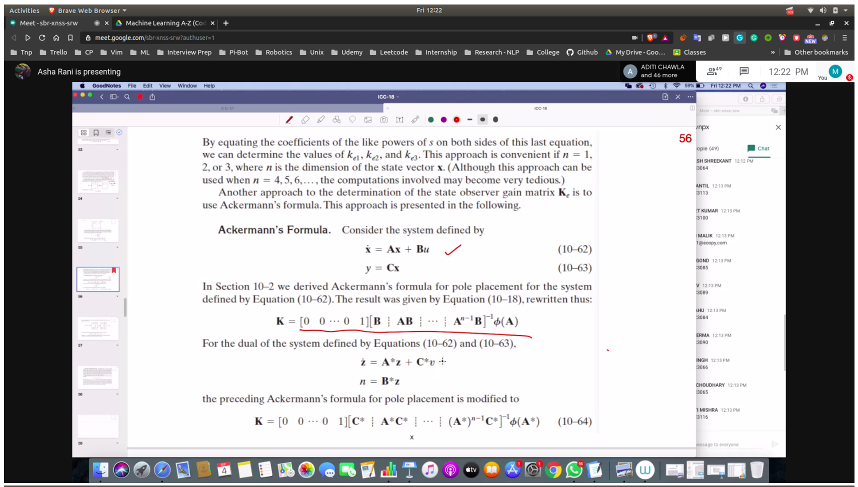
Finding Observability

Using Ackermann's Formula

- 

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By equating the coefficients of the like powers of s on both sides of this last equation, we can determine the values of k_{e1}, k_{e2}, and k_{e3}. This approach is convenient if $n = 1$, 2, or 3, where n is the dimension of the state vector $\mathbf{x}$. (Although this approach can be used when $n = 4, 5, 6, \ldots$, the computations involved may become very tedious.)

Another approach to the determination of the state observer gain matrix $\mathbf{K}_e$ is to use Ackermann's formula. This approach is presented in the following.

Ackermann's Formula. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \qquad (10\text{–}62)$$

$$y = \mathbf{C}\mathbf{x} \qquad (10\text{–}63)$$

In Section 10–2 we derived Ackermann's formula for pole placement for the system defined by Equation (10–62). The result was given by Equation (10–18), rewritten thus:

$$\mathbf{K} = [0 \quad 0 \cdots 0 \quad 1][\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}]^{-1}\phi(\mathbf{A})$$

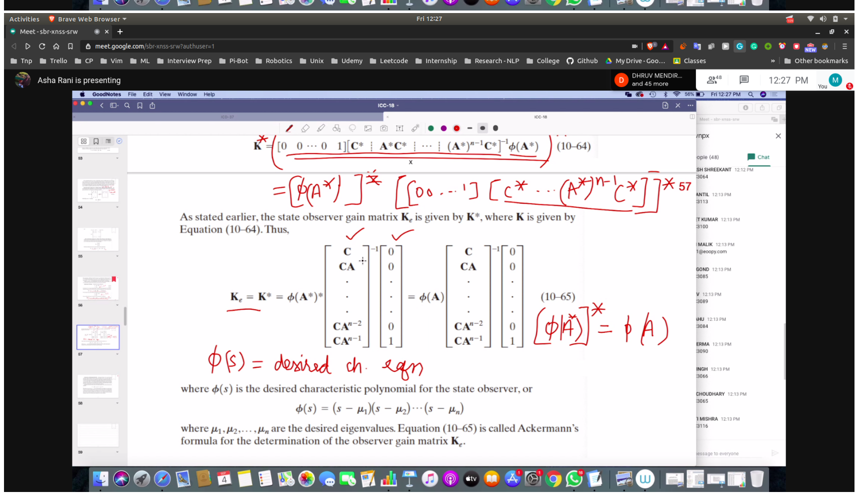
For the dual of the system defined by Equations (10–62) and (10–63),

$$\dot{\mathbf{z}} = \mathbf{A}^*\mathbf{z} + \mathbf{C}^*v$$

$$n = \mathbf{B}^*\mathbf{z}$$

the preceding Ackermann's formula for pole placement is modified to

$$\mathbf{K} = [0 \quad 0 \cdots 0 \quad 1][\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^* \mid \cdots \mid (\mathbf{A}^*)^{n-1}\mathbf{C}^*]^{-1}\phi(\mathbf{A}^*) \qquad (10\text{–}64)$$

- 

$$\mathbf{K}^* = \left([0 \quad 0 \cdots 0 \quad 1][\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^* \mid \cdots \mid (\mathbf{A}^*)^{n-1}\mathbf{C}^*]^{-1}\phi(\mathbf{A}^*)\right) \qquad (10\text{–}64)$$

$$= [\phi(A^*)]^* \left[[0\,0 \cdots 1][C^* \cdots (A^*)^{n-1}C^*]\right]^*$$

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As stated earlier, the state observer gain matrix $\mathbf{K}_e$ is given by $\mathbf{K}^*$, where $\mathbf{K}$ is given by Equation (10–64). Thus,

$$\mathbf{K}_e = \mathbf{K}^* = \phi(\mathbf{A}^*)^* \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{CA}^{n-2} \\ \mathbf{CA}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} = \phi(\mathbf{A}) \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{CA}^{n-2} \\ \mathbf{CA}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \qquad (10\text{–}65)$$

$[\phi(A^*)]^* = \phi(A)$

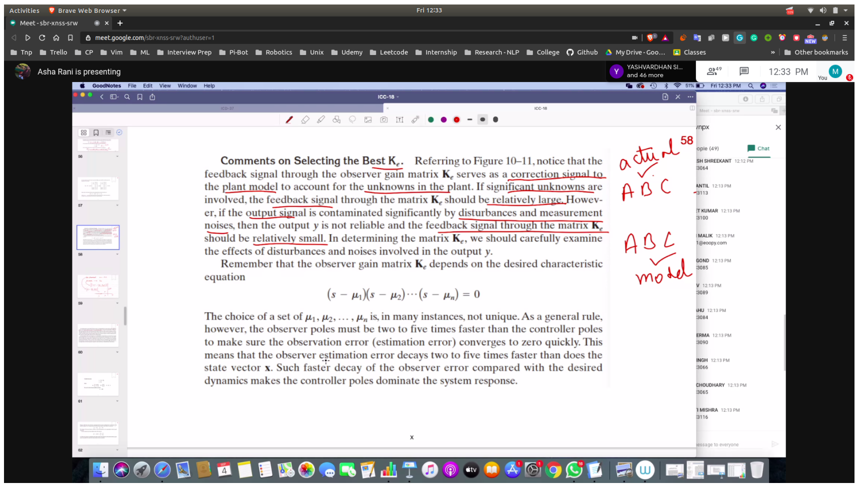
$\phi(s)$ = desired ch. eqn

where $\phi(s)$ is the desired characteristic polynomial for the state observer, or

$$\phi(s) = (s - \mu_1)(s - \mu_2)\cdots(s - \mu_n)$$

where $\mu_1, \mu_2, \ldots, \mu_n$ are the desired eigenvalues. Equation (10–65) is called Ackermann's formula for the determination of the observer gain matrix $\mathbf{K}_e$.

For finding K_e

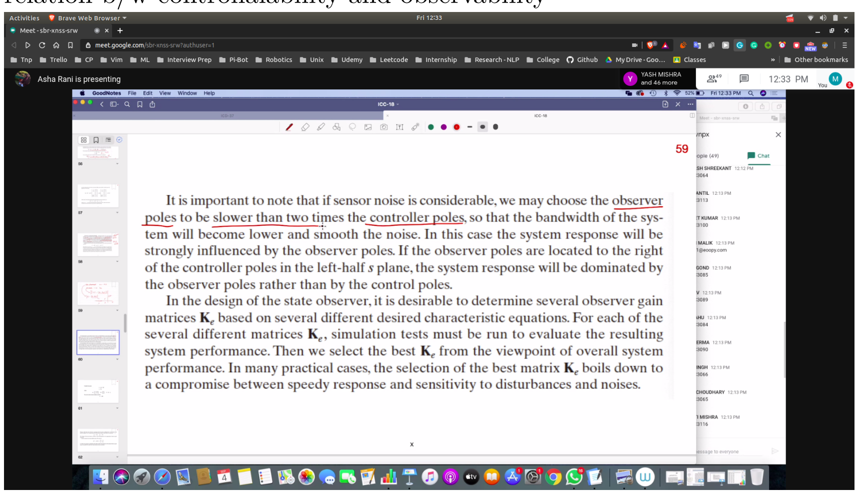
- 

Comments on Selecting the Best $\mathbf{K}_e$. Referring to Figure 10–11, notice that the feedback signal through the observer gain matrix $\mathbf{K}_e$ serves as a correction signal to the plant model to account for the unknowns in the plant. If significant unknowns are involved, the feedback signal through the matrix $\mathbf{K}_e$ should be relatively large. However, if the output signal is contaminated significantly by disturbances and measurement noises, then the output y is not reliable and the feedback signal through the matrix $\mathbf{K}_e$ should be relatively small. In determining the matrix $\mathbf{K}_e$, we should carefully examine the effects of disturbances and noises involved in the output y.

Remember that the observer gain matrix $\mathbf{K}_e$ depends on the desired characteristic equation

$$(s-\mu_1)(s-\mu_2)\cdots(s-\mu_n)=0$$

The choice of a set of $\mu_1, \mu_2, \ldots, \mu_n$ is, in many instances, not unique. As a general rule, however, the observer poles must be two to five times faster than the controller poles to make sure the observation error (estimation error) converges to zero quickly. This means that the observer estimation error decays two to five times faster than does the state vector $\mathbf{x}$. Such faster decay of the observer error compared with the desired dynamics makes the controller poles dominate the system response.

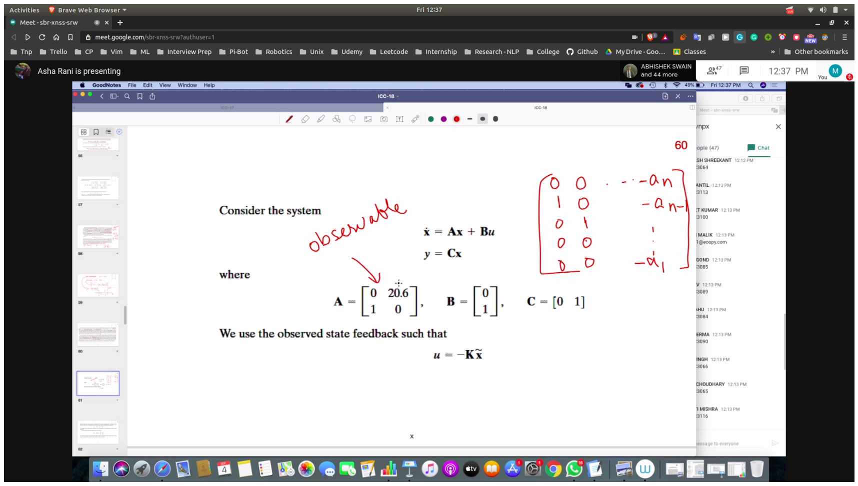
- relation b/w controllalability and observability
- 

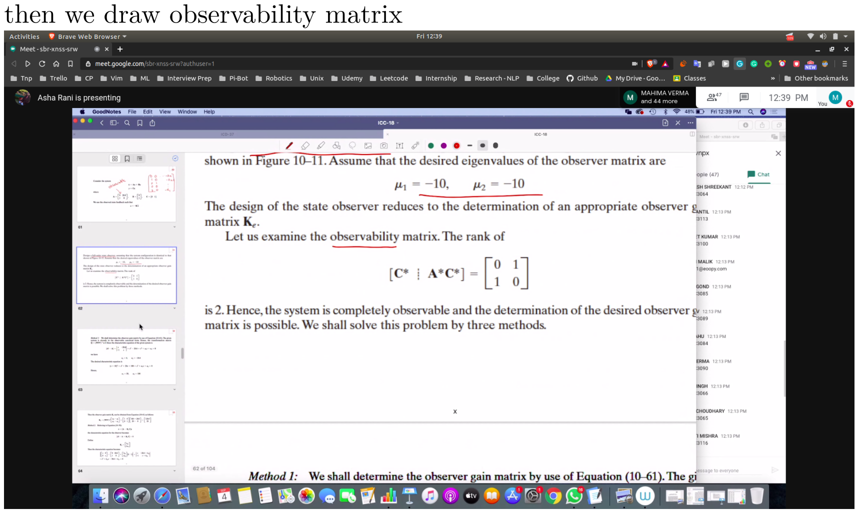
It is important to note that if sensor noise is considerable, we may choose the observer poles to be slower than two times the controller poles, so that the bandwidth of the system will become lower and smooth the noise. In this case the system response will be strongly influenced by the observer poles. If the observer poles are located to the right of the controller poles in the left-half s plane, the system response will be dominated by the observer poles rather than by the control poles.

In the design of the state observer, it is desirable to determine several observer gain matrices $\mathbf{K}_e$ based on several different desired characteristic equations. For each of the several different matrices $\mathbf{K}_e$, simulation tests must be run to evaluate the resulting system performance. Then we select the best $\mathbf{K}_e$ from the viewpoint of overall system performance. In many practical cases, the selection of the best matrix $\mathbf{K}_e$ boils down to a compromise between speedy response and sensitivity to disturbances and noises.

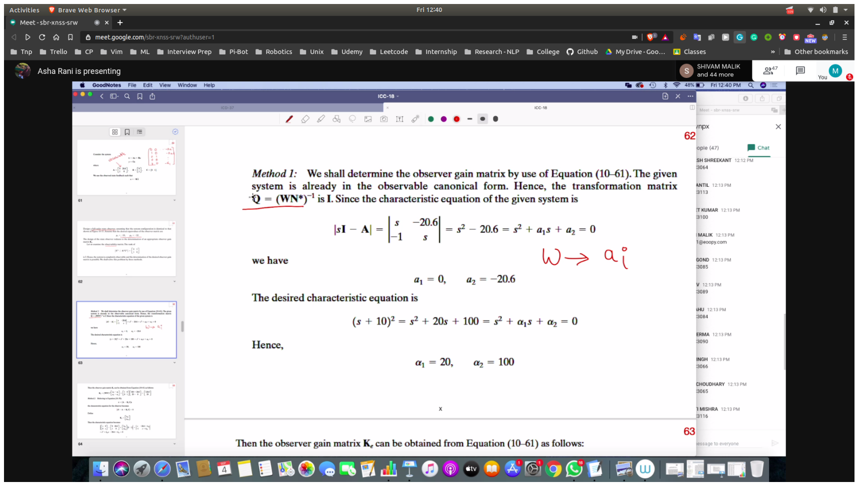
Process

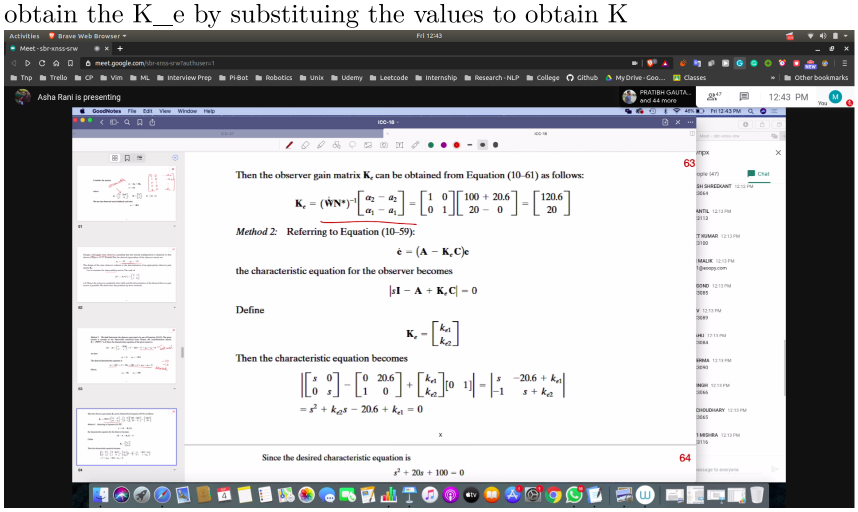
- given A,B,C
- u = -K (x (observed))

- 

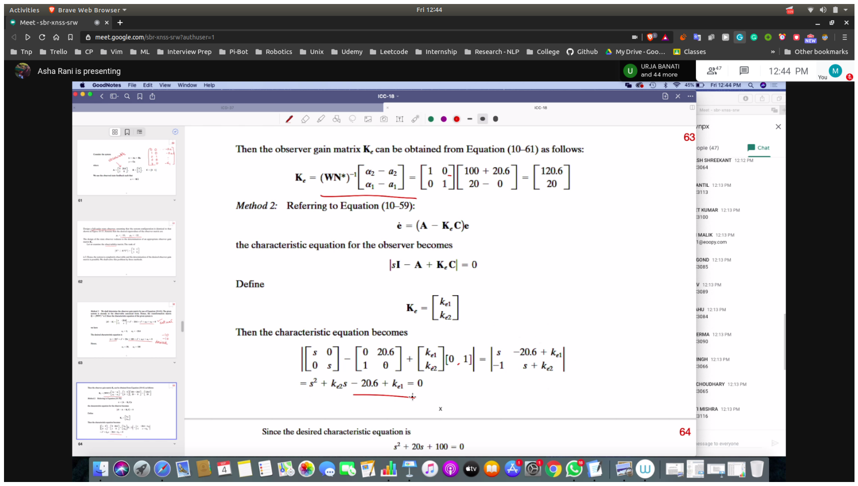
- ve are given desired eigen vector
- then we draw observability matrix
- 

First Method
- get transformation matrix
- compare to desired characteristic equation
- get the value of coeff

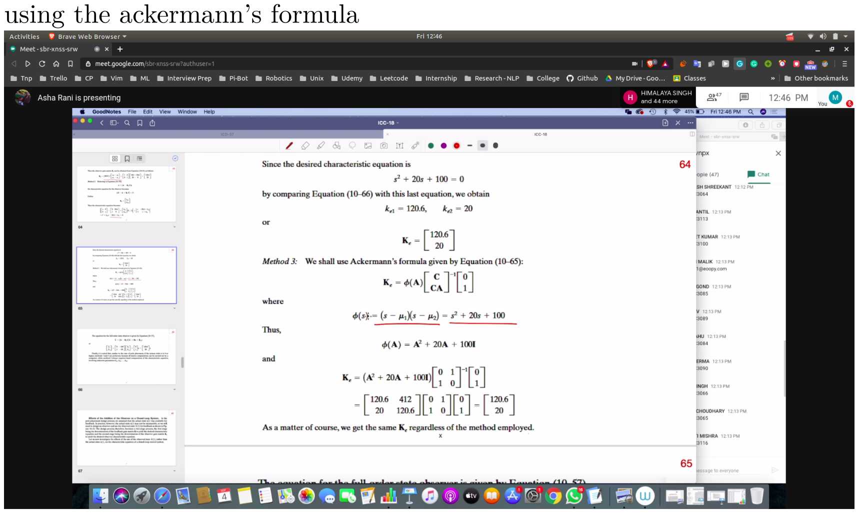
- 

- obtain the K_e by substituing the values to obtain K
- 

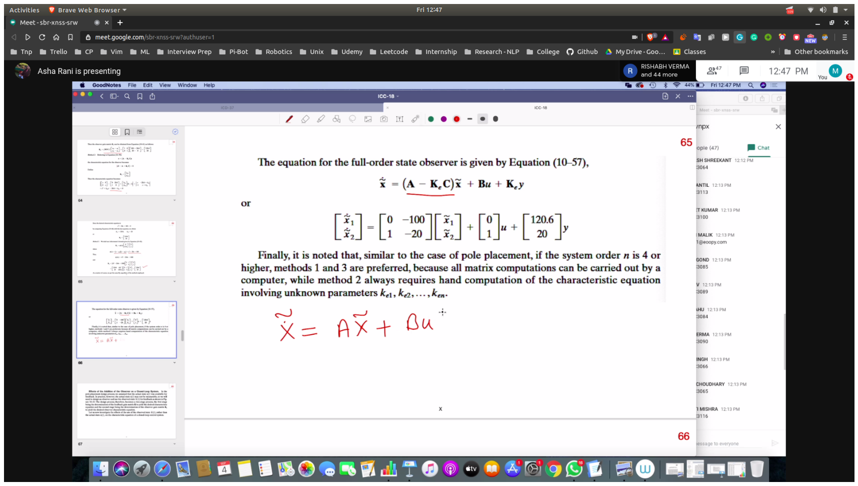
Second Method
- put ke values in e

- 

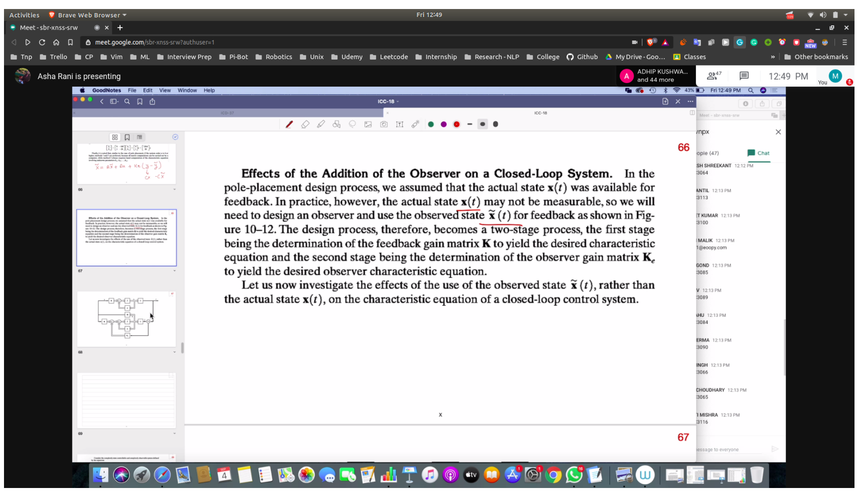
Third Method

- using the ackermann's formula
- 

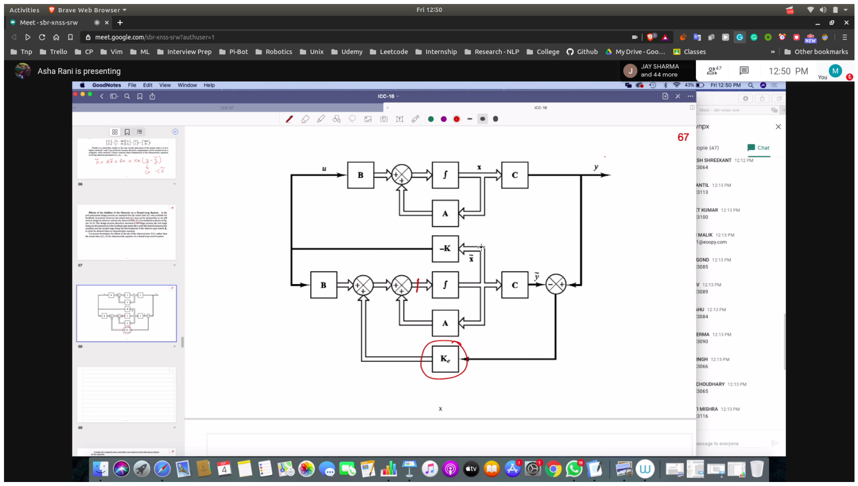
Finally obtain the full state observer eqn

- 

Effect of addition

- 

Full Integrator Circuit

- 
Asha Rani is presenting
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