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Control System-1

Nyquist Problem

Ques1. A unity feedback system has a loop transfer function $G(s) = 50/(s+1)(s+2)$
Use nyquist criterion to determine the system stability in the closed loop configuration. Is the open loop system stable.

Ans:) $G(s) = 50/(s+1)(s+2)$
Put $s = j\omega$
$$G(j\omega) = 50/(j\omega+1)(j\omega+2)$$

Write the equation for Magnitude $|G(j\omega)|$ and phase angle $\angle G(j\omega)$.

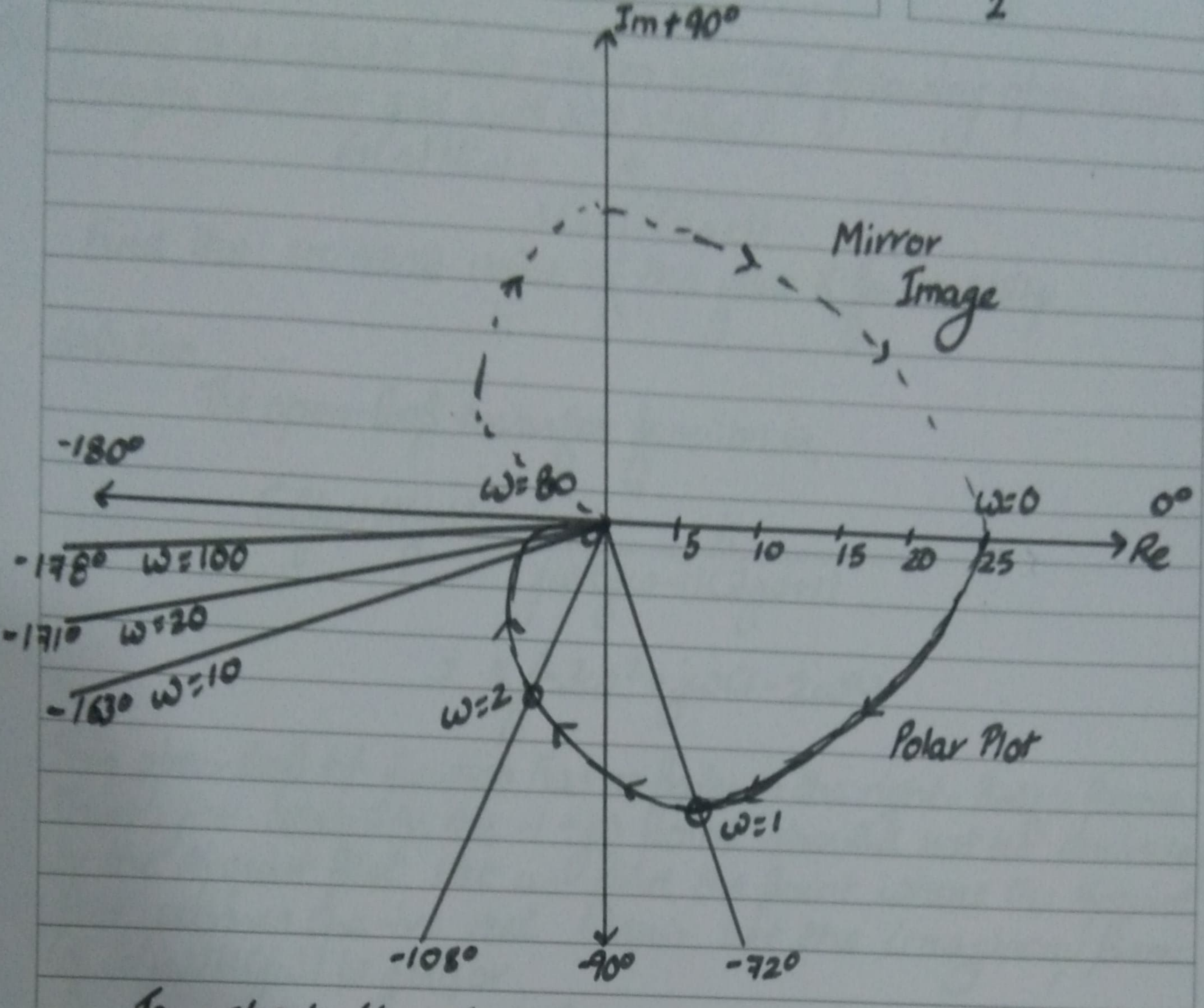
$$M = |G(j\omega)| = 50/\sqrt{1+\omega^2}\sqrt{4+\omega^2} \quad -(1)$$

$$\phi = \angle G(j\omega) = \tan^{-1}(0/50) - \tan^{-1}(\omega/1) - \tan^{-1}(\omega/2)$$

$$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2) \quad -(2)$$

ω varies from 0 to ∞

S.No	ω	$M = \lvert G(j\omega)\rvert$	$\angle G(j\omega) = \phi$
1.	0	25	0°
2.	1	16	-72°
3.	2	8	-108°
4.	10	0.5	-163°
5.	20	0.1	-171°
6.	100	0.005	-178°
7.	∞	0	-180°



To check the stability

$N = P - Z$

N = No of encirclements of point $-1 + j0$

$N = 0$

P = no. of poles of $G(s)H(s)$ that are on the right half of s-plane.

$P = 0$ $G(s) = 50/(s+1)(s+2)$; $s = -1, -2$

$Z = 0$ If $z = 0 \rightarrow$ closed loop system is stable

If $p = 0 \rightarrow$ open loop system is stable

$\left.\begin{matrix} P=0 \\ Z=0 \end{matrix}\right\}$ Open loop & closed loop system stable.

Q2) Below is a closed-loop system with the following open-loop transfer function and with $K=2$ stable?

$$G(s)H(s) = \frac{K}{s(s+1)(2s+1)}$$

Find the critical value of the gain K for stability.

Solution.

The open-loop transfer function is

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega-1)(2j\omega+1)}$$

$$= K / -3\omega^2 + j\omega(1-2\omega^2)$$

This open-loop t.f. function has no poles in the right-half's plane. Thus, for stability, the $-1+j0$ point should not be encircled by the Nyquist plot. Let us find the point where the Nyquist plot crosses the -ive real axis. Let the imaginary part of $G(j\omega)H(j\omega)$ be zero, or

$$1-2\omega^2=0$$

$$\Rightarrow \omega = \pm 1/\sqrt{2}$$

Substitutiong $\omega = 1/\sqrt{2}$ into $G(j\omega)H(j\omega)$, we obtain

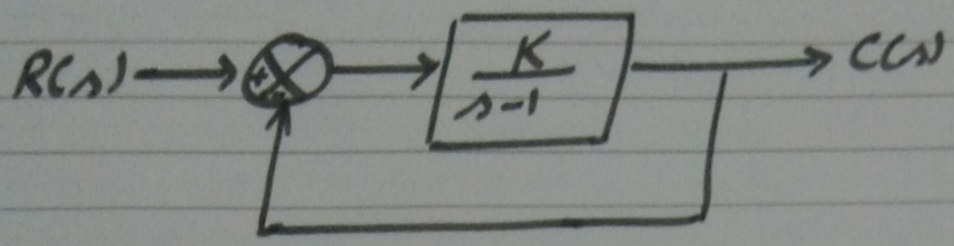
$$G\left(j\frac{1}{\sqrt{2}}\right)H\left(j\frac{1}{\sqrt{2}}\right), \text{we obtain} = \frac{-2K}{3}$$

The critical value of gain K is obtained by equating $-2K/3$ to -1, or

$$\frac{-2}{3}K = -1$$

Hence $K = 3/2 = 1.5$

The system is stable if $0<K<3/2$. Hence, the system with $K=2$ is unstabble



Consider the closed loop system as show.