Precautions should also be taken to minimise the effect of cable and stray capacitances on these circuits.

8.3 Inductive sensing elements

8.3.1 Variable inductance (variable reluctance) displacement sensors

In order to discuss the principles of these elements we must first introduce the concept of a **magnetic circuit**. In an electrical circuit an electromotive force (e.m.f.) drives a current through an electrical resistance and the magnitude of the current is given by

$$\text{e.m.f.} = \text{current} \times \text{resistance} \qquad [8.25]$$

A simple magnetic circuit is shown in Figure 8.11(a): it consists of a loop or core of ferromagnetic material on which is wound a coil of n turns carrying a current i. By analogy we can regard the coil as a source of magnetomotive force (m.m.f.) which drives a flux ϕ through the magnetic circuit. The equation corresponding to [8.25] for a magnetic circuit is:

$$\text{m.m.f.} = \text{flux} \times \text{reluctance} = \phi \times \mathcal{R} \qquad [8.26]$$

so that reluctance $\mathcal{R}$ limits the flux in a magnetic circuit just as resistance limits the current in an electrical circuit. In this example m.m.f. = ni, so that the flux in the magnetic circuit is:

$$\phi = \frac{ni}{\mathcal{R}} \text{ weber} \qquad [8.27]$$

This is the flux linked by a single turn of the coil; the total flux N linked by the entire coil of n turns is:

$$N = n\phi = \frac{n^2 i}{\mathcal{R}} \qquad [8.28]$$

By definition the self-inductance L of the coil is the total flux per unit current, i.e.

Self-inductance of a coil

$$\boxed{L = \frac{N}{i} = \frac{n^2}{\mathcal{R}}} \qquad [8.29]$$

The above equation enables us to calculate the inductance of a sensing element given the reluctance of the magnetic circuit. The reluctance $\mathcal{R}$ of a magnetic circuit is given by:

$$\mathcal{R} = \frac{l}{\mu\mu_0 A} \qquad [8.30]$$

where l is the total length of the flux path, μ is the relative permeability of the circuit material, μ_0 is the permeability of free space = $4\pi \times 10^{-7}$ H m^{-1} and A is the

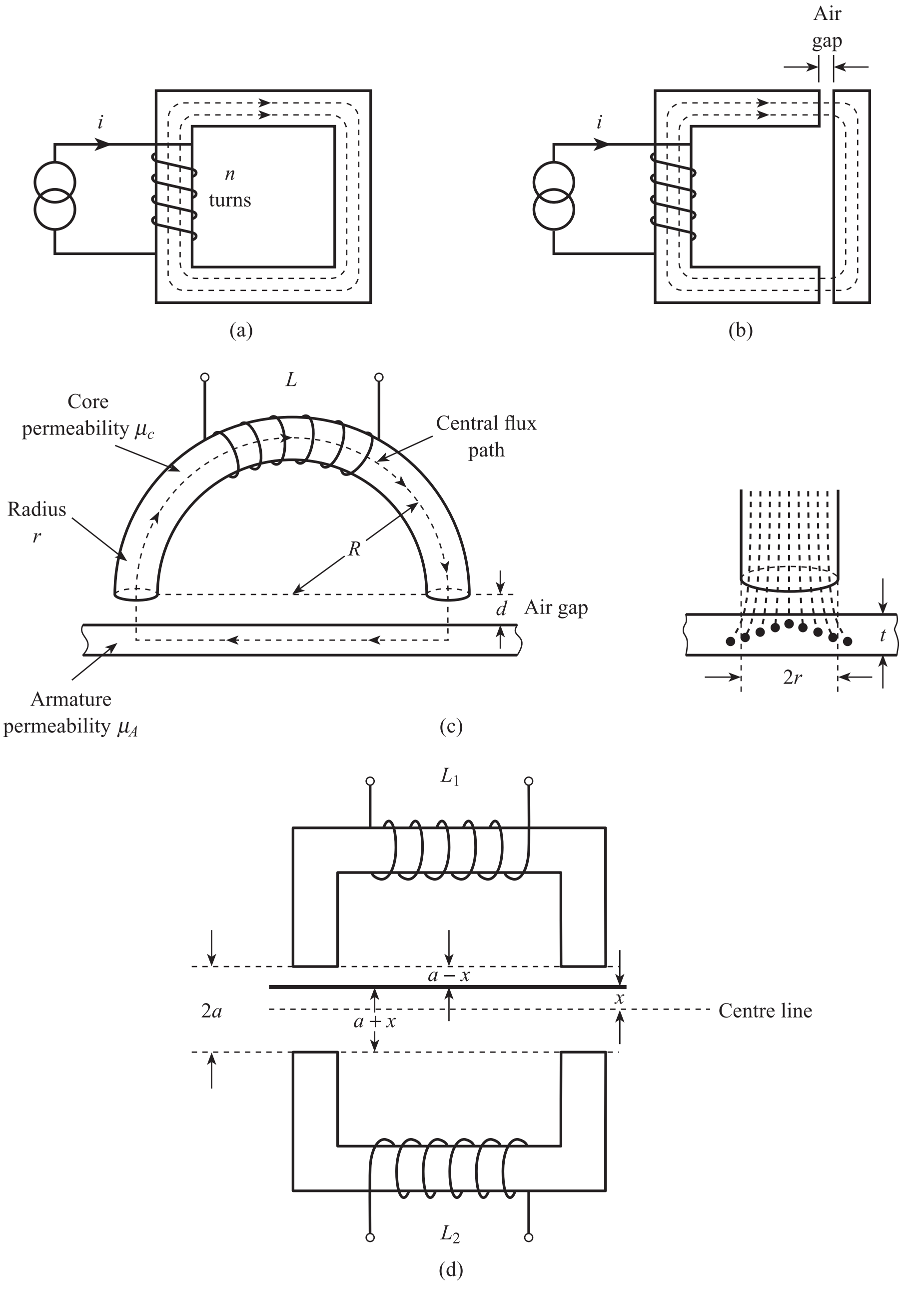


Figure 8.11 Variable reluctance elements: (a)(b) Basic principle of reluctance sensing elements (c) Reluctance calculation for typical element (d) Differential or push/pull reluctance displacement sensor.

cross-sectional area of the flux path. Figure 8.11(b) shows the core separated into two parts by an air gap of variable width. The total reluctance of the circuit is now the reluctance of both parts of the core together with the reluctance of the air gap. Since the relative permeability of air is close to unity and that of the core material many thousands, the presence of the air gap causes a large increase in circuit reluctance and a corresponding decrease in flux and inductance. Thus a small variation in air gap causes a measurable change in inductance so that we have the basis of an **inductive displacement sensor**.

Figure 8.11(c) shows a typical variable reluctance displacement sensor, consisting of three elements: a ferromagnetic core in the shape of a semitoroid (semicircular ring), a variable air gap and a ferromagnetic plate or armature. The total reluctance of the magnetic circuit is the sum of the individual reluctances, i.e.

$$\mathcal{R}_{\text{TOTAL}} = \mathcal{R}_{\text{CORE}} + \mathcal{R}_{\text{GAP}} + \mathcal{R}_{\text{ARMATURE}} \quad [8.31]$$

The length of an average, i.e. central, path through the core is πR and the cross-sectional area is πr^2, giving:

$$\mathcal{R}_{\text{CORE}} = \frac{\pi R}{\mu_0 \mu_C \pi r^2} = \frac{R}{\mu_0 \mu_C r^2} \quad [8.32]$$

The total length of the flux path in air is twice the air gap, i.e. $2d$; also if there is little bending or fringing of the lines of flux in the air gap, then the cross-sectional area of the flux path in air will be close to that of the core. Assuming the relative permeability of air is unity,

$$\mathcal{R}_{\text{GAP}} = \frac{2d}{\mu_0 \pi r^2} \quad [8.33]$$

The length of an average central flux path in the armature is $2R$; the calculation of the appropriate cross-sectional area is more difficult. A typical flux distribution is shown in Figure 8.11(c) and for simplicity we assume that most of the flux is concentrated within an area $2rt$, giving

$$\mathcal{R}_{\text{ARMATURE}} = \frac{2R}{\mu_0 \mu_A 2rt} = \frac{R}{\mu_0 \mu_A rt} \quad [8.34]$$

Thus

$$\mathcal{R}_{\text{TOTAL}} = \frac{R}{\mu_0 \mu_C r^2} + \frac{2d}{\mu_0 \pi r^2} + \frac{R}{\mu_0 \mu_A rt}$$

i.e.

$$\mathcal{R}_{\text{TOTAL}} = \mathcal{R}_0 + kd \quad [8.35]$$

where

$$\mathcal{R}_0 = \frac{R}{\mu_0 r}\left[\frac{1}{\mu_C r} + \frac{1}{\mu_A t}\right] = \text{reluctance at zero air gap}$$

$$k = \frac{2}{\mu_0 \pi r^2}$$

A typical element with $n = 500$ turns, $R = 2$ cm, $r = 0.5$ cm, $t = 0.5$ cm, $\mu_C = \mu_A = 100$, has $\mathcal{R}_0 = 1.3 \times 10^7\ \text{H}^{-1}$, $k = 2 \times 10^{10}\ \text{H}^{-1}\ \text{m}^{-1}$. This gives $L = 19$ mH at $d = 0$ (zero air gap) and $L = 7.6$ mH at $d = 1$ mm. From [8.29] and [8.35] we have

Inductance of reluctance displacement sensor

$$\boxed{L = \frac{n^2}{\mathcal{R}_0 + kd} = \frac{L_0}{1 + \alpha d}} \quad [8.36]$$

where $L_0 = n^2/\mathcal{R}_0$ = inductance at zero gap and $\alpha = k/\mathcal{R}_0$. Equation [8.36] is applicable to any variable reluctance displacement sensor; the values of L_0 and α depend on core geometry and permeability. We see that the relationship between L and d is non-linear, but this problem is often overcome by using the push-pull or differential displacement sensor shown in Figure 8.11(d). This consists of an armature moving between two identical cores, separated by a fixed distance $2a$. From eqn [8.36] and Figure 8.11(d) we have:

Differential reluctance displacement sensor

$$\boxed{L_1 = \frac{L_0}{1 + \alpha(a - x)}, \quad L_2 = \frac{L_0}{1 + \alpha(a + x)} \qquad [8.37]}$$

The relationship between L_1, L_2 and displacement x is still non-linear, but if the sensor is incorporated into the a.c. deflection bridge of Figure 9.5(b), then the overall relationship between bridge out of balance voltage and x is linear (eqn [9.25]). A typical sensor of this type would have an input span of 12 mm, a coil inductance (L_0) of 25 mH, a coil resistance of 70 Ω and a maximum non-linearity of 0.5%. Thus inductive sensors are not pure inductances but have an associated resistance R in series; this has an important influence on the design of oscillator circuits (Figure 9.25(a)).

8.3.2 Linear Variable Differential Transformer (LVDT) displacement sensor

This sensor is a transformer with a single primary winding and two identical secondary windings wound on a tubular ferromagnetic former (Figure 8.12). The primary winding is energised by an a.c. voltage of amplitude $\hat{V}_P$ and frequency f Hz; the two secondaries are connected in series opposition so that the output voltage $\hat{V}_{OUT} \sin(2\pi ft + \phi)$ is the difference $(V_1 - V_2)$ of the voltages induced in the secondaries. A ferromagnetic core or plunger moves inside the former; this alters the mutual inductance between the primary and secondaries. With the core removed the secondary voltages are ideally equal so that $\hat{V}_{OUT} = 0$. With the core in the former, V_1 and V_2 change with core position x, causing amplitude $\hat{V}_{OUT}$ and phase ϕ to change.

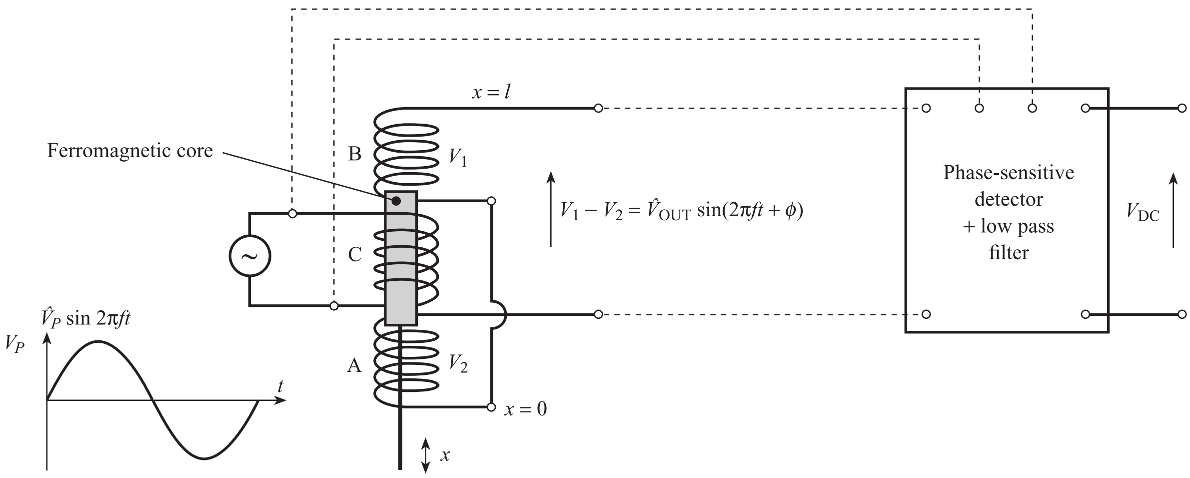
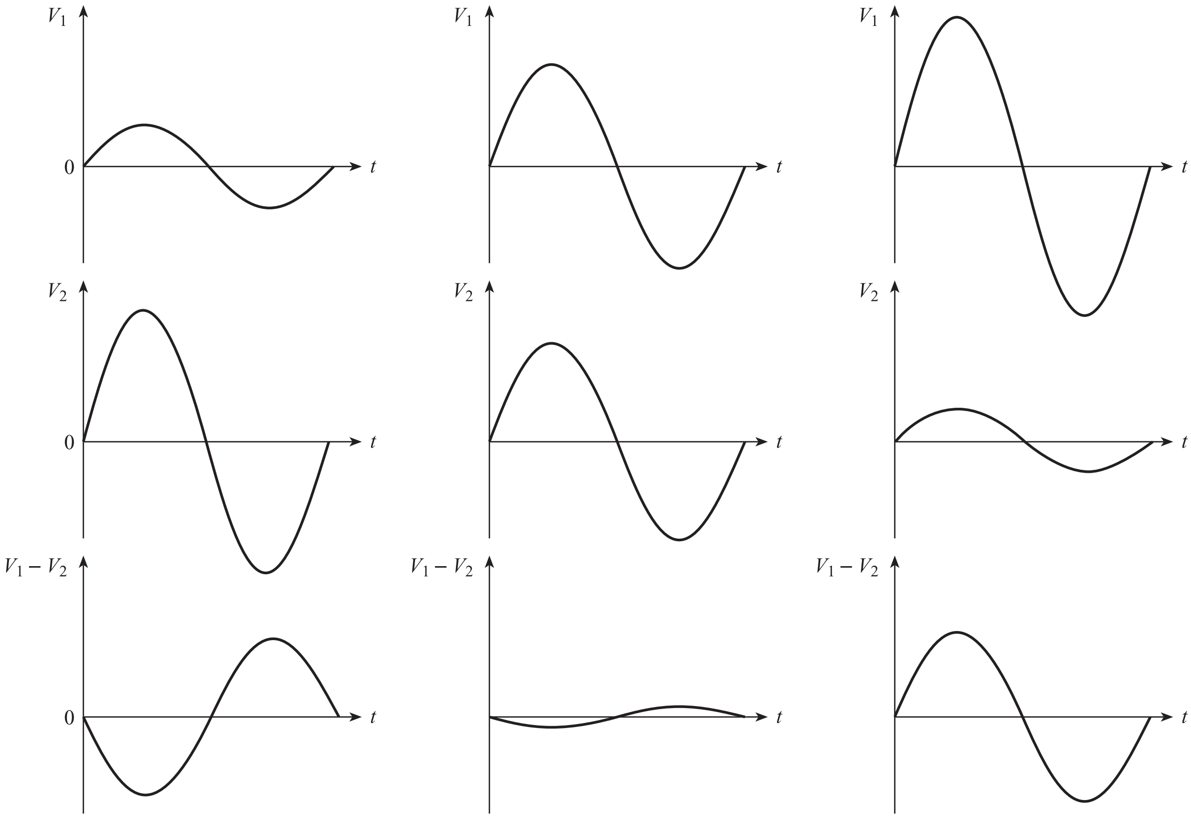


Figure 8.12 LVDT and connections to phase-sensitive detector.



(a) At A (b) At C (c) At B

Figure 8.13 LVDT secondary waveforms.

Figure 8.13 shows a.c. waveforms for secondary voltages V_1, V_2 and the difference $V_1 - V_2$ at three positions A, C and B. C is the halfway point at $x = \frac{1}{2}l$ and A and B are equidistant from C. Figure 8.13(a) shows the waveforms at A; here the lower secondary is strongly coupled to the primary and the upper secondary is weakly coupled to the primary; V_2 has therefore greater amplitude than V_1 and $V_1 - V_2$ is 180° out of phase with the primary voltage V_P. Figure 8.13(b) shows the waveforms at *C*; here the secondaries are equally coupled to the primary and V_1 and V_2 have approximately equal amplitudes; $V_1 - V_2$ has therefore minimum amplitude (ideally zero) and C is termed the **null point**. Figure 8.13(c) shows the waveforms at B; here the upper secondary is strongly coupled to the primary and the lower secondary is weakly coupled to the primary; V_1 has greater amplitude than V_2 and $V_1 - V_2$ is in phase with the primary voltage. Therefore $V_1 - V_2$ has the **same amplitude** at positions A and B but there is a **phase difference of 180°**.

The a.c. voltage $V_1 - V_2$ is converted to d.c. in a way which distinguishes between positions such as A and B either side of the null point C. This is done using a **phase sensitive detector or demodulator** (Section 9.3). This senses the 180° phase difference and gives a negative d.c. voltage at A and a positive d.c. voltage of equal magnitude at B. Figure 8.14 shows how the amplitude $\hat{V}_{OUT}$ and phase ϕ of the a.c. output and the d.c. voltage V_{DC} vary with displacement x. The relationship between V_{DC} and x is linear over the central portion of the range 0 to l, but non-linear effects

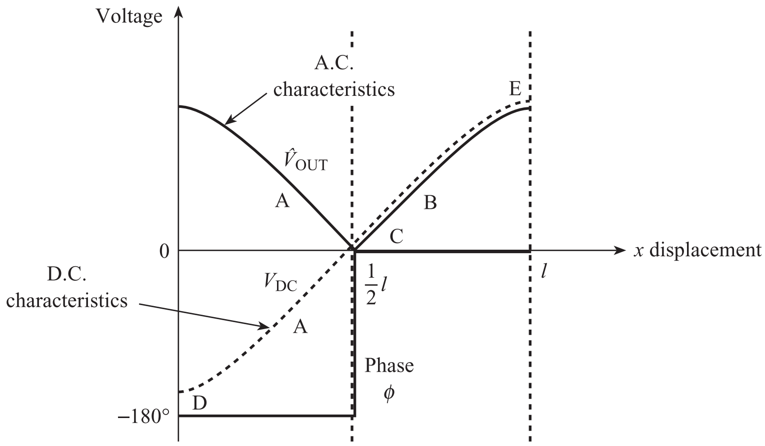


Figure 8.14 A.C. and D.C. characteristics of LVDT.

occur at either end (D and E) as the core moves to the edge of the former. This non-linearity can be reduced by using only a central portion of the available range.

LVDT displacement sensors are available to cover ranges from ±0.25 mm to ±25 cm. For a typical sensor of range ±2.5 cm, the recommended $\hat{V}_P$ is 4 to 6 V, the recommended f is 5 kHz (400 Hz minimum, 50 kHz maximum), and maximum non-linearity is 1% f.s.d. over the above range.

8.4 Electromagnetic sensing elements

These elements are used for the measurement of linear and angular velocity and are based on Faraday's law of electromagnetic induction. This states that if the flux N linked by a conductor is changing with time, then a back e.m.f. is induced in the conductor with magnitude equal to the rate of change of flux, i.e.

$$E = -\frac{dN}{dt} \quad [8.38]$$

In an electromagnetic element the change in flux is produced by the motion being investigated; this means that the induced e.m.f. depends on the linear or angular velocity of the motion. A common example of an electromagnetic sensor is the variable reluctance tachogenerator for measuring angular velocity (Figure 8.15). It consists of a toothed wheel of ferromagnetic material (attached to the rotating shaft) and a coil wound onto a permanent magnet, extended by a soft iron pole piece. The wheel moves in close proximity to the pole piece, causing the flux linked by the coil to change with time, thereby inducing an e.m.f. in the coil.

The magnitude of the e.m.f. can be calculated by considering the magnetic circuit formed by the permanent magnet, air gap and wheel. The m.m.f. is constant with time and depends on the field strength of the permanent magnet. The reluctance of the circuit will depend on the width of the air gap between the wheel and pole piece. When a tooth is close to the pole piece the reluctance is minimum but will increase as the tooth moves away. The reluctance is maximum when a 'gap' is adjacent to the pole