International Institute of Information Technology, Bangalore
CS 511 Algorithms: Practice Problems 2
16, August 2023.

1. If $T(n) = \Theta(1)$, for $n < 5$, write the solutions to the following recursions, by Masters Theorem.

   (a) $T(n) = 4T(n/2) + n^2$,
   (b) $T(n) = 16T(n/2) + n$,
   (c) $T(n) = 3T(n/3) + n \log n$
   (d) $T(n) = 2T(n/4) + \log n$
   (e) $T(n) = 4T(n/2) + n/\log n$
   (f) $T(n) = 9T(n/3) + n$
   (g) $T(n) = 3T(n/3) + n^2$
   (h) $T(n) = 2T(n/4) + n^{2/3}$
   (i) $T(n) = 3T(n/9) + n^{3/4}$
   (j) $T(n) = 8T(n/3) + n^2$
   (k) $T(n) = 3T(n/4) + n \log n$
   (l) $T(n) = 6T(n/3) + n^2 \log n$

2. What is the complexity of the following algorithms?

   (a)
   ```
   while(n > 0){
    for(i = 1; i < n; i = i * 2)c++;

   n = n/2;}
   ```

   (b)
   ```
   while(n > 0){
    for(i = 1; i < n; i++)c++;

   n = n/2;}
   ```

   (c)
   ```
   j = 1;
   while(j < n){
    for(i = 1; i < n; ++i)c++;

   j = 2 * j;}
   ```

   (d)
   ```
   while(n > 0){
    for(i = 1; i < n; i = i * 3)c++;

   n = n/3;}
   ```

(e)
```
$while(n > 0)\{$
  $for(i = 1; i < n; i++)c++;$

 $n = n/3; \}$
```

(f)
```
$j = 1;$
$while(j < n)\{$
  $for(i = 1; i < n; ++i)c++;$

 $j = 3 * j; \}$
```

3. Solution to which of the following recursion is linear ?

   (a) $T(n) = 3T(n/5) + T(n/4) + n$

   (b) $T(n) = 3T(n/9) + 8T(n/11) + n$

   (c) $T(n) = 3T(n/10) + 8T(n/8) + n$

   (d) $T(n) = 3T(n/7) + 4T(n/8) + n$

   (e) $T(n) = 2T(n/5) + 4T(n/7) + n$

   (f) $T(n) = 3T(n/3) + 2T(n/4) + n$

   (g) If $n = 3m, T(n) = n + 5/n \sum_{k=0}^{k=m-1} T(3k)$

   (h) $T(n) = n + 49/n \sum_{k=0}^{k=n/5} T(k)$

   (i) $T(n) = n + 15/n \sum_{k=0}^{k=n/3} T(k)$

4. Given an array of sorted integers and an integer $X > 0$, design a linear time algorithm to count the number of pair elements in the array such that $A[j] - A[i] > X$.

5. Given an array of integers , design a $\Theta(n^2)$ algorithm to decide if there is i, j, k such that $A[i] + A[j] = A[k]$.

6. Given an array of integers , design an efficient algorithm to decide if there is i, j, k, l such that $A[i] - 2A[j] = A[k] - 3A[l]$.

7. Given a stream of n (about 10^9) numbers, design an $O(n)$ time and $O(k)$ space algorithm to find an element of rank k.

8. Given a sequence of n numbers and an integer $k < n$, design a linear time algorithm to find k numbers, closest to the median.

9. Given two sorted arrays of size m and n respectively and an integer k, design an $O(\log k)$ algorithm to find an element of rank k in the merged array.

10. Design a linear time algorithm to sort n integers in the range 0 to $n^{10} - 1$.