

Bode Plot -

Ex-2.) Draw the Bode Plot of the unity feedback system with forward gain as:

$$G(s) = \frac{40}{(s+2)(s+5)}$$

Also determine the gain margin, phase margin and comment on the stability of the system.

Solution:

1.) Convert transfer function to constant form

$$G(s)H(s) = \frac{40}{(s+2)(s+5)} = \frac{4}{(1+0.5s)(1+0.2s)}$$

2.) Convert to sinusoidal form -

$$G(j\omega)H(j\omega) = \frac{4}{(1+0.5j\omega)(1+0.2j\omega)}$$

3.) Different part of the plot

(i) Constant term $K=4$

(ii) Type of system: 0, this means initial slope is 0 dB/dec, and initial magnitude is $20 \log K = 20 \log 4 = 12.04 \text{ dB}$

(iii) Corner frequencies: $\omega_{c1} = 1/0.5 = 2 \text{ rad/s}$,
 $\omega_{c2} = 5 \text{ rad/s}$

4.) Draw reference slopes and axes.

5.) Draw magnitude plot.

6.) Draw phase plot, $\phi = \angle G(j\omega)H(j\omega) = -\tan^{-1}(0.5\omega) - \tan^{-1}(0.2\omega)$
Using this expression, tabulate some values of ϕ w.r.t ω

ω	0.1	1	2	5	10	20	∞
ϕ	-40.08°	-37.87°	-66.8°	-113.2°	-142.1°	-160.25°	-180°

Step 7) Gain cross-over and phase cross-over ω and corresponding values of phase and magnitude

$$\omega_{gc} = 5.18 \text{ rad/s}, \phi(\omega_{gc}) = 115^\circ$$

$$\omega_{pc} = \infty \text{ rad/s}, M(\omega_{pc}) = -\infty \text{ dB}$$

Step 8) Stability margins

$$\text{Gain Margin, } GM = 0 - M(\omega_{pc}) = 0 - (-\infty) = +\infty \text{ dB}$$

$$\text{Phase Margin, } PM = 180^\circ - \phi(\omega_{gc}) = 180^\circ - 115^\circ = 65^\circ$$

The phase plot curve is asymptotic to -180° axis at high ω and the gain at high crossover is $-\infty$ such that the GM is $+\infty$. Since, both PM and GM are +ve, therefore, the system is stable. Further the GM is infinite, hence, the system inherently stable.

