Alternative Methods for checking controllability

1) Eigen Vector Method.

ALWAYS USING EIGEN VECTORS!!

Note:
Basically in this method we are checking if each unique dimension / state is controllable or not

$$\dot{x} = Ax + Bu$$

so here we will find a transformation matrix "P" so that it converts it into unique dimensions like.

$$P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & \lambda_n \end{bmatrix}$$

so the whole equation is modified as

$$\dot{z} = P^{-1}APx + \underbrace{P^{-1}Bu}$$

Now if an entire row of the matrix $P^{-1}B$ is zero then you would be able to completely control thee system

→ Hence if the system has unique eigen vectors and the $P^{-1}B$ matrix does not has any row zero, then the system is controllable.

But what happens if the system does not have distinct eigen vectors !!
so what you do is that

1) Pick up chunks of matrices when the eigen vector values are not same in the $P^{-1}AP$ matrix.

$$\begin{bmatrix} \boxed{\begin{matrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 1 \\ 0 & 0 & \lambda_1 \end{matrix}} & \\ & \boxed{\begin{matrix} \lambda_4 & 1 \\ 0 & \lambda_4 \end{matrix}} \end{bmatrix}$$

JORDAN BLOCKS !!
You consider them as one distinct value and then treat the controllability using this

controllability assumption

so now we assume that

$$\underline{S^{-1}AS} = \underset{\parallel}{J} \quad \text{(above matrix)}$$

then

$$\dot{z} = \underline{S^{-1}A}Sz + \underline{S}^{-1}Bu$$

and the conditions come out to be that

1) no two same jordan blocks should be associated with the same eigen vector (obv :))

2) the elements which correspond to the last row in $S^{-1}B$ to a jordan block should not be all zero

3) elements of each row in $S^{-1}B$ corresponding to each distinct Ĵordan block should not be zero.