# Assignment 1

June 2024

## 1 Solutions

**1)** Let n be a positive integer. Since $n > 0$,$n^2 > 0$ and $n^3 > 0$. We note first that if n>=5, then $n^3 >= 125$, i.e. $n^3 > 100$, and therefore $n^2 + n^3 > 100$. The only possible solutions are therefore n = 1, n = 2, n = 3, and n = 4.
We test each of those values separately:
i) n = 1 then $n^2 + n^3 = 2 \neq 100$. n = 1 is not a solution.
ii) n = 2 then $n^2 + n^3 = 12 \neq 100$. n = 2 is not a solution.
iii) n = 3 then $n^2 + n^3 = 36 \neq 100$. n = 3 is not a solution.
iv) n = 4 then $n^2 + n^3 = 80 \neq 100$. n = 4 is not a solution.

**2)** To prove $n^2 + 1 >= 2^n$ when n is a positive integer with $1 <= n <= 4$, we can evaluate all four possible cases:
i) n = 1 then $n^2 + 1 = 1^2 + 1 = 2$ and $2^n = 2^1 = 2$. Therefore, $n^2 + 1 >= 2^n$. So the inequality holds for n=1.
ii) n = 2 then $n^2 + 1 = 2^2 + 1 = 5$ and $2^n = 2^2 = 4$. Therefore, $n^2 + 1 >= 2^n$. So the inequality holds for n=2.
iii) n = 3 then $n^2 + 1 = 3^2 + 1 = 10$ and $2^n = 2^3 = 8$. Therefore, $n^2 + 1 >= 2^n$. So the inequality holds for n=3.
iv) n = 4 then $n^2 + 1 = 4^2 + 1 = 17$ and $2^n = 2^4 = 16$. Therefore, $n^2 + 1 >= 2^n$. So the inequality holds for n=4.

**3)** We are required to find a compound proposition involving propositional variables p,q,r, and s, which is true when exactly three of these variables are true and false otherwise. So we can find this proposition by making any three of the variables true and the last one as false and then taking the and of all.

So, The required compound proposition is given by:

W=(p ∧ q ∧ r∧ ¬s)∨(p ∧ q∧ ¬r ∧ s)∨(p ∧ ¬q ∧ r ∧ s)∨(¬p ∧ q ∧ r ∧ s)

**4)** Suppose that ∃x(P(x)→Q(x)) is true. This means that there is at least one value of x for which P(x)→Q(x) is true. If Q(x0) is true for some x0, then ∀x P(x)→∃x Q(x) is true, since P(x0) would be true for that value of x0.

Now, suppose that $\exists x(P(x) \to Q(x))$ is false. This would mean that for all values of x, $P(x) \to Q(x)$ is false. This would also mean that for all values of x, $P(x) \land \neg Q(x)$. Also, $\forall x\ P(x) \to \exists x\ Q(x)$ would be false.

Therefore, we can conclude that $\exists\ x\ (P(x) \to Q(x))$ and $\forall x\ P(x) \to \exists x\ Q(x)$ always have the same truth value.

**5)** Given: $P(A) \subset P(B)$
Let x be an element of A. Then $\{x\} \in P(A)$. If x is an element in a set S, then the set containing only that element x is set in the power set of S.
Since $P(A) \subset P(B)$,
$\{x\} \in P(B)$.Thus $x \in B$. So we have found that if $x \in A$ then $x \in B$. Thus $A \subset B$.

**6)** Proof: Given $A \subset B$

Let $x \in A \cap B$ which implies that $x \in A$ and $x \in B$. Hence we can imply that $A \cap B \subset A$

(We started with an element in $A \cap B$ and concluded that it is in A)

——(1)

Now, let $x \in A$ which implies $x \in B$ (as $A \subset B$)

Hence, $x \in A \cap B$, which in turn implies

$A \subset A \cap B$-(2)

From (1) and (2), $A \cap B = A$

Conversely, assume $A \cap B = A$ and we have to prove that $A \subset B$

Proof: Given $A \cap B = A$

Let $x \in A$ , then since $A \cap B = A$ , $x \in A \cap B$

This implies $x \in B$, hence in turn implies $A \subset B$ (We have an element in A and we proved that it is in B).

Hence proved!