T- Multipliers

Q: 1 $px - qy = y^2 - x^2$ $\Rightarrow$ $\frac{xdx + ydy + dz}{0}$

Sol^n $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{y^2 - x^2}$ $= \frac{\cancel{dx + dy + dz}}{}$

$\frac{xdx + ydy + dz}{0}$

→ taking first 2 fractions

$\cancel{ydx} = xdy$

$\frac{dx}{x} = \frac{dy}{-y}$

On integr'

$\log x = -\log y + \log c_1$

$\log x + \log y = \log c_1$

$\boxed{xy = c_1}$

→ ~~taking 2 last fractions~~ two

$\frac{dy}{y} = \frac{dz}{(y+x)(y-x)}$

$\frac{dx}{x} = \frac{xdx + ydy + dz}{0}$

$xdy + ydy + dz = 0$

$x^2 + y^2 + 2z = c_2$

$F(xy, x^2 + y^2 + 2z)$

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Q:02 Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$

Sol^n $\dfrac{dx}{z^2 - 2yz - y^2} = \dfrac{dy}{xy + zx} = \dfrac{dz}{xy - zx}$

using x, y, z as multipliers

each fraction $= \dfrac{x\,dx + y\,dy + z\,dz}{x\{(z^2 - 2yz - y^2) + y(xy + zx) + z(xy - zx)\}}$

$= \dfrac{x\,dx + y\,dy + z\,dz}{\cancel{xz^2} - \cancel{2yxz} - \cancel{xy^2} + \cancel{xy^2} + \cancel{xyz} + \cancel{xyz} - \cancel{z^2x}}$

$= \dfrac{x\,dx + y\,dy + z\,dz}{0}$

$x\,dx + y\,dy + z\,dz = 0$

Integrate : $\dfrac{x^2}{2} + \dfrac{y^2}{2} + \dfrac{z^2}{2} = c_1$

$x^2 + y^2 + z^2 = 2c_1$ (u)

taking las two fractions :

$\dfrac{dy}{xy + zx} = \dfrac{dz}{xy - zx}$

$\dfrac{dy}{y + z} = \dfrac{dz}{y - z}$

$(y - z)\,dy = (y + z)\,dz$

$ydy - zdy - ydz - zdz = 0$

$ydy - (ydz + zdy) - zdz = 0$

$ydy - d(yz) - zdz = 0$

Integrate $\frac{y^2}{2} - yz - \frac{z^2}{2} = C_2$

$y^2 - 2yz - z^2 = \cancel{2}C_2$

$f(u, v) = 0$

$f(x^2+y^2+z^2,\ y^2-z^2-2yz) = 0$

Que: 03 $p\tan x + q\tan y = \tan z$

Sol^n $\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$

$dx \cot x = dy \cot y = \cot z\, dz$

taking (I) & (II) fraction

$dx \cot x = \cot y\, dy$

Integrate:

$\log \sin x = \log \sin y + \log C_2$

$\frac{\sin x}{\sin y} = C_2$

taking last two fractions:

$dy \cot y = \cot z \, dz$

Integrate

$$\log \sin y = \log \sin z + \log c_2$$

$$\frac{\sin y}{\sin z} = \cancel{\log}\, c_2$$

$$f\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$$

Que:04 $y^2 p - xyq = x(z-2y)$

Sol^n

$$\boxed{p - xyq = x(z-2y)}$$ Subsidiary eq^n

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

taking first two fraction:

$$\frac{dx}{y^2} = \frac{dy}{-xy}$$

$$x\,dx = -y\,dy$$

On integrating: $\frac{x^2}{2} = -\frac{y^2}{2} + c_1$

$$x^2 + y^2 = c_1$$

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Taking last two fractions:

$$\frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

$$(z-2y)\,dy = -y\,dz$$

$$y\,dz + z\,dy - 2y\,dy = 0$$

$$d(yz) - 2y\,dy = 0$$

$\frac{2y^2}{2}$

Integrate

$$yz - y^2 = C_2$$

$$f(x^2+y^2+z^2,\; yz-y^2) = 0$$

Ques 05 $(x^2-y^2-z^2)p + 2xyq = 2xz$

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

Taking last 2 fractions:

$$\frac{dy}{\cancel{2x}y} = \frac{dz}{\cancel{2x}z}$$

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\log y = \log z + \log c_1$$

$$\frac{y}{z} = c_1$$

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using x, y, z multipliers

each fraction = $\dfrac{x\,dx + y\,dy + z\,dz}{x(x^2 - y^2 - z^2) + 2xy^2 + 2xz^2}$

$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 - \underbrace{xy^2} - \underbrace{xz^2} + \underbrace{2xy^2} + \underbrace{2xz^2}}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 + xy^2 + xz^2}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{x(x^2 + y^2 + z^2)}$$

Taking

$$\frac{dy}{2xy} = \frac{x\,dx + y\,dy + z\,dx}{x(x^2 + y^2 + z^2)}$$

$$\frac{dy}{y} = \frac{2(x\,dx + y\,dy + z\,dz)}{x^2 + y^2 + z^2}$$

$$\log y = \log(x^2 + y^2 + z^2) + \log c_2$$

$$c_2 = \frac{y}{x^2 + y^2 + z^2}$$

$$f\left(\frac{y}{z}, \frac{y}{x^2 + y^2 + z^2}\right) = 0$$

Que: 06 $p - q = \log(x+y)$

Sol^n $\underbrace{\frac{dx}{1} = \frac{dy}{-1}} = \frac{dz}{\log(x+y)}$

$dx = -dy$

Integrate

$\boxed{x+y = c_1}$

Taking first & last fractions:

$$\frac{dx}{1} = \frac{dz}{\log(x+y)}$$

$$dx = \frac{dz}{\log c_1}$$

$$(\log c_1)\, dx = dz$$

Integrate: $(\log c_1)\, x = z + c_2$

$$(\log(x+y))\, x = z + c_2$$

$$x \log(x+y) - z = c_2$$

$$f\left(x+y,\ x\log(x+y) - z\right) = 0$$

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Ques 07 $(x^2-yz)p + (y^2-zx)q = (z^2-xy)$

Solⁿ $\frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy}$

each fracⁿ $= \frac{dx+dy+dz}{x^2-yz+y^2-zx+z^2-xy}$

$= \frac{dx+dy+dz}{x^2+y^2+z^2-xy-yz-zx}$ —— (i)

multipliers x, y, z

each fracⁿ $= \frac{x\,dx+y\,dy+z\,dz}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}$ (ii)

(i) = (ii)

$\frac{x\,dx+y\,dy+z\,dz}{(x+y+z)\cancel{(x^2+y^2+z^2-xy-yz-zx)}} = \frac{dx+dy+dz}{x^2+y^2+\cancel{z^2-xy-yz}}$

$\frac{x\,dx+y\,dy+z\,dz}{x+y+z} = \frac{dx+dy+dz}{1}$

$x\,dx+y\,dy+z\,dz = (x+y+z)\,d(x+y+z)$

integrate

$\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2} = \frac{(x+y+z)^2}{2}+C_1$

$xy+yz+zx = C_1$

each fraction : $\dfrac{dx - dy}{(x^2 - yz) - (y^2 - zx)} = \dfrac{dy - dz}{(y^2 - zx) - (z^2 - xy)}$

$$\frac{d(x-y)}{(x-y)(x+y+z)} = \frac{d(y-z)}{(y-z)(x+y+z)}$$

$$\frac{d(x-y)}{(x-y)} = \frac{d(y-z)}{(y-z)}$$

integrate : $\log(x-y) = \log(y-z) + \log c_2$

$$\frac{x-y}{y-z} = c_2$$

$$f(u, v) = 0$$

$$f\left(xy + yz + zx, \frac{x-y}{y-z}\right) = 0$$

Ques 08 $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)}$$

using $\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{z^2}$ as multiplier.

each fraction = $\dfrac{\frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2}}{(y-z) + (z-x) + (x-y)} \to 0$

$$\frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2} = 0$$

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integrate : $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = C_1$ —— (iv)

using $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multiplier

each fraction = $\frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{x(y-z)+y(z-x)+z(x-y) \to 0}$ ⇒

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$$\log x + \log y + \log z = \log C_2$$

$$xyz = C_2 \quad \text{—— (v)}$$

$$f\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z},\ xyz\right) = 0$$

Que: 09 $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$

$$\frac{dx}{x^2(y^2-z^2)} = \frac{dy}{y^2(z^2-x^2)} = \frac{dz}{z^2(x^2-y^2)}$$

use x, y, z as multiplier

$$\frac{xdx + ydy + zdz}{\cancel{x^2y^2} - \cancel{x^2z^2} + \cancel{y^2z^2} - \cancel{x^2y^2} + \cancel{z^2x^2} - \cancel{z^2y^2} \to 0}$$

$$xdx + ydy + zdz = 0$$

integrate

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = C_1$$

$$x^2 + y^2 + z^2 = C_1$$

using $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multipliers

$$\frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{0}$$

Integrate

$$\log x + \log y + \log z = \log C_2$$

$$xyz = C_2$$

$$F(x^2 + y^2 + z^2, xyz) = 0$$