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* M circles are used to determine the magnitude of CLTF using OLTF.

* Applicable only for unity feedback systems

$$M(j\omega) = G(j\omega) / 1 + G(j\omega)$$

$$G(j\omega) = x + jy$$

$$|M(j\omega)| = \sqrt{x^2 + y^2} / \sqrt{(1+x^2) + y^2}$$

$$M^2(1 + x^2) + M^2 y^2 = x^2 + y^2$$

$$x^2(1 - M^2) + (1 - M^2)y^2 - 2M^2 x = M^2$$

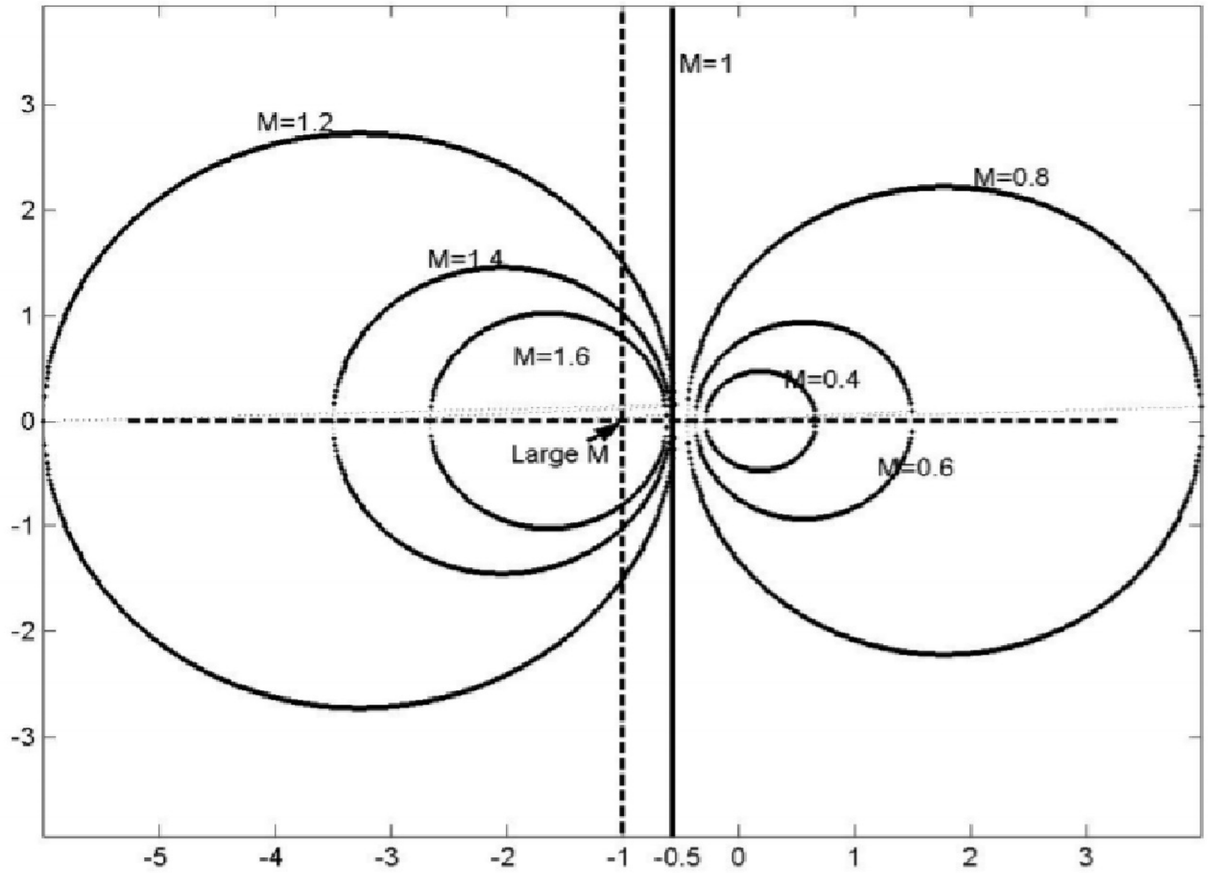
$$x^2 + y^2 - \frac{2M^2 x}{1 - M^2} x = \frac{M^2}{1 - M^2}$$

adding $\left(\frac{M^2}{1-M^2}\right)^2$ in both sides we get

$$\left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \left(\frac{M}{1-M^2}\right)^2$$

The above equation represents a family of circles with its center at $\left(\frac{M}{1-M^2}, 0\right)$ and radii

$\left|\frac{M}{1-M^2}\right|$



M=1
M=1.2
M=0.8
M=1.4
M=1.6
M=0.4
Large M
M=0.6
3
2
1
0
-1
-2
-3
-5
-4
-3
-2
-1
-0.5
0
1
2
3

N Circles-

(Constant phase angles loci)

* N- circles are used to determine the phase response of a closed loop system using open-loop transfer- function.

$$G(j\omega) = x + jy$$

$$C(j\omega)/R(j\omega) = G(j\omega)/1 + G(j\omega)$$

$$= x + jy/1 + x + jy$$

$$\phi = \tan^{-1}(y/x) - \tan^{-1}(y/1+x)$$

Consider tans

$$\tan\phi = \tan(\tan^{-1}(y/x) - \tan^{-1}(y/1+x))$$

$$\tan\phi = \frac{\tan(\tan^{-1}(y/x)) - (\tan^{-1}(y/1+x))\tan}{1 + \tan(\tan^{-1}(y/x))\tan(\tan^{-1}(y/1+x))}$$

$$\tan\phi = \frac{(y/x) - (y/1+x)}{1 + (y/x)(y/1+x)} = y/x^2+x+y^2$$

let $N = \tan\phi$

$$N = y/x^2+x+y^2$$

$$\Rightarrow x^2 + x + y^2 - y/N = 0$$

Add $1/4 + 1/4N$ on both side

$$x^2 + x + \frac{1}{4} + y^2 - y/N + 1/4N = \frac{1}{4} + \frac{1}{4N}$$

$$(x+1/2)^2+(y-1/2N)^2=\left(\sqrt{\frac{1}{4}+\frac{1}{4N}}\right)^2$$

Center $(-1/2,\ 1/2N)$

$$\text{Radious}=\sqrt{\frac{1}{4}+\frac{1}{4N^2}}$$