where γ is the coefficient of volumetric expansion and v the combined volume of the capillary and Bourdon tube.

$$\therefore \qquad (\Delta T)_{\text{error}} = \frac{16}{8} = 2°\text{C}$$

Thus the error due to ambient temperature rise is 2°C or alternatively the correction to be applied to the observed readings would be –2°C.

(c) The pressure of 60 cm column of Hg = $\rho_{\text{Hg}}\, gh$

$$= (13.6 \times 10^3)\,(9.81) \times \frac{60}{100} = 80.05 \text{ kPa}$$

Angular movement in radius caused by 80.05 kPa pressure in the pressure gauge

$$= \frac{270 \times \pi / 180 \times 80.05 \times 10^3}{6 \times 10^6} = 0.06287 \text{ rad}$$

$\therefore$ Error due to elevation effect in the pressure thermometer

$$= \frac{\text{angular movement due to elevation}}{\text{sensitivity of thermometer}}$$

$$= \frac{0.06287}{2.18 \times 10^{-2}} = 2.88°\text{C}$$

12.5 ■ ELECTRICAL METHODS

Electrical methods are in general preferred for the measurement of temperature as they furnish a signal which can be easily detected, amplified or used for control purposes. There are two main electrical methods used for measuring temperature. They are:

1. Thermo-resistive type i.e., variable resistance transducers and
2. Thermo-electric type i.e., emf generating transducers.

12.5.1 Electrical Resistance Thermometers

In resistance thermometers, the change in resistance of various materials, which varies in a reproducible manner with temperature, forms the basis of this important sensing technique. The materials in actual use fall in two classes namely, conductors (metals) and semiconductors. In general, the resistance of the highly conducting materials (metals) increases with increase in temperature and the coils of such materials are called *metallic resistance thermometers*. Whereas the resistance of semiconductor materials generally (not always) decreases with increase in temperature. Thermo-sensitive resistors having such negative temperature characteristics are commonly known as NTC thermistors. Figure 12.4 illustrates the typical variation of specific resistance of the metals (platinum for example) and the NTC thermistor.

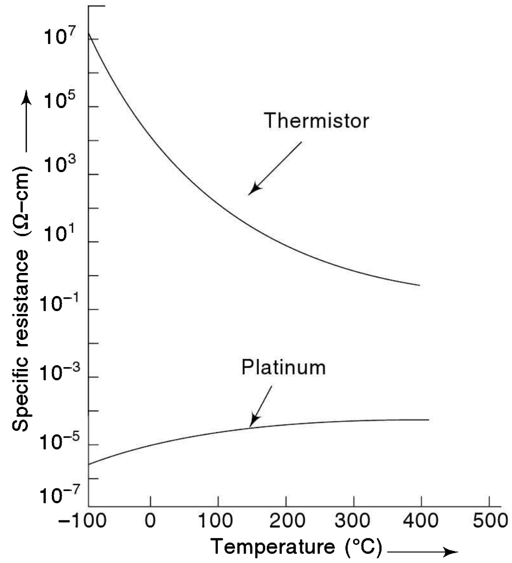


Fig. 12.4 *Resistance–temperature characteristics of platinum and a typical NTC thermistor*

Metallic Resistance Thermometers or Resistance—Temperature Detectors (RTDs) Metals such as platinum copper, tungsten and nickel exhibit small increases in resistance as the temperature rises because they have a positive temperature coefficient of resistance. Platinum is a very widely used sensor and its operating range is from 4K to 1064°C. Because it provides extremely reproducible output, it is used in establishing International Practical Temperature Scale from 13.81 K to 961.93°C. However for the measurement of lower temperatures up to 600°C, RTD sensor is made of nickel. Further, for ranges of temperature below 300°C, the sensing element is fabricated using pure copper wire. Metallic resistance thermometers are very suitable for both laboratory and industrial applications because of their high degree of accuracy as well as long-term stability. In addition, they have a wide operating range and have linear characteristics throughout the operating range. However, the limitations of the RTDs are low sensitivity, relatively higher cost as compared to other temperature sensors and their proneness to errors caused due to contact resistance, shock and accelerations.

Metallic resistance thermometers are constructed in many forms, but the temperature sensitive element is usually in the form of a coil of fine wire supported in a stress-free manner. A typical construction is shown in Fig. 12.5, where the wire of metal is wound on the grooved hollow insulating ceramic former and covered with protective cement. The ends of the coils are welded to stiff copper leads that are taken out to be connected in one of the arms of the Wheatstone bridge circuit. In some cases, this arrangement can be used directly in the medium whose temperature is being measured, thus giving a fast speed of response. However, in most applications, a protective metal sheath is used to provide rigidity and mechanical strength. Alternatively, RTD sensor's may be fabricated by depositing the thin films of platinum, nickel or copper on a ceramic substrate. These thin film sensors have the advantage of extremely low mass and consequently more rapid thermal response.

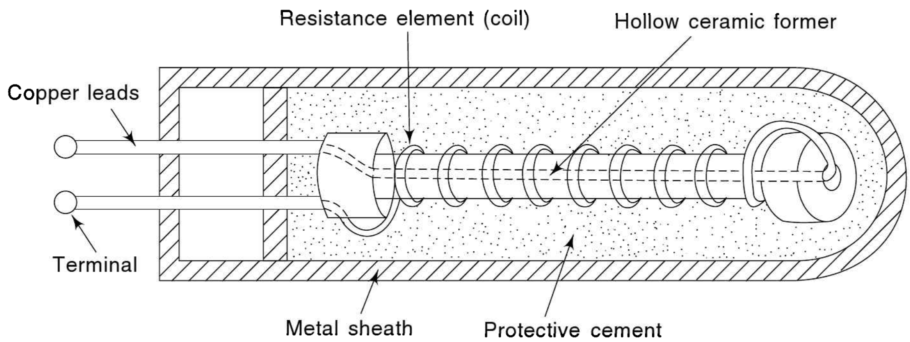


Fig. 12.5 *Construction of a platinum resistance thermometer (PRT)*

Platinum, in spite of its low sensitivity and high cost as compared to nickel and copper, is the most widely used material for metallic resistance element. This is because of the following:

1. The temperature–resistance characteristics of pure platinum are well defined and stable over a wide range of temperatures.
2. It has high resistance to chemical attack and contamination ensuring long-term stability.
3. It forms the most easily reproducible type of temperature transducer with a high degree of accuracy. The accuracy attainable with PRT is ±0.01°C up to 500°C and ±0.1°C up to 1200°C.

In general, the resistance relationship of most metals over a wide range of temperatures is given by the quadratic relationship:

$$R = R_0 \, [1 + aT + bT^2] \tag{12.6}$$

where R = resistance at absolute temperature T

R_0 = resistance at 0°C

a and b = experimentally determined constants.

However, over a limited temperature range around 0°C (273 K), the following linear relationship can be applied:

$$R_t = R_0\,(1 + \alpha\, t) \tag{12.7}$$

where α = the temperature coefficient of resistance of material in (Ω/Ω)/°C or °C^{-1}.

R_0 = resistance at 0°C

t = temperature relative to 0°C

Some typical values of α are

copper = 0.0043°C^{-1}

nickel = 0.0068°C^{-1}

platinum = 0.0039°C^{-1}

If a change in temperature from t_1 to t_2 is considered, Eq. (12.7) becomes:

$$R_2 = R_1 + R_0\,\alpha\,(t_2 - t_1)$$

Rearranging gives

$$t_2 = t_1 + \frac{R_2 - R_1}{\alpha R_0} \tag{12.8}$$

The variation of resistance of the sensing element is normally measured using some form of electrical bridge circuit which may employ either the deflection mode of operation or the null (manually or automatically balanced) mode. However, particular attention must be given to the manner in which the thermometer is connected into the bridge. Leads of same length appropriate to the situation are normally required and any resistance change therein due to any cause, including temperature, may be credited to the thermometer element. It is desirable, therefore, that the lead resistance be kept as low as possible relative to the element resistance. In addition, some modifications may be employed for providing the lead compensation. For more precise results, either the Siemen's three wire lead arrangement or Callender's four wire lead arrangement may be employed (Fig. 12.6). Further, it is essential that the thermo-electric emf's do not affect the system. These can be eliminated by utilising ac excitation or by manually varying the polarity of the dc supply.

Problem 12.2 *A platinum resistance thermometer has a resistance of 140.5 and 100.0 Ω at 100 and 0°C, respectively. If its resistance becomes 305.3 Ω when it is in contact with a hot gas, determine the temperature of the gas. The temperature coefficient of platinum is 0.0039°C^{-1}.*

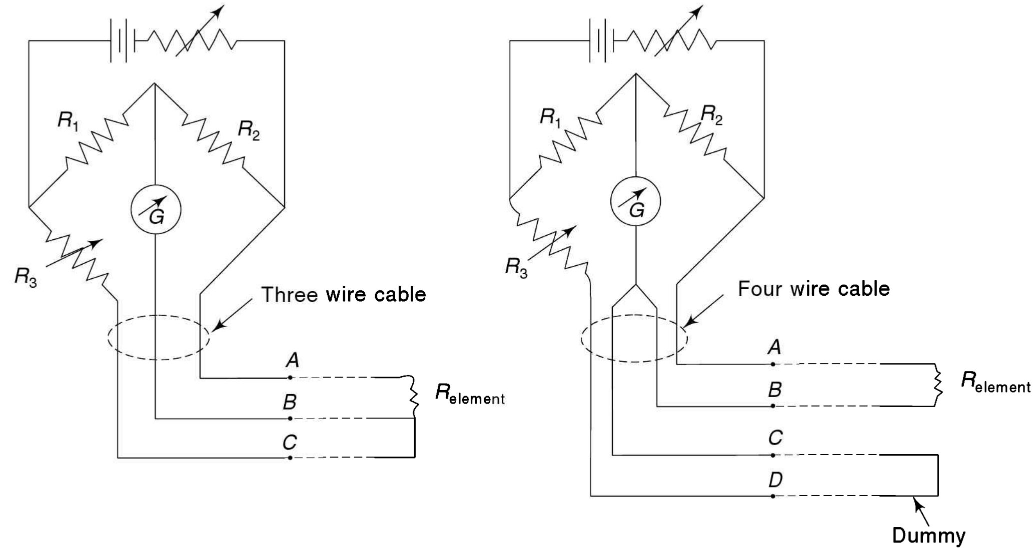
Solution The temperature-resistance relationship of platinum is given by:

$$R_2 = R_1 + R_0\,\alpha\,(t_2 - t_1)$$

$$\therefore \quad t_2 = 100 + \frac{305.3 - 140.5}{0.0039 \times 100}$$

$$= 100 + 422.56 = 522.56\text{°C}$$

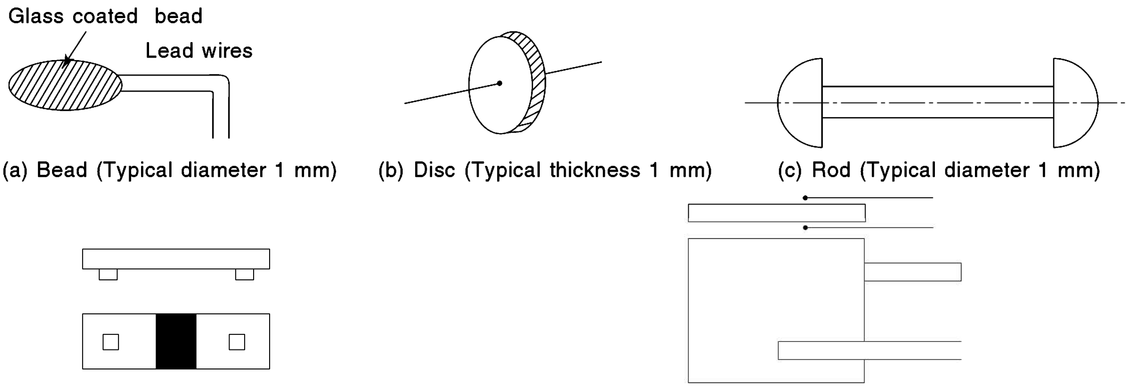
Semiconductor Resistance Sensors (Thermistors) Thermistor (shortened form of the words: thermal resistor) is a thermally sensitive variable resistor made of ceramic-like semiconducting materials. They are available in a greater variety of shapes and sizes having cold resistance ranging from a few ohms to mega ohms. The size can range from extremely small bead, thin disc, thin chip or wafer to a large sized



(a) Siemen's three wire lead arrangement (b) Callender's four wire lead arrangement

Fig. 12.6 *Cable compensation arrangements for platinum resistance thermometer*

rod as illustrated in Fig. 12.7. Unlike metals, thermistors respond negatively to temperature and their coefficient of resistance is of the order of 10 times higher than that of platinum or copper.



(a) Bead (Typical diameter 1 mm) (b) Disc (Typical thickness 1 mm) (c) Rod (Typical diameter 1 mm)

(d) I.C. Chip (Typical dimension 1.25 mm × 100 mm × 0.3 mm thickness) (e) Wafer (Typical dimension 1.25 mm to 15 mm × 0.3 mm thickness)

Fig. 12.7 *Range of thermistor forms*

Thermistors are fabricated from the semiconducting materials which include the oxides of copper, manganese, nickel, cobalt, lithium and titanium. These oxides are blended in a suitable proportion and compressed into desired shapes from powders and heat treated to recrystallise them, resulting in a dense ceramic body with the required resistance–temperature characteristics.

Thermistors have the following advantages for temperature measurements:

1. a large temperature coefficient which makes the thermistor an extremely sensitive device, thus enabling accuracy of measurement up ± 0.01°C with proper calibration,

2. ability to withstand electrical and mechanical stresses,
3. fairly good operating range which lies between –100 and 300°C,
4. fairly low cost and easy adaptability to the available resistance bridge circuits, and
5. the high sensitivity and the availability in extremely small sizes (of the size of a pin head) enable a fast speed of thermal response.

Thus, these devices are extremely useful for dynamic temperature measurements.

However, the *disadvantages* are a highly non-linear resistance–temperature characteristics and problems of self-heating effects which necessitate the use of much lower current levels than those with metallic sensors.

The temperature-resistance characteristics of a thermistor is of exponential type and is given by:

$$R = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] \tag{12.9}$$

where R_0 is the resistance at the reference temperature T_0 (kelvin)
R is the resistance at the measured temperature T (kelvin)
β is the experimentally determined constant for the given thermistor material.

The values of β usually lie between 3000 and 4400 K depending on the formulation or grade.

Using Eq. (12.9) we can obtain the temperature coefficient of resistance as:

$$\frac{\mathrm{d}R/\mathrm{d}T}{R} = -\frac{\beta}{T^2} \tag{12.10}$$

Assuming $\beta = 4000$ K and $T = 298$ K

we get, $$\frac{\mathrm{d}R/\mathrm{d}T}{R} = -0.045\ \mathrm{K}^{-1}$$

The value of $\dfrac{\mathrm{d}R/\mathrm{d}T}{R}$ for platinum is 0.0039 K^{-1}, indicating that the thermistor is at least 10 times more sensitive than the platinum resistance element.

Problem 12.3 *For a certain thermistor, $\beta = 3140$ K and the resistance at 27°C is known to be 1050 Ω. The thermistor is used for temperature measurement and the resistance measured is as 2330 Ω. Find the measured temperature.*

Solution The governing equation of the temperature-resistance characteristics of the thermistor is given by

$$R = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

The given data is

$$R_0 = 1050\ \Omega$$
$$T_0 = 273 + 27 = 300\ \mathrm{K}$$
$$\beta = 3140\ \mathrm{K}$$
$$R = 2330\ \Omega$$

Taking the logarithm of both sides of equation and rearranging we get,

$$\frac{1}{T} = \frac{\ln R - \ln R_0}{\beta} + \frac{1}{T_0}$$

$$= \frac{7.754 - 6.957}{3140} + \frac{1}{300} = 3.587 \times 10^{-3}$$

$$T = 278.77 \text{ K}$$
$$= 5.77°\text{C}$$

12.5.2 Thermo-electric Sensors

The most common electrical method of temperature measurement uses the thermo-electric sensor, also known as the *thermocouple* (TC). The thermocouple is a temperature transducer that develops an emf which is a function of the temperature between hot junction and cold junction. The construction of a thermocouple is quite simple. It consists of two wires of different metals twisted and brazed or welded together with each wire covered with insulation which may be either.

1. mineral (magnesium oxide) insulation for normal duty, or
2. ceramic insulation for heavy duty.

The basic principle of temperature measurement using a thermo-electric sensor was discovered by Seebeck in 1821 and is illustrated in Fig. 12.8. When two conductors of dissimilar metals, say A and B, are joined together to form a loop (thermocouple) and two unequal temperatures T_1 and T_2 are interposed at two junctions J_1 and J_2, respectively, then an infinite resistance voltmeter detects the electromotive force E, or if a low resistance ammeter is connected, a current flow I is measured. Experimentally, it has been found that the magnitude of E depends upon the materials as well as the temperature T_1 and T_2. Now, the overall relation between emf E and the temperatures T_1 and T_2 forms the basis for themo-electric measurements and is called the *Seebeck effect.* Thus, in practical applications, a suitable device is incorporated to indicate the emf E or the flow of current I. For convenience of measurements and standardisation, one of the two junctions is usually maintained at some known temperature. The measured emf E then indicates the temperature difference relative to the reference temperature, such as ice point which is very commonly used in practice.

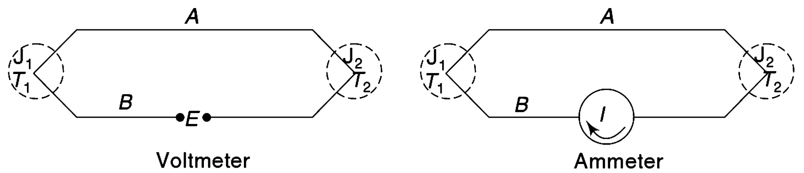


Fig. 12.8 *Basic thermo-electric circuit*

It may be noted that temperatures T_1 and T_2 of junctions J_1 and J_2 respectively are slightly altered if the thermo-electric current is allowed to flow in the circuit. Heat is generated at the cold junction and is absorbed from the hot junction thereby heating the cold junction slightly and cooling the hot junction slightly. This phenomenon is termed *Peltier effect.* If the thermocouple voltage is measured by means of potentiometer, no current flows and *Peltier* heating and cooling are not present. Further, these heating and cooling effects are proportional to the current and are fortunately quite negligible in a thermocouple circuit which is practically a millivolt range circuit. In addition, the junction emf may be slightly altered if a temperature gradient exists along either or both the materials. This is known as *Thomson effect.*

Again, the Thomson effect may be neglected in practical thermo-electric circuits and potentiometric voltage measurements are not susceptible to this error as there is no current flow in the circuit.

The actual application of thermocouples to the measurements requires consideration of the laws of thermo-electricity.

Law of Intermediate Temperatures This states that the emf generated in a thermocouple with junctions at temperatures T_1 and T_3 is equal to the sum of the emf's generated by similar themocouples, one acting between temperatures T_1 and T_2 and the other between T_2 and T_3 when T_2 lies between T_1 and T_2 (Fig. 12.9).

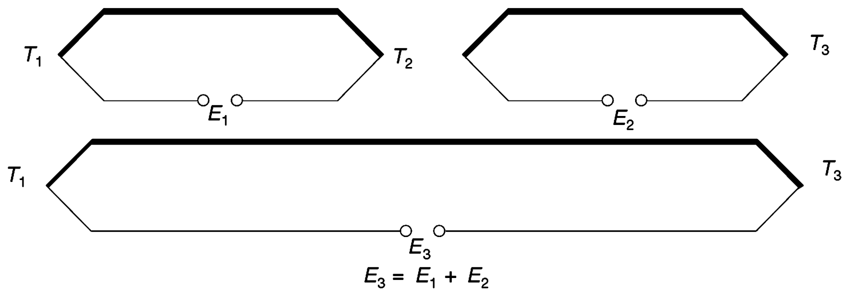


Fig. 12.9 *Law of intermediate temperatures*

This law is useful in practice because it helps in giving a suitable correction in case a reference junction temperature (which is usually an ice bath at 0°C) other than 0°C is employed. For example, if a themocouple is calibrated for a reference junction temperature of 0°C and used with junction temperature of say 20°C, then the correction required for the observation would be the emf produced by the thermocouple between 0 and 20°C.

Law of Intermediate Metals The basic thermocouple loop consists of two dissimilar metals A and B [Fig. 12.10 (a)]. If a third wire is introduced, then three junctions are formed as shown in Fig. 12.10(b). The emf generated remains unaltered if the two new junctions B–C and C–A are at the same temperature.

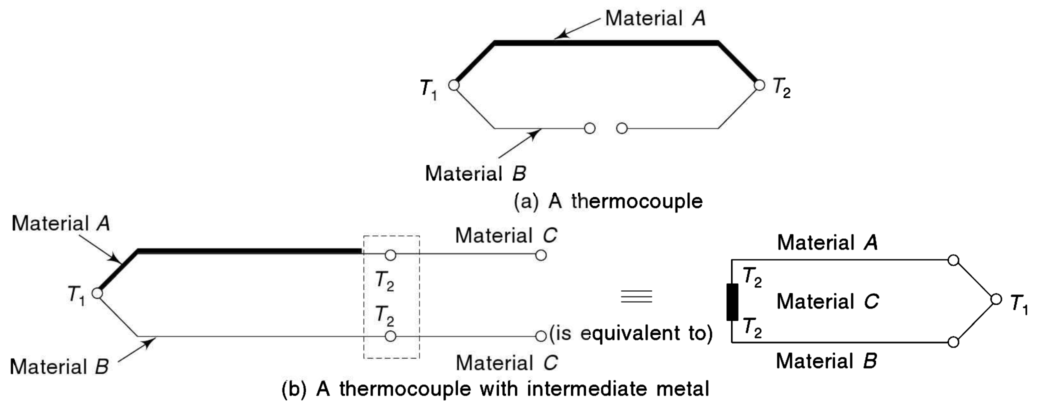
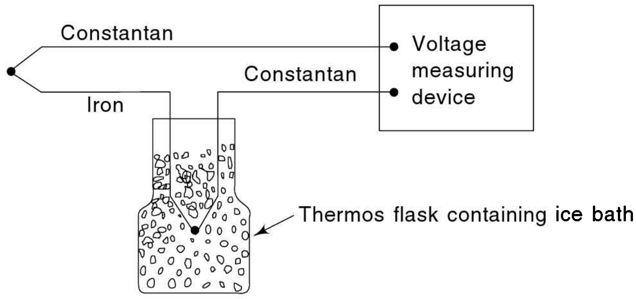
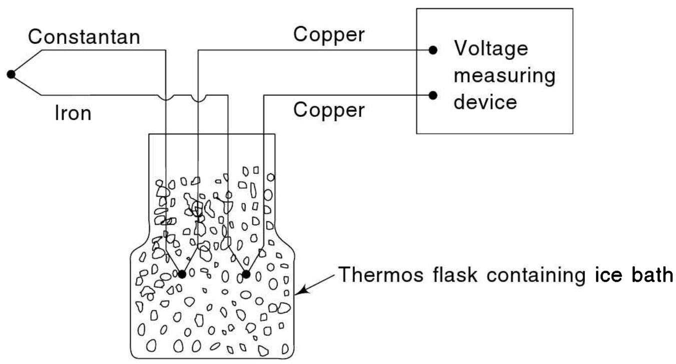


Fig. 12.10 *Law of intermediate metals*

It may be noted that extension wires are needed when the measuring instrument is to be placed at a considerable distance from the reference junction. Maximum accuracy is obtained when the leads are of the same material as the themocouple element [Fig. 12.11(a)]. However, this approach is not economical while using expensive themocouple materials. Further, a small inaccuracy is still possible if the binding post of the instrument is made of say copper and the two binding posts are at different temperatures. Therefore, it is preferable to employ the system shown in Fig. 12.11(b) to keep the copper-iron and copper-constantan junctions in the thermos flask at 0°C and provide binding posts of copper. This ensures maximum accuracy in the thermocouple operation.



(a) A thermocouple without extension leads



(b) Conventional method of establishing reference function temperature with copper extension leads

Fig. 12.11 *Schematics of Thermocouple circuits with and without extension leads in a typical iron–constantan thermocouple circuit*

Thermocouple Materials The choice of materials for thermocouples in governed by the following factors:

1. ability to withstand the temperature at which they are used,
2. immunity from contamination/oxidation, etc. which ensures maintenance of the precise thermo-electric properties with continuous use, and
3. linearity characteristics.

It may be noted that the relationship between thermo-electric emf and the difference between hot and cold junction temperatures is approximately of the parabolic form:

$$E = aT + bT^2 \tag{12.11}$$

Thermocouple can be broadly classified in two categories:

1. base-metal thermocouples, and
2. rare-metal thermocouple.

Base-metal thermocouples use the combination of pure metals and alloys of iron, copper and nickel and are used for temperature up to 1450 K. These are most commonly used in practice as they are more sensitive, cheaper and have nearly linear characteristics. Their chief limitation is the lower operating range because of their low melting point and vulnerability to oxidation. On the other hand, rare-metal thermocouples use a combination of pure metals and alloys of platinum for temperatures up to 1600°C and tungsten, rhodium and molybdenum for temperatures up to 3000°C.

Typical thermocouples with their temperature ranges and other salient operating characteristics, are given in Table 12.3.

Table 12.3 *Characteristics of Some Thermocouples*

S.No.	*Type*	*Thermocouples material**	*Approximate sensitivity in (μV/°C)*	*Useful temperature range (°C)*	*Approximate accuracy (%)*
1.	T	Copper–Constantan	20 – 60	–180 to +400	± 0.75
2.	J	Iron–Constantan	45 – 55	–180 to +850	± 0.75
3.	K	Chromel–Alumel	40 – 55	–200 to +1300	± 0.75
4.	E	Cheromel–Constantan	55 – 80	–180 to +850	± 0.5
5.	S	Platinum–Platinum/10% Rhodium	5 – 12	0 to +1400	± 0.25
6.	R	Platinum–Platinum/13% Rhodium	5 – 12	0 to +1600	± 0.25
7.	B	Platinum/30% Rhodium–Platinum/6% Rhodium	5 – 12	+100 to +1800	± 0.25
8.	W5	Tungston/5% Rhenium–Tungston/20% Rhenium	5 – 12	0 to +3000	± 0.15

*Constantan = copper/nickel; chromel = nickel/chromium; alumel = nickel/aluminium

For special purposes where high sensitivity is needed, thermocouples may be attached in series. The output is then the numerical sum of the voltages expected from each of the single couples. This is commonly known as *thermopile*. When connected in parallel, a group of thermocouples will give a reading that is the numerical average of the individual ones provided the resistance of each individual thermocouple being the same.

The following are the advantages of the TC sensors:

1. Thermocouple bead can be made of small size and consequently with low thermal capacity. In other words dynamic response of sensor is fairly good.
2. They cost considerably less as compared to other thermal sensors and further, they require no maintenance.
3. They are quite rugged type, i.e. they can withstands rough handling.
4. They cover wide range of temperature, i.e., from –200 to 3000°C.
5. Output signal is electrical and they can be used for indicating recording or microprocessor-based control systems.
6. Output signal, i.e., emf is independent of length or diameter of the wire.
7. They have good accuracy of the order of ±0.2 to ±0.75% of f.s.d.
8. They have excellent stability for a long period of time.
9. They can be conveniently mounted in a variety of temperature measurement situations.

The TC sensors, however, have the following limitations:

1. Inhomogeneity of composition of the thermocouple material and cold working of wires affect the sensitivity of the thermocouple.
2. The require insulation covering while using them in conducting fluids.
3. The output signal, i.e., emf requires amplification in most applications.

Problem 12.4 *A copper–constantan thermocouple was found to have linear calibration between 0 and 400°C with emf at maximum temperature (reference junction temperature 0°C) equal to 20.68 mV.*

(a) Determine the correction which must be made to the indicated emf if the cold junction temperature is 25°C.

(b) If the indicated emf is 8.92 mV in the thermocouple circuit determine the temperature of the hot junction.

Solution

(a) Sensitivity of the thermocouple$= \dfrac{20.68}{400-0} = 0.0517$ m V/°C

Since the thermocouple is calibrated at the reference junction of 0°C and is being used at 25°C, then the correction which must be made should be the thermo emf say E_{corr} between 0 and 25°C, that is:

$$E_{\text{corr}} = 0.0517 \times 25 = 1.293 \text{ mV}$$

(b) Indicated emf between the hot junction and reference junction at 25°C = 8.92 mV
Difference of temperature between hot and cold junctions

$$= \frac{8.92}{0.0517} = 172.53°\text{C}$$

Since the reference junction temperature is equal to 25°C
∴ hot junction temperature = 172.53 + 25 = 197.53°C

12.5.3 Solid State Temperature Sensors

Common I.C. devices like silicon diodes and transistors exhibit a stable and reproducible response to temperature. When a PN junction is forward biased by a constant current source, its governing equation between current and voltage is as follows:

$$V_{BE} = \frac{kT}{q} \ln\left(\frac{I_c}{I_{es}}\right) \tag{12.12}$$

where V_{BE} = base emitter voltage
I_c = collector current
I_{es} = emitter saturation current
K = Boltzmann constant (1.38×10^{-23} J/K)
q = electron charge (1.6×10^{-19} C)
T = absolute temperature (K).

Generally, the term within the parenthesis in Eq. (12.12) is constant and the emitter base voltage i.e., the output of the transducer becomes directly proportional to T which is the measured input. The *main advantage* of the solid state temperature sensors is their inherent linear operating characteristics with excellent accuracy of the order of ±1°C. In addition they have high levels of output signal which is capable of direct indication without any signal conditioning. The sensitivity of the silicon transistor within its useable range of –55 to 150°C is of the order of –2 mV/°C. Further, since the output is electrical, they have the capability of μp based control applications.

The *disadvantages* of these sensors are their limited temperature measuring range and their thermal mass which limits their response characteristics.

12.5.4 Quartz Thermometer

A piezo-electric crystal provides a highly accurate and sensitive method of temperature measurement based on the change in its resonant frequency which is directly proportional to the temperature change. Herein, the crystal is cut in the form of shear type *LC* cut, in which the change in resonant frequency is highly linear as well as repeatable. The associated electronic circuitry of this thermometer consists of frequency counters and digital read-out of the measured frequency.

The fundamental frequency f_0 depends on the thickness of the crystal and can be adjusted so as to give a sensitivity of the order of 1000 Hz for a temperature change of 1°C. In other words, the detection of change in frequency of oscillation of 1 Hz gives a resolution of 0.001°C. Further, temperature in the range of –40 to 230°C can be measured precisely and accurately by this method.

The advantages of the quartz thermometer are:

1. Highly linear output as the linearity error is ±0.5% of F.S.
2. Long-term stability and reliability.
3. High resolution of the order of 0.001°C.
4. Excellent repeatability in the measuring range of –40 to 230°C.

The limitations of the quartz thermometer are:

1. Limited measuring range i.e., –40 to 230°C.
2. Piezo-electric crystals have strong cross-sensitivity for pressure changes if they occur simultaneously in the temperature measuring systems.

12.6 ■ RADIATION METHODS (PYROMETRY)

All the temperature measuring deviced discussed so far i.e., pressure thermometer, thermistor or thermocouple, etc. require the thermometer to be brought into physical contact with the body whose temperature is to be measured. This means that the thermometer must be capable of withstanding this temperature. In the case of very hot bodies, the thermometer may melt at the high temperature. Secondly, for bodies that are moving, a non-contacting device for measuring the temperature is most convenient. Thirdly, if the distribution of temperature over the surface of an object is required, a non-contacting device can readily 'scan' the surface.

For temperatures above 650°C, the heat radiations emitted from the body are of sufficient intensity to be used for measuring the temperature. Instruments that employ radiation principles fall into three general classes: (a) total radiation pyrometer, (b) selective (or partial) radiation pyrometers, and (c) infrared (IR) pyrometer. The first is sensitive to all the radiation that enters the instrument and the second only to radiation of a particular wavelength. Further, the IR pyrometers employ the infrared portion of the spectrum by using a thermal detector to measure the temperature on the surface of the body.

12.6.1 Total Radiation Pyrometer

The total radiation pyormeter receives a controlled sample of the total radiation of a hot body (say a furnace) and focusses it on to a temperature sensitive transducer. The term 'total radiation' includes both visible (light) and invisible (infrared) radiations. It may be noted that the wave lengths of light in the visible range is from 0.3 to 0.72 μm, whereas the infrared radiations are associated with relatively large wavelengths of 0.72 to 1000 μm. They require special optical materials for focussing. Ordinary glass is unsatisfactory, as it absorbs infrared radiations. In fact, the practical radiation pyrometers are sensitive to a limited wavelength band of radiant energy, (i.e., from 0.32 to 40 μm) although theory indicates that they should be sensitive to the entire spectrum of energy radiated by the object.