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Q) Find solution of

$$\frac{\delta^2 u}{\delta x^2} = h^2 \frac{\delta u}{\delta t}$$

for which $u(0,t) = u(l,t) = 0$ $u(x,0) = \sin\frac{\pi x}{l}$ by the method of separation.

$$\frac{\delta u}{\delta t} = \frac{1}{h^2}\frac{\delta^2 u}{\delta x^2}$$

its solution is

$$u = (c_1 \cos px + c_2 \sin px)\, c_3 e^{-\frac{p^2 t}{h^2}} \quad -(1)$$

Put $x=0$, $u=0$ in (1) we have

$$0 = c_1 c_3 e^{-\frac{p^2 t}{h^2}} \Rightarrow c_1 = \infty$$

Put $c_1 = 0$ in (1)

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$$u = c_2 c_3 \sin px\, e^{-\frac{p^2 t}{h^2}} \quad -(2)$$

Put $x = l$, $u=0$ in (2) we have

$$0 = c_2 c_3 \sin pl\, e^{-\frac{p^2 t}{h^2}}$$

$$pl = n\pi \qquad \boxed{p = \frac{n\pi}{l}}$$

Then 2 becomes

$$u = b_n \sin\left[\frac{n\pi x}{l}\right] e^{-\frac{n^2\pi^2 t}{l^2 h^2}} \qquad (b_n = c_2 c_3)$$

General Equation

$$u = \sum_{n=1}^{\infty} b_n \sin\left[\frac{n\pi x}{l}\right]$$

$$\sin\frac{\pi x}{l} = b_1 \sin\left(\frac{\pi x}{l}\right) + b_2 \sin\left(\frac{2\pi x}{l}\right)$$

$$b_1 = 1$$

Hence 3 becomes

$$u = \sin\left[\frac{\pi x}{l}\right] e^{\frac{\pi^2 t}{l^2 h^2}}$$