Date.........

Ques 1 Determine the analytic func^n whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$

Sol^n) $e^{2x}(x\cos 2y - y\sin 2y)$

$$\frac{\delta u}{\delta x} = e^{x}\left(1(\cos 2y - 0\right) + 2e^{2x}(x\cos 2y - y\sin 2y)$$
$$= e^{x}\cos 2y$$

$$= e^{2x}(\cos 2y + 2x\cos 2y - 2y\sin 2y) = \phi_1(x,y)$$
$$\phi_1(z,0) = e^{-2z}(0) = 0$$

By milne thomson method.

$$F(z) = \int \phi_1(z,0)\,dz, \int \phi_2(z_2,0)\,dz + c$$

$$= \int e^{2z}(1+2z)\,dz - 0 + c$$

$$= (1+2z)\left(\frac{e^{2z}}{2}\right) - 2\left(\frac{e^{2z}}{4}\right) + c.$$

$$= \frac{e^{2z}}{2}(1+2z-1)+c$$

$$\boxed{f(z) = e^{2z} + c}$$

Que 3 02 find the regular function whose imaginary part is $\cos x \cosh y$.

$$\Rightarrow \cos x \cdot \cosh y \Rightarrow \cos x\left(\frac{e^{y}+e^{-y}}{2}\right)$$

$$\frac{\delta u}{\delta y} = \delta(\cos x \cosh y)$$

$$= \cos x\left(\frac{e^{y}-e^{-y}}{2}\right) \rightarrow 0$$

$$\frac{\delta u}{\delta x} = -\sin x\left(\frac{e^{y}+e^{-y}}{2}\right)$$

$x = z$, $y = 0$; $-\sin z$

$$\frac{dw}{dz} = \frac{\delta v}{\delta y} + i\frac{\delta v}{\delta x}$$

$$\frac{dw}{dz} = i \sin z$$

$$\int dw = \int -i\sin z\, dz$$

$$\boxed{w = i\cos z}$$

Que: 03 $f(z) = u + iv$ if

(i) $u - v = e^x(\cos y - \sin y)$

$u = e^x \cos y$

$v = e^x \sin y$

$f(z) = u + iv$

$= e^x \cos y + i e^x \sin y$

$= e^x [\cos y + i \sin y)$

by Euler's identity $\rightarrow$ $f(z) = e^x e^{(iy)}$

$f(z) = e^{(x+iy)}$

$$\boxed{f(z) = e^z}$$

(ii) $u + v = \dfrac{x}{x^2 + y^2}$, when $f(1) = 1$

$f(z) = u + iv$ — (1)

$i f(z) = iu - v$ — (ii)

(1) + (ii) gives $(1+i) f(z) = (u - v) + i(v + u)$

$F(z) = u + iv$

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v is given as $x/(x^2+y^2)$

$$\frac{dv}{dx} = \frac{(1\cdot(x^2+y^2)) - x(2y)}{(x^2+y^2)^2}$$

$$\left.\begin{matrix} x \to z \\ y \to 0 \end{matrix}\right\} \frac{\delta v}{\delta x} = \frac{z^2 - 2z^2}{z^4} = \frac{1}{z^2}$$

$$\frac{\delta v}{\delta y} = -2yx/x^2+y^2 = 0$$

$$\frac{dw}{dz} = 0 + \left(-1/z^2\right)$$

$$\int \delta w = -i \int \frac{1}{z^2}\, dz$$

$$w = i/z$$

$f(2) = \frac{1}{2}$ & $(i+1)\, f(2) = f(z)$

$$\boxed{f(z) = \left(\left(\frac{i}{i+1}\right)\frac{1}{z}\right)}$$

Que: 04

$$\omega = \frac{1}{z} \quad , \quad z = \frac{1}{\omega} \qquad \therefore \; z = \frac{1}{u+iv}$$

$$= \frac{u - iv}{u^2+v^2}$$

$$|z - 2i| = 2$$

$$|x + (y-2)i| = 2$$

$$x^2 + (y-2)^2 = 4$$

$$x^2 + y^2 - 2y = 0$$

$$x + iy = \frac{u - iv}{u^2+v^2}$$

$$x = \frac{u}{u^2+v^2} \quad , \quad y = \frac{-v}{u^2+v^2}$$

$$\frac{u^2}{(u^2+v^2)^2} + \frac{v^2}{(u^2+v^2)^2} - \frac{2v}{u^2+v^2} = 0$$

$$\frac{u^2 + v^2 - 2v(u^2+v^2)}{(u^2+v^2)^2} = 0$$

$$\cancel{u^2 + v^2 - 2vu^2 - 2v^3 = 0}$$

$$(u^2+v^2) + 2v(u^2+v^2) = 0$$

$$(u^2+v^2)(1+2v) = 0$$

$$1 + 2v = 0$$

$$\boxed{v = -\frac{1}{2}}$$

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Que : 05 Show that the transformation $\omega = \dfrac{2z+3}{z-4}$ maps the circle $x^2+y^2-4x=0$ onto the straight line $4u+3=0$

$\Rightarrow$

$$\omega = \frac{2z+3}{z-4} \qquad 4u+3=0$$

→ it is a bilinear transformation.

inverse transformation

$$z = \frac{4\omega+3}{\omega-2}$$

The eq^n of circle → $x^2+y^2-4x=0$

i.e. $|z|^2 - 4\,Re\,z = 0$

$$z\bar{z} - 2(z+\bar{z}) = 0$$

$\therefore \omega = u+iv$

$$\frac{4\omega+3}{\omega-2} \cdot \frac{4\bar{\omega}+3}{\bar{\omega}-2} - 2\left(\frac{4\omega+3}{\omega-2} + \frac{4\bar{\omega}+3}{\bar{\omega}-2}\right) = 0$$

$$(4w+3)(\cancel{w}4\bar{w}+3) - 2\{(4w+3)(\bar{w}-2) + (4\bar{w}+3)(w-2)\} = 0$$

i.e. $2(w+\bar{w}) + 3 = 0$

i.e. $4u + 3 = 0$

H.P.

Que: 06 find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$

Sol^n

$$w = \frac{az+b}{cz+d} \quad —(i)$$

put $z = 1$ & $w = i$

$$i = \frac{a+b}{c+d}$$

$$\boxed{ic + id = a + b \quad —(ii)}$$

put $z = i$ & $w = 0$

$$0 = \frac{ai+b}{c(i)+d} = ai + b = 0$$

$$\boxed{b = -ai} \quad (iii)$$

put $z = -1$ & $\omega = -i$

$$-i = \frac{a(-1)+b}{c(-1)+d}$$

$$ic - id = -a + b \quad — \text{iv}$$

add eq ② & (iv)

$$ic + id = a + b$$
$$ic - id = -a + b$$

we, get $2ic = 2b$

$$c = \frac{b}{i} = \frac{-ai}{i} = -a \rightarrow (v)$$

$$\therefore \quad \omega = \frac{az - ai}{-az - ai}$$

$$\therefore \quad \omega = \frac{-a(-z+i)}{-a(z+i)}$$

$$\Rightarrow \quad \frac{i-z}{z+i}$$ is the bilinear transformation.

Que: 07 $\int \frac{3z^2+z}{z^2-1}$ $|z-1|=1$

$z^2-1=0$

$z=+-1$

$|z-1|=1$

$z=1$

$r=1$ includes the point $z=1$

$$\oint \frac{3z^2+z}{z^2-1}\,dz = \int \frac{\left(\frac{3z^2+z}{z+1}\right)}{z-1}\,dz$$

$$= 2\pi i \left(\frac{3z^2+z}{z+1}\right)_{z=}$$

$$= \underline{\underline{4\pi i}}$$

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Que : 08

$$\frac{z^2-2z}{(z+1)^2(z^2+4)}$$

$z = -1, -1$

$z = \pm 2i$

Residue at -1

$$\frac{d}{dz}\left[(z-(-1))^2 f(z)\right]$$

$$\frac{(z+1)^2 \cdot (z^2-2z)}{(z+1)^2(z^2+4)}$$

$$\frac{d}{dz}\,\frac{(z^2-2z)}{(z^2+4)}$$

$$= \cancel{2z^3}+8z-2z^2-8-\cancel{2z^3}+4z^2$$

$$= \frac{2z^2+8z-8}{(z^2+4)^2}$$

$$= \frac{2-8-8}{25} = \frac{-14}{25}$$

Residue at $z = 2i$

$$\left|(z-2i)\, f(z)\right._{z=2i} = \frac{(z-2i)}{(z+1)^2} \cdot \frac{z^2-2z}{(z-2i)(z+2i)}$$

$$= \frac{z^2-2z}{(z^2+7+2z)(z+2i)}$$

$$= \frac{-4(1+i)}{-(3i+4)4} = \frac{1+i}{3i+4}$$

$$\boxed{= \frac{7+i}{25}}$$

Ans

Residue at $z = -2i$

$$= \frac{7-i}{25}$$