Control System

→ initial state + control vector → some state

⇓

controllable.

$\xrightarrow{\text{obs vector}}$

→ Some state → Initial state ⇒ observability.

Controllability

↓

complete state controllability

→ (Output) controllability

Control the output rather than the state of the system.

Controllability

Controllability Matrix

$[A \mid AB \mid AB^2 \mid AB \cdots \mid AB^{n-1}]$

↳ made up of linearly independent vectors

↳ rank of the matrix is $n \times n$

Alternate Method

→ eigen vector matrix

→ Jordan matrix

non zero rows

etc.

Output controllability

↳ $[CB \mid CAB \mid CA^2B \cdots \mid CA^{n-1}B \mid D]$

↳ this should be rank of matrix M and have linearly independent vectors

Observability

→ Why?

→ Sometime the state variables of a system are not directly measurable.

↳ It can be proved that IFF the system is "OBSERVABLE"

then

We can estimate these variables

and

Finally design the control signals.

Complete state observability:

↳ For this we consider the statement that the system is unforced

ie

$$\dot{x} = Ax$$
$$y = Cx.$$

no need to go into detail as to why is this

It can be shown that if

$$\begin{bmatrix} C \\ \hline CA \\ \hline \vdots \\ CA^{n-1} \end{bmatrix}$$

"OBSERVABILITY MATRIX"

= rank

↓

n then the system is observable.

"OBSERVABILITY MATRIX"

↳ can also be written as

$$\left[C^* \;\vdots\; A^* C^* \;\vdots\; (A^*)^2 C^* \;\middle|\; \cdots\cdots \; (A^*)^{n-1} C^* \right]$$

$=$