ICE-2

Ques-1: Determine the analytic func^n whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$

Sol^n:) $e^{2x}(x\cos 2y - y\sin 2y)$

$$\delta u/\delta x = e^{2x}(\cos 2y) - 0) + 2e^{2x}(x\cos 2y - y\sin 2y)$$

$$= e^{x}\cos(2y)$$

$$= e^{2x}(\cos 2y + 2x\cos 2y - 2y\sin 2y) = \phi_1(x,y)$$

$$\phi_1(z,0) = e^{-2z}(0+0+0) = 0$$

By. Milne's thomson method

$$F(z) = \int \phi_1(z,0)\,dz + i\int \phi_2(z_2,0)\,dz + k$$

$$= \int e^{2z}(1+2z)\,dz - 0 + c$$

$$= (1+2z)\left(\frac{e^{2z}}{2}\right) - 2\left(\frac{e^{2z}}{4}\right) + c$$

$$= \frac{e^{2z}}{2}(1+2z-1) + c$$

$$\boxed{f(z) = e^{2z} + c}$$

Que 302 Find the regular function whose imaginary part is $\cos x \cosh y$.

$\Rightarrow$ $\cos x \cosh y \Rightarrow \cos x \left(\dfrac{e^y + e^{-y}}{2} \right)$

$\dfrac{\delta u}{\delta y} = \delta (\cos x \cosh y)$

$= \cos x \left(\dfrac{e^y - e^{-y}}{2} \right) \Bigg\} \to 0$

$\dfrac{\delta u}{\delta x} = -\sin x \left(\dfrac{e^y + e^{-y}}{2} \right) \Bigg\} \begin{array}{l} x = z \\ y = 0 \end{array} ; -\sin z$

$\dfrac{d \omega}{dz} = \dfrac{\delta v}{\delta y} + i \dfrac{\delta v}{\delta x}$

$\dfrac{d\omega}{dz} = i \sin z$

$\int d\omega = \int -i \sin z \, dz$

$\boxed{\omega = i \cos z}$

Que:03 $f(z) = u + iv$ if

(i) $u - v = e^x(\cos y - \sin y)$

$u = e^x \cos y$

$v = e^x \sin y$

$f(z) = u + iv$

$\qquad = e^x \cos y + i e^x \sin y$

$\qquad = e^x [\cos y + i \sin y]$

by Euler's identity $\longrightarrow$ $f(z) = e^x e(iy)$

$f(z) = e^{(x+iy)}$

$$\boxed{f(z) = e^z}$$

(ii) $u + v = \dfrac{x}{x^2 + y^2}$, when $f(1) = 1$

$\qquad f(z) = u + iv \quad -①$

$\qquad i f(z) = iu - v \quad -②$

$\qquad ① + ②$ gives $\quad (1+i) f(z) = (u - v) + i(v + u)$

$\qquad\qquad\qquad f(z) = u + iv$

V is given as x/x^2+y^2

$$\frac{dv}{dx} = \frac{(1.6x^2+y^2) - x(2x))}{(x^2+y^2)^2}$$

$$\left.\begin{matrix} x\to 2 \\ y\to 0 \end{matrix}\right\} \quad \frac{\delta v}{\delta x} = \left(\frac{z^2-2z^2)}{z^4}\right) = -\frac{1}{z^2}$$

$$\frac{\delta v}{\delta y} = -2yx/x^2+y^2 = 0$$

$$\begin{matrix} x+2 \\ y+0 \end{matrix}$$

$$\frac{dw}{dz} = 0 + (-1/z^2)$$

$$\int \delta w = -i \int \frac{1}{z^2} \, dz$$

$$w = i/z$$

$$f(z) = \frac{1}{z} \quad \& \quad (1+i)f(z) = f(z)$$

$$\boxed{f(z) = \left(\left(\frac{i}{i+1}\right)\frac{1}{z}\right)}$$

Que: 04

$$\omega = \frac{1}{z} \quad , \quad z = \frac{1}{\omega} \qquad \& \quad z = \frac{1}{u + iv}$$

$$= \frac{u - iv}{u^2 + v^2}$$

$$|z - 2i| = 2$$

$$|x + (y-2)i| = 2 \qquad x + iy = \frac{u - iv}{u^2 + v^2}$$

$$x^2 + (y-2)^2 = 4 \qquad x = \frac{u}{u^2+v^2} \quad , \quad y = \frac{-v}{u^2+v^2}$$

$$x^2 + y^2 - 2y = 0$$

$$\frac{u^2}{(u^2+v^2)^2} + \frac{v^2}{(u^2+v^2)^2} - 2\frac{v}{u^2+v^2} = 0$$

$$\frac{u^2 + v^2 - 2v(u^2+v^2)}{(u^2+v^2)^2} = 0 \qquad\qquad 1 + 2v = 0$$

$$\boxed{v = -\frac{1}{2}}$$

$$\cancel{u^2 + v^2 - 2vu^2 - 2v^3 = 0}$$

$$(u^2 + v^2) + 2v(u^2 + v^2) = 0$$

$$(u^2 + v^2)(1 + 2v) = 0$$

Que:05   Show that the transformation $\omega = \dfrac{2z+3}{z-4}$

maps the circle $x^2 + y^2 - 4x = 0$ onto
the straight line $4u + 3 = 0$

⇒ $\omega = \dfrac{2z+3}{z-4}$ $4u + 3 = 0$

→ it is a bilinear transformation.

Inverse transformation

$$z = \frac{4\omega + 3}{\omega - 2}$$

the eq^n of circle → $x^2 + y^2 - 4x = 0$

i.e. $|z|^2 - 4\,\text{Re}\,z = 0$

$$z\bar{z} - 2(z + \bar{z}) = 0$$

$\therefore \omega = U + iv$

$$\frac{4\omega + 3}{\omega - 2} \cdot \frac{4\bar{\omega} + 3}{\bar{\omega} - 2} - 2\left(\frac{4\omega + 3}{\omega - 2} + \frac{4\bar{\omega} + 3}{\bar{\omega} - 2}\right) = 0$$

$$(4\omega+3)(4\bar{\omega}+3) - 2\{(4\omega+3)(\bar{\omega}-2) + (4\bar{\omega}+3)(\omega-2)\}$$
$$\sim 0$$

i.e. $2(\omega+\bar{\omega}) + 3 = 0$

i.e. $4u + 3 = 0$
$$= H.P.$$

Que: 06 find the bilinear transformation which maps the
points $z = 1, i, -1$ into the points $\omega = i, 0, -i$

Sol^n $\omega = \dfrac{az+b}{cz+d}$ — (i)

put $z=1$ & $\omega = i$

$$i = \dfrac{a+b}{c+d}$$

$$\boxed{ic + id = a + b} \quad — (ii)$$

put $z = i$ & $\omega = 0$

$$0 = \dfrac{ai+b}{c(i)+d} \qquad = \quad ai+b = 0$$

$$\boxed{b = -ai} (iii)$$

put $z = -1$ & $\omega = -i$

$$-i = \frac{a(-1) + b}{c(-1) + d}$$

$$ic - id = -a + b \quad - iv$$

add eq ② & ⓘⱽ

$$ic + id = a + b$$
$$ic - id = -a + b$$

we, get $2ic = 2b$

$$c = \frac{b}{i} = \frac{-ai}{i} = -a \longrightarrow (v)$$

$$\therefore \omega = \frac{az - ai}{-az - ai}$$

$$\therefore \omega = \frac{-a(-z + i)}{-a(z + i)}$$

$$\Rightarrow \frac{l - 2}{z + 1} \quad \text{is the bilinear transformation.}$$

Que:07 $\displaystyle\int \frac{3z^2+z}{z^2-1}$ $|z-1|=1$

$$z^2-1=0$$
$$z=+-1$$
$$|z-1|=1$$
$$z=1$$
$$r=1 \quad \text{includes the point } z=1$$

$$\oint \frac{3z^2+z}{z^2-1}\,dz = \int \frac{\left(\dfrac{3z^2+z}{z+1}\right)}{z-1}\,dz$$

$$= 2\pi i \left(\frac{3z^2+z}{z+1}\right)_{z=}$$

$$= 4\pi i$$

Que :08

$$\frac{z^2-2z}{(z+1)^2(z^2+4)}$$

$$z = -1, -1$$
$$z = \pm 2i$$

Residue at -1

$$\frac{d}{dz}\left[[z-(-1)]^2 f(z)\right]$$

$$\frac{d}{dz}\frac{(z^2-2z)}{(z^2+4)}$$

$$\frac{(z+1)^2 \cdot (z^2-2)}{(z+1)^2(z^2+4)}$$

$$, \quad 2z^3+8z-2z^2-8-2z^3+4z^2$$

$$= \frac{2z^2+8z-8}{(z^2+4)^2}$$

$$\doteq \frac{2-8-8}{25} = \frac{-14}{25}$$

Residue at $z = 2i$

$$\left|(z-2i)\, f(z)\right|_{z=2i} = \frac{(z-2i)}{(z+1)^2} \cdot \frac{z^2-2z}{(z-2i)(z+2i)}$$

$$= \frac{z^2-2z}{(z^2+1+2z)(z+2i)}$$

$$= \frac{+4(1+i)}{-(3i+4)4} = \frac{1+i}{3i+4}$$

$$\boxed{= \frac{7+i}{25}}\quad \text{Ans}$$

Residue at $z = -2i$

$$= \frac{7-i}{25}$$