Risk-Sensitive Energy Procurement with Uncertain Wind

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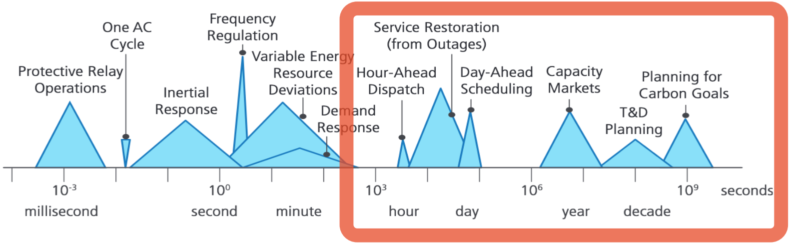
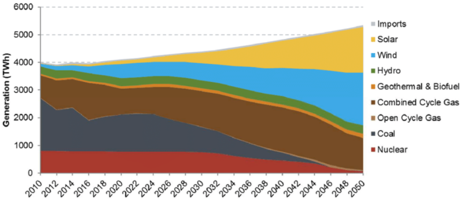
“Expecting uncertainty” in energy procurement



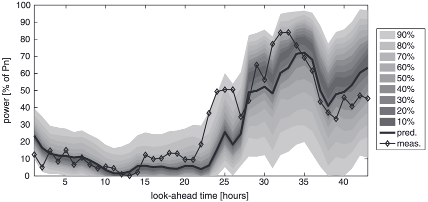
Source: Congressional Research Service

Wind energy is projected to grow rapidly.



Source: NREL 2017

Wind energy is inherently uncertain.



Source: Pinson 2009

The forward power procurement problem

- Minimize procurement costs subject to engineering constraints of grid assets and the underlying power network.

Forecasted wind supply

$$
\begin{aligned}
\underline{\mathcal{P}_{\text{det}}} : \underset{\boldsymbol{G}}{\text{minimize}} \quad & \boldsymbol{c}^{\mathsf{T}} \boldsymbol{G}, \\
\text{subject to} \quad & \mathbb{1}^{\mathsf{T}} (\boldsymbol{G} - \boldsymbol{D} + \boldsymbol{\omega}) = 0, \\
& \boldsymbol{H} (\boldsymbol{G} - \boldsymbol{D} + \boldsymbol{\omega}) \leq \boldsymbol{f}^{\mathsf{L}}, \\
& \underline{\boldsymbol{G}} \leq \boldsymbol{G} \leq \overline{\boldsymbol{G}},
\end{aligned}
$$

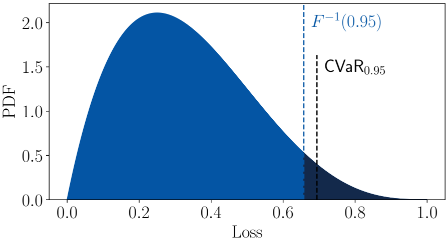
- With uncertain renewable supply, generators must respond in realtime. For simplicity, we consider affine recourse policies,

$$
\boldsymbol{G}(\boldsymbol{\omega}) := \boldsymbol{G}^0 + \boldsymbol{G}^{\omega} \boldsymbol{\omega}
$$

- Random costs and constraints require measurable functions for optimization. We utilize the conditional value at risk (CVaR).

Understanding the conditional value at risk

For continuous loss distributions, the conditional value at risk parameterized by α measures the expected tail loss in worst $(1-\alpha)$ fraction of outcomes.



$$\mathrm{CVaR}_\alpha[\chi] \coloneqq \mathbb{E}\left[\chi \mid \chi \geq F^{-1}(\alpha)\right]$$

Notice, CVaR_α is able to capture

- $\alpha = 0 \equiv$ expected value
- $\alpha \uparrow 1 \equiv$ worst-case $(\operatorname{ess\,sup})$

Additionally,

$$\mathrm{CVaR}_\alpha[\chi] \leq 0 \implies \mathbb{P}\{\chi \leq 0\} \geq 1-\alpha$$

For general loss distributions, CVaR can be written as

$$\mathrm{CVaR}_\alpha[\chi] \coloneqq \min_u \left\{ u + \frac{1}{1-\alpha} \mathbb{E}\left[\chi - u\right]^+ \right\}.$$

Rockafellar and Uryasev (2000, 2002)

The risk-sensitive forward energy procurement problem ($\mathcal{P}_{\text{risk}}$)

Incorporating the affine recourse policy for generation and CVaR for the procurement problem gives the following.

Constraint enforced by Kirchhoff's Laws.

$$
\begin{aligned}
\underline{\mathcal{P}_{\text{risk}}} : \underset{\boldsymbol{G}^0, \boldsymbol{G}^\omega}{\text{minimize}} \quad & \mathrm{CVaR}_\alpha \left[\boldsymbol{c}^\mathsf{T} (\boldsymbol{G}^0 + \boldsymbol{G}^\omega \boldsymbol{\omega}) \right], \\
\text{subject to} \quad & \mathbb{1}^\mathsf{T} (\boldsymbol{G}^0 + \boldsymbol{G}^\omega \boldsymbol{\omega} - \boldsymbol{D} + \boldsymbol{\omega}) = 0 \ \text{ a.s.}, \\
& \mathrm{CVaR}_{\beta^\mathsf{L}} \left[\boldsymbol{H} (\boldsymbol{G}^0 + \boldsymbol{G}^\omega \boldsymbol{\omega} - \boldsymbol{D} + \boldsymbol{\omega}) - \boldsymbol{f}^\mathsf{L} \right] \leq 0, \\
& \mathrm{CVaR}_{\beta^\mathsf{G}} \left[-\boldsymbol{G}^0 - \boldsymbol{G}^\omega \boldsymbol{\omega} + \underline{\boldsymbol{G}} \right] \leq 0, \\
& \mathrm{CVaR}_{\beta^\mathsf{G}} \left[\boldsymbol{G}^0 + \boldsymbol{G}^\omega \boldsymbol{\omega} - \overline{\boldsymbol{G}} \right] \leq 0.
\end{aligned}
$$

A generalization of $\mathcal{P}_{\mathbf{risk}}$ for optimization

$$\begin{aligned} \underline{\mathcal{P}^0} : \underset{\boldsymbol{x}\in\mathbb{X}}{\text{minimize}} \quad & \text{CVaR}_\alpha[f_{\boldsymbol{\omega}}(\boldsymbol{x})], \\ \text{subject to} \quad & \text{CVaR}_{\beta^i}[g^i_{\boldsymbol{\omega}}(\boldsymbol{x})] \le 0, \quad i = 1, \ldots, p. \end{aligned}$$

From the definition of CVaR we have

$$\text{CVaR}_\alpha[f_{\boldsymbol{\omega}}(\boldsymbol{x})] = \underset{u\in\mathbb{R}}{\text{minimum}} \;\; \mathbb{E}\underbrace{\left[u^f + \frac{1}{1-\alpha}\left[f_{\boldsymbol{\omega}}(\boldsymbol{x}) - u^f\right]^+\right]}_{:=\psi^f_\omega(\boldsymbol{x},u^f;\alpha)}.$$

A similar characterization for $\text{CVaR}^i_\beta[g^i_{\boldsymbol{\omega}}(\boldsymbol{x})]$ for $i = 1, \ldots p$, allows us to reformulate $\mathcal{P}^0$ as the following stochastic optimization problem

$$\begin{aligned} \underset{\boldsymbol{x},u^f,\boldsymbol{u}^g}{\text{minimize}} \quad & \mathbb{E}[\psi^f_{\boldsymbol{\omega}}(\boldsymbol{x}, u^f; \alpha)], \\ \text{subject to} \quad & \mathbb{E}[\psi^{g^i}_{\boldsymbol{\omega}}(\boldsymbol{x}, u^{g^i}; \beta^i)] \le 0, \; i = 1, \ldots, p. \end{aligned}$$

A primal-dual stochastic subgradient algorithm to solve $\mathcal{P}^0$

Avinash N. Madavan and Subhonmesh Bose (2019). *Subgradient Methods for Risk-Sensitive Optimization*. URL: `https://arxiv.org/abs/1908.01086`

Algorithm 1:

Initialization: Choose $\boldsymbol{x}_1 \in \mathbb{X}$, $\boldsymbol{z}_1 = 0$, and a positive sequence $\{\gamma_k\}$ and number of iterations K.

for $k = 1, \ldots, K$ **do**

Sample $\boldsymbol{\omega}_k \in \Omega$. Update $\boldsymbol{x}$, u^f, and $\boldsymbol{u^g}$ as

$$\boldsymbol{x}_{k+1} \leftarrow \operatorname*{argmin}_{\boldsymbol{x}\in\mathbb{X}} \left\langle \frac{\nabla f_{\boldsymbol{\omega}_k}(\boldsymbol{x}_k)}{1-\alpha}\,\mathbb{I}_{\{f_{\boldsymbol{\omega}_k}(\boldsymbol{x}_k)\geq u_k^f\}} + \sum_{i=1}^m \frac{z_k^i \nabla g^i_{\boldsymbol{\omega}_k}(\boldsymbol{x}_k)}{1-\beta_i}\,\mathbb{I}_{\{g^i_{\boldsymbol{\omega}_k}(\boldsymbol{x}_k)\geq u_k^f\}}, \boldsymbol{x}-\boldsymbol{x}_k \right\rangle + \frac{1}{2\gamma_k}\left\|\boldsymbol{x}-\boldsymbol{x}_k\right\|^2,$$

$$u^f_{k+1} \leftarrow \operatorname*{argmin}_{u^f\in\mathbb{U}^f} \left\langle 1 - \frac{1}{1-\alpha}\,\mathbb{I}_{\{f_{\boldsymbol{\omega}_k}(\boldsymbol{x}_k)\geq u_k^f\}}, u^f - u^f_k \right\rangle + \frac{1}{2\gamma_k}\left\|u^f - u^f_k\right\|^2,$$

$$\boldsymbol{u}^{\boldsymbol{g}}_{k+1} \leftarrow \operatorname*{argmin}_{\boldsymbol{u^g}\in\mathbb{U}^{\boldsymbol{g}}} \left\langle \mathbb{1} - \operatorname{diag}(\mathbb{1}-\boldsymbol{\beta})^{-1}\,\mathbb{I}_{\{\boldsymbol{g}_{\boldsymbol{\omega}_k}(\boldsymbol{x}_k)\geq \boldsymbol{u}^{\boldsymbol{g}}_k\}}, \boldsymbol{u^g} - \boldsymbol{u}^{\boldsymbol{g}}_k \right\rangle + \frac{1}{2\gamma_k}\left\|\boldsymbol{u^g} - \boldsymbol{u}^{\boldsymbol{g}}_k\right\|^2.$$

Sample $\boldsymbol{\omega}_{k+1/2} \in \Omega$. Update $\boldsymbol{z}$ as

$$\boldsymbol{z}_{k+1} \leftarrow \operatorname*{argmax}_{\boldsymbol{z}\in\mathbb{R}^m_+} \left\langle \boldsymbol{u^g} + \operatorname{diag}(\mathbb{1}-\boldsymbol{\beta})^{-1}[\boldsymbol{g}_{\boldsymbol{\omega}_{k+1/2}}(\boldsymbol{x}_{k+1}) - \boldsymbol{u^g}]^+, \boldsymbol{z}-\boldsymbol{z}_k \right\rangle - \frac{1}{2\gamma_k}\left\|\boldsymbol{z}-\boldsymbol{z}_k\right\|^2.$$

Implementation available at `https://github.com/amadavan/stuka`

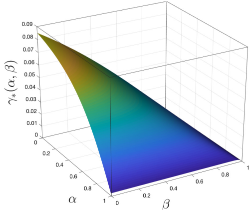
Theorem. Convergence guarantee of the algorithm

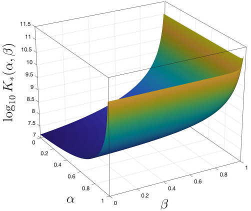
Given $\varepsilon > 0$ and let p_*^0 denote the optimal value of $\mathcal{P}^0$, and K_* and $\gamma_k = \gamma_* / \sqrt{K_*}$ for $k = 1, \ldots, K_*$, where K_* and γ_* satisfy the following with $y = 1 + \frac{P_2}{P_1 P_3}$,

$$\gamma_*^2 = \frac{2P_3^{-1}}{2 + y + \sqrt{y^2 + 8y}}, \qquad K_* = \frac{(P_1 + P_2\gamma_*^2)^2}{16\gamma_*^2(1 - P_3\gamma_*^2)^2\varepsilon^2}.$$

Letting $\bar{\boldsymbol{x}}_{K_*} = \frac{1}{K_*}\sum_{k=1}^{K_*} \boldsymbol{x}_k$, the iterates generated by the algorithm on $\mathcal{P}^0$ with parameters α and β satisfy

$$\mathbb{E}[\mathrm{CVaR}_\alpha(f_\omega(\bar{\boldsymbol{x}}_{K_*+1}))] - p_*^0 \le \varepsilon,$$
$$\mathbb{E}[\mathrm{CVaR}_{\beta^i}(g_\omega^i(\bar{\boldsymbol{x}}_{K_*+1}))] \le \varepsilon.$$



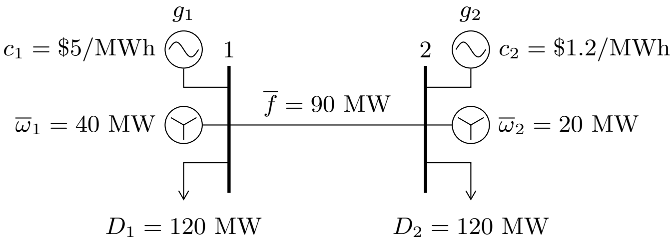


For $\alpha \approx 1$, the optimal iteration count grows as

$$K_*(\alpha, \beta) \sim \frac{1}{\varepsilon^2(1-\alpha)^2}$$

An illustrative two-bus example

Wind data, (ω_1, ω_2), was taken as samples from two nearby power plants in NREL's synthetic dataset for every 5 minutes in 2008-2011.



Aversion	α	β^{L}	β^{G}	Highest Dispatch Cost (\$)	Worst-Case Constraint Violation Line (MW)	Worst-Case Constraint Violation Generation (MW)	Iterations $(\times 10^9)$	Runtime (s)
None	0	0	0	400	25	24	2.9	3767
Line capacity	0	0.6	0.2	402	0	50	9.2	21400
Dispatch cost	0.6	0.2	0.2	352	13	0	4.6	9660

Conclusions and future work

- Provided a convex CVaR-sensitive energy procurement formulation that captures aversion towards constraint violation and high costs.[1]
- Proposed a rate-optimal primal-dual stochastic subgradient method with theoretical convergence guarantees.
- Observed high iteration complexity and has high runtime for even a small problem with low risk parameters.
- Wish to take advantage of smoothness or apply an iterative sample average approximation scheme to speed up the algorithm.

Questions?

[1] Avinash N. Madavan, Ye Guo, et al. (2019). "Risk-Sensitive Security-Constrained Economic Dispatch via Critical Region Exploration". In: *Power and Energy Society General Meeting*