ASSIGNMENT

Control System

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Q1) Draw the inverse polar plot for a feed back system $\overline{\omega}$ as open loop transfer function

$$G(s)\,H(s) = \frac{K}{s(1+Ts)}$$

Ans) $$\frac{1}{G(s)H(s)} = \frac{s(1+Ts)}{K}$$

Therefore, the sinusoidal TF is

$$\frac{1}{G(j\omega)\,H(j\omega)} = \frac{j\omega(1+Tj\omega)}{K} = \frac{-\omega^2 T + j\omega}{K}$$

At $\omega = -\infty$

$$\frac{1}{G(j\omega)H(j\omega)} = -\infty - j\infty$$

At $\omega = 0^-$ $$\frac{1}{G(j\omega)\,H(j\omega)} = -0 - j0$$

At $\omega = 0^+$ $$\frac{1}{G(j\omega)H(j\omega)} = -0 + j0$$

At $\omega = +\infty$ $$\frac{1}{G(j\omega)\,H(j\omega)} = -\infty + j\infty.$$

At $s = \lim_{\epsilon \to 0} \epsilon e^{j\theta}$ where θ varies from -90° to $+90^\circ$ is mapped on plane

$$\lim_{\epsilon \to 0} \left[\frac{\epsilon e^{j\theta}(\epsilon e^{j\theta}+1)}{k} \right] = \lim_{\epsilon \to 0} \frac{\epsilon e^{j\theta}}{k} = 0 e^{j\theta}$$

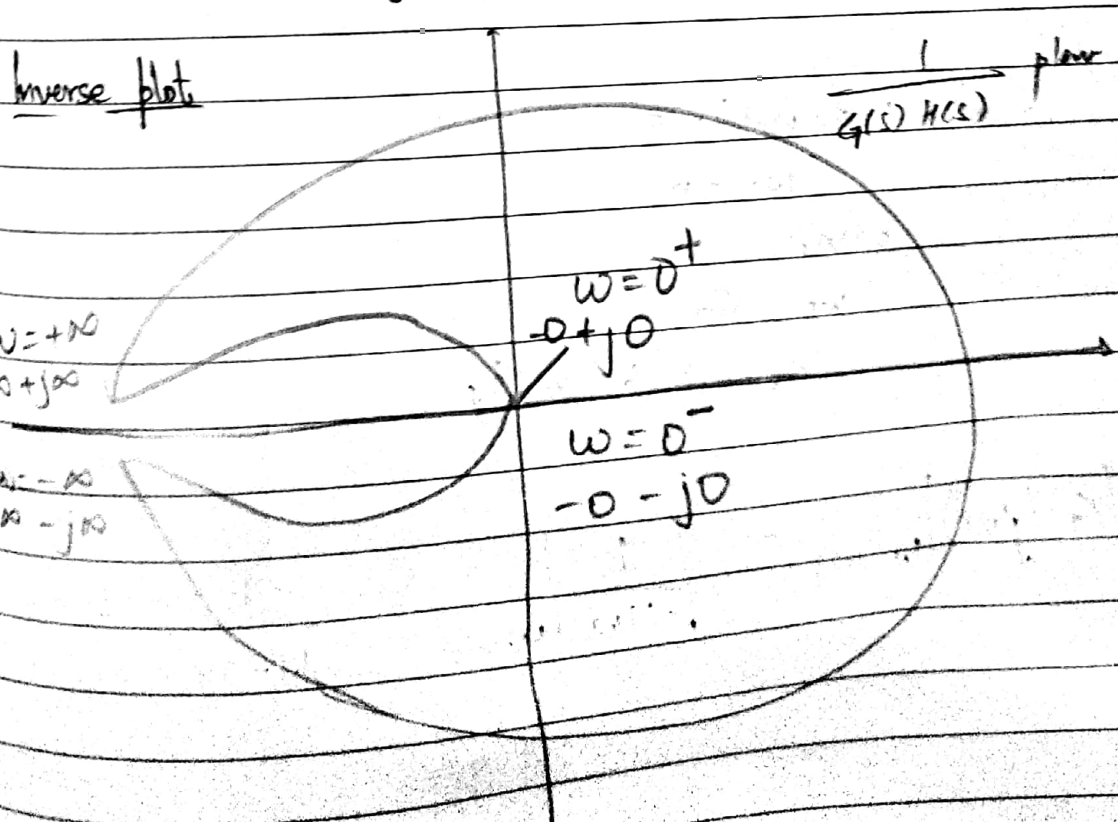
At $s = \lim_{R \to \infty} R e^{j\phi}$ where ϕ varies from $+90^\circ$ to -90° is mapped on plane

$$\lim_{R \to \infty} \left[\frac{R e^{j\phi}(R e^{j\phi}+1)}{k} \right] = \lim_{R \to \infty} \frac{R^2 e^{j2\phi}}{k} = \infty e^{j2\phi}$$

It is a circle of infinite radius varying from $+180^\circ$ to -180°

Inverse plot

$\frac{1}{G(s)H(s)}$ plane



Q2 The open loop function of a system is

$$G(s) = \frac{K}{s(1+0.1s)(1+s)}$$

Determine the value of K so that GM is 6dB

Soln $$G(s) = \frac{K}{s(1+0.1s)(1+s)}$$

Replacing $s = j\omega$

$$G(j\omega) = \frac{K}{j\omega(1+0.1j\omega)(1+j\omega)}$$

$$= \frac{K}{j\omega(1+j1.1\omega - 0.1\omega^2)} = \frac{K}{-1.1\omega^2 + j\omega(1-0.1\omega^2)}$$

$\omega_p(1-0.1\omega_p^2) = 0$

$\omega_p \neq 0$

$0.1\omega_p^2 = 1$

$\omega_p = \sqrt{10}$

$\omega_p = 3.162$ rad/s

$$|G(j\omega)|_{\omega=\omega_p} = \left|\frac{K}{-1.1\omega^2}\right|_{\omega=\omega_p} = \frac{K}{1.1\times 10} = 0.0909K$$

Given that $GM = 6dB$

$\Rightarrow 20\log GM = 6\ dB$

$\Rightarrow \log GM = 6/20 = 0.3$

$GM = 10^{0.3} = 1.9953$

$$GM = \frac{1}{|G(j\omega)|_{\omega=\omega p}}$$

$$\Rightarrow 1.9953 = \frac{1}{0.0909k}$$

$$\Rightarrow \boxed{k = \frac{1}{0.0909 \times 1.9953} = 5.513}$$ Ans

Q3 The open-loop T.F. of a unity feedback system is

$$G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$$

Determine the value of K so that PM is 40°

Sol Let $\omega = \omega_g$ (gain crossover frequency)

Then for a PM of 40°,

$$-90^\circ - \tan^{-1} 0.2\omega_g - \tan^{-1} 0.05\omega_g + 180^\circ = 40^\circ$$

$\Rightarrow \tan^{-1} 0.2\omega_g + \tan^{-1} 0.05\omega_g = 180 - 90 - 40 = 50^\circ$

Taking tan of both sides

$$\frac{0.2\omega_g + 0.05\omega_g}{1 - 0.2\omega_g \cdot 0.05\omega_g} = \tan 50^\circ = 1.2$$

$$\Rightarrow \frac{0.25\omega_g}{1 - 0.01\omega_g^2} = 1.2$$

$$\Rightarrow 1.2 - 0.012\omega_g^2 = 0.25\omega_g$$

$$\Rightarrow 0.012\omega_g^2 + 0.25\omega_g - 1.2 = 0$$

$$\omega_g = \frac{-0.25 \pm \sqrt{0.25^2 + 4 \times 0.012 \times 1.2}}{2 \times 0.012} \quad \text{--- (1)}$$

$$|G(j\omega)|_{\omega = \omega_g} = \frac{K}{\omega_g \sqrt{(1 + (0.2\omega_g)^2)} \sqrt{(1 + (0.05\omega_g)^2)}} = 1 \quad \text{--- (2)}$$

Solving eqn (1) for positive value

we get $\omega_g = 4$ rad/sec

Putting $\omega_g = 4$ rad/sec in (2)

we get $\boxed{K = 5.2}$ Ans