MATHS

ASSIGNMENT - 4

MANAN
2018UIC3087

HEAT EQUATION

Q1) A rod of length 'l' with insulated slides is initial at a temprature $u_0(x)$. Its ends are suddenly cooled to $0°C$ and are kept at that temperature. Prove that the temp fn $u(x,t)$ is given by $u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} e^{-\frac{c^2\pi^2 n^2 t}{l^2}}$ where b_n is determined from the equation $b_n = \frac{2}{l}\int_0^l u_0(x)\sin\frac{n\pi x}{l}\,dx$.

Sol: let $u(x,t) = (A\cos kx + B\sin kx)e^{-k^2c^2t}$ be the general solution of the heat eqn

$$\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial x^2}$$

since the ends $(0=x,\ x=l)$ are cooled to $0°c$ and kept at that temp throughout, we have

$u(0,t) = u(l,t) = 0$ for all t

$u(x,0) = u_0 x$ is the initial condition - ②

From ① & ②

$Ae^{-k^2c^2t} = 0 \quad A = 0$

and $B\sin kl\, e^{-k^2c^2t} = 0 \Rightarrow \sin kl = 0$.

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