Game Theory: Applications in Network Protocols

Q4) How does CDMA solves the near-far problem using Distributed Power Control
How does Wi-Fi manage interference in a shared network?

References:

1. Kevin Leyton-Brown and Yoav Shoham. (2008). Essentials of Game Theory. Morgan & Claypool Pool Publishers
2. Chapter 1 - What makes CDMA work on my Smartphone. In *Networked Life: 20 Questions and Answers*.

Game Theory

https://www.youtube.com/watch?v=qY0XKSzjBKI

- Game: played between two or more players
- Models of strategic interactions:
  - Outcomes depend on more than one player's decision
  - The decisions for one player cannot be separated from those of others!
- Networks: How to prevent users from overgrazing the Wi-Fi network capacity?
- Markets: What will happen if two companies are allowed to merge?
- E-commerce: How should an on-line auction be structured to maximize revenue?
- Legal: How should random audits of taxes be conducted?
- Sports: Should a soccer player adjust the fraction of time that s/he kicks penalty kicks to a goalies right or left based on the goalie?

Game Theoretic Models

- **Players**: who makes the decisions?
- **Strategies**: what are the actions available?
- **Timing**: who does what and when?
- **Information**: what do players know when choosing?
- **Payoffs**: what happens as a function of the actions? And what motivates players?
- The formal model
  - The main ingredients: Players, Strategies, Payoffs
- Normal Form games
  - Dominance
  - Equilibrium

GAME THEORY AND THE ECONOMICS OF COOPERATION

- ***Game theory*** **is the study of how people behave in strategic situations.**
  - **Strategic decisions are those in which each person, in deciding what actions to take, must consider how others might respond to that action.**

1. A set of **players** $\{1, 2, \ldots, N\}$
2. A **strategy space** A_i for each player
3. A **payoff function**, or utility function, U_i for each player to maximize (or a **cost function** to minimize). Function U_i maps each combination of all players' strategies to a real number, the payoff (or cost), to player i.

The Prisoners’ Dilemma

- **The *prisoners’ dilemma* provides insight into the difficulty in maintaining cooperation.**
  - **Often people (firms) fail to cooperate with one another even when cooperation would make them better off.**
  - **The prisoners’ dilemma is a particular “game” between two captured prisoners that illustrates why cooperation is difficult to maintain even when it is mutually beneficial.**

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Best Response

- ***Best Response* of A is the strategy of A that maximizes her profit given a specific response of the other actor B**

- Game: $(N, (A_i)_i, (u_i)_i)$

- a_i is a ***best response*** to a_{-i} (strategies of others) if
  $u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i})$ for all a_i'

- Always have a best response in a finite game
  (every finite set of numbers has a max)

Dominant strategy

- ***Dominant strategy* of A is the strategy of A that maximizes the payoffs irrespective of the strategic response of B**

  - A strategy a_i is a (weakly) dominant strategy for a player i

    $$u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \quad \text{for all } a_i' \textit{ and } a_{-i}$$

  - A strategy a_i is a *strictly* dominant strategy for a player i

    $$u_i(a_i, a_{-i}) > u_i(a_i', a_{-i}) \quad \text{for all } a_i' \textit{ and } a_{-i}$$

Nash Equilibrium

- Socially optimal equilibrium
  - Set of strategies that maximizes sum of utilities
- *Nash Equilibrium* -Named after John Nash
  - Also sometimes called Cournot-Nash equilibrium
- Nash equilibrium is stable
  - No player wishes to change her strategy if she knew what strategies the others are using

Definition 2.2.2 (Nash equilibrium). *A strategy profile* $s = (s_1, \ldots, s_n)$ *is a* Nash equilibrium *if, for all agents* i, s_i *is a best response to* s_{-i}.

$$U_1(a^*, b^*) \geq U_1(a, b^*), \quad \text{for any } a \in \mathcal{A},$$

$$U_2(a^*, b^*) \geq U_2(a^*, b), \quad \text{for any } b \in \mathcal{B}.$$

The Prisoners' Dilemma

Peeyush' s Decision

Pranjal's Decision	**Confess**	**Remain Silent**
Confess	Anirudh is fined Rs. 8,000 Kaushal is fined Rs. 8000	Anirudh is fined Rs. 20,000 Kaushal is let off free
Remain Silent	Anirudh is let off free Kaushal is fined Rs. 20,000	Anirudh is fined Rs. 1,000 Anirudh is fined Rs. 1,000

Prisoner's Dilemma

	C	D
C	a, a	b, c
D	c, b	d, d

Any $c > a > d > b$ define an instance of Prisoner's Dilemma.

The Prisoners’ Dilemma

- The *dominant strategy* is the best strategy for a player to follow regardless of the strategies chosen by the other players.
  - Cooperation is difficult to maintain, because cooperation is not in the best interest of the individual player.
  - Self-interest makes it difficult for the oligopoly to maintain a cooperative outcome with low production, high prices, and monopoly profits.

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Cooperation between players

- The ***prisoners' dilemma*** provides insight into the difficulty in maintaining cooperation.
  - Often people (firms) ***fail to cooperate*** with one another even when cooperation would make them better off.
  - The prisoners' dilemma is a particular "game" between two captured prisoners that illustrates why cooperation is difficult to maintain even when it is mutually beneficial
- Firms that care about future payoffs will cooperate in repeated games rather than cheating in a single game to achieve a one-time gain.

Co-ordination Game: Battle of the Sexes

		B	
A		Action Movie	Comedy Movie
	Action Movie	(2,1)	(0,0)
	Comedy Movie	(0,0)	(1,2)

Matching Pennies and Zero Sum Game

		B	
A		Head	Tail
	Head	(1, -1)	(-1,1)
	Tail	(-1,1)	(1, -1)

Arms Race: Nash Equilibrium

		Decision of	
Decision of		Arm	Disarm
	Arm	U.S. At Risk ---------------- N Korea at Risk	U.S. At Risk and Weak ---------------- N Korea Safe and Powerful
	Disarm	U.S. Safe and Powerful ---------------- N Korea at Risk and Weak	U.S. Safe ---------------- N Korea Safe

Why People Sometimes Cooperate

- Firms that care about future profits will cooperate in repeated games rather than cheating in a single game to achieve a one-time gain.
- Cooperation among oligopolists is undesirable from the standpoint of society as a whole because it leads to *production that is too low* and *prices that are too high*.

How does CDMA work: Relation to Game Theory

Cellular Networks

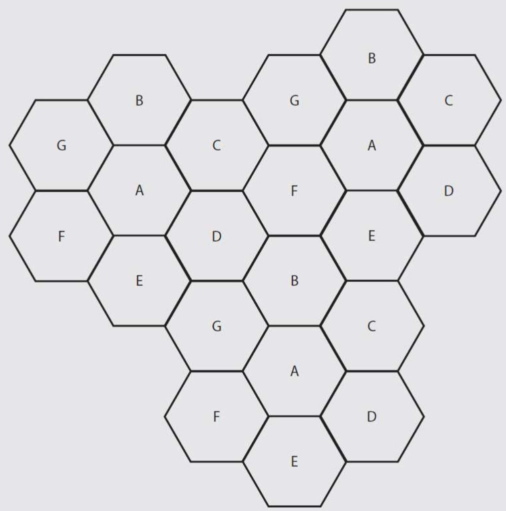
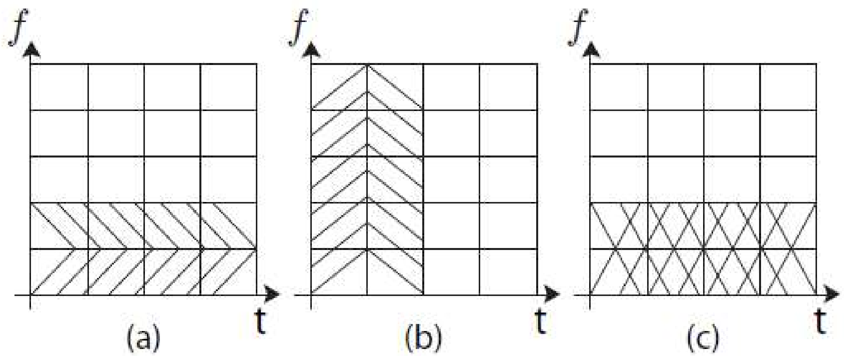


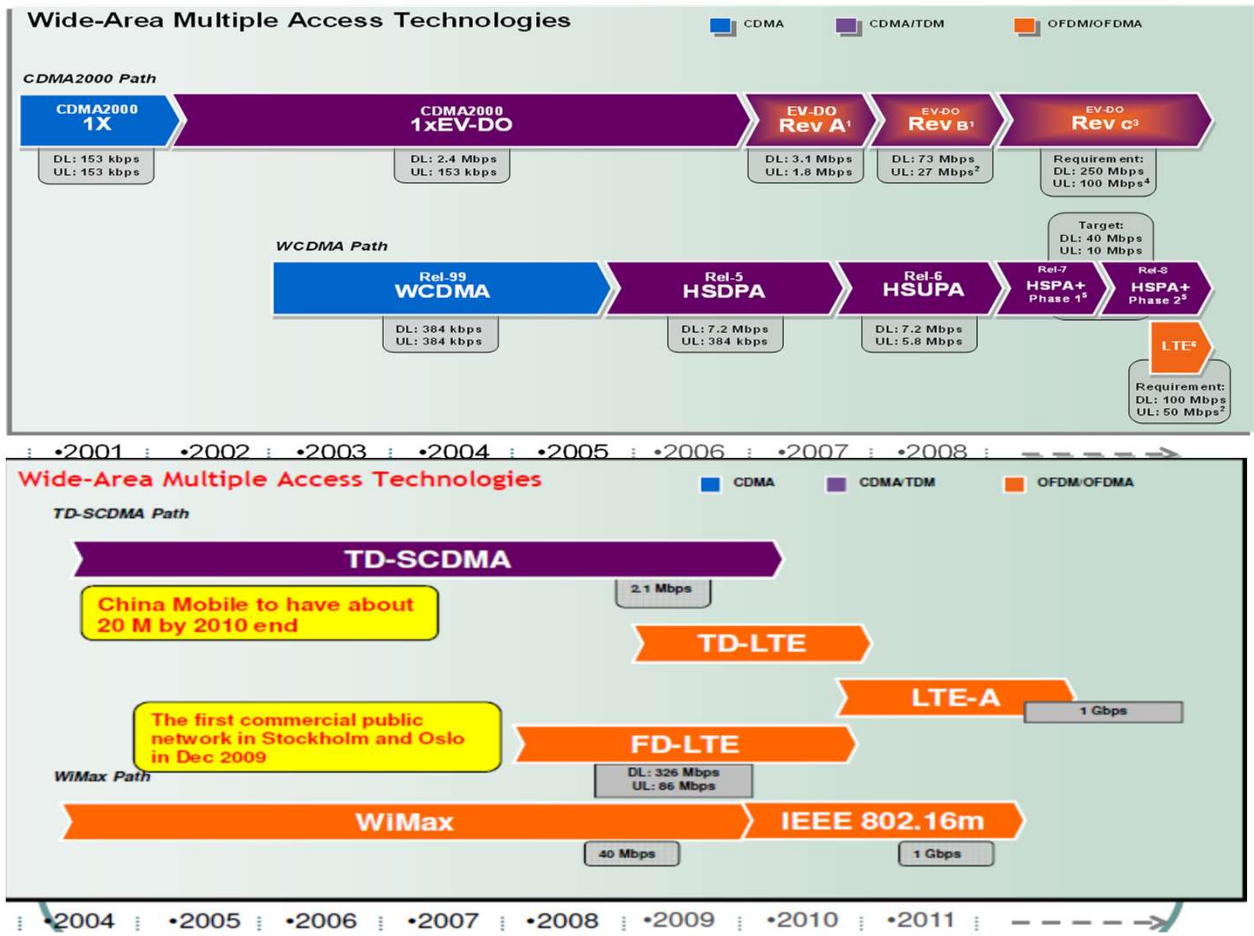
Figure 1.6 A typical cell pattern for reuse in cellular networks
Source: Authors' own.



Multiplexing and Access Techniques

- **Frequency Division Multiple Access (FDMA)**
  - frequency is divided among the participants
  - similar to people talking at different pitch and intensity, but all at the same time
- **Time Division Multiple Access (TDMA)**
  - each participant is allocated a time slot to participate
- **Code Division Multiple Access (CDMA)**
  - all the units talk at the same time
  - code word called *chipset* in each unit scrambles the conversation
  - similar to people talking in different languages in a room at the same time
  - more efficient than either FDMA or TDMA

2G, 3G and 4G Technologies: Increasing Spectral Efficiencies



Externalities

- **Negative externalities**
  - **Your conversation crowds out the others - > negative effect**
- **Tragedy of the Commons**
  - **If everyone talks, then everyone will be crowded out**
- **Near-Far problem**
  - **Nearer devices crowds out Farther devices**

Control Medium Access: *Traffic signal versus Stop signs*

www.alamy.com - F3FFCD

Which one is good for light traffic and which one for heavy traffic?
What are the trade-offs?

Notations

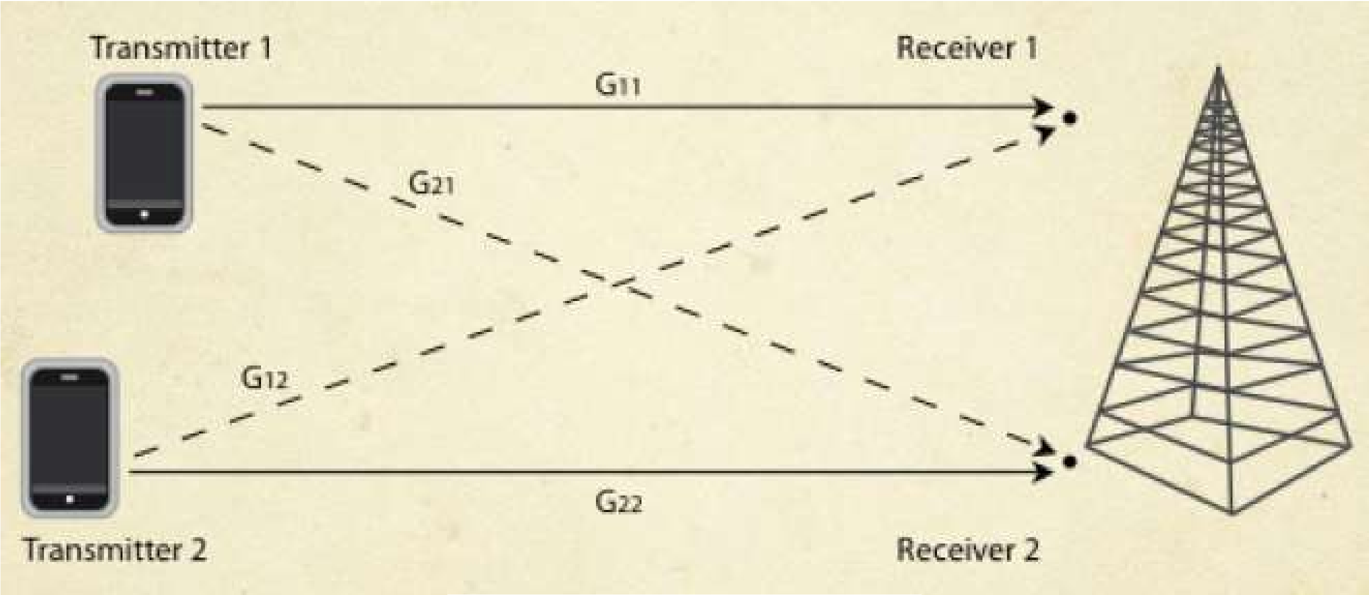
- $i \in I$: set of transmitters and receivers
- p_j: transmit power of device j
- G_{ij} : channel gain on link {i,j}
  - Depends on location of transmitter and receiver
  - Quality of channel in between
- N_i: noise at receiver i
- SNR_i : Signal to Noise Ratio at receiver i

Transmission Power Control(TPC)

- Receiver provides feedback to adjust the power of the transmitter if the threshold receiver power requirement is not met
    - Leads to Arms Race problem and the Nash Equilibrium is ..

$$p_j(t+1) = p_j(t) \times \{\gamma / [G_{ij} p_j(t)]\}$$

Uplink Interference Management



$$\mathrm{SIR}_i = \frac{G_{ii} p_i}{\sum_{j \neq i} G_{ij} p_j + n_i}.$$

Distributed Power Control (DPC)

- γ_i: Target SIR at receiver i
- **At convergence:** $SIR_i = \gamma_i$ **for all** i

$$p_i[t+1] = \frac{\gamma_i}{\mathrm{SIR}_i[t]} p_i[t], \quad \text{for each } i.$$

How is Wi-Fi different from cellular network protocols

- Basic Service Set (BSS) at each Access Point (AP)
- Characteristics of Wi-Fi
  - Operates in Unlicensed Industrial Scientific and Medical (ISM band
    - 2.4, 5.8, 60 GHz
  - Operate within a limited geographical area and hence limited power of operation
  - Interference in general is more
- Traffic signal or Stop sign??