# Swimming Of A Microorganism Inside A Drop Covered With Surfactant

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**Abstract**

In this work, we look at the swimming of a microorganism encapsulated within a cage which is covered in surfactant. Due to hydrodynamic interactions, the swimmer is able to propel the drop. This problem is solved purely by Analytical methods, using Regular Perturbation and Lamb's general solution. The effect of surfactant of its locomotion is investigated. We see that Scallop theorem breaks due to the presence of surfactant.We also see that surfactant makes the swimmer move slower. When the swimmer makes only tangential actuations, the drop is always slower than the swimmer. When both tangential and normal actuations are concerned, for a particular gait of the swimmer, we observe that swimmer and drop can co-swim.

## 1. Introduction

In recent times, medical science has been looking at micro-robots to revolutionize the way we treat various diseases. Drug delivery, which is one of its key applications, can use the microorganism to deliver pharmaceutical compounds to targeted cells. Side effects of drugs can be minimized, and overall efficiency can be improved. In such a scenario,a droplet encapsulating the microbe, functioning as its armor, will be of advantage. The question arises, whether the swimmer inside the drop will be able to propel the drop along with itself.

Microbial swimming is characterized by very low Reynold's number. *Re* is a measure of the relative importance of inertial and viscous forces. At low Reynold's numbers, inertia does not aid in swimming. As a result, most swimming mechanisms that we see around us, like the swimming of a man, or a boat propelling itself, do not work at this scale. The microorganism needs to change its surrounding in a time-irreversible manner to be able to swim. This is called the Scallop Theorem.

Previous works have shown that if the microbe deforms its surface in a time-irreversible manner, then it would swim. In the current work, we are trying to see if the presence of a surfactant breaks the Scallop theorem instead. The motivation for this approach is that impurities are omnipresent; there is no such thing as a clean interface. These impurities or surfactants can be made use of to propel the swimmer and the drop.

## 2. Problem Description

We consider the locomotion of a spherical micro-swimmer encapsulated in a droplet, in turn covered in surfactant, in the low Reynold's number regime. The micro-swimmer is concentric with the drop. We use an axisymmetric squirmer as our model swimmer.It achieves locomotion by generating tangential and normal velocities by 'squirming' its surface [1],[5]. This is a model used to describe microorganisms that deform their body or beat their densely packed cilia to swim. The non-dimensional radius of squirmer is denoted by $\chi$, while the non-dimensional radius of droplet is equal to 1. The fluid phases inside and outside the drop are marked as phase (1) and phase (2) respectively. Their viscosities are denoted by $\mu^{(1)}$ and $\mu(2)$, and $\lambda = \mu^{(1)}/\mu^{(2)}$ is the viscosity ratio. Since the problem is solved in the low Reynold's number regime, the creeping flow equations govern the fluid flow and pressure. The equations are $\nabla p^{(i)} = \mu^{(i)}\nabla^2\mathbf{v}^{(\mathbf{i})}$ and $\nabla.\mathbf{v}^{(\mathbf{i})}$, where i={1,2}

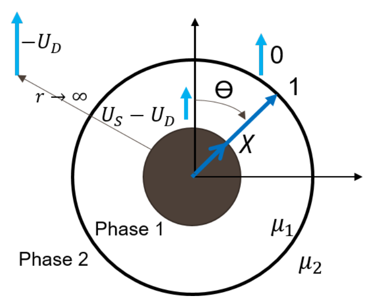



Figure 1: Fig. 1: Schematic for concentric squirmer and droplet swimming in the z direction.

A general outline of the derivation of equations employed in this study is as follows: All the governing equations and boundary conditions are listed in the dimensional form. These equations are then non-dimentionalized. The dimensionless quantity $Pe_s$, the Surface Peclet number appears in one of the equations. Our solution is for the case when $Pe_s$ is very small. Next, the variables in these non-dimensional equations are expanded as a regular perturbation in $Pe_s$. Substituting these variables back into the governing equations and boundary conditions, we now get a set of equations at {O(1), O($Pe_s$),O($Pe_s^2$),...}. Next, we solve the equations at O(1), O($Pe_s$) and O($Pe_s^2$), while truncating the variables in their expansions at O($Pe_s^2$).

### 2.1. *Dimensional Governing Equations and Boundary Conditions*

**Governing Equations:**

- Creeping flow equations for region 1: $\nabla p^{(1)} = \mu^{(1)}\nabla^2\mathbf{v}^{(1)}; \nabla \cdot \mathbf{v}^{(1)} = 0$
- region 2: $\nabla p^{(2)} = \mu^{(2)}\nabla^2\mathbf{v}^{(2)}; \nabla.\mathbf{v}^{(2)} = 0$

**Boundary Conditions:**

- $\mathbf{v}^{(1)} = \mathbf{U_S} - \mathbf{U_D} + \mathbf{u^S}$ : No-slip and no-penetration boundary condition over a spherical envelope of the swimmer; $U_S$ is the swimmer velocity, and $u^s$ is the velocity on the swimmer surface following the squirmer model.
- $\int\limits_{swimmer} \mathbf{n}.\mathbf{T}dS = 0$ : Assuming the density of the swimmer is the same as the surrounding fluid, the buoyant force is equal to its weight. Hence the net external force is zero. Since in low Reynold's number regime the inertia of the swimmer is zero, the net force acting on the swimmer must be zero. $\mathbf{F}_{net} = \mathbf{F}_{ext} + \mathbf{F}_{hydrodynamic}$. Hence $\mathbf{F}_{hydrodynamic} = 0$.
- $\mathbf{v}^{(2)} \to -\mathbf{U_D}$ as $r \to \infty$
- $\mathbf{u^S} = \sum\limits_{n=0}^{\infty} A_n P_n(cos(\theta))\mathbf{e_r} - \sum\limits_{n=1}^{\infty} \frac{2B_n}{(n^2+n)} V_n(cos(\theta))\mathbf{e}_\theta$
  {where $\frac{\partial P_n cos(\theta)}{\partial \theta} = -P_n^1 cos(\theta) = \frac{n(n+1)}{2} V_n$, and $P_n^1 cos(\theta)$ is the associated Legendre function of the first kind of order 1.[3]}

**On the drop,**

- $\mathbf{v}^{(1)} = \mathbf{v}^{(2)}$ : Continuity of velocity at the drop interface
- $\mathbf{n}.\mathbf{v}^{(1)} = \mathbf{n}.\mathbf{v}^{(2)} = 0$ : Kinematic boundary condition for a non-deforming drop
- $\mathbf{n}.(\mathbf{T}^{(2)} - \mathbf{T}^{(1)}).(\mathbf{I} - \mathbf{nn}) = RT\nabla_s\Gamma$ : Tangential stress balance at the surface of the drop; T is the stress tensor due to flow field, and $\Gamma$ is the surfactant concentration.
- $\frac{\partial\Gamma}{\partial t} + \nabla_s.(\Gamma\mathbf{v_s}) + \Gamma(\nabla_s.\mathbf{n}) - D_s(\mathbf{v}.\mathbf{n})\nabla^2\Gamma = (D^{(2)}\mathbf{n}.\nabla c(2) - D^{(1)}\mathbf{n}.\nabla c(1))_s$ : Mass balance of surfactant at the drop interface and in the fluid surrounding it [4]. The left hand side is the time rate of change of surfactant concentration at the drop surface and the right hand side is the net diffusion to the drop fro the bulk fluid on either side of the interface. D is the diffusion coefficient.
- $(D^{(2)}\mathbf{n}.\nabla c(2) - D^{(1)}\mathbf{n}.\nabla c(1))_s = \delta^{(2)}c_s^{(2)}(\Gamma_\infty - \Gamma) - \alpha^{(2)}\Gamma + \delta^{(1)}c_s^{(1)}(\Gamma_\infty - \Gamma) - \alpha^{(1)}\Gamma$ : Net diffusion of surfactant to the drop surface is equal to the net adsorption at the interface; $\delta$ is the adsorption coefficient and $\alpha$ is the desorption coefficient.

### 2.2. *Non-Dimentionalizing equations*

**Reference scales for non-dimentionalization:**

- $u_{ref} = U_{sq}$ (velocity of swimmer in unbounded quiescent fluid with only tangential squirming modes, $U_{sq} = 2B_1/3$
- $l_{ref} = a$ (radius of the drop)
- $t_{ref} = a/U_{sq}$ (assuming there is no other frequency driven processes)
- $P^{(1)}_{ref} = T^{(1)}_{ref} = \frac{\mu^{(1)}U_{sq}}{a}$ ; $P^{(2)}_{ref} = T^{(2)}_{ref} = \frac{\mu^{(2)}U_{sq}}{a}$
- $\Gamma_{ref} = \Gamma_{eq}$ (concentration of surfactant on the surface such that the net flux of surfactant is zero)
- $c^{(1)}_{ref} = c^{(2)}_{ref} = c_\infty$

**Hence the equations become:**

- $\nabla^* p^{*(1)} = \nabla^{*2}\mathbf{v}^{*(1)}; \nabla^*.\mathbf{v}^{*(1)} = 0$ inside the drop
- $\nabla^* p^{*(2)} = \nabla^{*2}\mathbf{v}^{*(2)}; \nabla^*.\mathbf{v}^{*(2)} = 0$ outside the drop
- $\mathbf{v}^{*(1)} = \mathbf{U_S}^* - \mathbf{U_D}^* + \mathbf{u}^{*\mathbf{s}}$ on the swimmer
- $\int\limits_{swimmer} \mathbf{n}.\mathbf{T}^{*(1)} dS^* = 0$
- $\mathbf{v}^{*(2)}(r \to \infty) = -\mathbf{U_D}*$

**On the drop**

- $\mathbf{v}^{*(1)} = \mathbf{v}^{*(2)}$
- $\mathbf{n}.\mathbf{v}^{*(1)} = \mathbf{n}.\mathbf{v}^{*(2)} = 0$
- $\int\limits_{drop} \mathbf{n}.\mathbf{T}^{*(2)} dS^* = 0$
- $\mathbf{n}.(\mathbf{T}^{*(2)} - \lambda \mathbf{T}^{*(1)}).(\mathbf{I} - \mathbf{nn}) = Ma\nabla^*_s\Gamma^*$ : $Ma$ is Marangoni number, a dimensionless quantity representative of the ratio of surface tension force to viscous force
- $\frac{a}{U_{sq}t_{ref}}\frac{\partial\Gamma^*}{\partial t^*} + \nabla^*_s.(\Gamma^*_s\mathbf{v_s}^*) + \Gamma^*(\nabla^*_s.\mathbf{n})(\mathbf{v}^*.\mathbf{n}) - \frac{1}{Pe_s}\nabla^{*2}_s\Gamma^* = \frac{c_\infty a}{\Gamma_{eq}}(\frac{1}{Pe^{(2)}}\mathbf{n}.\nabla c^{*(2)} - \frac{1}{Pe^{(1)}}\mathbf{n}.\nabla c^{*(1)})_s$: $Pe$ is Peclet number, a dimensionless quantity representative of the convective to diffusive surfactant transport rate, with $Pe_s$ is Peclet number evaluated at the surface, $Pe^{(1)}$ is that for region interior to the drop, and $Pe^{(2)}$ for the region exterior; $Bi$ is Biot number, which provides the importance of kinetic desorption relative to interfacial convection; $c_s$ is the bulk phase concentration as we approach the surface of the drop and $k^{(1)}$ and $k^{(2)}$ are the measures of the ratio of adsorption to desorption interior to the drop and exterior to it respectively.

- $\frac{c_\infty a}{\Gamma_{eq}}(\frac{1}{Pe^{(2)}}\mathbf{n}.\nabla c^{*(2)} - \frac{1}{Pe^{(1)}}\mathbf{n}.\nabla c^{*(1)})_s = Bi(k^{(2)}c_s^{*(2)}(\Gamma_\infty^* - \Gamma^*) - \Gamma^* + \frac{\alpha^{(1)}}{\alpha^{(2)}}(k^{(1)}c_s^{*(1)}(\Gamma_\infty^* - \Gamma^*) - \Gamma^*))$
  Assume surfactant is insoluble inside the drop, then on the drop:
  $\frac{a}{U_{sq}t_{ref}}\frac{\partial\Gamma^*}{\partial t^*} + \nabla_s^*.(\Gamma^*\mathbf{v_s}^*) + \Gamma^*(\nabla_s^*.\mathbf{n})(\mathbf{v}^*.\mathbf{n}) - \frac{1}{Pe_s}\nabla_s^{*2}\Gamma^* = \frac{c_\infty a}{\Gamma_{eq}}(\frac{1}{Pe^{(2)}}\mathbf{n}.\nabla c^{*(2)})_s$
  Also,
  $\frac{c_\infty a}{\Gamma_{eq}}(\frac{1}{Pe^{(2)}}\mathbf{n}.\nabla^* c^{(2)})_s = Bi(k^{(2)}c_s^{(2)}(\Gamma_\infty^* - \Gamma^*) - \Gamma_*)$
  Further, assume that either $Bi << O(1)$ or $\frac{c_\infty a}{\Gamma_e q}\frac{1}{Pe^{(2)}} << O(1)$ and assuming quasi-equilibrium conditions for spreading of $\Gamma^*$, we get
  $Pe_s\nabla_s^*.(\Gamma^* v^*) = \nabla_s^{*2}\Gamma^*$ on the drop

### 2.3. ***Regular Perturbation***

Removing stars from variables and taking all variables to be non-dimensional henceforth. Assume $Pe_s << O(1)$, then expand all the variables as regular perturbation in $Pe_s$.
$\mathbf{v}^{(1)} = \mathbf{v_0}^{(1)} + Pe_s\mathbf{v_1}^{(1)} + Pe_s^2\mathbf{v_2}^{(1)} + O(Pe_s^3)$
Similarly expand for $\{\mathbf{v}^{(2)}, P^{(1)}, \mathbf{T}^{(1)}, \mathbf{T}^{(2)}, \Gamma, \mathbf{U_S}, \mathbf{U_D}\} as \{\mathbf{v_0}^{(2)}, P_0^{(1)}, \mathbf{T_0}^{(1)}, \mathbf{T_0}^{(2)}, \Gamma_0, \mathbf{U_{S0}}, \mathbf{U_{D0}}\} + Pe_s\{\mathbf{v_1}^{(2)}, P_1^{(1)}, \mathbf{T_1}^{(1)}, \mathbf{T_1}^{(2)}, \Gamma_1, \mathbf{U_{S1}}, \mathbf{U_{D1}}\} + Pe_s^2\{\mathbf{v_2}^{(2)}, P_2^{(1)}, \mathbf{T_2}^{(1)}, \mathbf{T_2}^{(2)}, \Gamma_2, \mathbf{U_{S2}}, \mathbf{U_{D2}}\}$.
Assume $\mathbf{u^s}$ is O(1). Substitute these in the non-dimensional equation.

**Governing Equations and Boundary Conditions at O(1)**

- creeping flow equations for region 1: $\nabla p_0^{(1)} = \nabla^2\mathbf{v_0}^{(1)}; \nabla.\mathbf{v_0}^{(1)} = 0$
- region 2: $\nabla p_0^{(2)} = \nabla^2\mathbf{v_0}^{(2)}; \nabla.\mathbf{v_0}^{(2)} = 0$

  **Boundary Conditions:**

- $\mathbf{v_0}^{(1)} = \mathbf{U_{S0}} - \mathbf{U_{D0}} + \mathbf{u^S}$ on the swimmer
- $\int\limits_{swimmer} n.\mathbf{T_0}dS = 0$
- $\mathbf{v_0}^{(2)} \rightarrow -\mathbf{U_{D0}}$ as $r \rightarrow \infty$

  **On the drop,**

- $\mathbf{v_0}^{(1)} = \mathbf{v_0}^{(2)}$
- $\mathbf{n}.\mathbf{v_0}^{(1)} = \mathbf{n}.\mathbf{v_0}^{(2)} = 0$
- $\int\limits_{drop} n.\mathbf{T_0}^{(2)}dS$
- $\mathbf{n}.(\mathbf{T_0}^{(2)} - \lambda\mathbf{T_0}^{(1)}).(\mathbf{I} - \mathbf{nn}) = 0$ ; $\Gamma_0 = 1$

**Governing Equations and Boundary Conditions at $O(Pe_s)$**

- Creeping flow equations for region 1: $\nabla p_1^{(1)} = \nabla^2 v_1^{(1)}; \nabla.v_1^{(1)} = 0$

- region 2: $\nabla p_1^{(2)} = \nabla^2 \mathbf{v_1}^{(2)}; \nabla . \mathbf{v_1}^{(2)} = 0$

  **Boundary Conditions:**

- $\mathbf{v_1}^{(1)} = \mathbf{U_{S1}} - \mathbf{U_{D1}}$ On the swimmer
- $\int\limits_{swimmer} n.\mathbf{T_1} dS = 0$
- $\mathbf{v_1}^{(2)} \to -\mathbf{U_{D1}}$ as $r \to \infty$

  **On the drop,**

- $\mathbf{v_1}^{(1)} = \mathbf{v_1}^{(2)}$
- $\mathbf{n}.\mathbf{v_1}^{(1)} = \mathbf{n}.\mathbf{v_1}^{(2)} = 0$
- $\int\limits_{drop} \mathbf{n}.\mathbf{T_1}^{(2)} dS$
- $\mathbf{n}.(\mathbf{T_1}^{(2)} - \lambda \mathbf{T_1}^{(1)}).(\mathbf{I} - \mathbf{nn}) = Ma \nabla_s \Gamma_1$
- $\nabla_s . \mathbf{v_0} = \nabla_s^2 \Gamma_1$

**Governing Equations and Boundary Conditions at $O(Pe_s^2)$**

- Creeping flow equations for region 1: $\nabla p_2^{(1)} = \nabla^2 \mathbf{v_2}^{(1)}; \nabla . \mathbf{v_2}^{(1)} = 0$
- region 2: $\nabla p_2^{(2)} = \nabla^2 \mathbf{v_2}^{(2)}; \nabla . \mathbf{v_2}^{(2)} = 0$

  **Boundary Conditions:**

- $\mathbf{v_2}^{(1)} = \mathbf{U_{S2}} - \mathbf{U_{D2}}$ on the swimmer
- $\int\limits_{swimmer} n.\mathbf{T_2} dS = 0$
- $\mathbf{v_2}^{(2)} \to -\mathbf{U_{D2}}$ as $r \to \infty$

  **On the drop,**

- $\mathbf{v_2}^{(1)} = \mathbf{v_2}^{(2)}$
- $\mathbf{n}.\mathbf{v_2}^{(1)} = \mathbf{n}.\mathbf{v_2}^{(2)} = 0$
- $\int\limits_{drop} \mathbf{n}.\mathbf{T_2}^{(2)} dS$
- $\mathbf{n}.(\mathbf{T_2}^{(2)} - \lambda \mathbf{T_2}^{(1)}).(\mathbf{I} - \mathbf{nn}) = Ma \nabla_s \Gamma_2$
- $\nabla_s . (\Gamma_0 \mathbf{v_1} + \Gamma_1 \mathbf{v_0}) = \nabla_s^2 \Gamma_2$

## 3. The Solution

The solution methodology is based on Lamb's general solution of the stokes equations in spherical coordinates[2]. For a single-phase fluid with viscosity $\mu$, the fluid velocity field $\mathbf{v}$ can be expanded in spherical harmonics as
$\mathbf{v} = \sum_{n=-\infty}^{\infty} [\nabla\phi_n + \frac{n+3}{2\mu(n+1)(2n+3)} r^2 \nabla p_n - \frac{n}{\mu(n+1)(2n+3)} \mathbf{r} p_n]$ where $p_n$ and $\phi_n$ are solid spherical harmonics satisfying $\nabla^2 p_n$ and $\nabla^2 \phi_n = 0$ respectively. In axisymmetric case, $p_n$ and $\phi_n$ are expressed in terms of Legendre functions as $p_n(r,\zeta) = \tilde{p_n} r^n P_n(\zeta)$ and $\phi_n(r,\zeta) = \tilde{\phi_n} r^n P_n(\zeta)$, where $\zeta = cos(\theta)$, and $\tilde{p_n}$ and $\tilde{phi}_n$ are constants independent of r and $\zeta$.

The radial and tangential velocity components are then obtained as
$v_r = \mathbf{e_r}.\mathbf{v} = \sum_{n=0}^{\infty} [\bar{p}_n r^{n+1} + \bar{\phi}_n r^{n-1} + \bar{p}_{-n-1} r^{-n} + \bar{\phi}_{-n-1} r^{-n-2}] P_n(\zeta)$
$v_\theta = \mathbf{e}_\theta.\mathbf{v} = \sum_{n=0}^{\infty} [-\frac{n+3}{2} \bar{p}_n r^{n+1} - \frac{n+1}{2} \bar{\phi}_n r^{n-1} + \frac{n-2}{2} \bar{p}_{-n-1} r^{-n} + \frac{n}{2} \bar{\phi}_{-n-1} r^{-n-2}] V_n(\zeta)$
$T_{r\theta} = \sum_{n=1}^{\infty} -\frac{1}{r}[n(n+2)\bar{p}_n r^{n+1} + (n^2-1)\bar{\phi}_n r^{n-1} + (n^2-1)\bar{p}_{-n-1} r^{-n} + n(n+2)\bar{\phi}_{-n-1} r^{-n-2}] V_n(\zeta)$
where $\bar{p}_n = \frac{n}{2\mu(2n+3)} \tilde{p}_n$ , $\bar{\phi}_n = n\tilde{\phi}_n$.
On substituting the above expressions in the boundary conditions derived earlier, we get a system of linear equations which can be solved to evaluate $U_S, U_D$ and velocity flow field by solving for unknown constants $\bar{p}_n^{(i)}$, $\bar{\phi}_n^{(i)}$, $\bar{p}_{-n-1}^{(i)}$, $\bar{p}_{-n-1}^{(i)}$ (i=1,2).

### 3.1. *Solution at O(1)*

**Solving for Swimmer and Drop Velocity** Taking n={1,2} in the expansion for $v_r, v_\theta$ and $T_{r\theta}$ to evaluate the swimmer and droplet velocity, we get the following:

- $A_1 + U_{S0} - U_{D0} = \bar{\phi_1}^{(1)} + \chi^2 \bar{p}_1^{(1)} + \chi^{-3} \bar{\phi}_{-2}^{(1)} + \chi^{-1} \bar{p}_{-2}^{(1)}$
- $B_1 - U_{S0} + U_{D0} = -\bar{\phi_1}^{(1)} - 2\chi^2 \bar{p}_1^{(1)} + \frac{\bar{\phi}_{-2}^{(1)}}{2\chi^3} + \frac{\bar{p}_{-2}^{(1)}}{2\chi^1}$
- $0 = \bar{\phi_1}^{(1)} + \bar{p}_1^{(1)} + \bar{\phi}_{-2}^{(1)} - \bar{p}_{-2}^{(1)}$
- $\bar{\phi}_1^{(2)} = -U_{D0}$
- $0 = \bar{\phi}_1^{(2)} + \bar{\phi}_{-2}^{(2)} + \bar{p}_{-2}^{(2)}$
- $-\bar{\phi_1}^{(1)} - 2\bar{p}_1^{(1)} + \frac{\bar{\phi}_{-2}^{(1)}}{2} - \frac{\bar{p}_{-2}^{(1)}}{2} = -\bar{\phi_1}^{(2)} + \frac{\bar{\phi}_{-2}^{(2)}}{2} - \frac{\bar{p}_{-2}^{(2)}}{2}$
- $\bar{\phi}_{-2} - \lambda[\bar{p}_1^{(1)} + \bar{\phi}_{-2}^{(1)}] = 0$

From zero net hydrodynamic force conditions, and from $F = -4\pi\nabla[r^3 p_{-2}]$, we have $\bar{p}_{-2}^{(1)} = 0$ and $\bar{p}_{-2}^{(1)} = 0$. Hence we now have 9 equations and 9 unknowns, which need to be be solved for.

**Solving for disturbance flow field**

- $A_n = \chi^{n-1}\bar{\phi}_n^{(1)} + \chi^{n+1}\bar{p}_n^{(1)} + \chi^{-n-2}\bar{\phi}_{-n-1}^{(1)} + \chi^{-n}\bar{p}_{-n-1}^{(1)}$
- $B_n = -\frac{n+1}{2}\chi^{n-1}\bar{\phi}_n^{(1)} - \frac{n+3}{2}\chi^{n+1}\bar{p}_n^{(1)} + \frac{n}{2}\chi^{-n-2}\bar{\phi}_{-n-1}^{(1)} + \frac{n-2}{2}\chi^{-n}\bar{p}_{-n-1}^{(1)}$
- $0 = \bar{\phi}_n^{(1)} + \bar{p}_n^{(1)} + \bar{\phi}_{-n-1}^{(1)} + \bar{p}_{-n-1}^{(1)}$
- $0 = \bar{\phi}_{-n-1}^{(2)} + \bar{p}_{-n-1}^{(2)}$
- $-\frac{n+1}{2}\bar{\phi}_n^{(1)} - \frac{n+3}{2}\bar{p}_n^{(1)} + \frac{n}{2}\bar{\phi}_{-n-1}^{(1)} + \frac{n-2}{2}\bar{p}_{-n-1}^{(1)} = \frac{n}{2}\bar{\phi}_{-n-1}^{(2)} + \frac{n-2}{2}\bar{p}_{-n-1}^{(2)}$
- $(n^2-1)\bar{p}_{-n-1}^{(2)} + n(n+2)\bar{\phi}_{-n-1}^{(2)} - \lambda[n(n+2)\bar{p}_n^{(1)} + (n^2-1)\bar{\phi}_{-n-1}^{(1)} + (n^2 - 1)\bar{p}_{-n-1}^{(1)} + n(n+2)\bar{\phi}_{-n-1}^{(1)}] = 0$
  Hence we have 6 equations and 6 unknowns, which need to be solved for.

### 3.2. *Solution at* $O(Pe_s)$

**Solving for Swimmer and Drop Velocity**

Taking n={1,2} in the expansion for $v_r, v_\theta$ and $T_{r\theta}$ to evaluate the swimmer and droplet velocity, we get the following:

- $U_{S1} - U_{D1} = \bar{\phi_1}^{(1)} + \chi^2\bar{p}_1^{(1)} + \chi^{-3}\bar{\phi}_{-2}^{(1)} + \chi^{-1}\bar{p}_{-2}^{(1)}$
- $-U_{S1} + U_{D1} = -\bar{\phi_1}^{(1)} - 2\chi^2\bar{p}_1^{(1)} + \frac{\bar{\phi}_{-2}^{(1)}}{2\chi^3} + \frac{\bar{p}_{-2}^{(1)}}{2\chi^1}$
- $0 = \bar{\phi_1}^{(1)} + \bar{p}_1^{(1)} + \bar{\phi}_{-2}^{(1)} - \bar{p}_{-2}^{(1)}$
- $\bar{\phi}_1^{(2)} = -U_{D1}$
- $0 = \bar{\phi}_1^{(2)} + \bar{\phi}_{-2}^{(2)} + \bar{p}_{-2}^{(2)}$
- $-\bar{\phi_1}^{(1)} - 2\bar{p}_1^{(1)} + \frac{\bar{\phi}_{-2}^{(1)}}{2} - \frac{\bar{p}_{-2}^{(1)}}{2} = -\bar{\phi_1}^{(2)} + \frac{\bar{\phi}_{-2}^{(2)}}{2} - \frac{\bar{p}_{-2}^{(2)}}{2}$
- $\bar{\phi}_{-2} - \lambda[\bar{p}_1^{(1)} + \bar{\phi}_{-2}^{(1)}] = \frac{Ma\tilde{\Gamma}_{1,1}}{3}$
- $\bar{p}_{-2}^{(1)} = 0$ and $\bar{p}_{-2}^{(1)} = 0$

Where $\tilde{\Gamma}_{1,1}$ is -$\frac{5(A_1+B_1)\lambda\chi^2}{(2\lambda-2)\chi^5+3\lambda+2}$. Hence we have 9 equations and 9 unknowns.

**Solving for disturbance flow field**

- $A_n = \chi^{n-1}\bar{\phi}_n^{(1)} + \chi^{n+1}\bar{p}_n^{(1)} + \chi^{-n-2}\bar{\phi}_{-n-1}^{(1)} + \chi^{-n}\bar{p}_{-n-1}^{(1)}$
- $B_n = -\frac{n+1}{2}\chi^{n-1}\bar{\phi}_n^{(1)} - \frac{n+3}{2}\chi^{n+1}\bar{p}_n^{(1)} + \frac{n}{2}\chi^{-n-2}\bar{\phi}_{-n-1}^{(1)} + \frac{n-2}{2}\chi^{-n}\bar{p}_{-n-1}^{(1)}$

- $0 = \bar{\phi}_n^{(1)} + \bar{p}_n^{(1)} + \bar{\phi}_{-n-1}^{(1)} + \bar{p}_{-n-1}^{(1)}$

- $0 = \bar{\phi}_{-n-1}^{(2)} + \bar{p}_{-n-1}^{(2)}$

- $-\frac{n+1}{2}\bar{\phi}_n^{(1)} - \frac{n+3}{2}\bar{p}_n^{(1)} + \frac{n}{2}\bar{\phi}_{-n-1}^{(1)} + \frac{n-2}{2}\bar{p}_{-n-1}^{(1)} = \frac{n}{2}\bar{\phi}_{-n-1}^{(2)} + \frac{n-2}{2}\bar{p}_{-n-1}^{(2)}$

- $(n^2-1)\bar{p}_{-n-1}^{(2)} + n(n+2)\bar{\phi}_{-n-1}^{(2)} - \lambda[n(n+2)\bar{p}_n^{(1)} + (n^2-1)\bar{\phi}_{-n-1}^{(1)} + (n^2-1)\bar{p}_{-n-1}^{(1)} + n(n+2)\bar{\phi}_{-n-1}^{(1)}] = \frac{n(n+1)}{2}Ma\tilde{\Gamma}_{1,n}$

  Where $\tilde{\Gamma}_{1,n}$ is $2\frac{(n+1)\bar{\phi}_{n,0}^{(1)}+(n+3)\bar{p}_{n,0}^{(1)}-n\bar{\phi}_{-n-1,0}^{(1)}-(n-2)\bar{p}_{-n-1,0}^{(1)}}{n(n+1)}$. The '-,0' in subscript indicates that this term is from the solution of O(1) problem. Hence we have 6 equations and 6 unknowns, which need to be solved for.

### *3.3. **Solution at** $O(Pe_s^2)$*

**Solving for Swimmer and Drop Velocity**

Taking n={1,2} in the expansion for $v_r, v_\theta$ and $T_{r\theta}$ to evaluate the swimmer and droplet velocity, we get the following:

- $U_{S2} - U_{D2} = \bar{\phi_1}^{(1)} + \chi^2\bar{p}_1^{(1)} + \chi^{-3}\bar{\phi}_{-2}^{(1)} + \chi^{-1}\bar{p}_{-2}^{(1)}$

- $-U_{S2} + U_{D2} = -\bar{\phi_1}^{(1)} - 2\chi^2\bar{p}_1^{(1)} + \frac{\bar{\phi}_{-2}^{(1)}}{2\chi^3} + \frac{\bar{p}_{-2}^{(1)}}{2\chi^1}$

- $0 = \bar{\phi_1}^{(1)} + \bar{p}_1^{(1)} + \bar{\phi}_{-2}^{(1)} - \bar{p}_{-2}^{(1)}$

- $\bar{\phi}_1^{(2)} = -U_{D2}$

- $0 = \bar{\phi}_1^{(2)} + \bar{\phi}_{-2}^{(2)} + \bar{p}_{-2}^{(2)}$

- $-\bar{\phi_1}^{(1)} - 2\bar{p}_1^{(1)} + \frac{\bar{\phi}_{-2}^{(1)}}{2} - \frac{\bar{p}_{-2}^{(1)}}{2} = -\bar{\phi_1}^{(2)} + \frac{\bar{\phi}_{-2}^{(2)}}{2} - \frac{\bar{p}_{-2}^{(2)}}{2}$

- $\bar{\phi}_{-2} - \lambda[\bar{p}_1^{(1)} + \bar{\phi}_{-2}^{(1)}] = \frac{Ma\tilde{\Gamma}_{2,1}}{3}$

- $\bar{p}_{-2}^{(1)} = 0$ and $\bar{p}_{-2}^{(1)} = 0$

  Where $\tilde{\Gamma}_{2,1}$ is $\frac{2}{15}\tilde{u}_{0,1}^2 + \frac{1}{70}\tilde{u}_{0,2}\tilde{u}_{0,3} - \tilde{u}_{1,1}$. Hence we have 9 equations and 9 unknowns.

**Solving for disturbance flow field**

- $A_n = \chi^{n-1}\bar{\phi}_n^{(1)} + \chi^{n+1}\bar{p}_n^{(1)} + \chi^{-n-2}\bar{\phi}_{-n-1}^{(1)} + \chi^{-n}\bar{p}_{-n-1}^{(1)}$

- $B_n = -\frac{n+1}{2}\chi^{n-1}\bar{\phi}_n^{(1)} - \frac{n+3}{2}\chi^{n+1}\bar{p}_n^{(1)} + \frac{n}{2}\chi^{-n-2}\bar{\phi}_{-n-1}^{(1)} + \frac{n-2}{2}\chi^{-n}\bar{p}_{-n-1}^{(1)}$

- $0 = \bar{\phi}_n^{(1)} + \bar{p}_n^{(1)} + \bar{\phi}_{-n-1}^{(1)} + \bar{p}_{-n-1}^{(1)}$

- $0 = \bar{\phi}_{-n-1}^{(2)} + \bar{p}_{-n-1}^{(2)}$

- $-\frac{n+1}{2}\bar{\phi}_n^{(1)} - \frac{n+3}{2}\bar{p}_n^{(1)} + \frac{n}{2}\bar{\phi}_{-n-1}^{(1)} + \frac{n-2}{2}\bar{p}_{-n-1}^{(1)} = \frac{n}{2}\bar{\phi}_{-n-1}^{(2)} + \frac{n-2}{2}\bar{p}_{-n-1}^{(2)}$

- $(n^2-1)\bar{p}_{-n-1}^{(2)} + n(n+2)\bar{\phi}_{-n-1}^{(2)} - \lambda[n(n+2)\bar{p}_n^{(1)} + (n^2-1)\bar{\phi}_{-n-1}^{(1)} + (n^2-1)\bar{p}_{-n-1}^{(1)} + n(n+2)\bar{\phi}_{-n-1}^{(1)}] = \frac{n(n+1)}{2}Ma\tilde{\Gamma}_n$

  Where $\tilde{\Gamma}_n$ is $2\frac{(n+1)\bar{\phi}_{n,0}^{(1)}+(n+3)\bar{p}_{n,0}^{(1)}-n\bar{\phi}_{-n-1,0}^{(1)}-(n-2)\bar{p}_{-n-1,0}^{(1)}}{n(n+1)}$. The ',0' in subscript indicates that this term is from the solution of O(1) problem. Hence we have 6 equations and 6 unknowns, which need to be solved for.

## 4. Results

- $U_{D0} = \frac{10\chi^3\lambda(A_1+B_1)}{(6\lambda-6)\chi^5+9\lambda+6}$

- $U_{S0} = \frac{-12(\lambda-1)(A_1+\frac{1}{2}B_1)\chi^5+10\chi^3(A_1+B_1)(\lambda-1)-3(\lambda+\frac{2}{3})(A_1-2B_1)}{(6\lambda-6)\chi^5+9\lambda+6}$

- $U_{D1} = \frac{5}{6}\frac{Ma\lambda(A_1+B_1)\chi^3(\chi^5-1)}{((\lambda-1)\chi^5+\frac{3}{2}\lambda+1)^2}$

- $U_{S1} = \frac{25}{12}\frac{Ma\chi^3\lambda(\chi^2-1)(A_1+B_1)}{((\lambda-1)\chi^5+\frac{3}{2}\lambda+1)^2}$

- $U_{D2} = \frac{2}{3}\frac{Ma(1-\chi^5)\Gamma_{1,1}}{(2\lambda-2)\chi^5+3\lambda+2}$

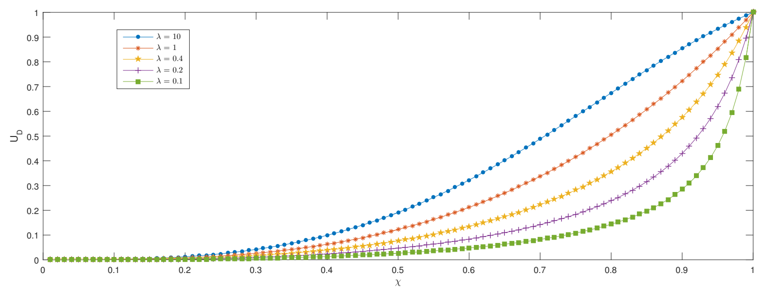
- $U_{S2} = \frac{5}{3}\frac{Ma(1-\chi^2)\Gamma_{1,1}}{(2\lambda-2)\chi^5+3\lambda+2}$

- We see from figure 2 that $U_D/U_S$ is always smaller than 1 for all feasible values of $\chi$. Hence for purely tangential actuation, concentric configuration is never stable.

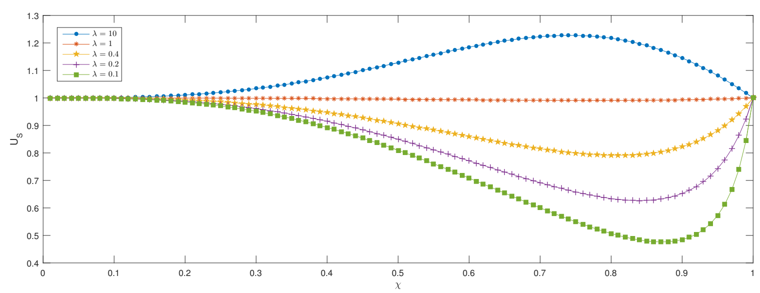
- In figure 3, swimmer and droplet velocity is described at different Marangoni numbers. As discussed previously, Marangoni number represents the ratio of surface tension stress to viscous stress. Since we modeled the surface tension as linear in surfactant concentration, figure 3 is actually plotting swimmer and drop velocities for different surfactant concentrations. Interestingly, even though the surfactant is present on the drop, and only the boundary conditions at the drop change due to presence of surfactant, it has negligible effect on the drop velocity, while the swimmer velocity is seen to decrease with increasing surfactant concentration.

- We see from figure 4 that the drop causes the far field flow to reverse its nature as compared to that in an unbounded fluid. This result agrees both qualitatively and quantitatively with the results in [6]

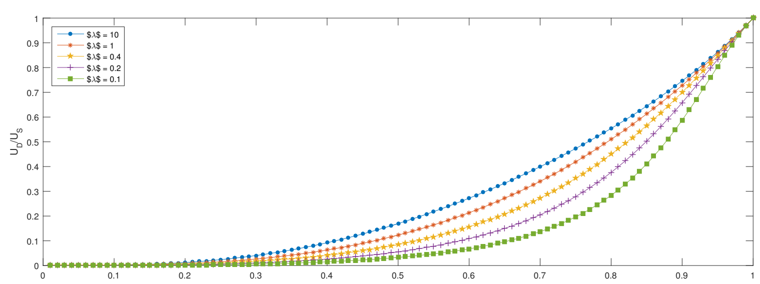
- In figure 6, surfactant concentration is the highest at the rear end of the drop with respect to the swimming direction, as expected. Surprisingly though, there is a local maxima at the anterior tip of the drop, with a local minima around the middle.




(a) Variation of drop velocity with $\chi$ for Ma=1

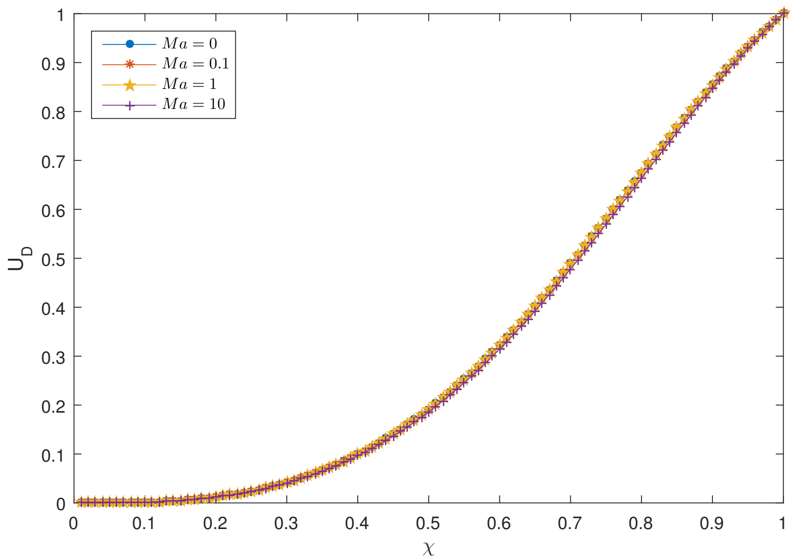



(b) Variation of swimmer velocity with $\chi$ for Ma=1

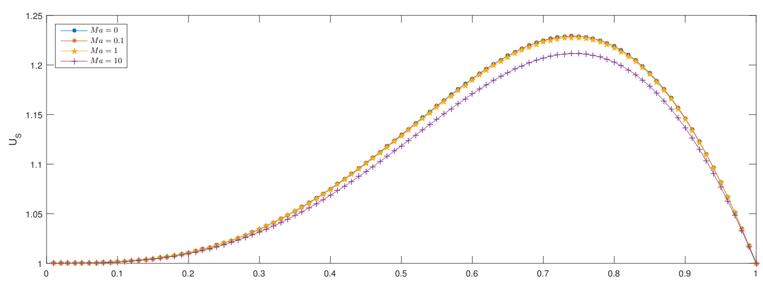



(c) Variation of the ratio of drop velocity to swimmer velocity with $\chi$ for Ma=1

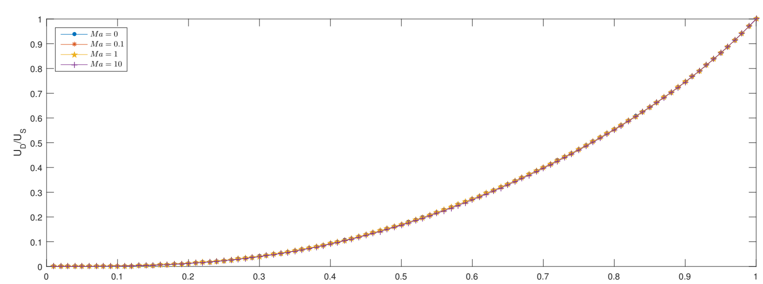
Figure 2: Drop velocity, swimmer velocity and its ratio varying as a function of size ratio $\chi$ for different viscosity ratios $\lambda$ and for Ma=1




(a) Variation of drop velocity with $\chi$ for $\lambda = 1$

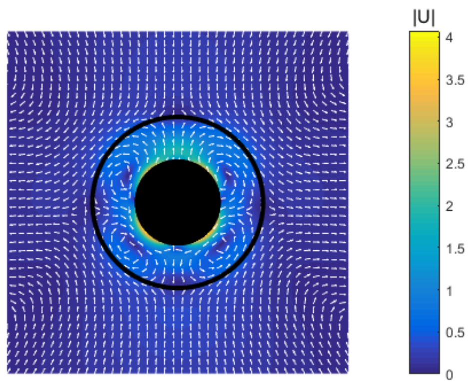



(b) Variation of swimmer velocity with $\chi$ for $\lambda = 1$

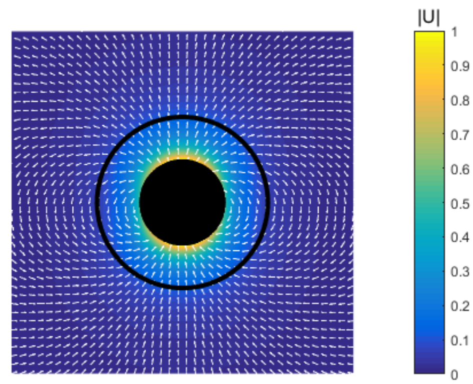



(c) Variation of the ratio of drop velocity to swimmer velocity with $\chi$ for $\lambda = 1$

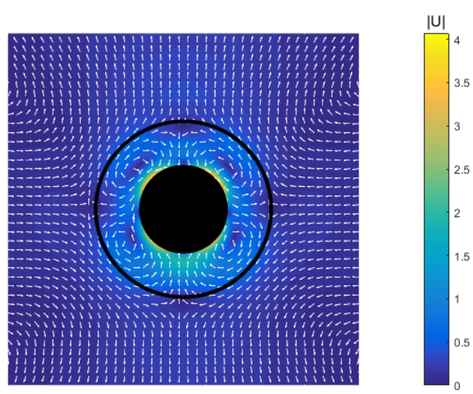
Figure 3: Drop velocity, swimmer velocity and its ratio varying as a function of size ratio $\chi$ for different Marangoni numbers $\lambda$ and for $\lambda = 1$




(a) Velocity flow field for pusher ($\beta = -5$)

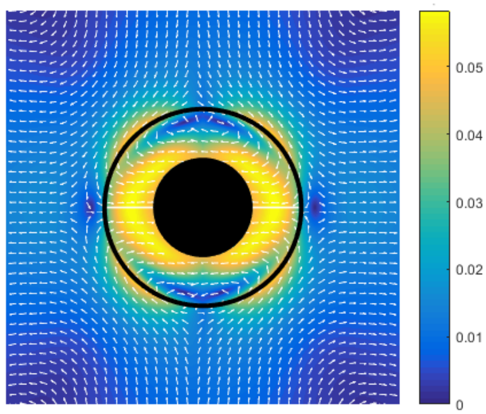



(b) Velocity flow field for neutral swimmer ($\beta = 0$)

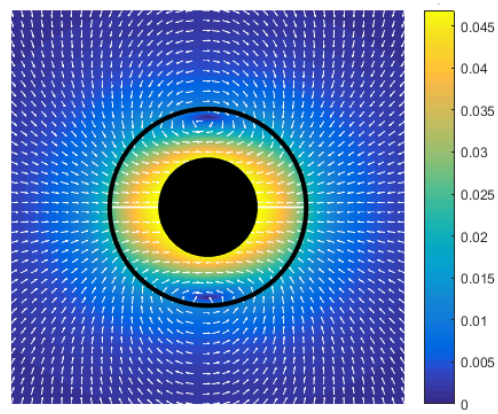



(c) Velocity flow field for puller ($\beta = 5$)

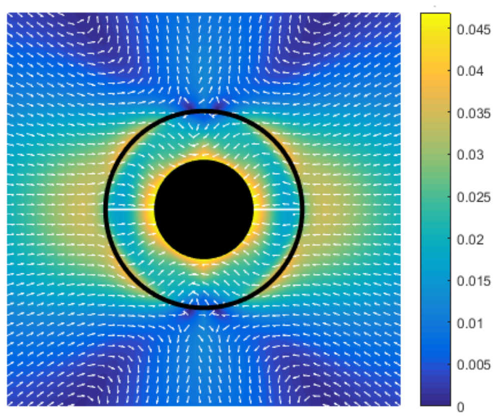
Figure 4: Velocity flow field for the O(1) solution for only tangential actuation of squirmer, with modes $B_3$ and above identically equal to zero




(a) Velocity flow field for pusher ($\beta = -5$)




(b) Velocity flow field for neutral swimmer ($\beta = 0$)




(c) Velocity flow field for puller ($\beta = 5$)

Figure 5: Velocity flow field for the $O(Pe_S)$ solution for only tangential actuation of squirmer, with modes $B_3$ and above identically equal to zero

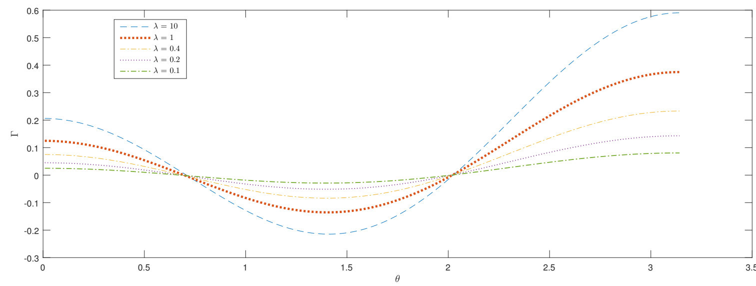
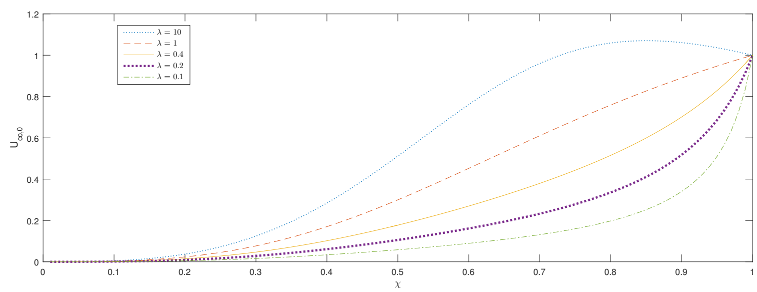
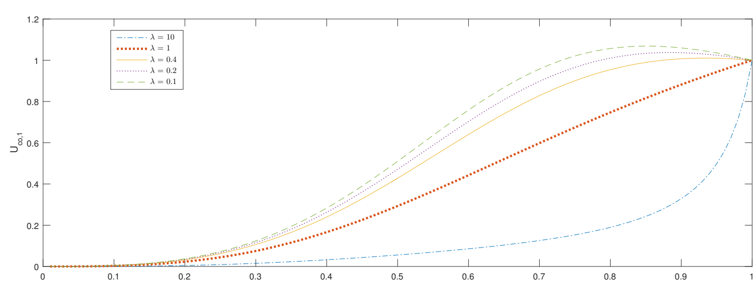



Figure 6: Concentration of surfactant as a function of $\theta$




(a) Co swimming velocity for solution till O(1)




(b) Co swimming velocity for solution till $O(Pe_s)(\beta = 5)$

Figure 7: Co-swimming velocity varying with theta at different viscosity ratios

*4.1.* ***Co-swimming modes***

- For the case when the microorganism swims only by tangential actuation, the drop is always slower than the swimmer. This is an unsteady situation in which the swimmer will eventually collide with the drop.
- However, if radial and tangential actuation are both employed, for a particular gait, the swimmer and drop can co-swim, i.e since they start in the concentric configuration, they continue to be concentric while moving together with the same speed.
- Co-swimming gait has been evaluated for speeds calculated up to O(1), $O(Pe_s)$ and $O(Pe_s^2)$ by evaluating $\alpha_{co} = A_1/B_1$ evaluated by equating drop and swimmer velocity. All $A_n$ modes higher than $A_1$ are assumed to be zero, and all $B_n$ modes higher than $B_2$ are assumed to be zero. $\beta B_2/B_1$ is known a-priori.
- When $\alpha_{co}$ is substituted back into the expression for $U_S$ or $U_D$, we get the co swimming velocity.
- $U_{co}$ obtained from velocity evaluated up to $O(Pe_s^2)$ $(U_{co,2})$ contains a non-linear expression in $B_1$.
- Since $B_1$ is a periodic function with zero time averaged value, $U_{co}$ evaluated up to $O(Pe_s)$ $(U_{co,1})$ has a time averaged value as zero, while $U_{co,2}$ has a non-zero average value
- Hence we see that scallop theorem indeed breaks due to the presence of surfactant

## 5. References


[1] JR Blake. A spherical envelope approach to ciliary propulsion. *Journal of Fluid Mechanics*, 46(1):199–208, 1971.

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[5] MJ Lighthill. On the squirming motion of nearly spherical deformable bodies through liquids at very small reynolds numbers. *Communications on Pure and Applied Mathematics*, 5(2):109–118, 1952.

[6] Shang Yik Reigh, Lailai Zhu, Francois Gallaire, and Eric Lauga. Swimming with a cage: low-reynolds-number locomotion inside a droplet. *Soft Matter*, 13:3161–3173, 2017.