Q) What is a canonical form?

$$\dot{\bar{x}} = \bar{A}x(t) + \bar{B}u(t)$$
$$\bar{y}(t) = \bar{C}x(t) + \bar{D}u(t)$$

Q) Diagonal canonical form,

$$x(t) = T\bar{x}(t)$$
$$\bar{x}(t) = T^{-1}x(t)$$
$$\dot{\bar{x}}(t) = T^{-1}\dot{x}(t)$$
$$= T^{-1}(A \cdot x(t) + Bu(t))$$
$$= T^{-1}Ax(t) + T^{-1}Bu(t)$$
$$= T^{-1}AT\bar{x}(t) + T^{-1}Bu(t)$$

$$y(t) = CT\bar{x}(t) + \bar{D}u(t)$$
$$\bar{c} = CT$$

$$\bar{A} = T^{-1}AT \qquad \bar{B} = T^{-1}B$$

$T = P_1, P_2, P_3 \ldots\ldots P_n$

- If the A matrix is controllable canonical form and A has distinct eigen values
- The diagonal canonical matrix T is wander-mand matrix

$$T = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_n^{n-1} \end{bmatrix}$$

where $\lambda_1, \lambda_2, \ldots \lambda_n$ are eigen values of matrix A

Q) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

eigen vectors of A:

$-1, -2, -3$

$T = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$

$T^{-1} = \begin{bmatrix} 3 & 2.5 & 0.5 \\ -3 & -4 & -1 \\ 1 & 1.5 & 0.5 \end{bmatrix}$

$\bar{A} = T^{-1}AT = \begin{bmatrix} 2 & 6 & 9 \\ -3 & -8 & -9 \\ 1 & 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

$\bar{B} = T^{-1}B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ $\qquad \bar{C} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = CT$

$$\bar{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \\ \bar{x}_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \begin{bmatrix} u_1(t) & u_2(t) & u_3(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ x_3 \end{bmatrix}$$