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ICE - Section 2

Ivariant to Transformation

(i) $|sI-\bar{A}| = |sI - P^{-1}AP| = |sP^{-1}P - P^{-1}AP|$

$= |P^{-1}(sI-A)P| = |P^{-1}|\,|sI-A|\,|P| = |sI-A|$

(ii) $\bar{G}(s) = \bar{C}(sI-\bar{A})^{-1}\bar{b} + d$

$= CP(sI - P^{-1}AP)^{-1}P^{-1}b + d$

$= CP(sP^{-1}P - P^{-1}AP)^{-1}P^{-1}b + d$

$= CP[P^{-1}(sI-A)P]^{-1}P^{-1}b + d$

$= CPP^{-1}(sI-A)^{-1}PP^{-1}b + d$

$= C(sI-A)^{-1}b + d = G(s)$

- So the system is invariant to the transformation!

- System o/p to unit response

$\bar{C}(sI-\bar{A})^{-1}\bar{x}(t_0) = CP(sI - P^{-1}AP)^{-1}P^{-1}x(t_0)$

Thus the I/O behaviour $: C(sI-A)^{-1}x(t_0)$ is invariant under transformation.

- Checking for position control system. (Just an eg)

$$|sI-A| = \begin{vmatrix} s & -1 & 0 \\ 0 & s+1 & -1 \\ 0 & 1 & s+1 \end{vmatrix} = s(s^2+11s+11)$$

adjoint matrix transpose

- $G(s)$ will come out to be $\frac{10}{s(s^2+11s+11)}$ using $G(s) = \frac{C\,(sI-A)^{adj}\,b}{|sI-A|}$

- Can also be done using integrator circuit → later

Conversion of Transfer function to $\underbrace{\text{cannonical}}_{\text{simpler matrices}}$ state variables

why?? { Given a running system, if we don't know the state variables
BUT WE WILL HAVE T.F BECAUSE I/O is given. ?
Then using T.F we will be able to get state variable model }

So given TF is

$$G(s) = \frac{\beta_0 s^m + \beta_1 s^{m-1} + \dots \beta_m}{s^n + \alpha_1 s^{n-1} + \dots \alpha_n}$$

$m \leq n$

ONLY POSSIBLE TO OBTAIN STATE VARIABLES IF RATIONAL.

$\alpha_0 = 1$

α_1 & β_1 .. real constant scalar

- Putting extra co-eff in $G(s)$ and putting co-efficients to zero

$$G(s) = \frac{\beta_0 s^n + \beta_1 s^{n-1} \dots + \beta_n}{s^n + \alpha_1 s^{n-1} \dots \alpha_n}$$

SIMPLE CANONICAL FORM.

1st decompose into 2 parts

$$U(s) \rightarrow \boxed{\frac{1}{s^n + \alpha_1 s^{n-1} \dots \alpha_n}} \xrightarrow{Z(s)} \boxed{\beta_0 s^n + \beta_1 s^{n} \dots + \beta_n} \rightarrow \text{output}$$

$$(s^n + \alpha_1 s^{n-1} + \dots \alpha_n)\, Z(s) = U(s).$$

co-rresponding diff eq^n using Inverse Laplace

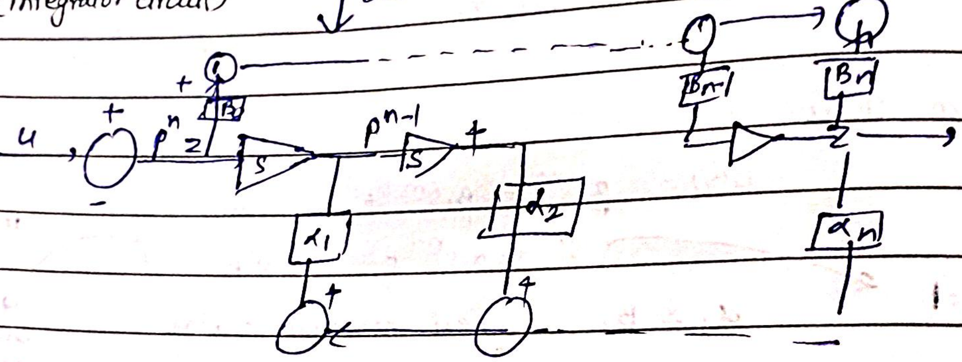
$$p^n z(t) + \alpha_1 p^{n-1} z(t) \cdot \ldots + \alpha_n (z(t)) = u(t)$$

$$p^\alpha z(t) = \frac{d^k z(t)}{dt^k}$$

$$p^n z(t) = u(t) - \left(\alpha_1 p^{n-1}(z(t)) + \alpha_2 p^{n-2}(z(t)) - \ldots\right)$$

= (integrator circuit)

BLOCK DIAGRAM REPRESENTATION



similarly $y(t) = B_0 p^n z(t) + B_1 p^{n-1} z(t) + \ldots B_n z(t)$.

so $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$ ---- $\dot{x}_{n-1}$

$\dot{x}_n = -\alpha_n x_1 - \alpha_{n-1} x_2 + \ldots \alpha_1 x_n + u$.

$y = (B_n - \alpha_n B_0) x_1 + (B_{n-1} - \alpha_{n-1} B_0) x_2 \ldots - (B_1 - \alpha_1 B_0) x_n + B_0 u$

using integrator circuit