# Many-Electron Effects on Donor States in a Two-Dimensional Electron Gas in a Strong Magnetic Field

Pawel Hawrylak
*Institute for Microstructural Sciences, National Research Council of Canada, Ottawa, Canada K1A 0R6*
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The many-electron effects on donor-bound intra- and inter-Landau-level excitation spectrum in a two-dimensional electron gas in a strong magnetic field are investigated. At $\nu=2$ the donor related far infrared (FIR) spectrum shows a blueshift and an additional structure below cyclotron energy. At $\nu=1$ the spin polarized ground state is unstable against the formation of donor-bound intra-Landau-level spin flip excitations (spin waves). The FIR spectrum of this new ground state is calculated and shown to be distinct from either the paramagnetic ($\nu=2$) or the ferromagnetic ($\nu=1$) ground states. These findings provide a qualitative understanding of recent experiments by Cheng *et al.*

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Many-electron effects in optical transitions from filled to empty bands in solids are one of the fundamental aspects of condensed matter physics. A unique system to study these many-electron effects is a two-dimensional electron gas in a strong magnetic field. The application of the magnetic field results in creation of filled and empty Landau bands separated by the cyclotron energy. Unfortunately, due to Kohn's theorem, in a clean translationally invariant system optical transitions from filled to empty Landau levels are unaffected by electron-electron interactions. The many-electron effects can, however, be observed by purposely introducing charged impurities directly into the electron gas and a significant experimental effort in this direction has been reported recently [1, 2]. Alternatively, one can think of photoexcited valence holes as impurities introduced in interband magneto-optics experiments and a number of interesting effects and high magnetic field anomalies due to free carriers interacting with the hole have been recently reported [3,4]. Hence it is important to understand the effect of a positively charged impurity on an interacting electron gas in a strong magnetic field. This effect has been carefully studied using far infrared (FIR) spectroscopy by Cheng *et al.* [2].

Cheng *et al.* [2] measured the effect of free carriers on the FIR absorption spectrum associated with electrons bound to donors in semiconductor quantum wells. When the concentration of electrons was approximately double that of donors in a quantum well they observed transitions with frequency larger than the cyclotron frequency involving two electrons bound to a donor ($D^-$). By further increasing the number of free carriers in the well, Cheng *et al.* [2] observed three effects: (a) the energy $E(B)$ of the original $D^-$ transition changed slope at magnetic fields $B$ corresponding to filling factors $\nu=2$ and $\nu=1$ of the lowest Landau level, (b) the transition energy $E(B)$ *increased* as a function of free carriers (blueshift), and (c) a new transition below cyclotron energy appeared. The blueshift of the transition energy is a counterintuitive result as we would expect the screening by free carriers to reduce the transition energy (redshift). The change of the slope of the transition energy could be understood in terms of the oscillation with the magnetic field in the screening of the $D^-$ transition by free carriers [5]. However, the puzzle of the blueshift of the transition energy and the origin of the new transition below the cyclotron frequency remains. We present here an answer to this puzzle.

Since the blueshift is largest for filled spin states of the lowest Landau level it is enough to restrict ourselves to the study of donor-bound excitations from a filled Landau level. For a translationally invariant system, charge (magnetorotons) and spin (spin wave, spin flip) excitations from a filled Landau level have been studied using diagrammatic techniques by several authors [6–9]. Numerical Hartree and local density approximation calculations of the effect of an impurity on electronic states in a magnetic field have also been carried out [10–12]. We present here *exact* semianalytical calculations in the strong magnetic field limit of the donor-bound inter-Landau-level and intra-Landau-level excitations. These are simply excitons in the interacting electron system. At small filling factors and small spin gap, the donor attracts a second electron with reversed spin ($D^-$) and a hole left in an otherwise spin polarized ground state. The inter-Landau-level magnetoexcitons from such donor-bound spin flip excitations are discussed. A clear picture of the physical processes involved and the origin of the blueshift of the donor related transition is given.

We consider a quantum well of area $A$ in the shape of a disk containing $N_e$ electrons and the same number of positively charged donors distributed in the barrier at a distance $d$ from the well. We shall replace the actual distribution of positive charge by a homogeneous distribution, to be specified later. Our reference Hamiltonian $H_0$ describes electrons interacting with each other and with a compensating positive background. We next transfer a *single* positively charged donor from the barrier into the center of the well, leaving behind a localized negative charge missing from the homogeneous background. Hence the potential of the "donor" $U(r)$ is actually the potential of a charge neutral dipole.

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We apply a magnetic field $B$ normal to the plane. The cyclotron energy is $\omega_c = eB/mc$, the magnetic length is $l = (1/m\omega_c)^{1/2}$, and the spin gap is $\delta E_z = g\mu B$ (we set $\hbar = 1$, $m$ is the electron effective mass, and $g$ is the electron effective $g$ factor). In the symmetrical gauge centered at the origin and in the strong magnetic field limit the single particle energies $E(N,M,\sigma) = (N+\frac{1}{2})\omega_c + g\mu S_z$ and the single particle states $|N,M\rangle$ are those of bulk Landau levels. Here $N$ is the Landau-level index, $M$ is the intra-Landau-level index, $L = M - N$ is the angular momentum, and $S_z$ is the spin. In the lowest Landau level electrons are localized on rings with radius $\approx M^{1/2}$ surrounding the donor. The number of states in the lowest Landau level $N=0$ is $n = A/2\pi l^2$. When the number of electrons $N_e = 2n$ both spin states of the lowest Landau level ($|0M\rangle$, $M < M_{\max}$) are filled and filling factor $\nu = 2$. When $N_e = n$ the system is spin polarized and $\nu = 1$. It costs energy $\omega_c$ to promote electrons into the second Landau level $N=1$ and energy $\delta E_z$ to flip its spin. The effect of the donor is to shift the energy of single-electron states $|N,M\rangle$ and remove their degeneracy. This is illustrated in Figs. 1(a) and 1(b) where the spin split single energies in the $N=0$ and $N=1$ Landau levels are shown for $\nu=2$ and $\nu=1$ spin polarized (SP) state. In Fig. 1(c) we indicate the instability of the spin polarized ground state toward spin flip excitations.

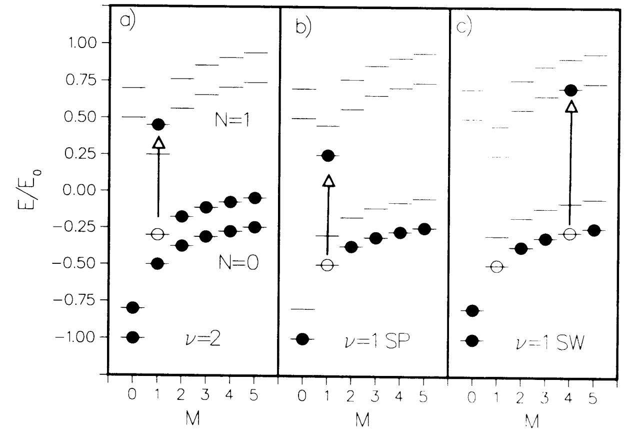


FIG. 1. A schematic picture of single particle energies of states $|N,M\rangle$ in the $N=0$ and $N=1$ Landau levels for (a) filling factor $\nu=2$, (b) filling factor $\nu=1$, and (c) spin wave (SW) state at $\nu=1$. $E_0$ is the binding energy of the donor, and for the purpose of illustration we have chosen the cyclotron gap $\omega_c = E_0$ and spin gap $\delta E_z = 0.20 E_0$. Black circles indicate occupied states and empty circles indicate holes, while arrows indicate FIR active transitions.

Let us denote by $a^\dagger_{M,\sigma}, b^\dagger_{M,\sigma}$ the creation operators for electrons in the $N=0$ and $N=1$ Landau levels. Neglecting Landau-level mixing and, for the moment, interaction with the positive charge, the Hamiltonian can be written as

$$\begin{aligned} H = & \sum_{M,\sigma} E(0,M,\sigma) a^\dagger_{M,\sigma} a_{M,\sigma} + \sum_{M,\sigma} E(1,M,\sigma) b^\dagger_{M,\sigma} b_{M,\sigma} + \sum_{M,M'\sigma} \langle 0,M'|U|0,M\rangle a^\dagger_{M',\sigma} a_{M,\sigma} + \sum_{M,M'\sigma} \langle 1,M'|U|1,M\rangle b^\dagger_{M',\sigma} b_{M,\sigma} \\ & + \frac{1}{2} \sum_{M_1 M_2 M_3 M_4 \sigma,\sigma'} \langle 0M_1 0M_2|V|0M_3 0M_4\rangle a^\dagger_{M_1,\sigma} a^\dagger_{M_2,\sigma'} a_{M_3,\sigma'} a_{M_4,\sigma} \\ & + \frac{1}{2} \sum_{M_1 M_2 M_3 M_4 \sigma,\sigma'} \langle 1M_1 1M_2|V|1M_3 1M_4\rangle b^\dagger_{M_1,\sigma} b^\dagger_{M_2,\sigma'} b_{M_3,\sigma'} b_{M_4,\sigma} \\ & + (+) \sum_{M_1 M_2 M_3 M_4 \sigma,\sigma'} \langle 1M_1 0M_2|V|0M_3 1M_4\rangle b^\dagger_{M_1,\sigma} b_{M_4,\sigma} a^\dagger_{M_2,\sigma'} a_{M_3,\sigma'} \\ & + (-) \sum_{M_1 M_2 M_3 M_4 \sigma,\sigma'} \langle 1M_1 0M_2|V|1M_3 0M_4\rangle b^\dagger_{M_1,\sigma} b_{M_3,\sigma'} a^\dagger_{M_2,\sigma'} a_{M_4,\sigma} , \end{aligned} \quad (1)$$

where $\langle V\rangle$ are two-body Coulomb matrix elements and $\langle U\rangle$ are donor matrix elements defined in Ref. [5]. Note that there are two types of inter-Landau-level scattering processes [the last two terms in Eq. (1)]: direct (+) and exchange (−).

In the strong magnetic field approximation the gap in the excitation spectrum $\omega_c$ is much larger than the donor binding energy $E_0 = \langle 00|U|00\rangle$ ($E_0 = \mathrm{Ry}\sqrt{2\pi} a_0/l$, where Ry is the effective Rydberg and $a_0$ is the effective Bohr radius). Hence the ground state wave function at $\nu=2$ is a simple Slater determinant of all filled spin down and up states $|0M\rangle$ $(M < M_{\max})$, i.e., $|G\rangle = \prod_{M<M_{\max},\sigma} a^\dagger_{M,\sigma}|0\rangle$, and it remains unchanged when the potential of the donor is switched on.

The ground state energy, of course, does change in the presence of the donor. In this approximation there is no extra charge on the donor site. The situation is identical for a spin polarized state at $\nu=1$ providing that both the cyclotron gap $\omega_c$ and spin gap $\delta E_z$ are much larger than $E_0$ [13]. However, at $B = 10$ T in GaAs quantum wells we have approximately $\omega_c = 17$ meV, $E_0 = 17$ meV, and $\delta E_z = 0.3$ meV, i.e., the spin gap is orders of magnitude smaller than the donor binding energy.

We can now simply construct excited states of the system. The excited states involve the promotion of an electron from a state $|0M_0\rangle$ in the $N=0$ Landau level to the state $|1M_1\rangle$ in the $N=1$ Landau level, i.e., the creation of an electron-hole pair $|M_0 M_1\rangle$. The change in the total angular momentum $L$ of the $N_e$ particle system carried by the electron-hole pair is $\delta L = M_1 - M_0 - 1$. Because both spin states are occupied at $\nu=2$ we may construct spin singlet (+) and spin triplet (−) excitations $|M_0M_1\rangle=\{b^\dagger_{M_1,+}a_{M_0,+}\pm b^\dagger_{M_1,-}a_{M_0,-}\}|G\rangle/\sqrt{2}$. For the $\nu=1$ spin polarized state we may construct spin triplet magnetoexcitons $|M_0M_1\rangle_t=b^\dagger_{M_1,-}a_{M_0,-}|G\rangle$, intra-Landau-level spin flip excitons [spin waves (SW)] $|M_0'M_0\rangle_{\rm SW}=a^\dagger_{M_0',+}a_{M_0,-}|G\rangle$, and inter-Landau-level spin flip (SF) excitons $|M_1M_0\rangle_{\rm SF}=b^\dagger_{M_1,+}a_{M_0,-}|G\rangle$. We diagonalize the Hamiltonian $H$ in the space of electron-hole pair states $|M_0,M_1\rangle$ for a fixed value of $\delta L=M_1-M_0-1$. The magnetoexciton energy consists of potential and kinetic energy to promote an electron from the $N=0$ to $N=1$ Landau level in the potential of the donor, the differences in Hartree-Fock energies ($\Sigma^{\rm HF}_{M_0}=-\Sigma_{M\,\text{all occupied}}\langle 0M_0 0M|V|0M_0 0M\rangle$) of initial and final states, and the mixing of magnetoexcitations via Coulomb interactions (vertex corrections). The detailed discussion of the paramagnetic ($\nu=2$) and ferromagnetic ($\nu=1$) case will be given elsewhere [14]. Here we concentrate on the spin flip excitations.

For the $\nu=1$ spin polarized state it is possible to lower the energy by flipping one electron's spin in a state $|0M_h\rangle$, and putting it on an empty spin up state $|0M_e\rangle$ on the donor, creating a spin singlet state on the donor and a hole in a state $|0M_h\rangle$ with angular momentum $\delta M=M_h$. This is illustrated in Fig. 1(c). This results in gain of potential energy in the field of the donor or the hole, but loss of exchange and Zeeman energy. In addition, such a state is mixed by Coulomb interactions with all such spin flip states with identical angular momentum $\delta M=M_h$ in the form of a spin wave state. If a spin flip electron involves the second Landau level we have a spin flip state. Hence, expanding our true states $|p,\delta M\rangle$ in terms of spin wave excitons $|M_hM_e\rangle_{\rm SW}=a^\dagger_{M_e,+}a_{M_h,-}|G\rangle$ as $|p,\delta L\rangle=\Sigma A^p_{M_e}|M_hM_e\rangle$ (for a given $\delta M=M_h-M_e$) leads to the Wannier equation for SW exciton amplitudes $A^p_{M_e}$:

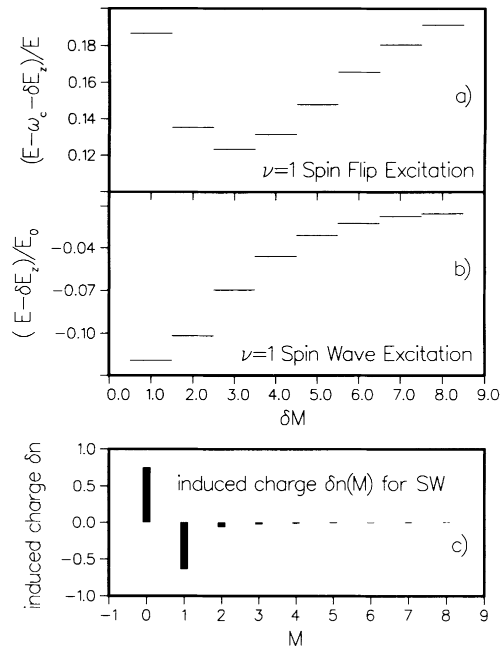


FIG. 2. (a) Energy spectrum $E(\delta M)$ as a function of angular momentum $\delta M$ of donor-bound spin flip ($N=0$ to $N=1$) excitations, (b) energy spectrum $E(\delta M)$ as a function of angular momentum $\delta M$ of donor-bound spin wave ($N=0$ to $N=0$) excitations, and (c) induced charge $\delta n(M)$ for the $\delta M=1$ lowest energy SW state.

$$\{\delta E_z+\langle M_e|U|M_e\rangle-\langle M_h|U|M_h\rangle-\Sigma^{\rm HF}_{M_h}\}A^p_{M_e}-\sum_{S_e}\langle M_e,S_h|V|M_h,S_e\rangle A^p_{S_e}=EA^p_{M_e}\,. \tag{2}$$

In Fig. 2 we show the spin flip and the spin wave excitation spectrum as a function of $\delta M$. As a good approximation we can view different $\delta M$ states as a spin singlet $D^-$ state and a hole bound to it in an otherwise filled spin polarized state. Clearly in the presence of the donor the spin wave states, which are excited states of the spin polarized ground state, are lower in energy, with $\delta M=1$ becoming a new SW ground state. That this is a collective state is illustrated in Fig. 2(c) where we show the net induced charge $\delta n(M)$ in the new $\delta M=1$ SW ground state. This indicates that only about 75% of an electron is transferred into $D^-$, and the hole, while concentrated on the $M=1$ state, is nevertheless spread on different states. The calculation of FIR active magnetoexcitons from the new SW ground states involves diagonalization of the Hamiltonian in the space of simultaneously present SW ($X_S$) and inter-Landau-level ($X_M$) excitations:

$$|X_{M_0}X_{S_0}\rangle =[\{b^\dagger_{M_0,+}a_{M_0,+}+b^\dagger_{M_0,-}a_{M_0,-}\}/\sqrt{2}]a^\dagger_{S_0,+}a_{S_0+\delta M,-}|G\rangle$$

(for a fixed $\delta M$). We expand our magnetoexciton states as $|p,\delta M\rangle=\Sigma B^p_{M_0}A^0_{S_0}|X_{M_0}X_{S_0}\rangle$ where $A^0_{S_0}$ are the amplitudes of individual SW excitons in the SW ground state [from Eq. (3)]. Defining the induced charge density in the SW state as $\delta n(S_0)=|A^0_{S_0}|^2-|A^0_{S_0+\delta M}|^2$ and using the special orthogonality relations for our biexciton states $\langle X_{S'}X_{M'}|X_MX_S\rangle=\delta_{MM'}\delta_{SS'}\{1+\delta_{MS}-\delta_{M,S+\delta M}\}$, we arrive at the effective equation for the inter-Landau-level magnetoexcitons amplitudes $B^p_M$:

$$\sum_{M'}\left[\sum_{S,S'}(A^0_S)^*\langle X_SX_M|H|X_{M'}X_{S'}\rangle(A^0_{S'})\right]B^p_{M'} =E\{[1+\delta n(M)]/2\}B^p_M\,.$$

A rather lengthy calculation of matrix elements will be described elsewhere [14], while here we give only final results.

In the dipole approximation the FIR radiation couples to the center of mass coordinate operator $R=\sum_{M,\sigma}b^\dagger_{M,\sigma}\times a_{M,\sigma}+a^\dagger_{M,\sigma}b_{M,\sigma}$. To extract the information independent of the size of the disk we sum over the number of states in the bulk of the disk and vary the disk size. In

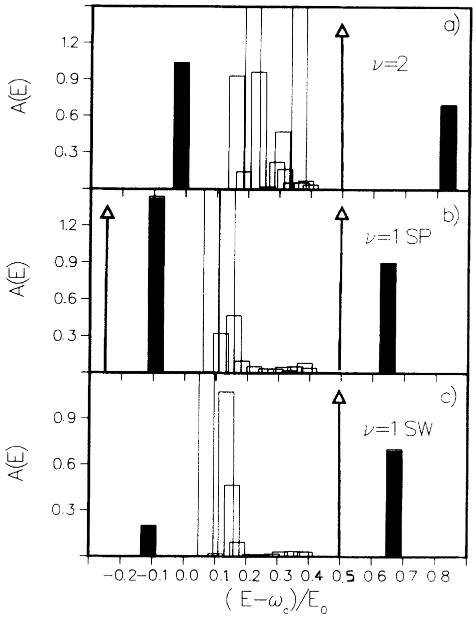


FIG. 3. Absorption spectrum of $D^-$ for (a) completely filled Landau level ($\nu=2$), (b) spin polarized state ($\nu=1$), and (c) spin wave state $\delta M=1$; arrows indicate an isolated $D^-$ ion transition.

Fig. 3 we show the effect of electron-electron interaction on the renormalized absorption spectrum $A(E)$ of the $D^-$ transition $A(E)=\Sigma_p|\Sigma_M B_M^p|^2\delta(E_p-E)$ for the $\nu=2$ paramagnetic state [Fig. 3(a)], $\nu=1$ ferromagnetic state (SP) [Fig. 3(b)], and spin wave ground state [Fig. 3(c)]. Arrows indicate $D^-$ transitions in the absence of free carriers and bars indicate transitions originating from the unrenormalized $D^-$ transitions. In addition, transitions which originate from states away from the donor site are shown, and should be interpreted as impurity shifted and broadened cyclotron resonance. At $\nu=2$, the original $D^-$ transition [5,15] at $\omega_c+\frac{1}{2}E_0$ is blueshifted by $\Delta E=0.334E_0$ and a new spin singlet transition below $\omega_c$ appears, which corresponds to spin singlet excitons from the $M=1$ state. At $\nu=1$ spin polarized state, both the original spin triplet $D^-$ transitions at $\omega_c+\frac{1}{2}E_0$ and $\omega_c-\frac{1}{4}E_0$ are blueshifted, with the higher energy transition shifted by $\Delta E=0.148E_0$. The transition from the SW ground state is slightly higher in energy, with a blueshift of $\Delta E=0.171E_0$. Because this initial state involves a hole in the $M=1$ state [see Figs. 1(c) and 2(c)], the transition originating from the $M=1$ state below $\omega_c$ is significantly suppressed in comparison to the spin polarized state. The blueshift at $\nu=2$ is higher from the blueshift at $\nu=1$ SP due to additional cross-spin exchange vertex correction in a spin singlet exciton.

In summary, we studied donor-bound inter-Landau-level excitations from the lowest filled Landau level in a strong magnetic field limit. When both spin states of the lowest Landau level are occupied ($\nu=2$) the donor related inter-Landau far infrared $D^-$ spectrum shows both a blueshift and an additional structure below the cyclotron energy. When only one spin state of the lowest Landau level ($\nu=1$) is occupied in the presence of a positively charged impurity, the spin polarized ground state is unstable against the formation of the spin singlet $D^-$ state and a hole in the spin polarized state. This collective state has a distinct FIR spectrum which should be easily identified via its temperature dependence. We hope that these calculations will help in the understanding of recent FIR [2] and magneto-optical experiments [3,4].

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*Note added.*— After submission of this manuscript, we became aware of a similar work by Dzyubenko *et al.* [16].

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