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ICE-2

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Maths assignment
(Heat equation)

Q Find solution of

$$\frac{\delta^2 u}{\delta x^2} = h^2 \frac{\delta u}{\delta t}.$$

for which
$u(0,t) = u(l,t) = 0$, $u(x,0) = \sin\frac{\pi x}{l}$.

By method of seperable

$$\frac{\delta u}{\delta t} = \frac{1}{h^2}\frac{\delta^2 u}{\delta x^2}$$

It's solution is

$$u = (c_1 \cos px + c_2 \sin px)\, c_3 e^{-\frac{p^2 t}{h^2}} \quad -(1)$$

Put $x=0$, $u=0$ in (1), we have

$$0 = c_1 c_3 e^{-\frac{p^2 t}{h^2}} \Rightarrow c_1 = 0$$

Put $c_1 = 0$ in (1)

$$u = c_2 c_3 \sin px\, e^{-\frac{p^2 t}{h^2}} \quad -(2)$$

Put $x=l$, $u=0$ in (2), we have

$$0 = c_2 c_3 \sin pl\, e^{-\frac{p^2 t}{h^2}}$$

$$pl = n\pi \qquad \boxed{p = \frac{n\pi}{l}}$$

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Then (2) becomes

$$u = b_n \sin\left(\frac{n\pi x}{l}\right) e^{-\frac{n^2\pi^2 t}{l^2 h^2}} \quad (b_n = c_2 c_3)$$

∴ General equation

$$u = \sum_{n=1}^{\infty} b_n \sin\left[\frac{n\pi}{l}\right]x \; e^{-\frac{n^2\pi^2 t}{l^2 h^2}} \quad \text{—(3)}$$

put $t=0$, and $u = \sin\left(\frac{\pi x}{l}\right)$

$$\sin\left(\frac{\pi x}{l}\right) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{\pi x n}{l}\right)$$

$\cancel{b_n = 1}$

$$\sin\frac{\pi x}{l} = b_1 \sin\left(\frac{\pi x}{l}\right) + b_2 \sin\left(\frac{2\pi x}{l}\right) \ldots$$

$$\boxed{b_1 = 1}$$

Hence (3) becomes

$$\boxed{u = \sin\left[\frac{\pi x}{l}\right] e^{-\frac{\pi^2 t}{l^2 h^2}}}$$

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