

## Second Order Systems

- The second order equation can be written as:

Second Order Systems :- (Quadratic Lag).

→ Systems which can be modelled with the help of a second order differential equation.

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

$$\frac{a_2}{a_0} \frac{d^2 y}{dt^2} + \frac{a_1}{a_0} \frac{dy}{dt} + y = \frac{b}{a_0} f(t)$$

$$\frac{a_2}{a_0} = T_p^2 ; \quad \frac{a_1}{a_0} = 2\gamma T_p ; \quad \frac{b}{a_0} = K_p$$

- Here  $t_p$  ==> Natural Period of oscillations
- $\gamma$  ==> Damping Coefficient
- $k_p$  ==> Process Gain

## Converting into Laplace

- Taking Laplace Transform , we will get

Taking Laplace Transform....

$$T_p^2 [s^2 Y(s) - s Y(0)] + 2\gamma T_p [s Y(s)] + Y(s) = K_p F(s)$$

$$\Rightarrow T_p^2 s^2 Y(s) + 2\gamma T_p s Y(s) + Y(s) = K_p F(s)$$

$$\Rightarrow \boxed{\frac{Y(s)}{F(s)} = \frac{K_p}{T_p^2 s^2 + 2\gamma T_p s + 1}}$$

J. Fn. for 2<sup>nd</sup> order function.

Figure 1: laplace\_transform

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- Slides in the today's folder only

## How to make 2nd order using the First Order

Check in the NPTEL Examples Side for each of these Examples

- By cascading the 1st order in series the to get the second order system
- If propotional integration is applied to the first order system it will become a second order system
- Inherent Systems
- No need to derive these equation , just know examples from high level

## Examples

Check in the NPTEL Examples Side for each of these Examples

- Spring Mass Damper
- Manometer
- CSTRs in series (NPTEL Slides)
  - Also go in the calculation for this system!
  - Try to cascade and get the calculation

## Damping

- Discussed Damping – Overdamped , Critically Damped , Underdamped

The screenshot shows a presentation slide with the following content:

Step Response of a 2<sup>nd</sup> order system

- ①  $\zeta > 1 \Rightarrow$  (Exponential Nature)  $\Rightarrow$  Overdamped.  
 $\rightarrow$  Roots (poles) are real but unequal.
- ②  $\zeta = 1 \Rightarrow$  (exponential + time)  $\Rightarrow$  Critically damped.  
 $\rightarrow$  Roots (poles) are real and equal.
- ③  $\zeta < 1 \Rightarrow$  (sinusoid)  $\Rightarrow$  Underdamped.  
 $\rightarrow$  Roots are complex conjugate.

Overdamped System :- ( $\zeta > 1$ )

## References

- NPTEL Slides (Also in the Folder)
- Unacademy Slides (Also in the Folder)