CS 736 Assignment 3 Report

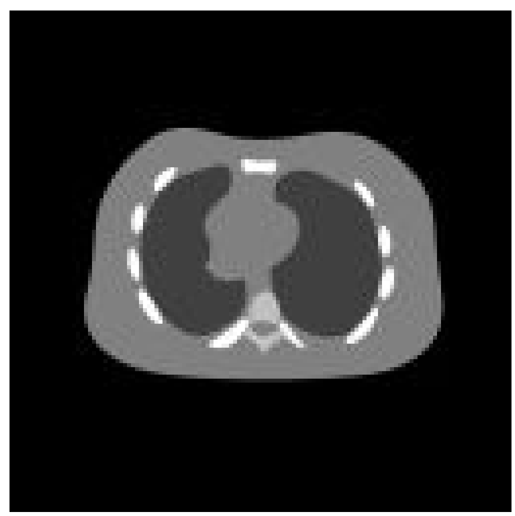
Simulated Data Acquisition and Reconstruction

a) Strategy for building the system matrix

Our idea follows the simple observation that for the linear system **Ax = b,** ($A \in \mathbb{R}^{mxn}$ and $x \in \mathbb{R}^n, b \in \mathbb{R}^m$), if we replace x with $e_1, e_2, \ldots\ldots e_n$ where e_i is the i^{th} basis vector of $\mathbb{R}^n$, we get in form of b, the columns of **A,** i.e, if **x = e$_i$** then **b = A(: , i)**. Now we know that for a vectorized (stretched) image, **x**, **A** works as the transformation matrix, which transforms **x** to it's radon transform. Hence we replace **x** with the basis vectors of the image-vector dimensional space and then using MATLAB™'s 'radon( )' function on it to calculate the value of **b**, which serves as the columns of **A**. Since the image is of dimension 128x128, **x** is a vector of dimension 16384. For the image of 128x128, radon() uses the bins of size 185 to calculate the discrete radon transform vector for one scalar value of theta. Since we are using 180 values (0:179), we get a radon transform of dimensions 185x180, which is stretched to form a vector of length 33300. Hence the dimension of **A** is 33300x16384. A useful observation that helps reduce our computational costs is that **A** is sparse. This is implemented in file, 'formSystemMatrix.m'.

PLEASE NOTE: The RRMSE values reported will change every time the code is run due to the randomness incurred while corrupting the forward projection

b) The following image (left) is reconstructed using the Ram-Lak filter in FBP using the gaussian corrupted(S.D. 2%) forward-projection of the ChestPhantom image. The original image is reported on the right.



The RRMSE of these images is: 0.4925

c) Using a gaussian prior, Tikhonov regularization

Here a zero mean gaussian is assumed on each component of **x**, with the same variance for all. This has a closed-form solution, $\mathbf{x} = (\mathbf{A}^T\mathbf{A} + \alpha\mathbf{I})^{-1}\mathbf{A}^T\mathbf{b}$ but is unfortunately highly ineffective computationally owing to the fact that generally, the inverse of a sparse matrix is not sparse. Hence iterative strategies are used. Ideas of the paper 'Kaczmarz Algorithm

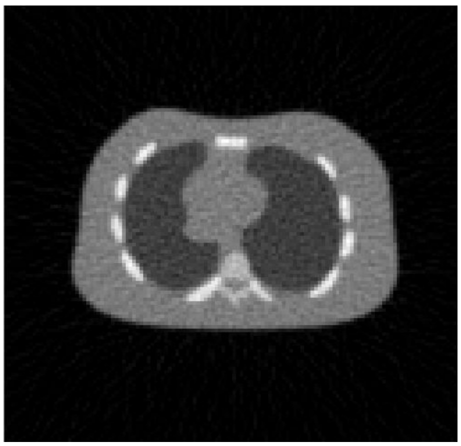
For Tikhonov Regularization Problem', by Ivanov and Zhdanov have been implemented, which state, that a pixel wise update is:

$$x^{i+1}(j) = x^{i}(j) + lambda^* (A(:,j)^T(b - A^*x) - alpha^*x(j))/(norm(A(:,j)) + alpha)$$

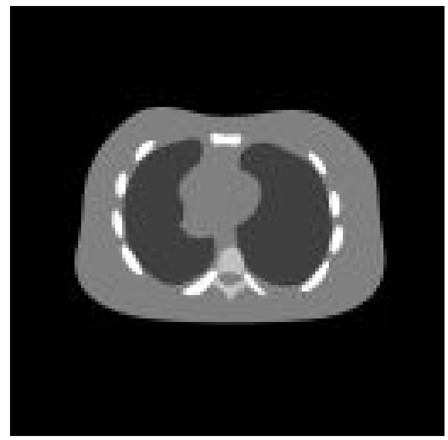
Where x(j) is the value of j^{th} pixel of x.

The hyperparameter alpha was optimized.
The optimal value is, alpha = 70
RRMSE at this value is 0.1084

Reconstructed image using this alpha

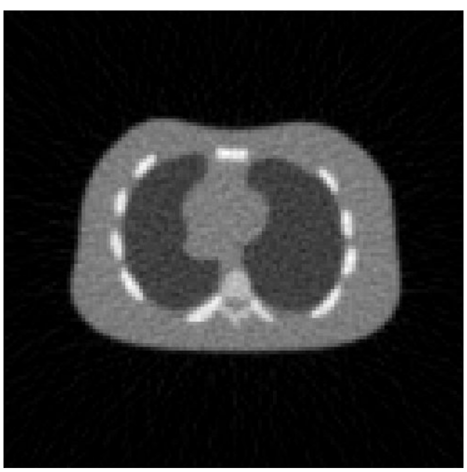


Original Image

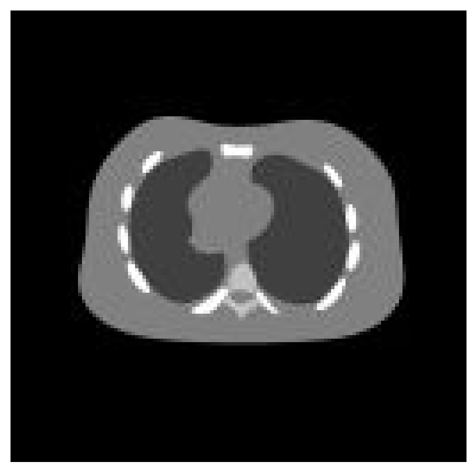


1.2*alpha = 84
RRMSE at this value 0.1099

Reconstructed image using this alpha



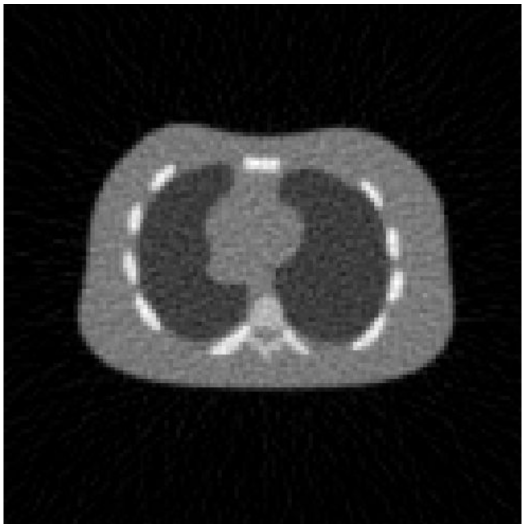
Original Image

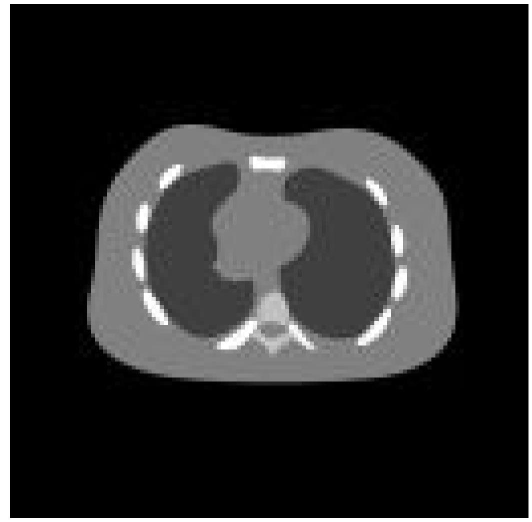


0.8*alpha = 56
RRMSE at this value = 0.1089

Reconstructed Image using this alpha

Original Image





Note: at these values image appears noisy but RRMSE is lower but at high alpha (~150-200) images are more visually similar and smoother but RRMSE is high

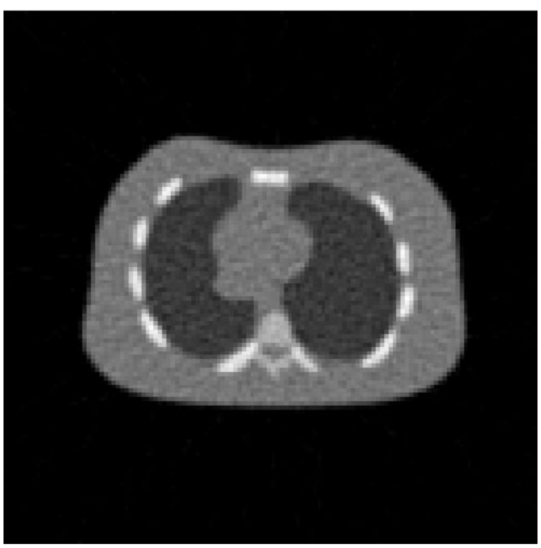
d) Using an MRF prior

Only hyperparameters to optimize are the gamma values for Huber and adaptive discontinuity functions since we are not weighing likelihood and prior.

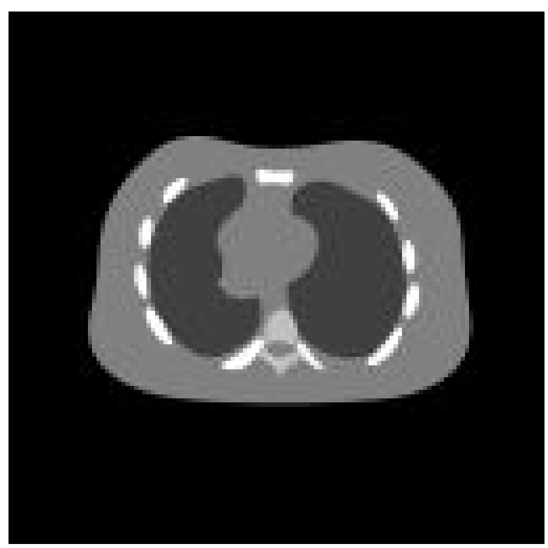
For Quadratic MRF Prior

RRMSE = 0.0857

Reconstructed Image



Original Image



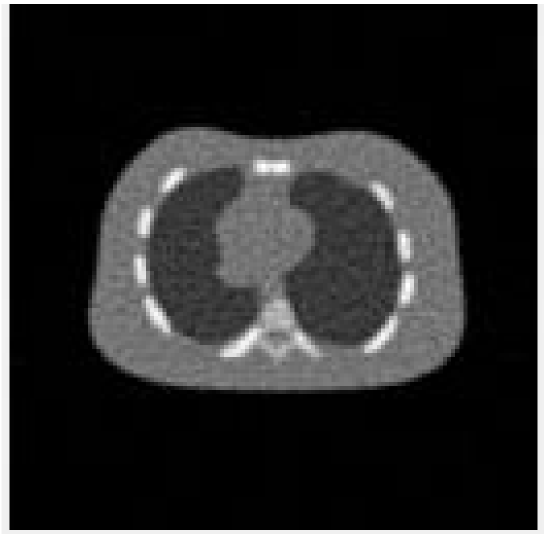
For Huber MRF Prior

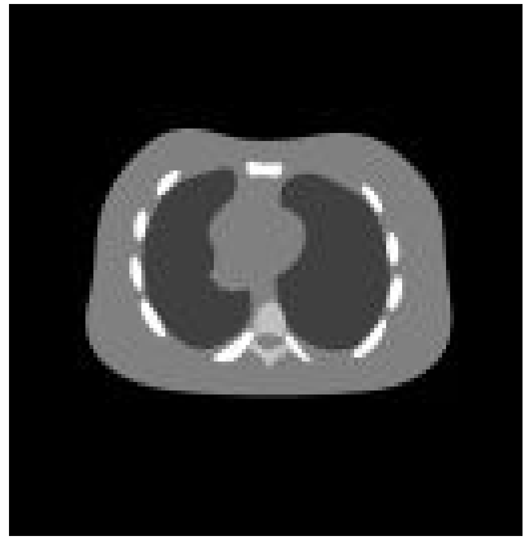
Optimal Gamma = 6.8

RRMSE at this gamma is: 0.0860

Image reconstructed using this gamma

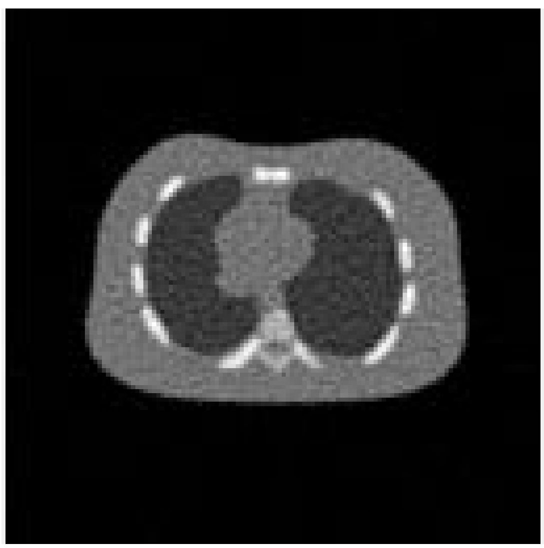
Original image

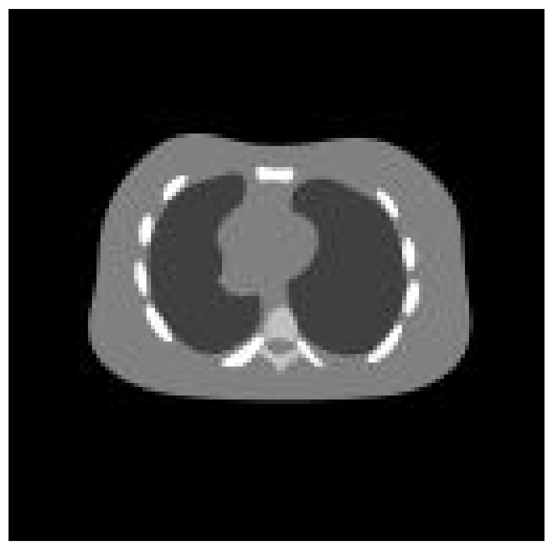




1.2*Optimal Gamma = 8.16
RRMSE at this gamma is: 0.0883
Image reconstructed using this gamma

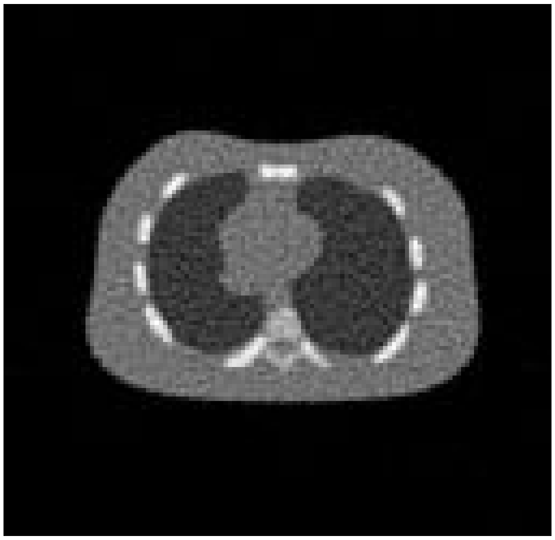
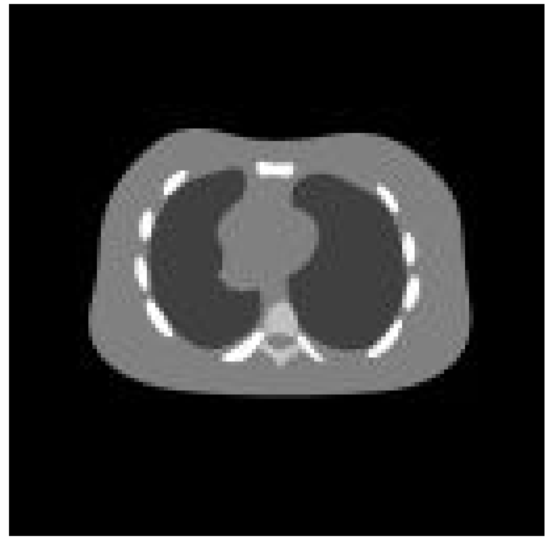
Original image





0.8*Optimal Gamma = 5.44
RRMSE at this gamma is: 0.0949
Image reconstructed using this gamma

Original image

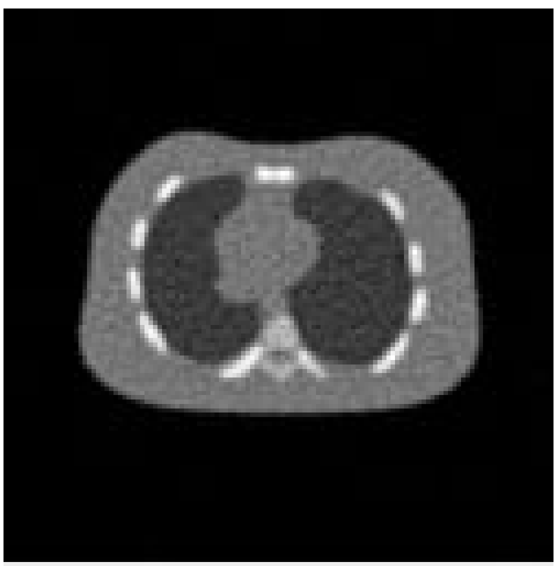
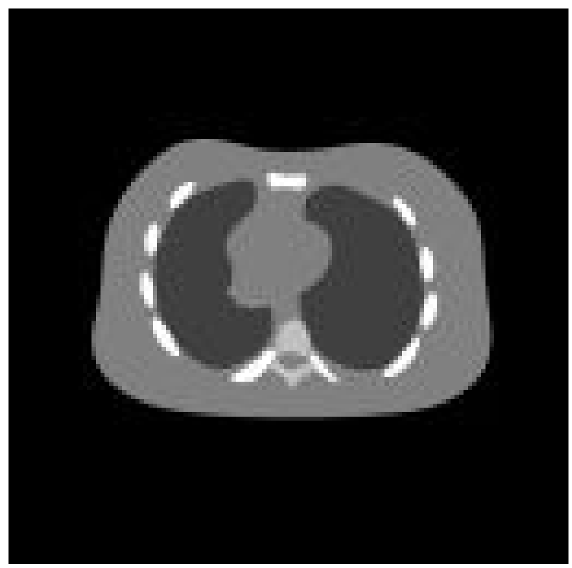
 

For Adaptive-Discontinuity MRF Prior

Optimal Gamma = 4.0

RRMSE at this gamma is: 0.0855

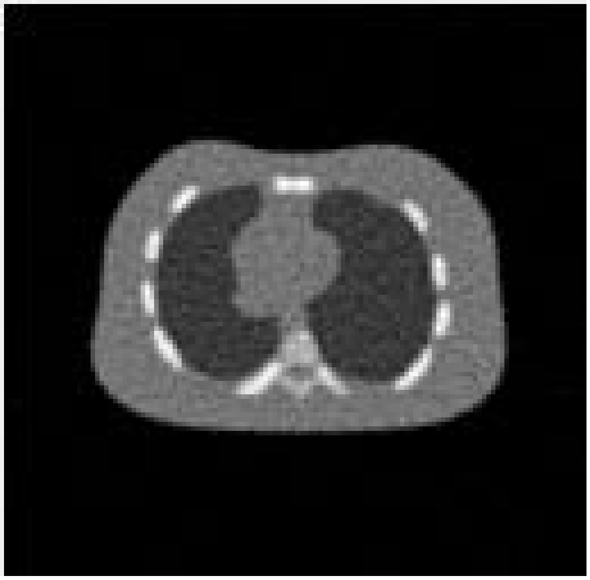
Image reconstructed using this gamma Original image

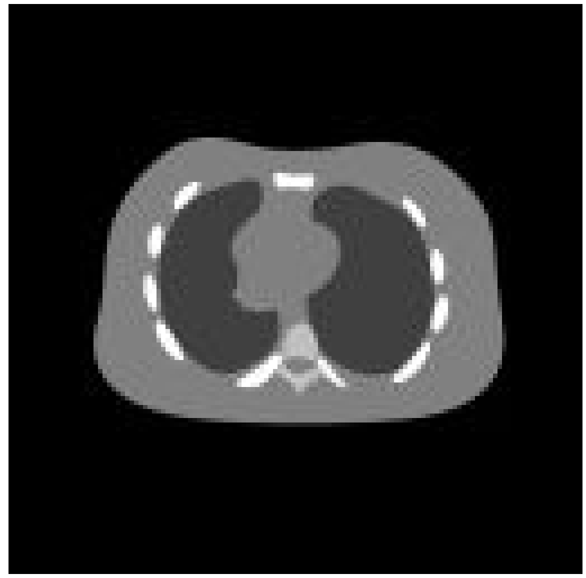
 

1.2*Optimal Gamma = 4.8

RRMSE at this gamma is: 0.0859

Image reconstructed using this gamma Original image





0.8*Optimal Gamma = 3.2
RRMSE at this gamma is: 0.0881
Image reconstructed using this gamma

Original image