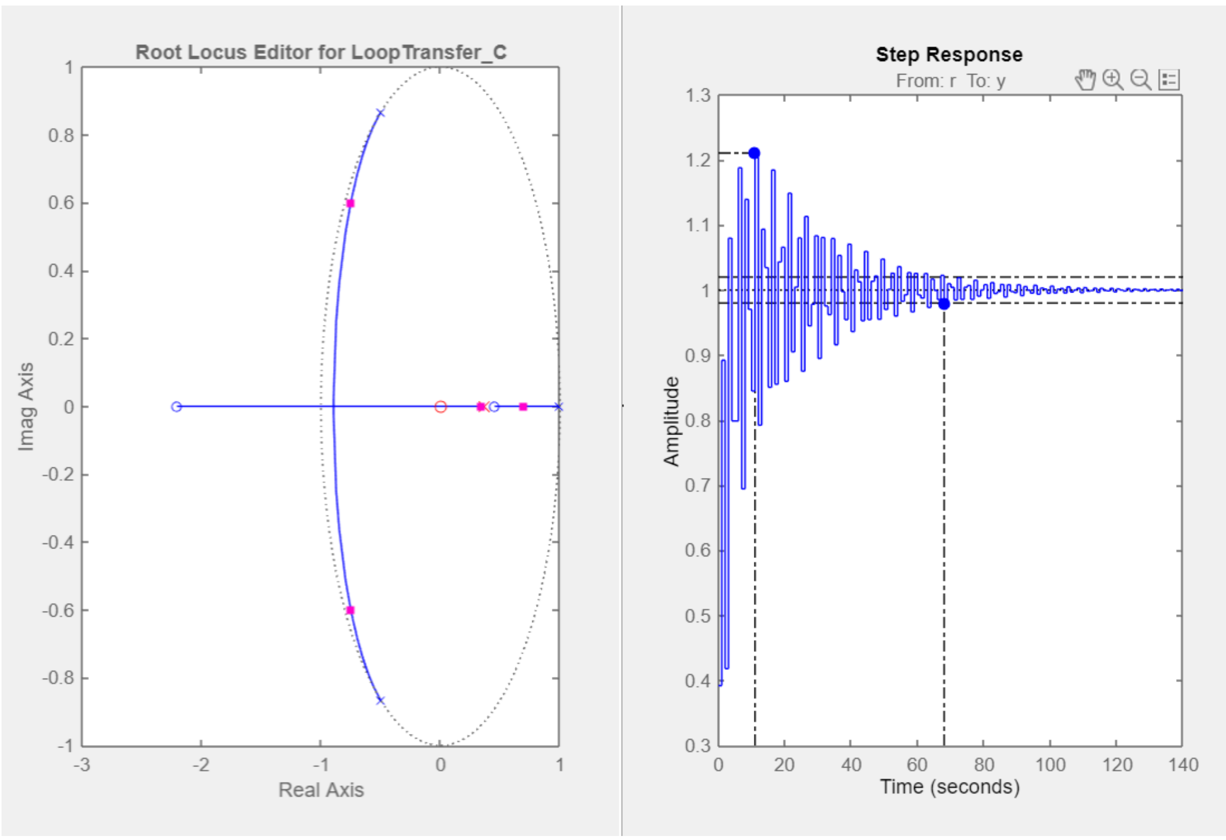
Settling Time = 690 s

The susceptibility to gain is not very high, however one must note that this compensator is not a new design - this is just a variation of the original design (K = 1, single zero at 0.01) with a low closed-loop gain. For a large increase in K the system again gets second order dynamics and the oscillations reappear, imitating the original controller.

Double Zero and Single Pole

Placing two zeros at 0.01 and a pole at 0.368:

There is an intuitive reason to choose this set of values for zeros and poles. Firstly, we have already analysed the effect of a zero at 0.01, and this zero has the best settling time among all discussed so far. Secondly, introducing a pole and a zero closely inside the unit circle means that the open loop pole ends up at the zero at 0.01, remaining inside the unit circle and thus the system remains stable at all times.



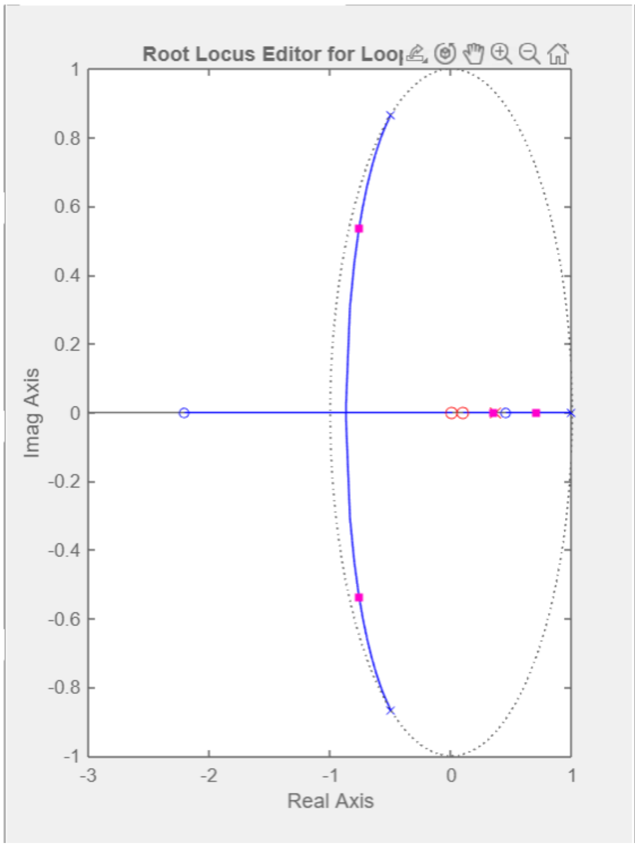
Settling Time = 68 s

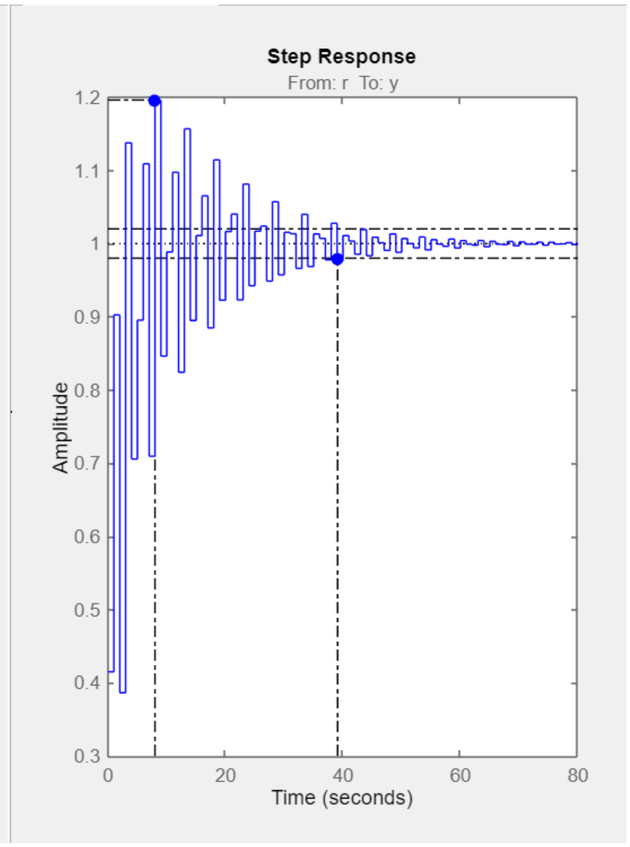
Peak Overshoot = 0.21

This particular controller gives the least peak overshoot, and is thus a major improvement in this regard. The settling time is decent as compared to the rest of the compensators (however still not as good as the controller with a single zero at 0.01). **The compensator is resilient to gain variations up to K = 2.3.** A general trend is that increase in gain decreases the settling time while increasing the peak overshoot.

Placing a zero at 0.01, zero at 0.1 and a pole at 0.368:

This set was obtained by tweaking the zero to find better system parameters.





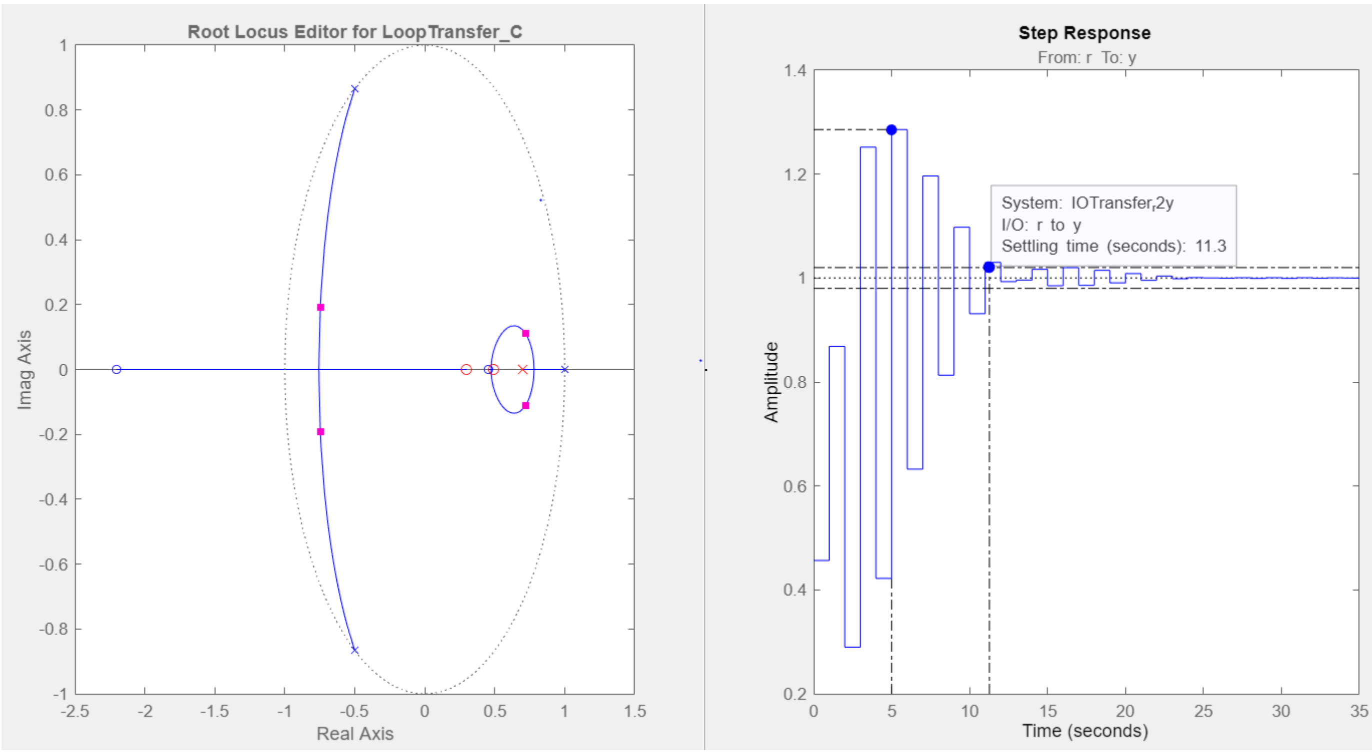
Settling Time = 39 s

Peak Overshoot = 0.19

This performs better than the previous controllers, with the settling time reduced to half and a marginal decrease in peak overshoot.

The compensator is resilient to gain variations up to K = 1.9.

Placing a zero at 0.3, zero at 0.49 and a pole at 0.7:



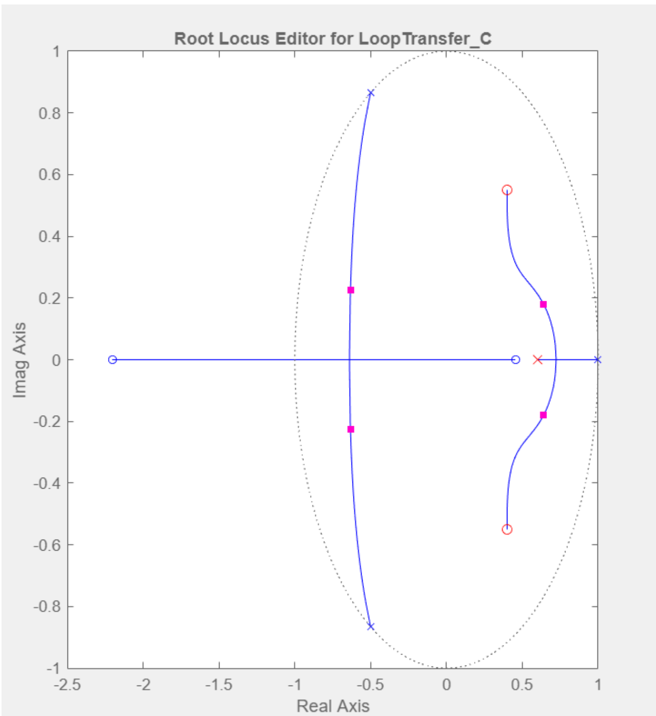
Settling Time = 11.3 s

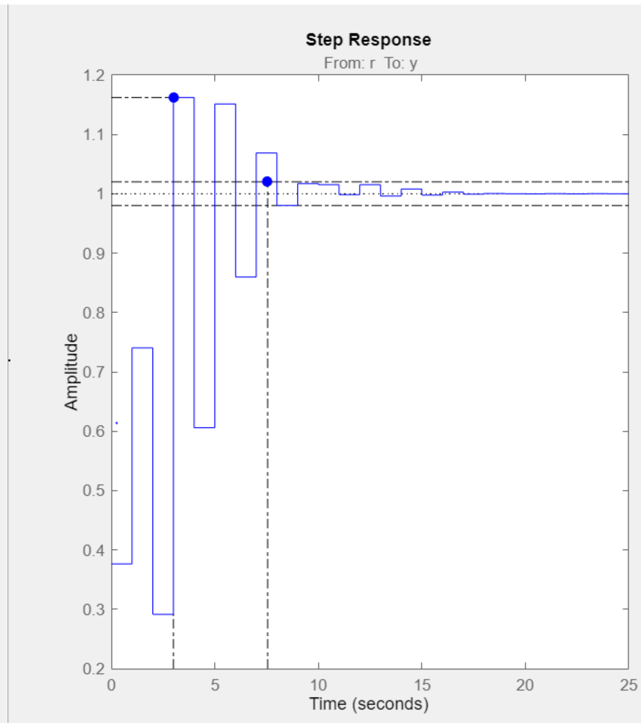
Peak Overshoot = 0.29

This controller vastly improves the settling time, but is **very susceptible to gain variations, with the system becoming unstable even for K = 1.1.**

Placing a zeroes at 0.4+0.55i, 0.4-0.55i and a pole at 0.6

This is the best controller we obtained, with complex zeros. We obtained the approximate locations of optimal poles and zeros through our observations of the previous case in which all zeros and the pole are real.





Settling Time = 7.55 s

Peak Overshoot = 0.16

This compensator is stable up to K = 1.3. Controllers which are less susceptible to gain variations have been studied before, however this gives excellent values for settling time and peak overshoot, with a decent stability limit, making this controller a good fit for our system.

TABULATION:

All the compensators designed are tabulated below:

Type	**Zeros**	**Poles**	**Gain (K)**	**Settling Time (in s)**	**Peak Overshoot**	**Sensitivity (to increase in K)**
Single Zero	0.01	None	1	15	0.42	Stable up to K = 1.2
Single Zero	0.01	None	0.01	690	*(Lag Type Response)*	Stable
Single Zero	-0.34	None	1	950	0.3	Stable up to K = 2
Single Zero	0.066	None	1	1870	0.69	Unstable *(even at K = 1.1)*
Double Zero, Single Pole	0.01, 0.01	0.368	1	68	0.21	Stable up to K = 2.3
Double Zero, Single Pole	0.01, 0.1	0.368	1	39	0.19	Stable up to K = 1.9
Double Zero, Single Pole	0.3, 0.49	0.7	1	11.3	0.29	Unstable *(even at K = 1.1)*
Double Zero, Single Pole	0.4 - 0.55i, 0.4 + 0.55i	0.6	1	7.55	0.16	Stable up to K = 1.3

Clearly there is a trade-off between the settling time, peak overshoot and sensitivity. The exact choice of compensator to be used depends on the desired specification of the system. The last compensator has an excellent settling time and the smallest peak overshoot, with a decent sensitivity limit.

CONCLUSION:

In this project, we were able to successfully design a cascade controller for the given discrete transfer function, using digital root loci technique. We tried out various permutations of poles, zeros and gain to get a stable step response with the best possible parameters (Peak Overshoot and Settling time).

While the performance of even a simple Single Zero controller was fairly decent, the best controller we obtained had 2 complex zeros at 0.4+0.55i and 0.4-0.55i and a real pole at 0.6 with the smallest peak overshoot and the best settling time among all the cases. A key takeaway is that we can get better parameters at the cost of a more complex controller design.