RQS(A, l, r)

if (l < r){ goodpivot(A, l, r); $O(n)$

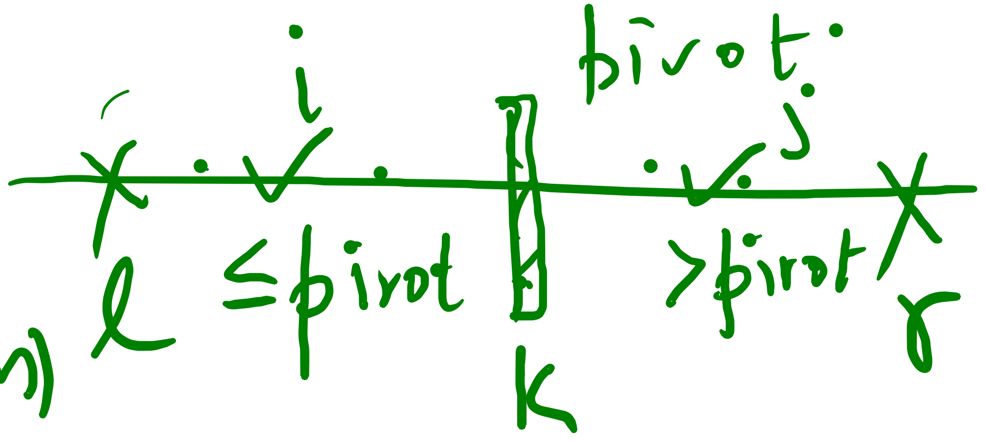
p = rand() % (r-l+1) + l;

swap(A, l, p)

K = Partition(A, l, r);

RQS(A, l, K-1);

RQS(A, K+1, r);



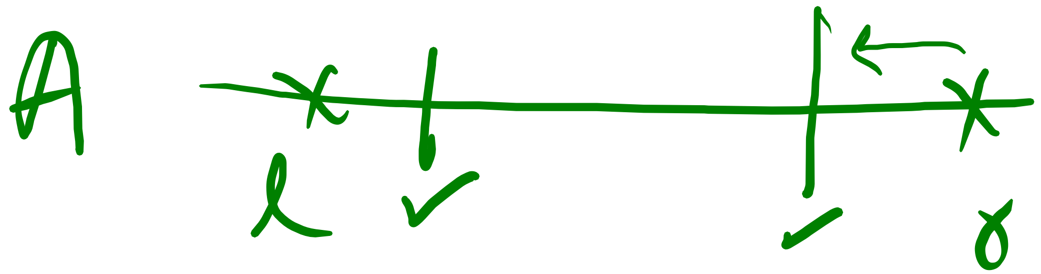
p   0   r-l

l   r

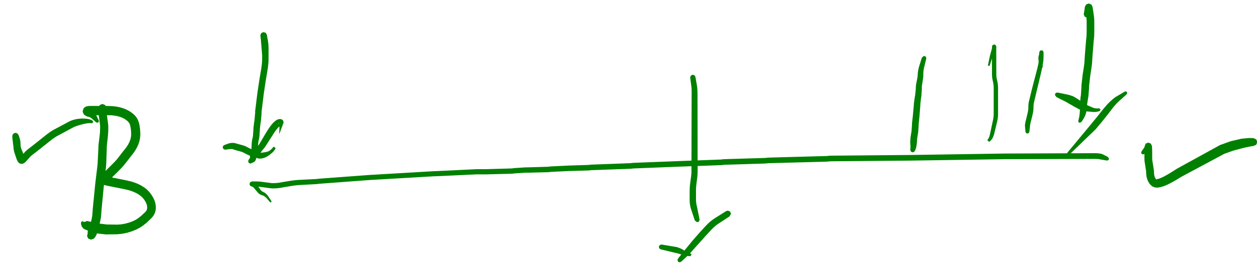
pivot = A[0];

A[i] ≤ pivot < A[j]

```
Partition(A, l, r)
  pivot = A[l];
   i = l+1;  j = r;
 while(i ≤ j) {
  while(i≤j & A[i] ≤ pivot)
                 i++;
  while(i≤j & A[j] > pivot)
                 j--;
    if(i≤j)
      { Swap(A,i,j); i++; j--; }.
 } i--; Swap(A,l,i);
```



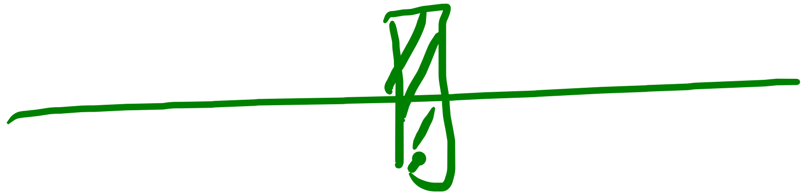
pivot.



return i;

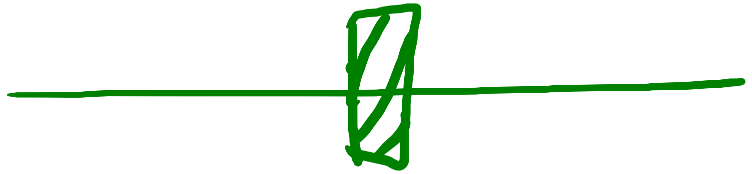
$T(n) = 1 + n + T(k) + T(n-1-k)$ $\quad 0 \le k \le n-1$

worst case. $\theta(n)$



$T(n) = n + 1 + T(n-1)$

$= \theta(n^2)$



$T(n) = n + 1 + 2T(n/2)$ $\quad k = n/2$

$= \theta(n \log n)$

$$T(n) = \theta(n) + n + 2T(n/2)$$

$$= \underline{\theta(n \log n)}.$$

$$T(n) = 1 + n + T(k) + T(n-k-1)$$

$$0 \le k \le n-1 \qquad \frac{1}{n}$$

$$\underline{\underline{T(n)}} = \sum_{k=0}^{n-1} \frac{1}{n} \left[(n+1) + T(k) + T(n-k-1) \right] \checkmark$$

$$n\,T(n) = n(n+1) + \sum_{k=0}^{n-1} \left(T(k) + T(n-k-1) \right)$$

$$= n(n+1) + 2 \sum_{k=0}^{n-1} T(k)$$

$T(0)$ $T(n-1)$

$T(1)$ $T(n-2)$

$T(2)$ $T(n-3)$

$T(n-1)$ $T(0)$

$$nT(n) = n(n+1) + 2\sum_{k=0}^{n-1} T(k) \quad \text{—} \quad (1)$$

$$(n-1)T(n-1) = (n-1)n + 2\sum_{k=0}^{n-2} T(k) \quad \text{—} \quad (2)$$

$$nT(n) - (n-1)T(n-1) = \underline{n(n+1) - n(n-1)} + 2T(n-1)$$

$$nT(n) = 2n + (n+1)T(n-1)$$

$$\frac{T(n)}{n+1} = \frac{2}{n+1} + \frac{T(n-1)}{n}$$

$$\frac{T(n)}{n+1} = \frac{2}{n+1} + \cancel{\frac{T(n-1)}{n}}$$

$$\cancel{\frac{T(n-1)}{n}} = \frac{2}{n} + \cancel{\frac{T(n-2)}{n-1}}$$

$$\cancel{\frac{T(n-2)}{n-1}} = \frac{2}{n-1} + \cancel{\frac{T(n-3)}{n-2}}$$

$$\vdots$$

$$\cancel{\frac{T(1)}{2}} = \frac{2}{2} + \frac{T(0)}{1}$$

$$\frac{T(n)}{n+1} = \frac{T(0)}{1} + 2\sum_{i=1}^{n} \frac{1}{i+1}$$

$$T(n) = (n+1) + 2(n+1) \sum_{i=1}^{n} \frac{1}{i+1}$$

$$= \theta(n \log n)$$

$$\boxed{\text{If } \sum_{i=1}^{n} \frac{1}{i+1} = \theta(\log n)}$$

Th:

$$S = \sum_{i=1}^{n} f(i) \qquad I = \int_{x=1}^{n} f(x)\,dx$$

$x \le y$

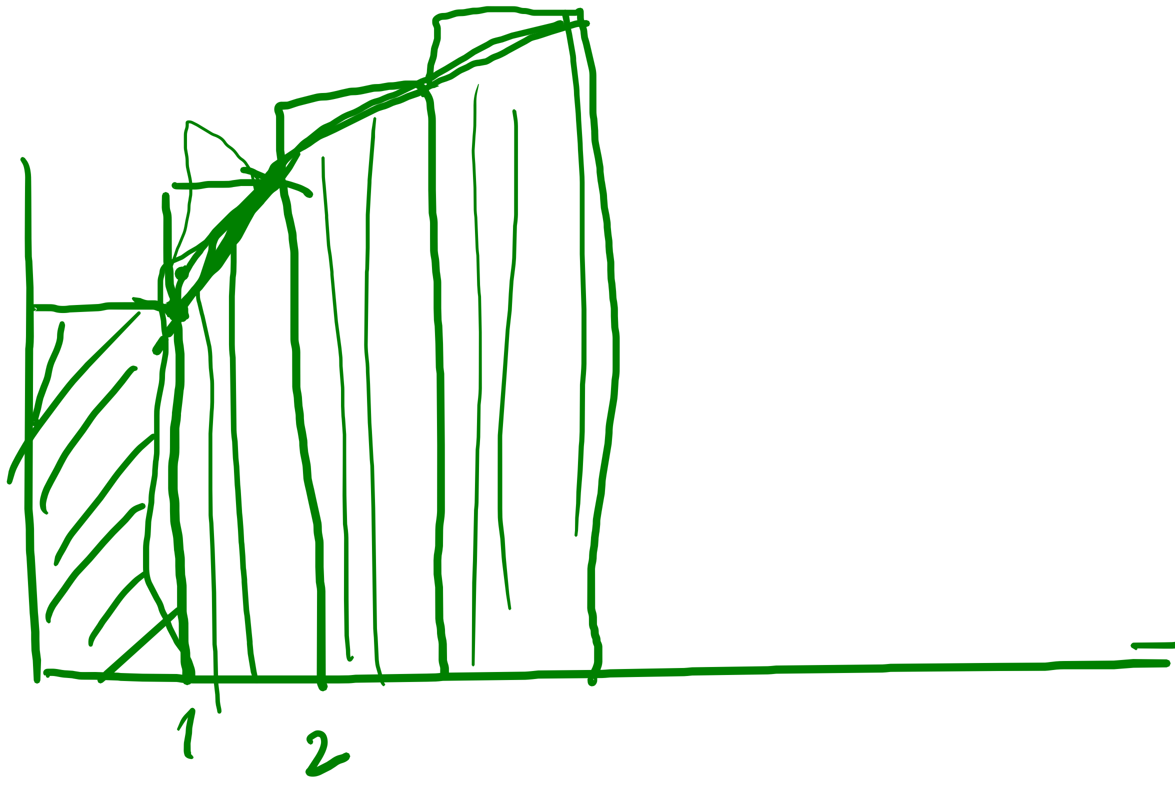
① If f is a increasing function. $f(x) \le f(y)$

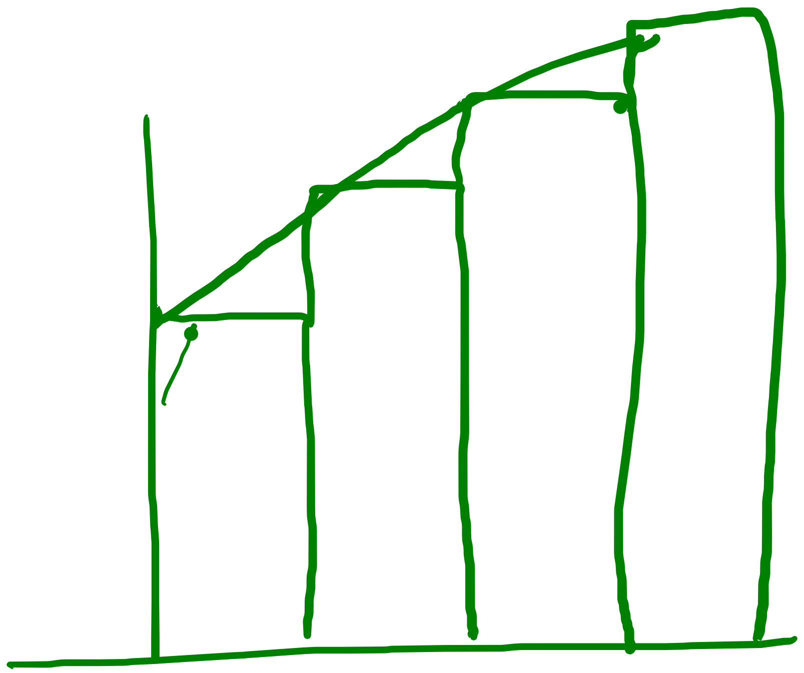
$$I + f(1) \le S \le I + f(n)$$

② If f is a decreasing function $\quad x \le y$

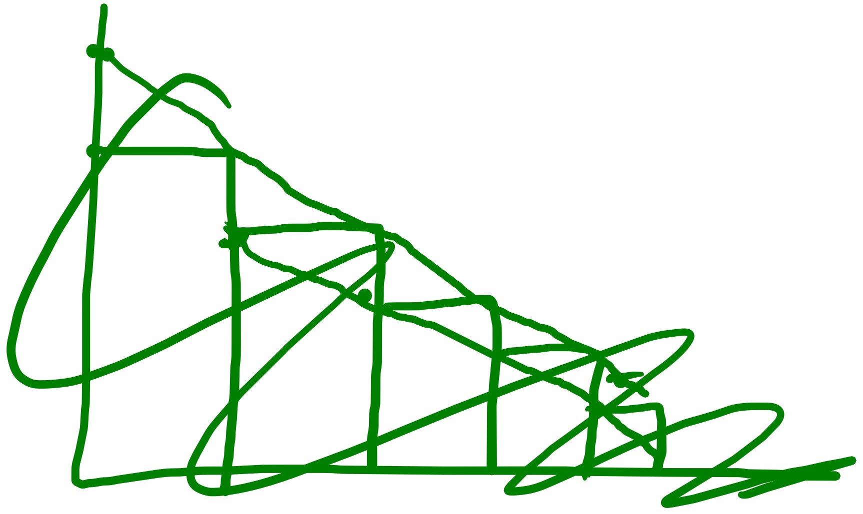
$$I + f(n) \le S \le I + f(1) \qquad f(y) \le f(x)$$

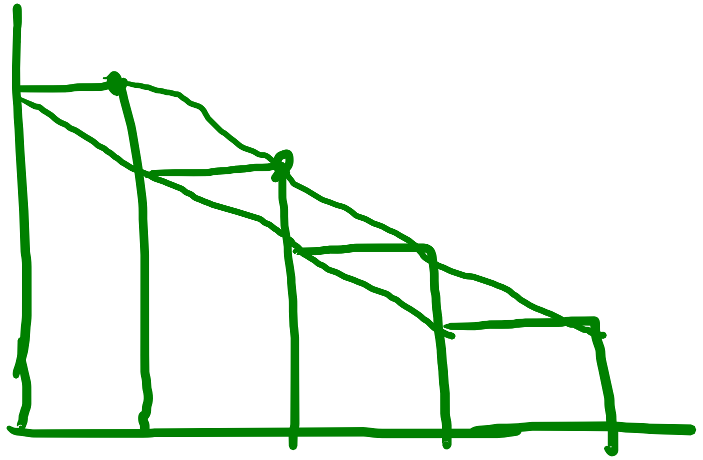
$$S = \sum_{i=1}^{n} f(i)$$





$$I + f(1) \leq S \leq I + f(n)$$





$$\int_{1}^{n} f(x)\, d(x)$$

$$\cancel{I + f(1) \leq S \leq I + f(n)}$$

$$I + f(1) \geq S \geq I + f(n)$$

$$S = \sum_{i=1}^{n} \frac{1}{i+1}$$

$$\underline{= \Theta(\log n)}.$$

$$I = \int_{1}^{n} \frac{1}{x+1}\,dx = \int_{2}^{n+1} \frac{1}{x}\,dx$$

$$= \log x+1 \Big|_{1}^{n} = \log n+1 - \log 2$$

$$I + f(n) \le S \le I + f(1)$$

$$\log(n+1) - \log 2 + \frac{1}{n+1} \le S \le \log(n+1) - \log 2 + \frac{1}{2}$$

```
Swap(A, l, r);
{ t = A[l];
  A[l] = A[r];
  A[r] = t;
}
```

$a_0 \; - \; - \; - \; \cdot \; a_{n-1}$

0 1 0 1 1 1 1

```
while(1)
{ i = rand() % n;
if(A[i])
  return i;
}
```

$T(n) \rightarrow \infty$ Los vegas.

$E[T(n)] = \theta(1)$ 2

$1 - \frac{1}{10^{30}}$

$j \rightarrow 0$ to 100 $k = \log n$

i = rand() % n

if(A[i]) return i,

Failed; Monte Carlo

$T(n) = 100 = O(1)$

$\frac{1}{2^{100}} = \frac{1}{(10^3)^{10}} = \frac{1}{10^{30}}$

Los Vegas.

$T(n)$ is not bounded.

Always gives correct answer.

$< n/2 + 1$ $\quad \leq n/2$

Monte Carlo:

$T(n)$ is bounded.

$O(1)$, $O(\log n)$

Algorithm is correct with high probability.

$\Omega(n)$

a_0 — — — - a_{n-1}

$n = 10^9$

$1 - \frac{1}{n}$

$\underline{\underline{\log n}}$

$\underline{\underline{n = 30}}$

$\underline{\underline{K}} = \log n.$

$\Theta(\log n)$

$$\frac{1}{2^K} = \frac{1}{n}$$

$1 - 1/n$

$$\sum_{i=1}^{\infty} i \frac{1}{2^i} = 2$$

$$\frac{1}{2^{i-1}} \frac{1}{2}$$

$$\sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1-\frac{1}{2}} = \underline{\underline{2}}$$

$$\sum_{i=1}^{\infty} \frac{1}{2^i} = 1$$

$$S = 1 + x + x^2 + \cdots$$

$$\underline{\underline{x < 1}}$$

$$xS = x + x^2 + x^3 + \cdots$$

$$S(1-x) = 1$$

$$S = \frac{1}{1-x}$$

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$

$$\sum x^i = \frac{1}{1-x}$$

$$\sum i x^{i-1} = \frac{1}{(1-x)^2}$$

$$\sum_{i=0}^{\infty} i x^i = \frac{x}{(1-x)^2}$$

$$\sum \frac{i}{2^i} = \frac{1/2}{(1/2)^2}$$

$$= \underline{\underline{2}}$$