Gaurav Gupta 2018UIC3093 ICE-2

M-Circles (constant magnitude Loci)

* M-Circles are used to determine the magnitude of C.L.T.F using OLTF.
* Applicable only for unity feedback systems.

$$M(j\omega) = G(j\omega)/1 + G(j\omega)$$

$$G(j\omega) = x + jy$$

$$|M(j\omega)| = \sqrt{x^2+y^2}/\sqrt{(1+x)^2+y^2}$$

$$M^2(1+x)^2 + M^2y^2 = x^2+y^2$$

$$x^2(1-M^2) + (1-M^2)y^2 - 2M^2x = M^2$$

$$x^2+y^2 - \frac{2M^2}{1-M^2}x = \frac{M^2}{1-M^2}$$

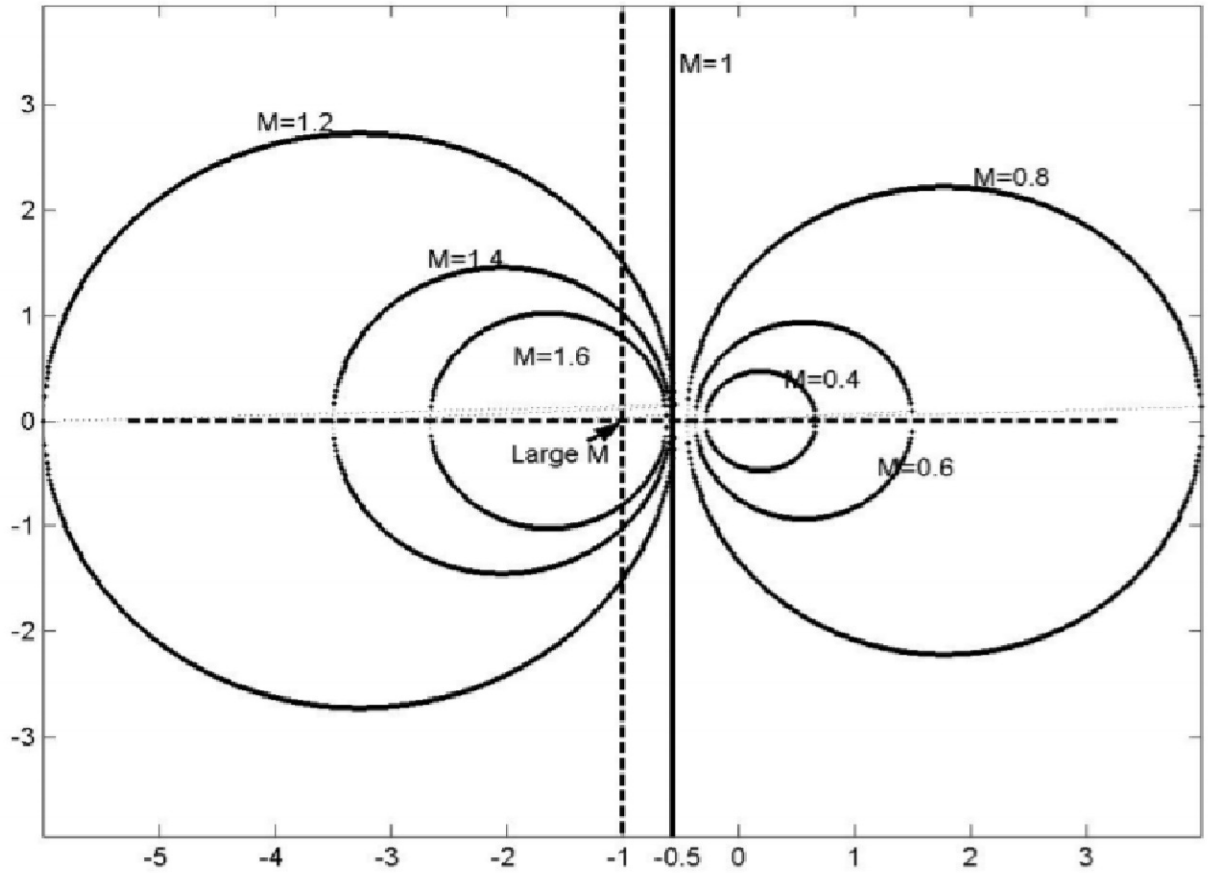
adding $\left(\frac{M^2}{1-M^2}\right)^2$ in both sides, we get

$$\left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \left(\frac{M}{1-M^2}\right)^2$$

The above equation represents a family of circles with its center at $\left(\frac{M^2}{1-M^2}, 0\right)$ and radius $\left|\frac{M}{1-M^2}\right|$.

* Family of M-Circles -

Family of M-circles corresponding to the close loop magnitudes (M) of a unit feedback system, constant M-circles for unity feedback system.



M=1
M=1.2
M=0.8
M=1.4
M=1.6
M=0.4
Large M
M=0.6
3
2
1
0
-1
-2
-3
-5
-4
-3
-2
-1
-0.5
0
1
2
3

N Circles-

(Constant phase angles loci)

* N- circles are used to determine the phase response of a closed loop system using open-loop transfer- function.

$$G(j\omega) = x + jy$$

$$C(j\omega)/R(j\omega) = G(j\omega)/1 + G(j\omega)$$

$$= x + jy / 1 + x + jy$$

$$\phi = \tan^{-1}(y/x) - \tan^{-1}(y/1+x)$$

Consider tans

$$\tan\phi = \tan(\tan^{-1}(y/x) - \tan^{-1}(y/1+x))$$

$$\tan\phi = \frac{\tan(\tan^{-1}(y/x)) - (\tan^{-1}(y/1+x))\tan}{1 + \tan(\tan^{-1}(y/x))\tan(\tan^{-1}(y/1+x))}$$

$$\tan\phi = \frac{((y/x) - (y/1+x))}{1 + (y/x)(y/1+x)} = y/x^2+x+y^2$$

let $N = \tan\phi$

$$N = y/x^2+x+y^2$$

$$\Rightarrow x^2 + x + y^2 - y/N = 0$$

Add $1/4 + 1/4N$ on both side

$$x^2 + x + \frac{1}{4} + y^2 - y/N + 1/4N = \frac{1}{4} + \frac{1}{4N}$$

$$(x+1/2)^2+(y-1/2N)^2=\left(\sqrt{\frac{1}{4}+\frac{1}{4N}}\right)^2$$

Center $(-1/2,\ 1/2N)$

Radious $=\sqrt{\frac{1}{4}+\frac{1}{4N^2}}$