Draw the polar plot

Q1) $G(s) = \frac{1}{(s)(s+1)(s+2)}$

$s = j\omega \qquad G(j\omega) = \frac{1}{(j\omega)(j\omega+1)(j\omega+2)}$

$M = |G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2}\ \sqrt{\omega^2+4}}$

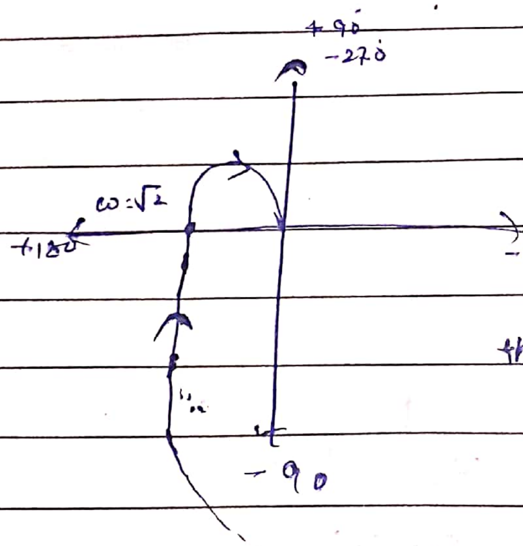
$\phi = -90 - \underset{0}{\tan^{-1}(\omega)} - \underset{0}{\tan^{-1}(\omega/2)}$

$\omega = 0$

$M = \infty, \ \phi = -90'$

$\omega = \infty$ → type 1, order 3 system

$M = 0, \ \phi = -270'$



this axis is touching -ve real axis at ω = 180° in Phase by that well get $\omega = \sqrt{2}$.

Q2) $G(s) = \frac{1}{1+sT}$ $\qquad M = |G(s)| = \frac{1}{1+sT}$

$= \frac{1}{\sqrt{1+\omega^2T^2}}$

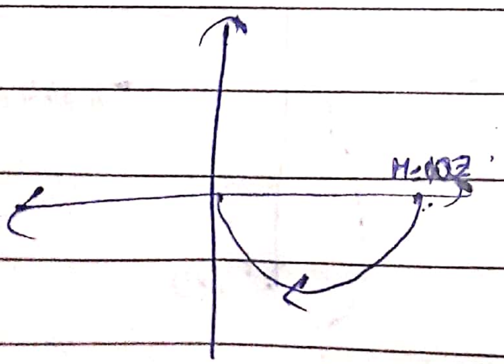
$\phi = -\tan^{-1}\left(\frac{\omega T}{1}\right)$

Consider $\omega = 0$

$M = 1, \ \phi = 0^\circ$

$1\angle 0^\circ$

$\omega = \infty \qquad , 0\angle -90^\circ$



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Type 0, Order 2

Q3)

$$G(S) = \frac{1}{(1+ST_1)(1+ST_2)}$$

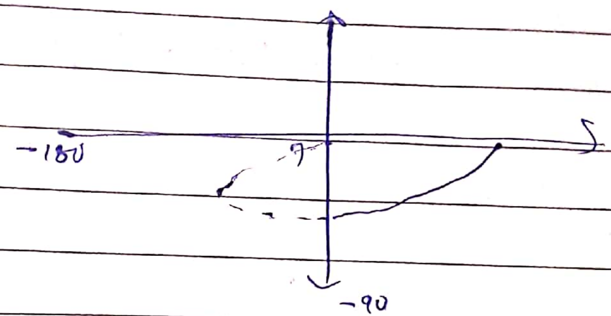
$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$M = \frac{1}{\sqrt{1+\omega^2 T_1^2}\ \sqrt{1+\omega^2 T_2^2}}$$

$$\phi = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

$\omega = 0 \rightarrow 1\angle 0^\circ$

$\omega = \infty \Rightarrow 0\angle -180^\circ$



type 1, order 2

Q4)

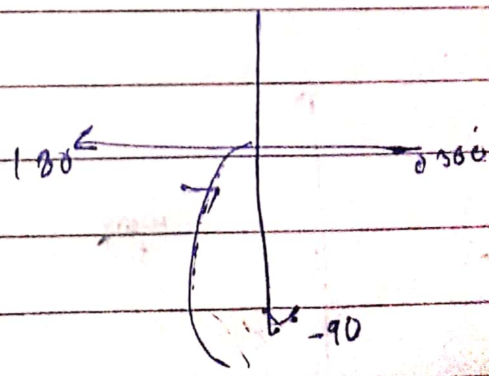
$$G(S) = \frac{1}{(S)(1+ST)}$$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)} \quad \frac{1}{\omega\sqrt{1+\omega^2 T^2}}$$

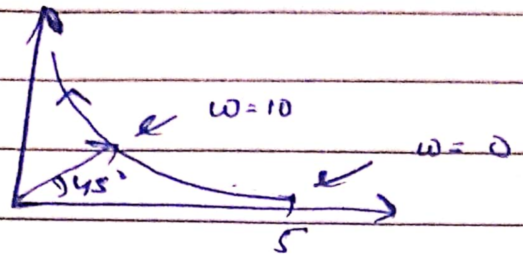
$$\phi = -90^\circ - \tan^{-1}(\omega T/1)$$

$\omega = 0 \Rightarrow \infty\angle -90^\circ$

$\omega = \infty \Rightarrow 0\angle 180^\circ$



Q5)



$\omega = 0 \quad 5\angle 0$

$\omega = \infty \quad \infty \angle 90°$

find out $G(s)$

$K = G(j\omega)\big|_{\omega=0} = G(j0) = 5$

$G(s) = K(1 + sT)$

no we need to find out T

$5(1 + j10T) = 45°$

$\tan^{-1}\left(\frac{10T}{1}\right) = 45°$

$\boxed{T = \frac{1}{10}}$

$G(s) = 5\left(1 + \frac{s}{10}\right)$

$G(s) = 5(1 + 0.1s)$

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2018UI3093 Gaurav Gupta

ICE-2

Ques-1: Determine the analytic func^n whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$

Sol^n:) $e^{2x}(x\cos 2y - y\sin 2y)$

$\delta u/\delta x = e^{2x}((\cos 2y) - 0) + 2e^{2x}(x\cos 2y - y\sin 2y)$

$= e^{x}\cos(2y)$

$= e^{2x}(\cos 2y + 2x\cos 2y - 2y\sin 2y) = \phi_1(x,y)$

$\phi_1(z,0) = e^{-2z}(0+0+0) = 0$

By. Milne's thomson method

$F(Z) = \int \phi_1(z,0)\,dz, -i\int \phi_2(z_1,0)\,dz + c$

$= \int e^{2z}(1+2z)\,dz - 0 + c$

$= (1+2z)\left(\frac{e^{2z}}{2}\right) - 2\left(\frac{e^{2z}}{4}\right) + c$

$$= \frac{e^{2z}}{2}(1+2z-1)+c$$

$$\boxed{f(z) = e^{2z} + c}$$

Que 3 02 Find the regular function whose imaginary part is $\cos x \cosh y$.

$$\Rightarrow \quad \cos x \cosh y \Rightarrow \cos x\left(\frac{e^{y}+e^{-y}}{2}\right)$$

$$\frac{\delta u}{\delta y} = \delta(\cos x \cosh y)$$

$$= \cos x\left(\frac{e^{y}-e^{-y}}{2}\right) \rightarrow 0$$

$$\frac{\delta u}{\delta x} = -\sin x\left(\frac{e^{y}+e^{-y}}{2}\right) \quad x = z,\ y = 0 \ ; \ -\sin z$$

$$\frac{dw}{dz} = \frac{\delta v}{\delta y} + i\frac{\delta v}{\delta x}$$

$$\frac{dw}{dz} = i\sin z$$

$$\int dw = \int -i\sin z\,dz$$

$$\boxed{w = i\cos z}$$

Que: 03 $f(z) = u + iv$ if

(i) $u - v = e^x(\cos y - \sin y)$

$u = e^x \cos y$

$v = e^x \sin y$

$f(z) = u + iv$

$= e^x \cos y + i e^x \sin y$

$= e^x [\cos y + i \sin y)$

by Euler's identity $\rightarrow$ $f(z) = e^x e^{(iy)}$

$f(z) = e^{(x+iy)}$

$\boxed{f(z) = e^z}$

(ii) $u + v = \dfrac{x}{x^2 + y^2}$, when $f(1) = 1$

$f(z) = u + iv$ — (1)

$i f(z) = iu - v$ — (ii)

(1) + (ii) gives $(1+i) f(z) = (u - v) + i(v + u)$

$f(z) = u + iv$

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v is given as x/x^2+y^2

$$\frac{dv}{dx} = \frac{(1.(x^2+y^2) - x(2x))}{(x^2+y^2)^2}$$

$$\left.\begin{matrix} x \to z \\ y \to 0 \end{matrix}\right\} \frac{\delta v}{\delta x} = \frac{(z^2 - 2z^2)}{z^4} = \frac{-1}{z^2}$$

$$\frac{\delta v}{\delta y} = -2yx/x^2+y^2 = 0$$

$x \to z$
$y \to 0$

$$\frac{dw}{dz} = 0 + (-1/z^2)$$

$$\int \delta\omega = -i \int \frac{1}{z^2} dz$$

$$\omega = i/z$$

$$f(z) = \frac{1}{z} \quad \delta \quad (1+i) f(z) = f(z)$$

$$\boxed{f(z) = \left(\left(\frac{i}{i+1}\right) \frac{1}{z}\right)}$$

Que: 04

$\omega = \frac{1}{z}$, $z = \frac{1}{\omega}$

$|z - 2i| = 2$

$|x + (y-2)i| = 2$

$x^2 + (y-2)^2 = 4$

$x^2 + y^2 - 2y = 0$

$z = \frac{1}{u+iv}$

$= \frac{u - iv}{u^2+v^2}$

$x + iy = \frac{u - iv}{u^2+v^2}$

$x = \frac{u}{u^2+v^2}$, $y = \frac{-v}{u^2+v^2}$

$$\frac{u^2}{(u^2+v^2)^2} + \frac{v^2}{(u^2+v^2)^2} - \frac{2v}{u^2+v^2} = 0$$

$$\frac{u^2 + v^2 - 2v(u^2+v^2)}{(u^2+v^2)^2} = 0$$

$\cancel{u^2 + v^2 - 2vu^2 - 2v^3 = 0}$

$(u^2+v^2) + 2v(u^2+v^2) = 0$

$(u^2+v^2)(1+2v) = 0$

$1 + 2v = 0$

$\boxed{v = -\frac{1}{2}}$

Que : 05 Show that the transformation $\omega = \frac{2z+3}{z-4}$ maps the circle $x^2+y^2-4x=0$ onto the straight line $4u+3=0$

$\Rightarrow$

$$\omega = \frac{2z+3}{z-4} \qquad 4u+3=0$$

→ it is a bilinear transformation.

Inverse transformation

$$z = \frac{4\omega+3}{\omega-2}$$

The eq^n of circle → $x^2+y^2-4x=0$

i.e. $|z|^2 - 4\,\mathrm{Re}\,z = 0$

$$z\bar{z} - 2(z+\bar{z}) = 0$$

$\therefore \omega = u+iv$

$$\frac{4\omega+3}{\omega-2} \cdot \frac{4\bar{\omega}+3}{\bar{\omega}-2} - 2\left(\frac{4\omega+3}{\omega-2} + \frac{4\bar{\omega}+3}{\bar{\omega}-2}\right) = 0$$

$$(4\omega+3)\cdot(\omega 4\bar{\omega}+3) - 2\{(4\omega+3)(\bar{\omega}-2)+(4\bar{\omega}+3)(\omega-2)\} = 0$$

i.e. $2(\omega+\bar{\omega})+3=0$

i.e. $4u+3=0$

H.P.

Que: 06 find the bilinear transformation which maps the points $z=1, i, -1$ into the points $\omega = i, 0, -i$

Sol^n

$$\omega = \frac{az+b}{cz+d} \quad \text{— (i)}$$

put $z=1$ & $\omega = i$

$$i = \frac{a+b}{c+d}$$

$$\boxed{ic+id = a+b \quad \text{— (ii)}}$$

put $z=i$ & $\omega=0$

$$0 = \frac{ai+b}{c(i)+d} \quad = \quad ai+b=0$$

$$\boxed{b=-ai} \quad \text{(iii)}$$

put $z = -1$ & $\omega = -i$

$$-i = \frac{a(-1)+b}{c(-1)+d}$$

$$ic - id = -a + b \quad \text{— iv}$$

add eq (2) & (IV)

$$ic + id = a + b$$
$$ic - id = -a + b$$

we, get $2ic = 2b$

$$c = \frac{b}{i} = \frac{-ai}{i} = -a \rightarrow (\text{v})$$

$$\therefore \quad \omega = \frac{az - ai}{-az - ai}$$

$$\therefore \quad \omega = \frac{-a(-z+i)}{-a(z+i)}$$

$$\Rightarrow \quad \frac{i - z}{z + i} \text{ is the bilinear transformation.}$$

Que: 07 $\int \frac{3z^2+z}{z^2-1}$ $|z-1|=1$

$z^2-1=0$

$z=+-1$

$|z-1|=1$

$z=1$

$r=1$ includes the point $z=1$

$$\oint \frac{3z^2+z}{z^2-1}\,dz = \int \frac{\left(\frac{3z^2+z}{z+1}\right)}{z-1}\,dz$$

$$= 2\pi i \left(\frac{3z^2+z}{z+1}\right)_{z=}$$

$$= 4\pi i$$

Que : 08

$$\frac{z^2-2z}{(z+1)^2(z^2+4)}$$

$z = -1, -1$

$z = \pm 2i$

Residue at -1

$$\frac{d}{dz}\left[(z-(-1))^2 f(z)\right]$$

$$\frac{(z+1)^2 \cdot (z^2-2z)}{(z+1)^2(z^2+4)}$$

$$\frac{d}{dz}\frac{(z^2-2z)}{(z^2+4)}$$

$$= \cancel{2z^3}+8z-2z^2-8-\cancel{2z^3}+4z^2$$

$$= \frac{2z^2+8z-8}{(z^2+4)^2}$$

$$= \frac{2-8-8}{25} = \frac{-14}{25}$$

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Residue at $z = 2i$

$$\lim_{z=2i} (z-2i) f(z) = \frac{(z-2i)}{(z+1)^2} \cdot \frac{z^2-2z}{(z-2i)(z+2i)}$$

$$= \frac{z^2-2z}{(z^2+1+2z)(z+2i)}$$

$$= \frac{\cancel{-}4(1+i)}{\cancel{-}(3i+4)4} = \frac{1+i}{3i+4}$$

$$\boxed{= \frac{7+i}{25}}$$

Ans

Residue at $z = -2i$

$$= \frac{7-i}{25}$$

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