2018UIC3087
Manan Madan.

Control system - II :

change in x_1
↓

- $\dot{x}_1(t) = a_{11}\, x_1(t) + a_{12}\, x_2(t) + a_{1n}\, x_n(t) + b_1 r(t)$
- $\dot{x}_2(t) = a_{21}\, x_1(t) + a_{22}\, x_2(t) + \cdots a_{2n} x_n(t) + b_2 r(t).$

$\vdots$

$\dot{x}_n(t) = \cdots\cdots$

(rate of change of internal variable dependent upon state variable and input

- $y(t) = c_1 x_1(t) + c_2 x_2(t) \cdots c_n x_n(t) + d r(t)$

$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & & a_{nn} \end{bmatrix} \quad \begin{bmatrix} b_1 \\ b_2 \\ \vdots \end{bmatrix}$

$\dot{x}(t) = A x(t) + b\, r(t)$

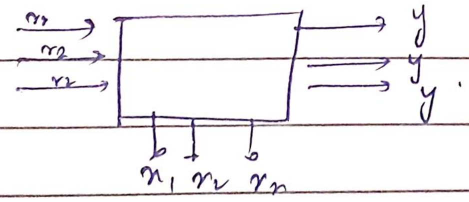
↳ differential of the state vector

- $y(t) = C x(t) + (d\, r(t))$ output eqn

- $A \to n \times n$, $b \to n \times 1$, $C = 1 \times n$ d = constant scalar .

MIMO State

- ip {



$r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_r \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_2 \\ \vdots \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

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- How will the state change according to MIMO system ?

- ~~Ex~~ Now $b_1 r_1 + b_2 r_2 + b_3 r_3 \ldots$

b will be rectangular , c will also be rectangular matrix.....

$\dot{X}(t) = A X(t) + B r(t)$ state eq^n.
$y(t) = C X(t) + D r(t)$ output eq^n.

$A \rightarrow n \times n$ $B \rightarrow n \times p$ $C \rightarrow q \times n$ $D \rightarrow q \times p$
constant Matrices.

H.W
Q Write the equation and upload

Formula for state eqn and output eq^n.

- $\dot{x}_1(t) = a_{11} x_1(t) + a_{12} x_2(t) + \ldots a_{1n} x_n(t) + b_{11} r_1(t) + b_{12} r_2(t) \ldots b_{1n} r_n(t)$

$\dot{x}_2(t) = a_{21} x_1(t) + a_{22} x_2(t) \ldots + a_{2n} x_n(t) + b_{21} r_1(t) + b_{22} r_2(t) \ldots b_{2n} r_n(t)$

$\dot{x}_3(t) = a_{31} x_1(t) + a_{32} x_2(t) \ldots + a_{3n} x_n(t) + b_{31} r_1(t) + b_{32} r_2(t) \ldots b_{3n} r_n(t)$

$\vdots$

$\dot{x}_n(t) = a_{n1} x_1(t) + a_{n2} x_2(t) \ldots + a_{nn} x_n(t) + b_{n1} r_1(t) + b_{n2} r_2(t) \ldots b_{nn}(r_n(t))$

- $y(t) = c_1 x_1(t) + c_2 x_2(t) \ldots c_n x_n(t) + D_1 r_1(t) + D_2 r_2(t) \ldots D_n r_n(t).$

$\dot{X}(t) = A X(t) + B r(t)$
$y(t) = C X(t) + D r(t)$

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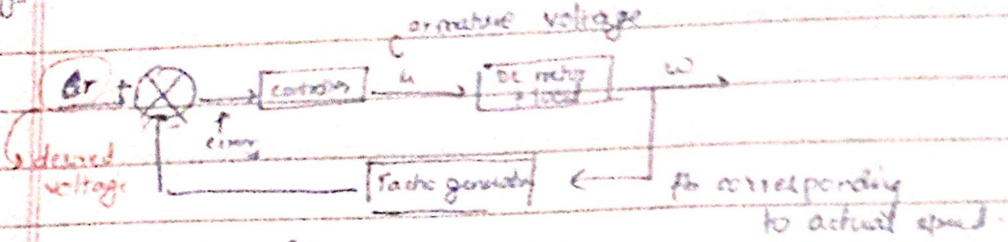
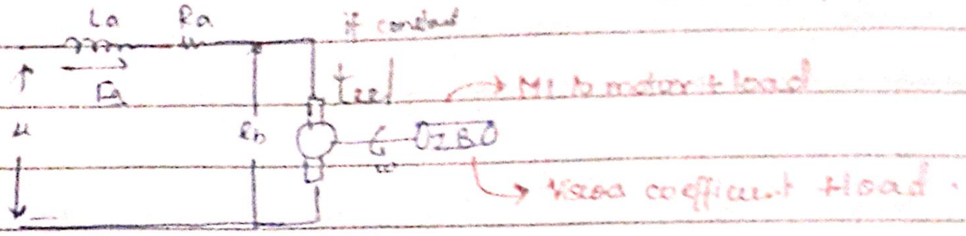
$A \rightarrow n \times n$ $B \rightarrow n \times p$ $C \rightarrow q \times n$ $D \rightarrow q \times p$

Eg. armature controlled DC motor





Voltage eqⁿ:

back emf

$$u(t) = L_a \frac{di_a(t)}{dt} + R_a i_a(t) + e_b(t)$$

τ (input torque)

$$T_m(t) = J\frac{d\omega(t)}{dt} + B\,\omega(t)$$

flux is fixed so:

$$T_m(t) = K_T I_a(t) \checkmark$$

$$e_b(t) = K_b\,\omega(t) \checkmark$$

Further solving:

$$\frac{di_a(t)}{dt} = -\frac{R_a}{L_a} i_a(t) + \frac{e_b(t)}{L_a} + \frac{1}{L_a} u(t)$$

$$\frac{di_a(t)}{dt} = -\frac{R_a}{L_a} i_a(t) - \frac{K_b}{L_a}\omega(t) + \frac{1}{L_a} u(t)$$

$$\frac{d\omega(t)}{dt} = \frac{K_T}{J} i_a(t) - \frac{B}{J}\omega(t).$$

$x_1(t) = \omega(t)$ $x_2(t) = i_a(t)$ state variables

$y(t) = \omega(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -B/J & K_T/J \\ -K_b/L_a & -R_a/L_A \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_a \end{bmatrix} u(t)$$

$y(t) = x(t)$.

$\dot{x}(t) = Ax(t) + bu(t)$

$y(t) = C\,x(t)$ most of the system's will have $D \to 0$

as there is no direct dependence of output on inputs

Manan