D.C. MOTOR STUDY

1. OBJECTIVE

To study the torque-speed characteristics and determine the transfer function of a d.c. motor.

2. EQUIPMENT DESCRIPTION

D.C. motors are the most commonly used actuators in electro-mechanical control systems or servomechanisms. Compared to actuators like 2-phase a.c. motor and stepper motor, the d.c. motor has the advantage of higher torque and simpler driving circuit. However the presence of a commutator and a set of brushes with the problems of sparking make the d.c. motor somewhat less durable. This of course is not true for a present day well designed d.c. servomotor.

The study of the dynamic characteristics of the d.c. motor is important because the overall performance of the control system depends on it. A standard analysis procedure is to model the various subsystems and then combine these to develop the model of the overall system.

This experiment is designed to obtain the torque-speed characteristics, compute the various parameter and finally determine the transfer function of a d.c. motor.

The various sections of the unit are described below in some detail.

(a) ***Mechanical Section*** : It comprises of the experimental permanent magnet d.c. motor (approx. 8W) coupled to a small d.c. generator (approx. 2W), which serves the twin purposes of,

- electrical loading of the motor, and
- transient response signal pick-up.

Further, a slotted disk mounted on the common shaft produces 6 pulses per revolution through an opto- interrupter, which is used in a 4-digit speed display in r.p.m.

The specifications of the main experimental d.c. motor are:

- Operating Voltage :12Vdc
- No Load Current: 0.09A
- Full Load Current: 1.0A
- Torque: 30mN-m/ 300g-cm

(b) ***Motor Power Supply*** : The operating voltage of the motor is 12 volt d.c. while the current, depending on loading, is around 120-650 mA. A built-in variable voltage source (2-14Vd.c.) provides this power and two 3½ digit DPMs are available to monitor the armature voltage and armature current of the motor.

(c) ***Transient Response Timing Section*** : When the power is suddenly switched ON, the motor speed increases gradually and finally reaches a steady value. This process takes a few tens of milliseconds and is therefore too slow for a CRO display with repeated ON/OFF of the motor. Although a storage CRO could be used to freeze the transient response and compute the time constant, an alternative using digital circuits is provided in the unit. A 3-digit time count display enables the user to measure the time constant without an expensive storage CRO. This is explained in section 3.3.

(d) ***Power Supplies*** : All the circuits are powered through built-in I.C. regulated power supply of appropriate capacities.

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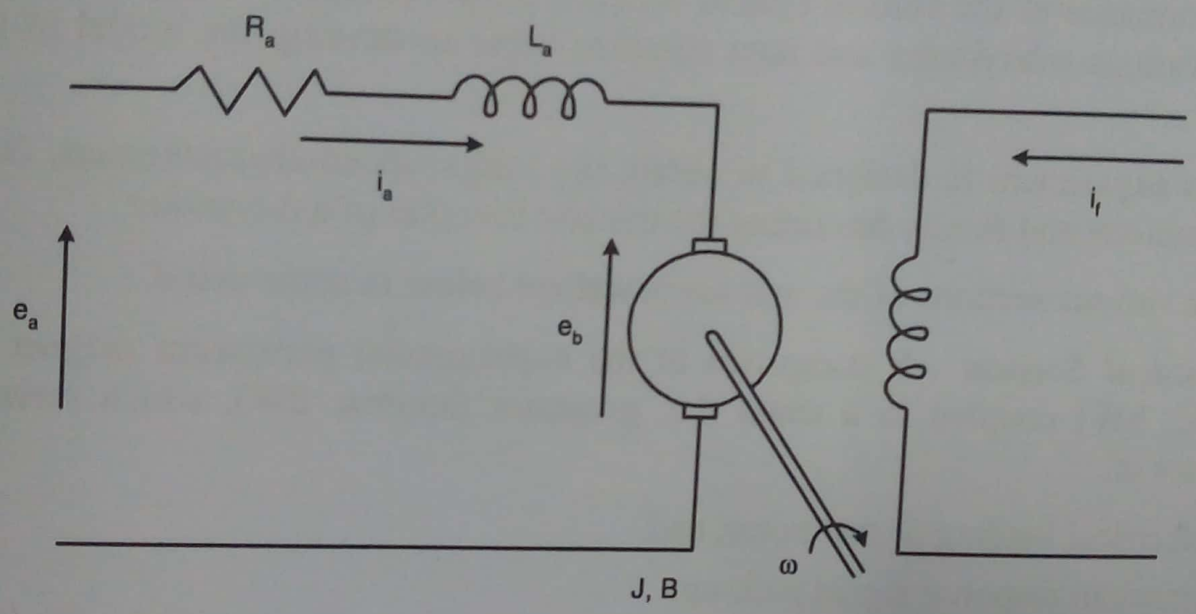

Fig. 1 Schematic diagram

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3. BACKGROUND SUMMARY

3.1 D.C. Motor Model [1]

The schematic diagram of a d.c. motor is shown in Fig.1 wherein the following notations are used

e_a : armature voltage (volts)

i_a : armature current (amp.)

R_a : armature resistance (ohms)

L_a : armature inductance (henrys)

e_b : back emf (volts)

i_f : field current (amp.)

T_M : motor torque (newton-m)

T_L : load torque (newton-m)

ω : angular velocity (rad/sec)

J : moment of inertia of the rotor including external loading if any (newton-m/rad/sec^2)

B : viscous friction coefficient including external loading if any (newton-m/rad/sec)

Upper case notations E_a, I_a, E_b, I_f are used for steady state values of the respective variables e_a, i_a, e_b and i_f

In the present set-up a permanent magnet d.c. motor is used, the field winding is thus absent and the air gap flux is constant. The input drive may therefore be applied to the armature only, that is, only armature controlled operation is possible.

The mathematical equations in this operating mode are,

$T_M = K_T\, i_a$; $\quad$ K_T : torque constant

$e_b = K_b \omega$; $\quad$ K_b : back emf constant

$L_a \dfrac{di_a}{dt} + R_a i_a + e_b = e_a$; $\quad$ armature circuit model

$J \dfrac{d\omega}{dt} + B\omega + T_L = T_M$; $\quad$ mechanical model

Taking Laplace Transform and rearranging the terms,

$$\frac{\omega(s)}{E_a(s)} = \frac{K_T}{(sL_a + R_a)(sJ + B) + K_T K_b}$$

Assuming the inductance of the armature circuit to be very small*, the motor transfer function may be written as,

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* For the motor used $R_a \cong 4\Omega$ and $L_a \cong 2.9$mH. Thus even operating at 10 Hz, $\omega L_a = 0.182$, which can be neglected in comparison to R_a.

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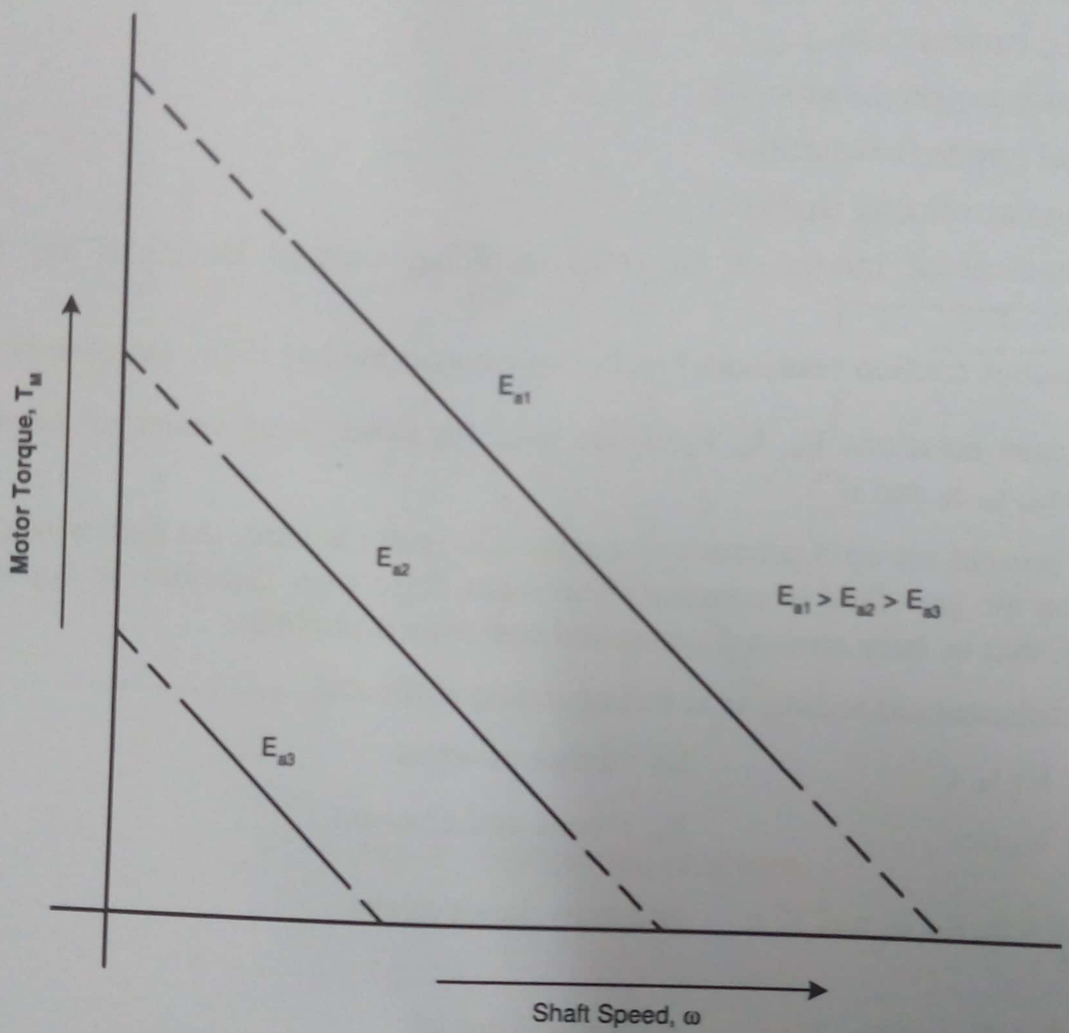

Fig. 2 Typical torque - speed curve

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$$G_M(s) \triangleq \frac{\omega(s)}{E_a(s)} \approx \frac{K_T/R_a}{Js+B+\frac{K_T K_b}{R_a}} = \frac{K_M}{s\tau_m+1} \quad \text{... (1)}$$

where,

$$K_M = \frac{K_T}{R_a B + K_T K_b} \quad : \text{ Motor gain constant}$$

$$\tau_m = \frac{R_a J}{R_a B + K_T K_b} \quad : \text{ Motor time constant}$$

The armature controlled motor therefore has a first order type-0 transfer function as shown in eqn. (1) and the two constant K_M and τ_m depend upon motor parameters and external loading, if any.

If the transfer function is defined with respect to shaft position (θ), rather than velocity (ω=dθ/dt), the transfer function may be written as,

$$G'_M(s) \triangleq \frac{\theta(s)}{E(s)} = \frac{K_M}{s(s\tau_m+1)}$$

This form would be appropriate if one is interested in shaft position as the output, e.g., in a position control system like the D.C. Position Control, DCP-01 manufactured by M/s Techno Instruments, Roorkee.

3.2 Torque – Speed Curves

As a mechanical actuator the magnitude of the steady state torque produced by the motor with a given armature voltage is of interest to an user. With a simple rearrangement of terms the motor torque may be written as,

steady state armature current, $I_a = \frac{E_a - E_b}{R_a} = \frac{E_a}{R_a} - \frac{K_b\omega}{R_a}$

steady state torque generated, $T_M = K_T\ I_a = -\frac{K_T K_b}{R_a}\cdot\omega + \frac{K_T}{R_a}\cdot E_a$ ... (2)

Here T_M, E_a, E_b, I_a and ω are the steady state values of the motor torque, applied armature voltage, back emf, armature current and angular velocity of the shaft.

A typical plot of the above equation is shown in Fig. 2. This assumes a linear torque-speed behaviour as indicated in eqn. (2). A practical d.c. motor/d.c. servomotor will show nonlinearity to varying extent depending upon the design and manufacturing aspects. Since a linear model (transfer function) is used in the current experiment, salient features may be studied using any good quality d.c. motor. The plotting of Fig. 2 from experimental data is a little involved because of the difficulty in experimentally determining K_T. A relationship between K_b and K_T developed here makes the task simpler.

As the motor runs at constant speed,

Electrical power input, $P_{in} = E_a \times I_a$ watts

Power lost in $R_a = R_a \times I_a \times I_a$

Power available in the armature, $P_{arm} = (E_a - I_a R_a)\, I_a$

$= E_b \times I_a$

$= K_b \times \omega \times I_a$

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Mechanical power developed, $P_{mech.} = T_M \times \omega$ newton-m rad/sec

$= K_T \times I_a \times \omega$

Assuming 100% conversion of power from electrical input to mechanical output, the above two expressions can be equated to get

$$K_b \left(\frac{\text{volts}}{\text{rad / sec}}\right) = K_T \left(\frac{\text{newton} - \text{m}}{\text{amp.}}\right)$$

Thus, the numerical values of K_T and K_b may be assumed to be identical.

The torque may then be expressed as,

$$T_M = -\frac{K_b^2}{R_a} \cdot \omega + \frac{K_b}{R_a} \cdot E_a \qquad \text{... (3)}$$

This equation may be obtained experimentally with ease since it is very simple to determine K_b.

When the motor is loaded, the speed decreases which reduces the back emf. This increases armature current i_a so that the motor develops more torque in order to supply the load. The operation for a constant voltage E_a is represented as in a straight line in Fig. 2.

At steady state (ω = constant) the load torque equation must read as

$$T_M = B\omega + T_L, \qquad T_L \; : \; \text{load torque} \qquad \text{... (4)}$$

In the experimental work T_L is increased in steps by loading the motor with the help of the coupled generator and the values of T_M and ω are recorded. While ω is computed from the speed N, in rpm, as displayed on the motor unit, the following expression is used to compute the motor torque T_M at a constant value of E_a,

$$T_M = K_T I_a = K_b I_a = \frac{E_b}{\omega} \cdot I_a = \frac{E_a - I_a R_a}{\omega} \cdot I_a \qquad \text{... (5)}$$

This is essentially same as (3) while avoiding the explicit computation of K_b.

Note that equations (3) and (4) both give the variation of T_M with ω for a constant armature voltage and are therefore basically the same, subject to the assumption that $K_T = K_b$. In the experiment however eqn. (4) is not used since it involves measurement of load torque T_L.

The value of B, coefficient of viscous friction, may be seen as the negative of the slope of torque speed curve eqn. (4) and K_b may be computed from the expression

$$K_b = \frac{E_b}{\omega} = \frac{E_a - I_a R_a}{\omega}$$

Two motor parameters, B and K_b, may therefore be determined from the Torque-Speed Characteristics obtained under steady state conditions or constant speed operation of the motor

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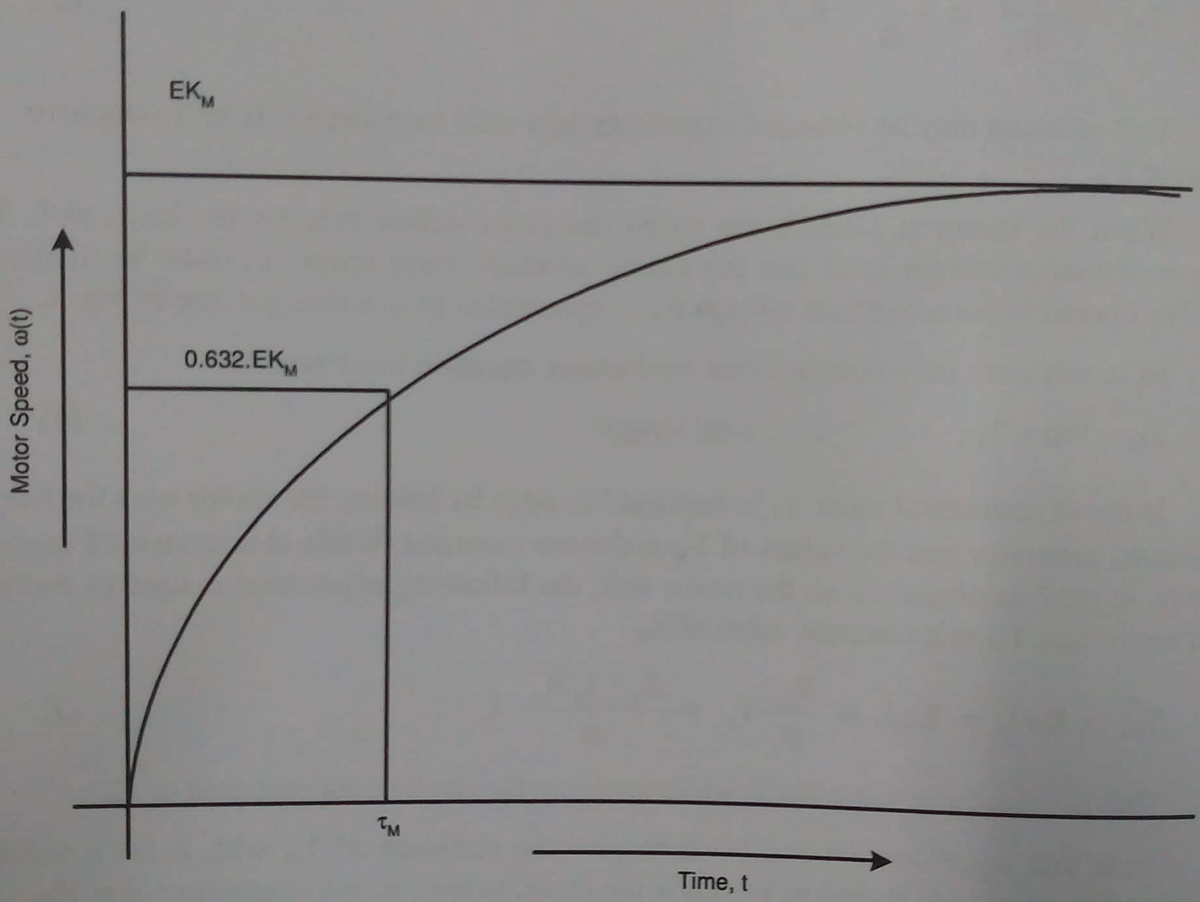

Fig. 3 Step response of the motor

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3.3 Transient Response

The transfer function of the motor was obtained in eqn.(1) as,

$$G_M(s) = \frac{\omega(s)}{E_a(s)} = \frac{K_T/R_a}{Js + B + \frac{K_T K_b}{R_a}} = \frac{K_M}{s\tau_m + 1} \qquad \ldots (6)$$

where,

$$K_M = \frac{K_T}{R_a B + K_T K_b} \quad : \text{Motor gain constant}$$

$$\tau_m = \frac{R_a J}{R_a B + K_T K_b} \quad : \text{Motor time constant}$$

In response to a step input, $e_a(t) = E.u(t)$, i.e. $E_a(s) = \frac{1}{s} \cdot E$, the motor speed will follow the expression,

$$\omega(t) = E.K_M\left(1 - e^{-\frac{t}{\tau_m}}\right)$$, as shown in Fig. 3

The step response is very similar to that of an RC circuit charging from a step voltage input. The parameters of interest, viz., EK_M and τ_m are indicated in Fig. 3. One can easily measure the steady state speed, N, and hence compute K_M,

$$\omega(t)\big|_{t\to\infty} = \omega_{ss} = E\,K_M$$

$$\text{or, } K_M = \frac{N}{E}\ \text{rpm/volt} = \frac{\pi N}{30E_a}\ \frac{\text{rad/sec}}{\text{volt}}$$

Measurement of τ_m, the time taken by the motor speed to rise from zero and attain 63.2% of ω_{ss}, is a little difficult because of the following facts:

(a) The motor being a mechanical system, takes a long time (approx. 500 msec.) to reach near ω_{ss}. If a steady trace on the CRO is to be displayed, the motor must be switched ON and OFF at around 1Hz. This is too low a frequency for convenient viewing.

(b) Even if the above scheme was possible, the time constants during ON and OFF would be different, since the back emf would be absent in the latter case (armature circuit is disconnected). The switching frequency will then need to be still lower.

It is of course possible to use a storage CRO or pen recorder to freeze the transient and make measurements at a later time. In the present unit, a class room experiment, a 3-digit timer has been provided which gives the time elapsed in milli seconds between starting the motor and reaching a preset speed, which would be set to 63.2% of the final speed. This then directly gives the time constant value with a least count of 1 msec.

From the motor time constant τ_m obtained experimentally, the value of coefficient of inertia (J) may be computed using eqn. (1) as,

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$$J = \tau_m \left(B + \frac{K_b^2}{R_a} \right) \qquad \ldots(7)$$

Also an explicit expression for the motor transfer function may than be written.

The transient response study gives the value of K_M and τ_m. All the motor parameters and its transfer function may then be calculated using these and the parameters obtained in section 3.2.

4. EXPERIMENTAL WORK

This section deals with the details of the suggested experimental work, typical results and calculations. It must be emphasized here that the typical results given below are based on the measurements made on an unit randomly picked up from our assembly line. Although these results are indicative of the general characteristics, they cannot be expected to be exactly duplicated on other units.

4.1 Armature Circuit Parameters

R_a and L_a are the two parameters in the armature circuit, which may be measured by any standard method. Although R_a is required for calculations in the next section, L_a has been neglected as explained in sec 3.1.

4.2 Motor and Generator Characteristics

At no load (load step at 0) the motor is supplied with varying armature voltages, E_a = 3, 4, 5, ...12. For each E_a, the motor current I_a, speed N, and generator voltage E_g are recorded. Straight line approximation of the E_a vs. speed and E_g vs. speed yield the motor and generator constants K_M (rpm/volt) and K_G (volts/rpm). Referring to the panel diagram on page 11, the following steps are suggested :

- Set 'MOTOR' switch to 'ON'. Set 'RESET' switch to 'RESET'. Set 'LOAD' switch to 0 position.
- Vary E_a in small steps and take readings as under (Table 1)

Table -1

S.No.	E_a, volts	I_a, amp.	N, rpm	E_g, volts
1.	3			
2.	4			
3.				
4.				

- Plot N vs. E_a and E_g vs. N. Obtain the slopes and compute K_M and K_G. {Ref. Fig. 5(a) and 5(b)}.

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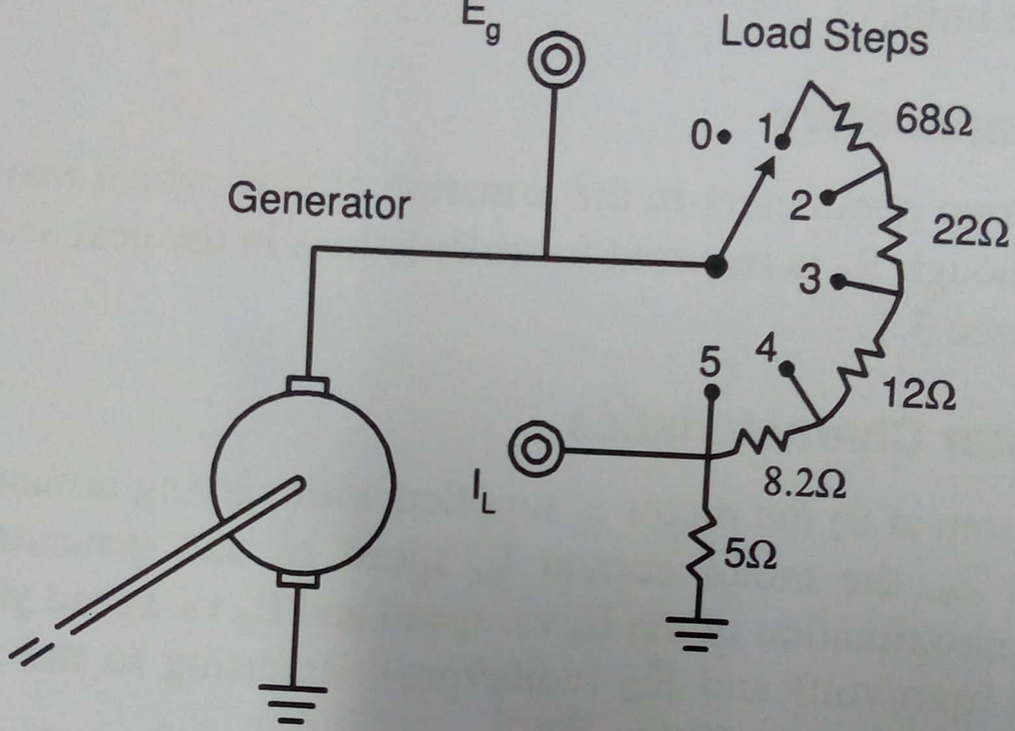

Fig. 4 Loading system Schematic

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4.3 Torque Speed Characteristics

To obtain the torque-speed characteristics, the motor is supplied with a fixed armature voltage and its speed is recorded for varying external loading (Fig. 4). This loading is effected by electrically loading the coupled generator. Referring to the panel diagram on page 11, following steps are suggested:

- Set 'MOTOR' switch to 'OFF'. Set 'RESET' switch to 'RESET'. Set 'LOAD' switch to 0 position.
- Connect E_a to the voltmeter and set $E_a = 6V$
- Shift the 'MOTOR' switch to 'ON'. Measure armature input (E_a), motor current (I_a) and motor speed in rpm. Record the readings (at S. No. 1 in the Table-2)
- Set the 'LOAD' switch to 1, 2, …5 and take readings as above. (S.Nos. 2, 3, … in the Table 2)
- Complete the table below with the calculated values
  Motor Voltage $E_a = 6$ volts; $R_a = 4\Omega$

Table – 2

S.No.	Load Step	I_a, mA	N, rpm	$\omega = \frac{2\pi N}{60} = \frac{N\pi}{30}$ rad/sec.	$E_b = E_a - I_aR_a$ Volts	$K_b = \frac{E_b}{\omega}$	$T_M = K_bI_a$ newton-m
1.	0						
2.	1						
3.	2						
4.	3						
5.	4						
6.	5						

- Plot Torque vs. Speed curves on a graph paper (approximated straight line plots)
- Compute B from the slope of Torque-Speed curve and average K_b from the table [Fig. 6].
- Repeat above for $E_a = 8$, $E_a = 10$, $E_a = 12$ and record the average values of motor parameters B and K_b.

4.4 Step Response

The dynamics of the motor is studied with the help of its step response. The various steps of this experiment are given below.

- Set 'MOTOR' switch to 'OFF'. Set 'RESET' switch to 'RESET'. Set 'LOAD' switch to 0 position.
- Connect E_a to the voltmeter and set it to 8V.
- Switch 'ON' the motor and measure E_g and the speed in rpm. These are the steady state generator voltage E_g and steady state motor speed N, respectively
- Set E_s to 63.2% of E_g measured above. This is the generator voltage at which the counter will stop counting.
- Switch 'OFF' the motor. Set 'RESET' switch to 'READY'.

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- Now switch the motor 'ON'. Record the counter reading as time constant in milliseconds.
- Repeat above with E_a = 10 V, E_a= 12 V and tabulate the results as shown below in Table 3.

Table - 3

S.No.	E_a, volts	E_g, volts	N, rpm	$E_s = 0.632.E_g$ volts	Time Constant τ_m msec	Gain Constant, $K_M = \frac{\pi N}{30E_a}$
1.						
2.						
3.						

- Substitute the values of K_M and τ_m in eqn. (6) and write down the motor transfer function.
- Using the average values of τ_m, B, K_b and R_a, calculate the motor inertia from eq. (7),

$$J = \tau_m \left(B + \frac{K_b^2}{R_a} \right)$$

4.5 Additional Experimentation

- It is also possible to get the data points for plotting the complete step response by setting $E_s = 0.1\ E_g, 0.2\ E_g, \ldots\ldots.0.9\ E_g$, and obtaining the time count to reach these values.
- Obtain τ_m at E_a = 10 volts with load switch set to 3. Note down the modified time constant and justify.
- The socket marked I_L on the motor unit may be used to measure the generator load current in terms of the voltage drop across a 5Ω resistance. This alongwith the open circuit generator voltage measurement, may be used to compute the actual electrical loading in watts and hence T_L. Further calculations may be made to establish the validity of the assumption, $K_b = K_T$, within reasonable experimental errors.

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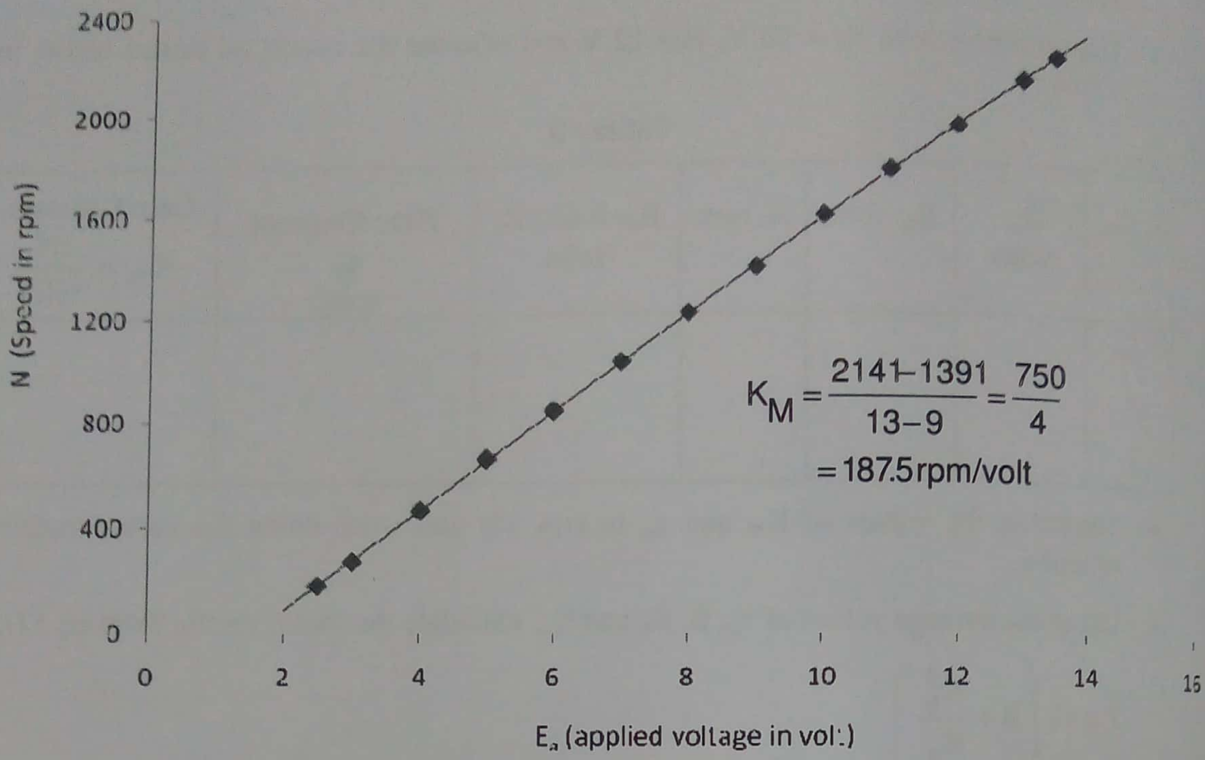

Fig 5(a) Motor Characteristics

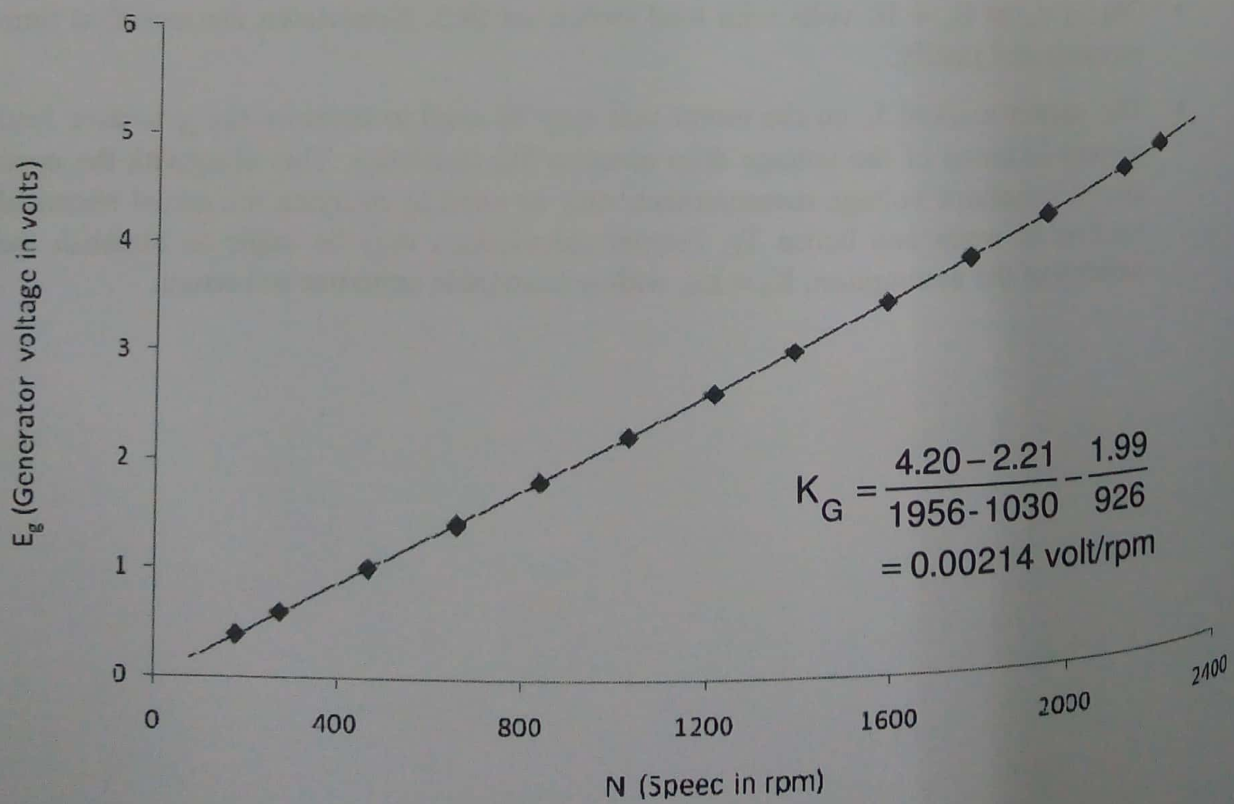

Fig 5(b) Generator Characteristics

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5. RESULTS

Results obtained on a typical unit are given in Table 4 below for guidance only. Actual readings may vary from unit to unit.

(a) Armature Resistance, $R_a = 10.0\Omega$, Armature Inductance = 14.65 mH

(b) Motor and Generator Characteristics

S.No.	E_a, volts	I_a, mA	N, rpm	E_g, volts
1.	2.5	100	181	0.38
2.	3.0	100	275	0.59
3.	4.0	97	466	1.00
4.	5.0	97	658	1.40
5.	6.0	97	839	1.80
6.	7.0	97	1030	2.21
7.	8.0	97	1217	2.61
8.	9.0	97	1391	2.99
9.	10.0	97	1594	3.43
10.	11.0	97	1777	3.82
11.	12.0	96	1956	4.20
12.	13.0	96	2141	4.60
13.	13.51	95	2236	4.81

From the plots of Fig 5(a) and 5(b), average values of motor and generator constants are:

K_M = 187.5 rpm/volt

K_G = 0.00214 volt/rpm

(c) Torque-Speed Characteristics

$E_a = 10V$, $R_a = 10.0\Omega$

S.No.	Load Step	I_a, mA	N, rpm	$\omega = \frac{2\pi N}{60} = \frac{N\pi}{30}$ rad/sec.	$E_b = E_a - I_aR_a$ volts	$K_b = \frac{E_b}{\omega}$	$T_M = K_bI_a$ newton-m
1.	0	94	1614	168.9	9.06	0.05364	0.005042
2.	1	106	1580	165.3	8.94	0.05408	0.005732
3.	2	119	1543	161.5	8.81	0.05455	0.006491
4.	3	131	1505	157.5	8.69	0.05517	0.007227
5.	4	146	1456	152.3	8.54	0.05607	0.008186
6.	5	166	1396	146.1	8.34	0.05708	0.009475

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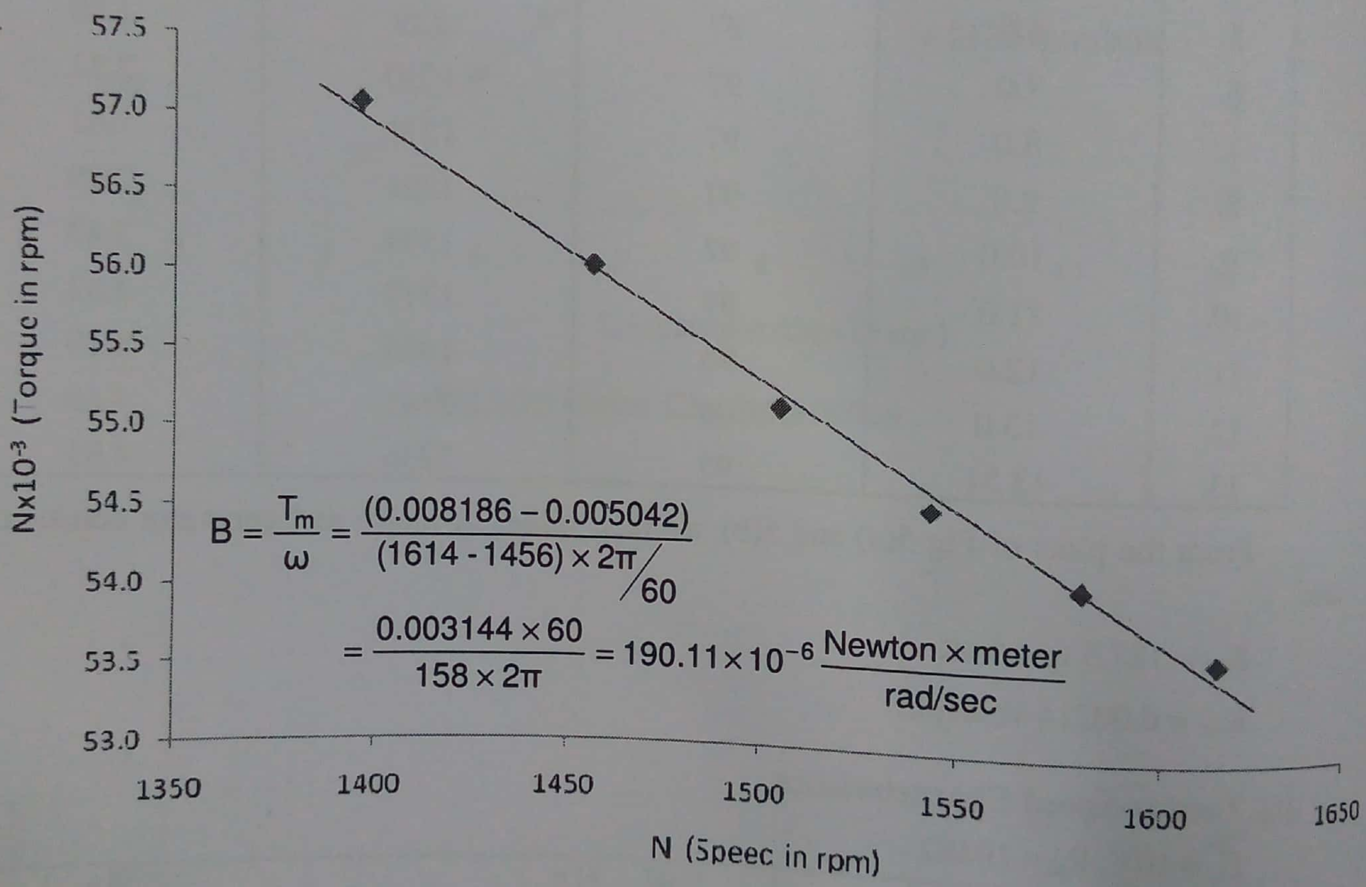

$$B = \frac{T_m}{\omega} = \frac{(0.008186 - 0.005042)}{(1614 - 1456) \times 2\pi / 60}$$

$$= \frac{0.003144 \times 60}{158 \times 2\pi} = 190.11 \times 10^{-6} \frac{\text{Newton} \times \text{meter}}{\text{rad/sec}}$$

Fig 6 Torque Speed Plot

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Average $K_b = 55.09 \times 10^{-3} \dfrac{\text{volts}}{\text{rad/sec}}$

(From the slope of the curve) Fig. 6, coefficient of viscous friction,

$B = 190.11 \times 10^{-6} \dfrac{\text{newton} - \text{m}}{\text{rad/sec}}$

(d) Step response study $E_a = 10V$, $E_g = 3.46$ volts, and, $\tau_m = 56$ msec

S.No.	E_a, volts	E_g, volts	N, rpm	$E_s = 0.632.E_g$ volts	Time Constant τ_m msec	Gain Constant, $K_M = \dfrac{\pi N}{30 E_a}$
1.	8					
2.	10	3.46V	1610	2.18V	56msec	16.85
3.	12					

$E_s = 0.632 \times 3.46 = 2.18$ V and $K_M = \dfrac{\pi}{30} \cdot \dfrac{N}{E_a} = \dfrac{\pi}{30} \cdot \dfrac{1610}{10} = 16.85$ rad/sec.

Thus, $J = \tau_m \left(B + \dfrac{K_b^2}{R_a} \right)$

$$= 0.056 \left[190.11 \times 10^{-6} + \frac{(55.09 \times 10^{-3})^2}{10} \right]$$

$$= 10.18 \times 10^{-6} \frac{\text{N.m}}{\text{rad/sec}^2}$$

and motor transfer function is

$$G(s) = \frac{\omega(s)}{E(s)} = \frac{K_M}{s\tau_m + 1} = \frac{16.850}{0.056s + 1} = \frac{300.89}{s + 17.85}$$

6. REFERENCES

[1] Control system Engineering - I.J. Nagrath and M. Gopal, Wiley Eastern Limited.

[2] Modern Control Engineering - K. Ogata, Prentice Hall of India Pvt. Ltd.

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