IV.6

Tank 1 : $A_1 s\bar{h}_1(s) = \bar{F}_1(s) - \bar{F}_2(s)$

$$A_2 s\bar{h}_2(s) = \bar{F}_2(s) - \bar{F}_3(s)$$

(a) F_1, manipulation

$F_2 = \alpha_1 F_1$; $F_3 = \alpha_2 h_2$

&

$\bar{F}_2(s) = \alpha_1 \bar{h}_1(s)$

$\bar{F}_3(s) = \alpha_2 \bar{h}_2(s)$

F_2 manipulation

$\bar{F}_3(s) = \alpha_2 \bar{h}_2(s)$

$\bar{h}_1(s) = \dfrac{\bar{F}_1(s)}{A_1 s} - \dfrac{\bar{F}_2(s)}{A_1 s}$

$\bar{h}_2(s) = \dfrac{(1/\alpha_1)}{T_1 s+1} \bar{F}_2(s)$

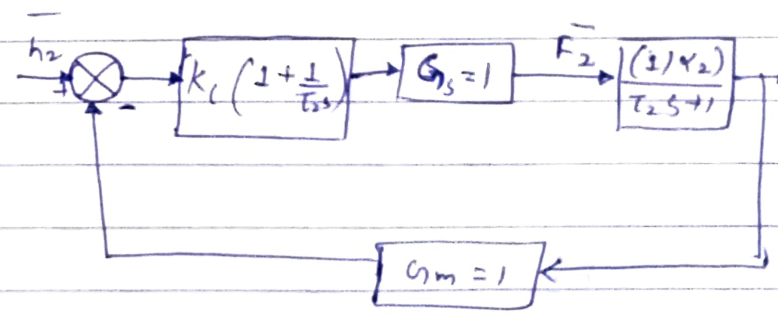
F_3 manipulation

$\bar{F}_2(s) = \alpha_1 \bar{h}_1(s)$

$\bar{h}_1(s) = \dfrac{(1/\alpha_1)}{T_1 s+1} \bar{F}_1$

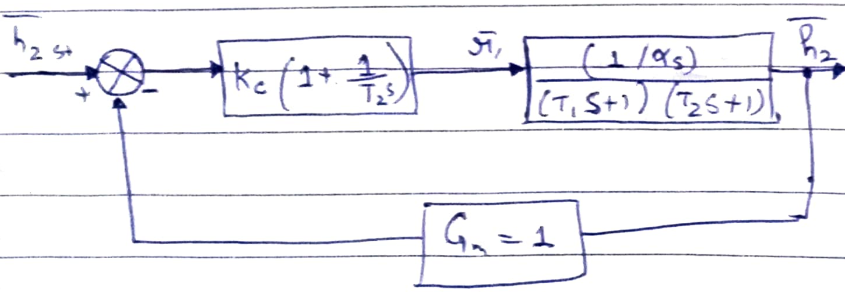
then,

$$\bar{h}_1(s) = \frac{(1/\alpha_1)}{\left(\frac{A_1}{\alpha_1}\right)s+1} \bar{F}_1(s)$$



$$\bar{h}_2(s) = \frac{(\alpha_1/\alpha_2)}{\left(\frac{A_2}{\alpha_2}\right)s+1} \bar{h}_1(s)$$

$$= \frac{(1/\alpha_2)}{(T_1 s+1)(T_2 s+1)} \cdot \bar{F}_1(s)$$



$T_1 = A_1/\alpha_1$ $T_2 = A_2/\alpha_2$

Page No.:
Date:

(b) F_1 : manipulation

$$\bar{h}_2(s) = \frac{K_c\left(1+\frac{1}{T_1 s}\right)\frac{(1/\alpha_2)}{(T_1 s+1)(T_2 s+1)}}{1+K_c\left(1+\frac{1}{T_2 s}\right)\left(\frac{(1/\alpha_2)}{(T_1 s+1)(T_2 s+1)}\right)} \cdot \bar{h}_{2,st}$$

$=$

F_2 ; manipulation

$$\bar{h}_2(s) = \frac{K_c\left(1+\frac{1}{T_1 s}\right)\frac{(1/\alpha_2)}{T_2 s+1}}{1+K_c\left(1+\frac{1}{T_2 s}\right)\frac{(1/\alpha_2)}{T_2 s+1}} \quad \bar{h}_{2,st}(s)$$

F_3 , manipulation

$$\bar{h}_2(s) = \frac{K_c\left(1+\frac{1}{T_2 s}\right)\frac{-1}{A_2 s}}{1+K_c\left(1+\frac{1}{T_1 s}\right)\frac{-1}{A_2 s}} \cdot \bar{h}_{2,sp}(s) + \frac{\frac{1}{A_2 s(T_1 s+1)}}{1+K_c\left(1+\frac{1}{T_2 s}\right)\frac{-1}{A_2 s}} \cdot \bar{F}_1$$

(c) Set $s \to 0$

F_1; manipulation : Closed loop gain $= 1$

F_2; " : " " " $= 1$

F_3; " : " " $(s/\omega \cdot h_2, F_{2\,sp}) = 1$

" " " $($ " $h_2, F_1) = 0$

(d) for the cases of F_1 or F_2 as manipulation we have only G_{sp}.

for the case of F_3 :

Page No.:
Date:

$$G_{load} = \frac{1/[A_2 s(\tau_1 s+1)]}{1 + K_c\left(1+\frac{1}{\tau_1 s}\right)\frac{(-1)}{A_2 s}}$$

$$G_{sp} = \frac{K_c\left(1+\frac{1}{\tau_1 s}\right)(-1/A_2 s)}{1 + K_c\left(1+\frac{1}{\tau_1 s}\right)(-1/A_2 s)}$$

IV.11:

(a) $\bar{y}(s) = \frac{5\cdot1\cdot2/[(s+1)(5s+1)]}{1+5\cdot1\cdot2/[(s+1)(5s+1)]} \cdot \frac{2}{s}$

(b) $\bar{y}(s) = \frac{20}{s(3s^2+4s+11)} = \frac{20/11}{s} + (0.91 + j\,0.3376)\cdot \frac{1}{s - \frac{-4+\sqrt{116}j}{6}} + (-0.91 + j\,0.337)\frac{1}{s - \frac{-4-\sqrt{116}j}{6}}$

Invert $\Rightarrow$ $y(t) = \frac{20}{11} - 1.9412\, e^{-2t/3}\cdot \sin\left[\frac{\sqrt{116}}{6}t + \tan^{-1}(2.69)\right]$

(c) from

$$\bar{y}(s) = \frac{20}{s(3s^2+4s+11)} = \frac{20/11}{s\left(\frac{3}{11}s^2+\frac{4}{11}s+1\right)}$$

we include that the response is 2nd order with $\tau^2 = 3/11$ & $2\zeta\tau = 4/11$

i.e $\tau = \sqrt{3/11}$ & $\zeta = \frac{1}{2}\cdot\frac{4}{11}\cdot\sqrt{\frac{11}{3}} = 2\sqrt{\frac{1}{33}} = 0.348$

Page No.:
Date:

<u>ultimate</u> <u>value</u>: $y(t \to \infty) = 20/11$ f

overshoot = (maximum − ultimate)/(ultimate) = 0.312

$\max y(t) = (1.312)(\text{ultimate}) = 2.38$

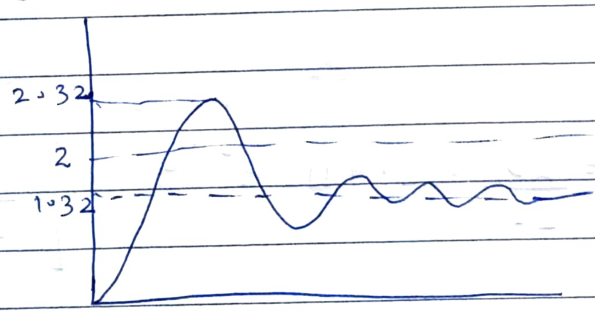
at $t = t^*$

$$Y(t^*) = 2.38 = \frac{20}{11} - 1.9412 \cdot e^{-2t^*/5} \cdot \sin\left[\frac{\sqrt{116}}{16} t^* + \tan^{-1}(2.69\right]$$

(d) offset : $2 - \frac{20}{11} = 2 - 1.82 = 0.18$

(e) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{116}/6} = 3.49$

(f)



(IV.5) $\quad G_{CL} = \dfrac{K_c \frac{5}{s+1} \cdot \frac{K_m}{T_m s+1}}{1 + K_c \cdot \frac{5}{s+1} \cdot \frac{K_m}{T_m s+1}}$; $G_{sp}(s) = \dfrac{(T_m s+1)K_m}{\frac{2T_m}{5K_cK_m} s^2 + \frac{T_m+2}{5K_cK_m} s + 1}$

$$\tau = \sqrt{\frac{2T_m}{5K_cK_m}} \qquad \zeta = \frac{1}{2}\,\frac{T_m+2}{5K_cK_m}\cdot\sqrt{\frac{5K_cK_m}{2T_m}}$$

$$= \frac{T_m+2}{2}\sqrt{\frac{1}{10T_mK_cK_m}}$$

(a) as K_m ↑es ζ decreases

e.g. $T_m = 1$ $\quad K_m = 1$, $\zeta = 0.474$ $\quad K_m = 5$, $\zeta = 0.4743$ $\quad K_m = 10$, $\zeta = 0.15$

& response becomes more underdamped.

(b) let $K_m = 1$

$T_m = 0.01$	$T_m = 0.1$	$T_m = 1$	$T_m = 5$
$\zeta = 3.18$	$\zeta = 1.05$	$\zeta = 0.474$	$\zeta = 0.49$

T_m ↑es the ζ ↓es

(c) for constant T_m, an increase in K_m leads to more oscillatory behaviour. ~~offset = 1 - y(t-~~

offset $= 1 - y(t\to\infty) = 1 - \frac{1}{K_m}$

increases for constant K_m as increase in T_m.