

Diagonalization of the matrix

- We want to obtain the trans. mat (P) to diagonalize the matrix
- Find the eigen vectors of the matrix A
- then obtain the modal matrix M by placing the eigenvectors(columns)

Handwritten notes on the slide:

Hady eigenvectors m_1 and m_2 associated with $\lambda_1 = -2$ & $\lambda_2 = -3$

The modal matrix M obtained by placing the eigenvectors (columns) together is given by

$$M = [m_1 \ m_2] = \begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix}$$

from which we find

$$M^{-1} = \frac{1}{5} \begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix}$$

$\therefore A = M^{-1} A M = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$ which is a diagonal matrix with eigenvalues of A as its diagonal elements

transformation

<https://forms.gle/aBybSmxVFc9wVsF7>

- then do $P(\text{inverse}) * A * P$ to obtain the matrix

This is independent of the fact of multiplicity

- if that comes out to be zero - then use the second row - if that comes out to be zero - then use the third row

Case when eig vector are same

- say λ_1 ($\lambda_2 = \lambda_3$)
- find the eigen vector with λ_1
- find the eigen vector with λ_2
- Now to obtain the third eigen factors
 - differentiate the elements in co-factors matrix of 2 to get the third vector

To obtain eigenvectors associated with repeated eigenvalue of $\lambda=2$ we construct the matrix

$$(A\lambda I - A) = \begin{bmatrix} \lambda - 4 & -1 & 3 \\ -1 & \lambda - 2 & -2 \\ -1 & 1 & \lambda - 3 \end{bmatrix}$$

for $\lambda=2$ rank of 3×3 matrix $(A\lambda I - A)$ is 2, \therefore one independent eigenvector associated with $\lambda=2$ can be obtained, it is given by

$$m_2 = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix} = \begin{bmatrix} A_{11}(\lambda-2) + 4 \\ A_{12}(\lambda-2) + 2 \\ A_{13}(\lambda-2) + 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 1 \end{bmatrix}$$

The vector m_3 for the modal matrix $M = [m_1, m_2, m_3]$ may be generated from the independent eigen vector m_1 as follows

$$m_3 = \begin{bmatrix} \frac{d}{dx} c_{11} \\ \frac{d}{dx} c_{12} \\ \frac{d}{dx} c_{13} \end{bmatrix}_{\lambda=2} = \begin{bmatrix} 2\lambda-3 \\ 1 \\ 1 \end{bmatrix}_{\lambda=2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- Then put the value $[\lambda_1 \mid \lambda_2 \mid \lambda_3]$
- We will not be able to form full diagonal
- We will form jordan blocks

The eigenvector m_2 is generalized eigenvector. The modal matrix M is then given by

$$M = \begin{bmatrix} 0 & 2 & 3 \\ 8 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

The modal matrix M now transforms A to the Jordan matrix i.e.

$$M^{-1}AM = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix} = J$$

↑
Jordan blocks

In general if an eigenvalue λ has multiplicity p and the rank of $n \times n$ matrix $(\lambda I - A)$ is $(n-1)$ i.e. there is only one independent eigenvector.

When multiplicity > 2

- matrix can be obtained as follows

$m = \begin{bmatrix} c_{q1} \\ c_{q2} \\ \vdots \\ c_{qn} \end{bmatrix}$ associated with the eigenvalue λ_1 ,
 then the remaining $(q-1)$ vectors for the modal matrix are the
 generalized eigenvectors given below

$$[m_1 \ m_2 \ \dots \ m_q] = \begin{bmatrix} c_{q1} \frac{d}{dt} - \lambda_1 & \frac{1}{1!} \frac{d^2}{dt^2} c_{q1} & \dots & \frac{1}{(q-1)!} \frac{d^{q-1}}{dt^{q-1}} c_{q1} \\ c_{q2} \frac{d}{dt} - \lambda_1 & \frac{1}{1!} \frac{d^2}{dt^2} c_{q2} & \dots & \frac{1}{(q-1)!} \frac{d^{q-1}}{dt^{q-1}} c_{q2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{qn} \frac{d}{dt} - \lambda_1 & \frac{1}{1!} \frac{d^2}{dt^2} c_{qn} & \dots & \frac{1}{(q-1)!} \frac{d^{q-1}}{dt^{q-1}} c_{qn} \end{bmatrix}$$

 The matrix $M^{-1}AM$ will have a $q \times q$ Jordan block corresponding
 to the eigenvalue λ_1 .
 When A is known in the companion form and has
 eigenvalues $\lambda_1, \lambda_1, \dots, \lambda_1, \lambda_{m+1}, \dots, \lambda_n$ (eigenvalue λ_1 of multiplicity
 m), then the transformation matrix is the modified Vander
 Monde matrix.

State Model Using Integrator Circuit

Substituting Equation (10-2) into Equation (10-1) gives
 $\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{BK})\mathbf{x}(t)$
 The solution of this equation is given by
 $\mathbf{x}(t) = e^{(\mathbf{A} - \mathbf{BK})t}\mathbf{x}(0)$ (10-3)
 where $\mathbf{x}(0)$ is the initial state caused by external disturbances. The stability and transient-
 response characteristics are determined by the eigenvalues of matrix $\mathbf{A} - \mathbf{BK}$. If matrix

$y = Cx + Du$
 $Bu + Ax = y$
 $= y$

Block diagram showing a feedback control system with blocks A, B, C, D, K, and an integrator (1/s). The input u is fed into block B and block D. The output of block B is fed into a summing junction, and the output of block D is also fed into the same summing junction. The output of the summing junction is fed into the integrator block. The output of the integrator is fed into block A, and the output of block A is fed back into block K, which is then fed back into the summing junction. The output of the system is y.

Pole Placement

What are poles?

- in TF put Den to zero to obtain char. eqn
- char eqn. can be obtained using $|\lambda I - A| = 0$
 - A is the state space model of the function
 - Why?
 - Explained by the method using
 - sininus

- Poles define the behaviour of the system
 - They define the dynamics of the system
 - like fast or slow
 - like rise time etc