Process Dynamics & Control

For GATE-2019

Chemical Engineering

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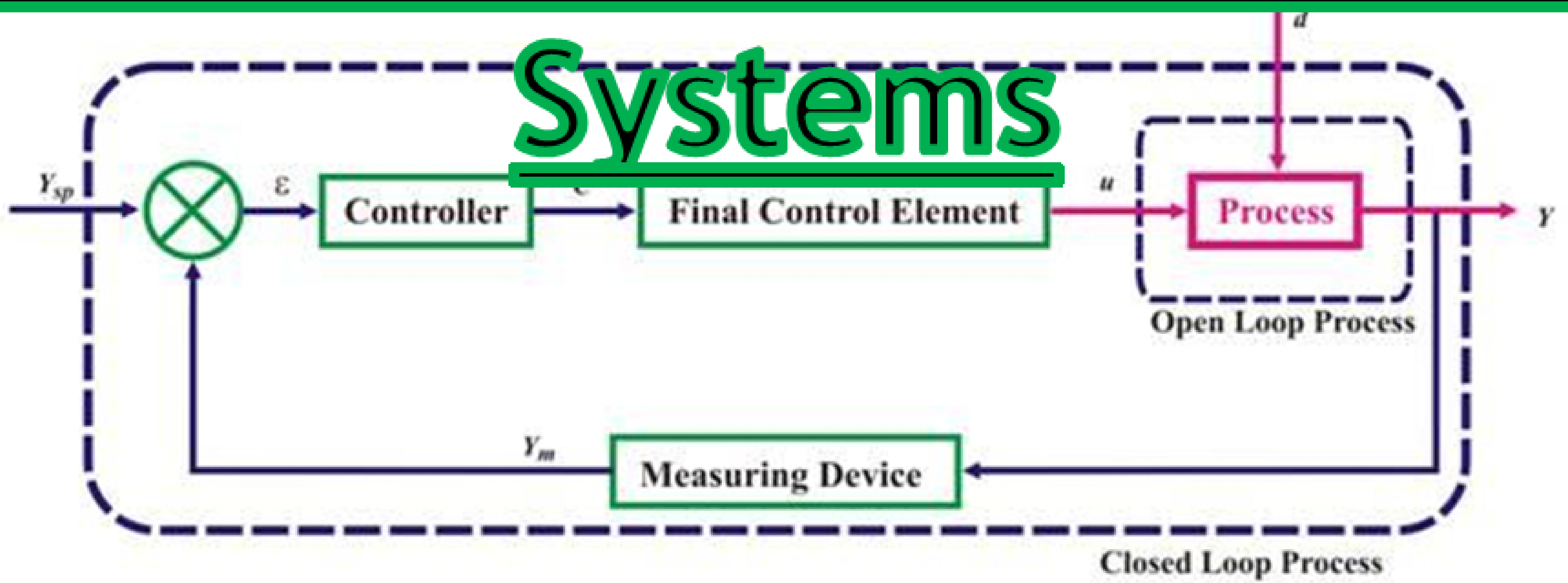
Process Dynamics & Control

Lesson 10: Sinusoidal Input Response of a First Order Systems



#My Courses on Unacademy

- Process Calculations for GATE (Chemical Engineering)-2019
- Preparation Strategy for GATE (Chemical Engineering)-2019 with most important topics.
- Heat Exchangers
- Radiation Heat Transfer for GATE-2019 exam.
- Transportation and Metering of Fluids for PSU Interviews -2018.
- Non-Ideal Reactors for GATE-2019.
- Mass Transfer Equipment for PSU Interviews -2018.
- Chemical Reaction Engineering- Part 1
- How to get Best Rank in GATE 2019 Chemical Engineering

Target Audience

All undergraduate Chemical Engineering Students

GATE- (Chemical Engineering) aspirants

Sinusoidal Response :-

$$Y(s) = \left(\frac{K_p}{\tau_p s+1}\right)\left(\frac{A\omega}{s^2+\omega^2}\right)$$

Doing partial fractions,

$$\frac{A\omega K_p}{(\tau_p s+1)(s^2+\omega^2)} = \frac{B}{\tau_p s+1} + \frac{Cs+D}{s^2+\omega^2}$$

where, B, C & D are constants.

from RHS,

$$Bs^2+B\omega^2+Cs^2\tau_p+Cs+D\tau_p s+D.$$

$$\Rightarrow s^2(B+C\tau_p)+s(C+D\tau_p)+(D+B\omega^2)$$

Comparing coefficients, we get,

$$B = \frac{A K_p \omega \tau_p^2}{1+\omega^2\tau_p^2} \; ; \; C = \frac{-A\omega K_p \tau_p}{1+\omega^2\tau_p^2} \; ; \; D = \frac{A\omega K_p}{1+\omega^2\tau_p^2}$$

$$Y(s) = \frac{\omega A K_p \tau_p^2}{(1+\omega^2\tau_p^2)(\tau_p s+1)} + \frac{(-A\omega K_p \tau_p s + A\omega K_p)}{(1+\omega^2\tau_p^2)(s^2+\omega^2)}$$

$$= \frac{A\omega K_p}{1+\omega^2\tau_p^2}\left[\frac{\tau_p^2}{\tau_p s+1} - \frac{\tau_p s}{s^2+\omega^2} + \frac{1}{s^2+\omega^2}\right]$$

ILT $\downarrow$

$$Y(t) = \frac{A\omega K_p \tau_p}{(1+\omega^2\tau_p^2)} e^{-t/\tau_p} - \frac{A\omega K_p \tau_p}{1+\omega^2\tau_p^2}\cos\omega t + \frac{A\omega K_p}{1+\omega^2\tau_p^2}\sin\omega t$$

After sometime, the e^{-t/τ_p} factor decays & we have,

$$Y(t) = \underset{\uparrow}{a} \cos\omega t + \underset{\uparrow}{b} \sin\omega t.$$

$$a = \frac{-A\omega\tau_p}{1+\tau_p^2\omega^2} \qquad b = \frac{A}{\tau_p^2\omega^2+1} \qquad \{\text{for } K_p = 1\}.$$

To get one term from above equation :-

Let $a = r\sin\phi$

$b = r\cos\phi$.

$$Y(t) = r\sin(\theta + \phi) \qquad \theta \to \omega t$$

$$r = \sqrt{a^2+b^2} \qquad \phi = \tan^{-1}(a/b)$$

$$r = \frac{A\,K_p}{\sqrt{1+\tau_p^2\omega^2}} \quad ; \quad \phi = \tan^{-1}(-\omega\tau_p)$$

$$X(t) = A \sin(\omega t - 0)$$ magnitude of input

$$\boxed{Y(t) = \frac{A\,K_p}{\sqrt{1+\tau_p^2\omega^2}} \sin\left[\omega t + \tan^{-1}(-\omega\tau_p)\right]}$$

- output magnitude is always less than the input magnitude for any value of ω & τ_p.
- Input is ahead in phase or output lags behind the input by time $\tan^{-1}(\omega\tau_p)$

Phase Difference : $\tan^{-1}(-\omega\tau_p)$

Phase Lag : $\tan^{-1}(\omega\tau_p)$

Amplitude Ratio = $\dfrac{\text{output Amplitude}}{\text{input Amplitude}}$.

$$AR = \frac{K_p}{\sqrt{1+\omega^2\tau_p^2}}$$

AR < 1 (always) → for first order systems

→ Attenuated (means system whose output amplitude is less than the input magnitude).

Range of ϕ

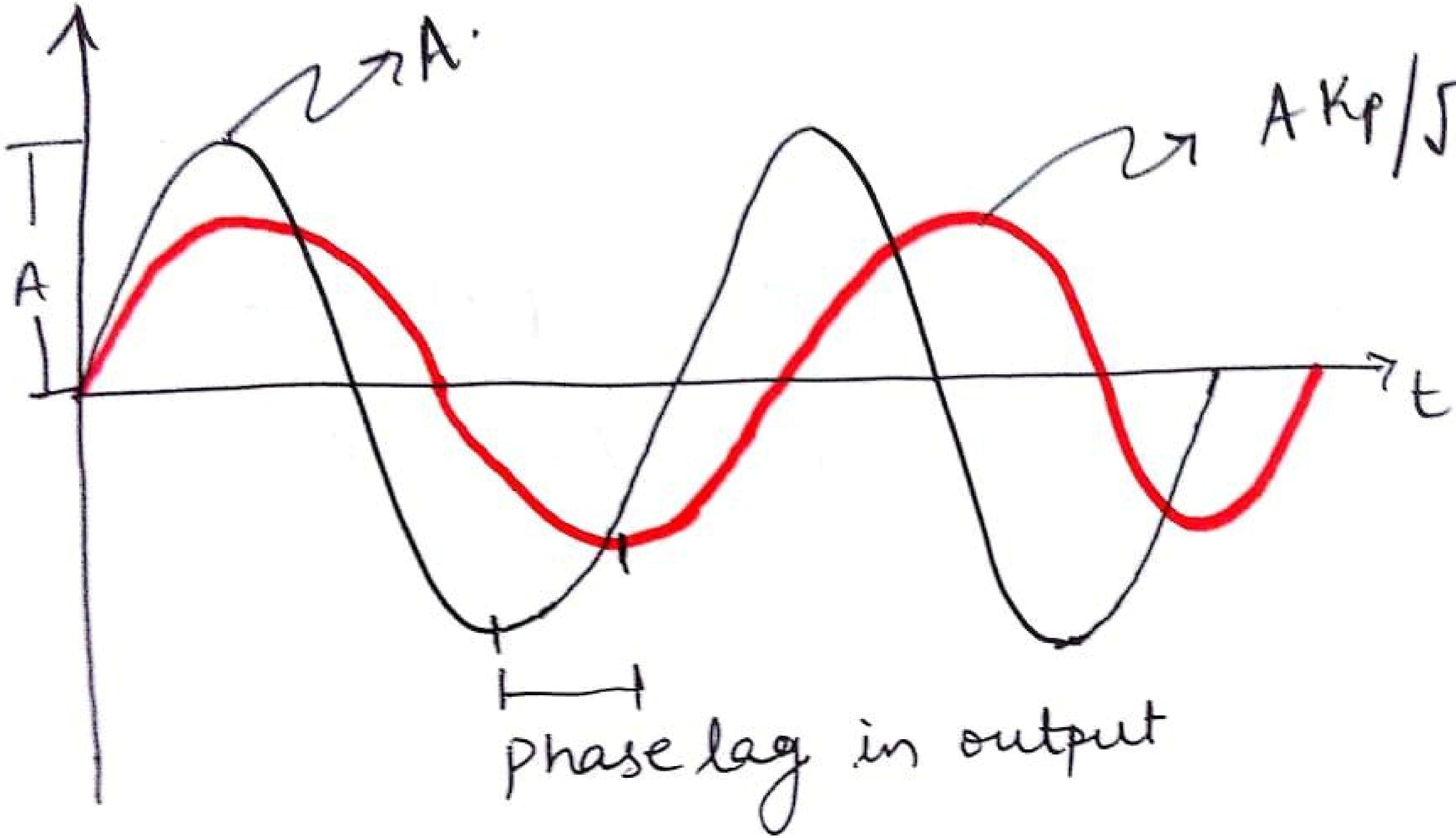
$$\phi \rightarrow (0, -\pi/2)$$

Response is always in degree.

$$\& \quad \tan^{-1}(\underbrace{-\omega\tau_p})$$

Domain of trigonometric inverse is always in radian.

Ex → $\tan^{-1}(-\infty) = -\pi/2$



A
A
$AK_p/\sqrt{1+\tau_p^2\omega^2}$
t
phase lag in output

Thanks!

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Any questions?