

Name: Manan Madan Roll No: 2018UIC3087

Why do we want to do pole placement

- They describe the behaviour of the system
- For eg
 - in second order system rise time , peak time etc.
 - can be explained by zeta and omega_n
 - which are nothing but indicate the position of the poles

Dominant and Non Dominant Roots

- To modify system only 2 root are dominant
- Other n-2 will be non dominant roots

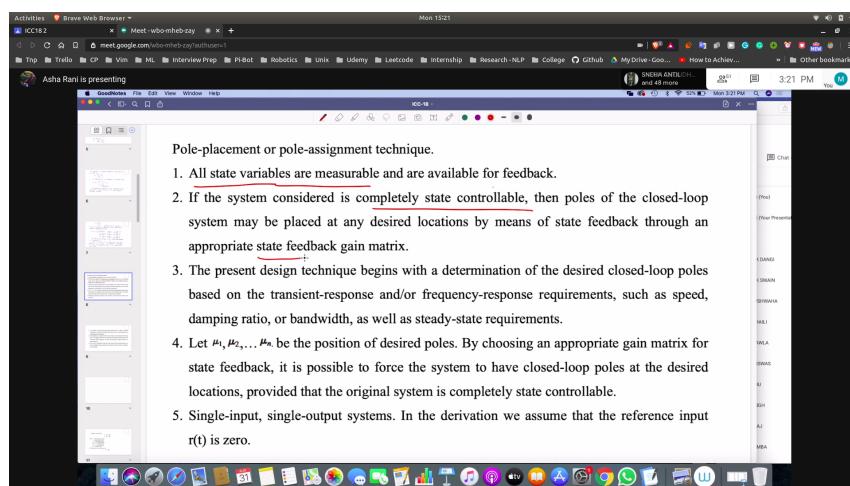
Solution of $|SI - A|$

- The solution of the characteristic equation are the eigen values

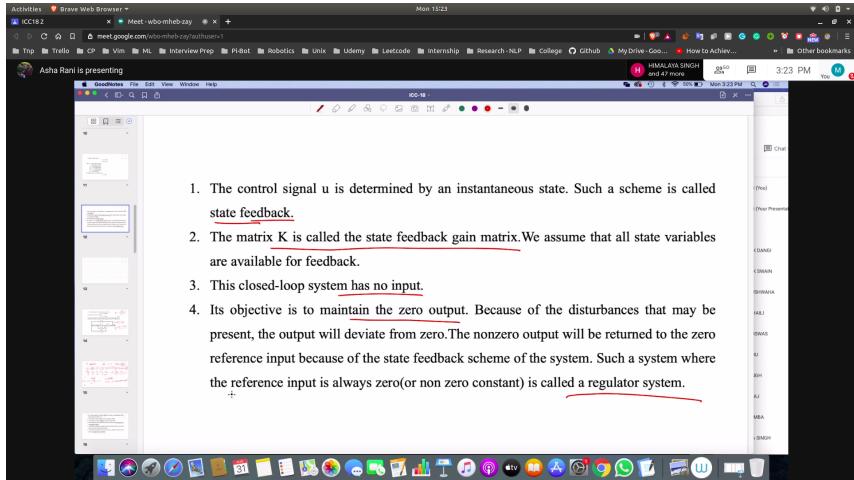
Value of U

- U is written as $-k * x(\text{input})$ to control the pole of the system

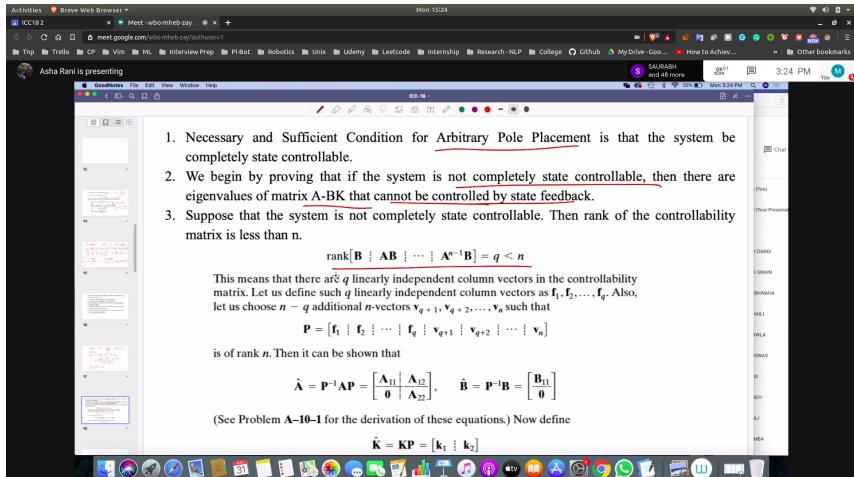
Assumptions for pole placement



Terms

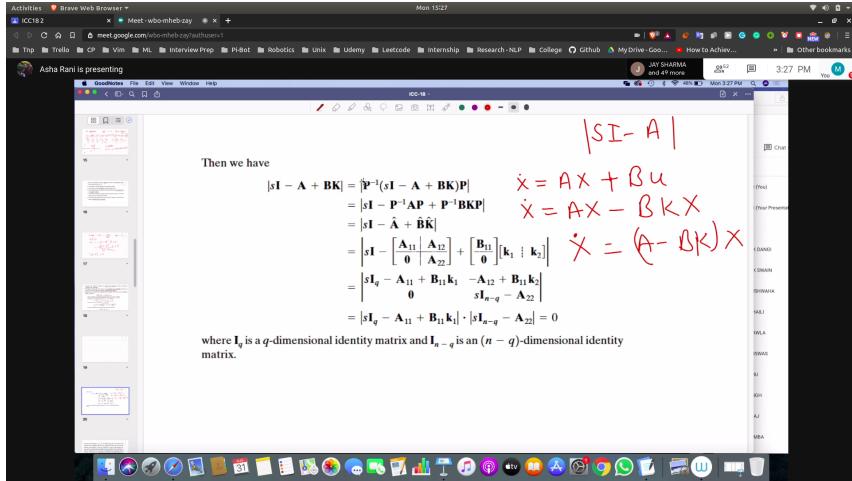


Condition for arbitrary pole placement



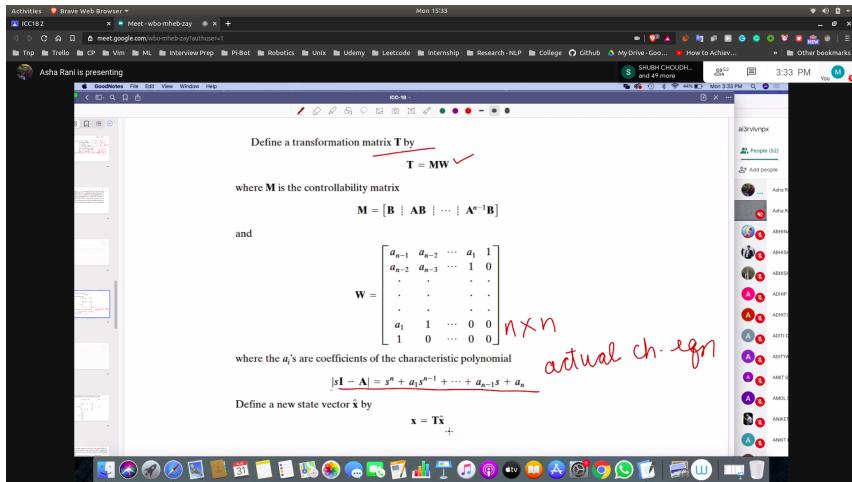
Here somewhere there is a proof that matrix should be controllable for pole placement

Characteristic Eqn for system with state feedback



- In this q poles will be controllable
- Then n-q poles will not be controllable
- So proved that we need to have full controllable matrix

How to do pole placement?



- Transform the system using T
- the system will be in a controllable form

Char eqn in controllable form

- This is the actual char eqn

Now let us simplify the characteristic equation of the system in the controllable canonical form. Referring to Equations (10-8), (10-9), and (10-11), we have

$$\begin{aligned}
 & |sI - T^{-1}AT + T^{-1}BKT| \\
 &= |sI - \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} [\delta_n \ \delta_{n-1} \ \cdots \ \delta_1] | \\
 &= |sI - \begin{bmatrix} s & -1 & \cdots & 0 \\ 0 & s & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ a_n + \delta_n & a_{n-1} + \delta_{n-1} & \cdots & s + a_1 + \delta_1 \end{bmatrix}| \\
 &= s^n + (a_1 + \delta_1)s^{n-1} + \cdots + (a_{n-1} + \delta_{n-1})s + (a_n + \delta_n) = 0 \quad (10-12)
 \end{aligned}$$

- a1 + delta1 = alpha1
- a2 + delta2 = alpha2

Desired Char Eqn

Equation (10-7) is in the controllable canonical form. Thus, given a state equation, Equation (10-1), it can be transformed into the controllable canonical form if the system is completely state controllable and if we transform the state vector x into state vector by use of the transformation matrix T given by Equation

Let us choose a set of the desired eigenvalues as $\mu_1, \mu_2, \dots, \mu_n$. Then the desired characteristic equation becomes

$$(s - \mu_1)(s - \mu_2) \cdots (s - \mu_n) = s^n + \underbrace{\alpha_1 s^{n-1}}_{+} + \cdots + \underbrace{\alpha_{n-1} s}_{+} + \underbrace{\alpha_n}_{=} = 0 \quad (10-10)$$

desired

Let us write

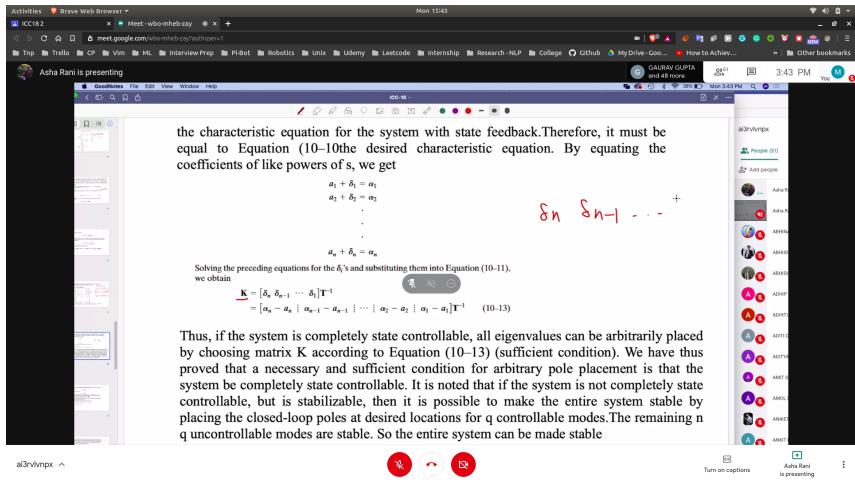
$$KT = [\delta_n \ \delta_{n-1} \ \cdots \ \delta_1] \quad (10-11)$$

When $u = -KT\dot{x}$ is used to control the system given by Equation (10-7), the system equation becomes

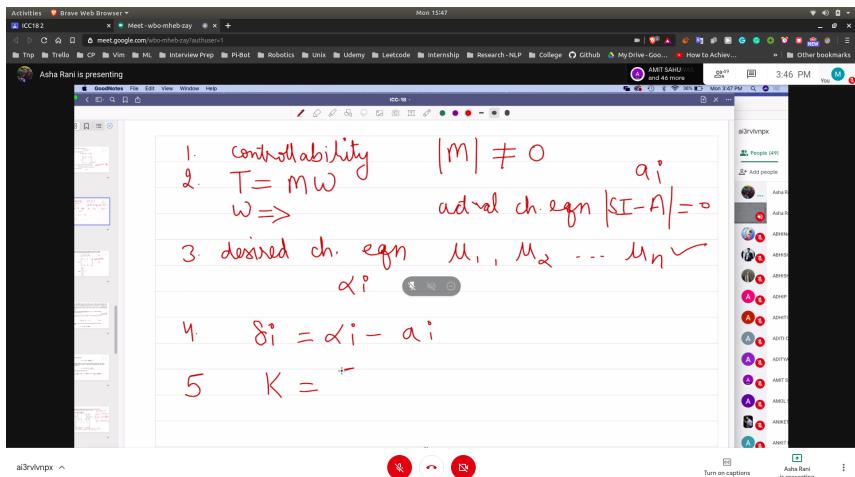
$$\dot{x} = T^{-1}AT\dot{x} - T^{-1}BKT\dot{x}$$

To make Desired == Equal

- Find K using delta i K = [.....]



Summary



- Find M $|M| \neq 0$
- Find T = MW
- Desired char eqn
- Calculate delta i from above
- Calculate K using delta values