Module 5: DC-AC Converters

Lecture 14: DC-AC Inverter for EV and HEV Applications

DC-AC Inverter for EV and HEV Applications

Introduction

The topics covered in this chapter are as follows:

- DC-AC Converters
- Principle of Operation of Half Bridge DC-AC Inverter (R Load)
- Half Bridge DC-AC Inverter with L Load and R-L Load
- Single Phase Bridge DC-AC Inverter with R Load
- Single Phase Bridge DC-AC Inverter with R-L Load

DC-AC Converters

In **Figure 1** a configuration of an EV. In this figure it can be seen that the traction motor requires AC input. The main source of electrical power is the battery which is a DC source. The DC output of the battery is bucked or bossted according to the requirement and then converted into AC using a ***DC-AC inverter***. The function of an inverter is to change a dc input voltage to a symmetric ac output voltage of desired magnitude and frequency. The output voltage waveforms of ideal inverters should be sinusoidal. However, the waveforms of practical inverters are non-sinusoidal and contain certain harmonics.

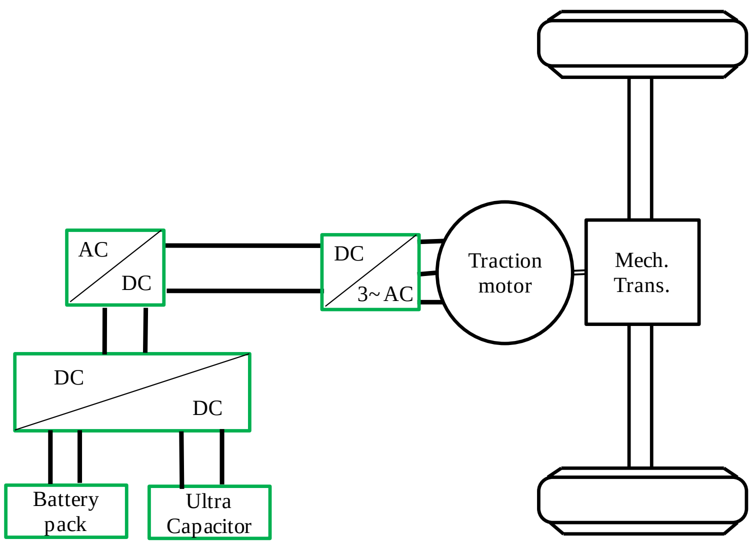

Figure 1: Configuration of electric vehicle [1]

Principle of Operation of Half Bridge DC-AC Inverter (R Load)

A single phase inverter is shown in **Figure 2**. The analysis of the DC-AC inverters is done taking into account the following assumptions and conventions:

- The current entering ***node a*** in **Figure 2** is considered to be positive.
- The switches S_1 and S_2 are unidirectional, i.e. they conduct current in one direction.
- The current through S_1 is denoted as i_1 and the current through S_2 is i_2.

The switching sequence is so design (**Figure 3**) that switch S_1 is on for the time duration $0 \le t \le T_1$ and the switch S_2 is on for the time duration $T_1 < t \le T_2$. When switch S_1 is turned on, the instantaneous voltage across the load is

$$v_o = \frac{V_{in}}{2} \tag{1}$$

When the switch S_2 is only turned ***on***, the voltage across the load is

$$v_o = -\frac{V_{in}}{2} \tag{2}$$

The waveform of the output voltage and the switch currents for a resistive load is shown in (**Figure 3**).

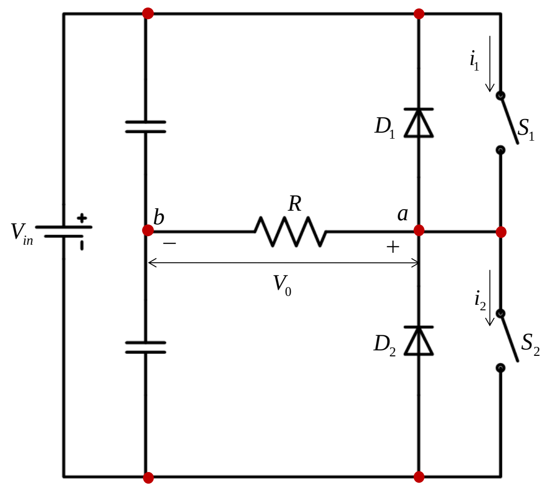

Figure 2: Basic DC-AC inverter

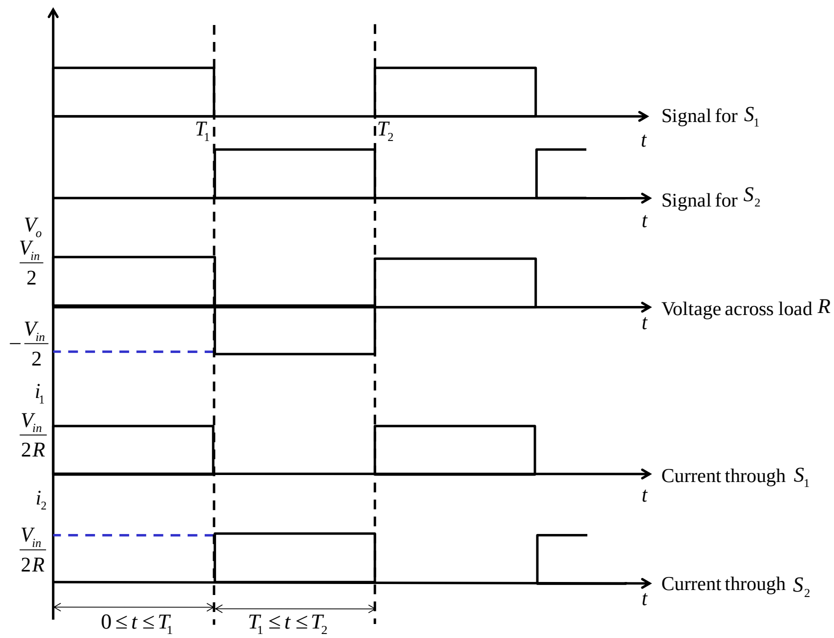

Figure 3: Current and voltage waveforms for DC-AC inverter

The r.m.s value of output voltage v_o is given by

$$V_{o,rms} = \left(\frac{1}{T_1} \int_0^{T_1} \frac{V_{in}^2}{4} dt \right) = \frac{V_{in}}{2} \tag{3}$$

The instantaneous output voltage (v_o) is rectangular in shape (**Figure 3**). The instantaneous value of v_o can be expressed in Fourier series as:

$$v_o = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) \tag{4}$$

Due to the quarter wave symmetry along the time axis (**Figure 3**), the values of a_0 and a_n are zero. The value of b_n is given by

$$b_n = \frac{1}{\pi} \left[\int_{\frac{-\pi}{2}}^{0} \frac{-V_{in}}{2} d(\omega t) + \int_0^{\frac{\pi}{2}} \frac{V_{in}}{2} d(\omega t) \right] = \frac{2V_{in}}{n\pi} \tag{5}$$

Substituting the value of b_n from **equation 5** into **equation 4** gives

$$v_o = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_{in}}{n\pi} \sin(n\omega t) \tag{6}$$

The current through the resistor (i_L) is given by

$$i_L = \sum_{n=1,3,5,...}^{\infty} \frac{1}{R}\frac{2V_{in}}{n\pi}\sin(n\omega t) \tag{7}$$

For $n=1$, **equation 6** gives the r.m.s value of the fundamental component as

$$V_{o1} = \frac{2V_{in}}{\sqrt{2}\pi} \approx 0.45V_{in} \tag{8}$$

Half Bridge DC-AC Inverter with *L* Load and *R*-*L* Load

The DC-AC converter with inductive load is shown in **Figure 4**. For an inductive load, the load current cannot change immediately with the output voltage. The working of the Dc-AC inverter with inductive load is as follow is:

Case* 1:** In the time interval $0 \le t \le T_1$ the switch S_1 is ***on and the current flows through the inductor from points ***a*** to ***b***. When the switch S_1 is turned ***off*** (case 1) at $t = T_1$, the load current would continue to flow through the capacitor C_2 and diode D_2 until the current falls to ***zero***, as shown in **Figure 5**. ***Case* 2:** Similarly, when S_2 is turned ***off*** at $t = T_2$, the load current flows through the diode D_1 and the capacitor C_1 until the current falls to ***zero***, as shown in **Figure 6**.

When diodes D_1 and D_2 conduct, energy is fed back to the dc source and these diodes are known as ***feedback diodes***. These diodes are also known ad ***freewheeling diodes***. The current for purely inductive load is given by

$$i_L = \sum_{n=1,3,5,...}^{\infty} \frac{1}{\omega nL}\frac{2V_{in}}{n\pi}\sin\left(n\omega t - \frac{\pi}{2}\right) \tag{9}$$

Similarly, for the $R-L$ load. The instantaneous load current is obtained as

$$i_L = \sum_{n=1,3,5,...}^{\infty} \frac{2V_{in}}{n\pi\sqrt{R^2+\left(n\omega L\right)^2}}\sin\left(n\omega t - \theta_n\right)$$

where (10)

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$$

The instantaneous voltage (v_o) across $R-L$ load and the instantaneous current (i_L) through it are shown in **Figure 7**.

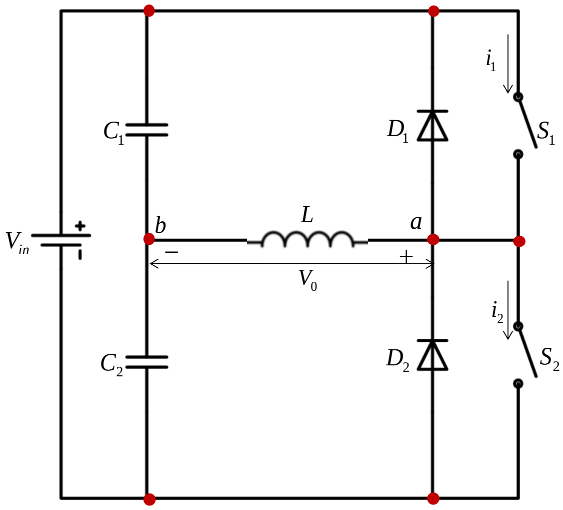

Figure 4: DC-AC inverter with inductive load

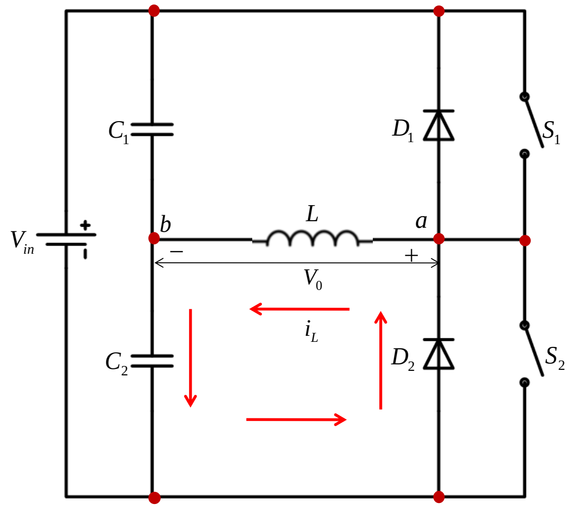

Figure 5: Load current in case 1

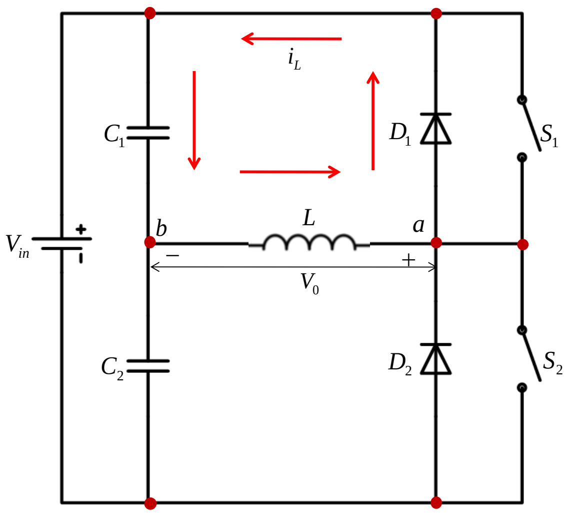

Figure 6: Load current in case 2

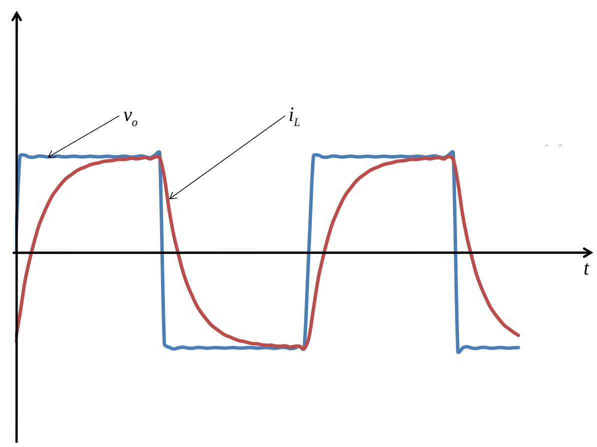

Figure 7: Instantaneous output voltage and load current through R-L load

Single Phase Bridge DC-AC Inverter with *R* Load

A single phase bridge DC-AC inverter is shown in **Figure 8**. The analysis of the single phase DC-AC inverters is done taking into account following assumptions and conventions:

- The current entering ***node a*** in **Figure 8** is considered to be positive.
- The switches S_1, S_2, S_3 and S_4 are unidirectional, i.e. they conduct current in one direction.

When the switches S_1 and S_2 are turned ***on*** simultaneously for a duration $0 \le t \le T_1$, the input voltage V_{in} appears across the load and the current flows from point ***a*** to ***b***. If the switches S_3 and S_4 are turned ***on*** for a duration $T_1 \le t \le T_2$, the voltage across the load is reversed and the current through the load flows from point ***b*** to ***a***. The voltage and current waveforms across the resistive load are shown in **Figure 9**. The instantaneous output voltage can be expressed in Fourier series as

$$v_o = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) \tag{11}$$

Due to the square wave symmetry along the ***x-axis*** (as seen in **Figure 9**), both a_o and a_n are ***zero***, and b_n is obtained as

$$b_n = \frac{1}{\pi}\left[\int_{\frac{-\pi}{2}}^{0} \frac{-V_{in}}{2} d(\omega t) + \int_{0}^{\frac{\pi}{2}} \frac{V_{in}}{2} d(\omega t)\right] = \frac{4V_{in}}{n\pi} \tag{12}$$

Substituting the value of b_n from **equation 12** into **equation 11** gives

$$v_o = \sum_{n=1,3,5,...}^{\infty} \frac{4V_{in}}{n\pi} \sin(n\omega t) \tag{13}$$

The instantaneous current through the resistive load is given by

$$i_L = \sum_{n=1,3,5,...}^{\infty} \frac{1}{R} \frac{4V_{in}}{n\pi} \sin(n\omega t) \tag{14}$$

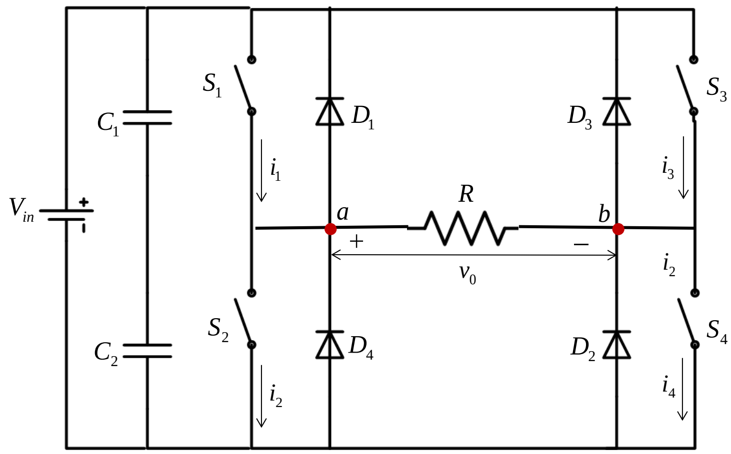

Figure 8: Full bridge DC-AC inverter with resistive load

Single Phase Bridge DC-AC Inverter with *R-L* Load

The function of the inverter in case of $R-L$ load can be explained as follows:

Case 1: At time $t=T_1$, the switches S_1 and S_2 are turned ***off*** and the pair of switches S_3 and S_4 are turned ***on***. Due to the inductive load, the current through the load (i_L) will not change its direction at $t=T_1$ and will continue to flow through the load from point ***a*** to ***b***, through the diodes D_3 and D_4, till it becomes ***zero*** as shown in **Figure 10a**. Once, $i_L=0$, S_3 and S_4 start conducting and the load current i_L builds up in opposite direction (point ***b*** to ***a***).

Case 2: At time $t=T_2$, the switches S_1 and S_2 are turned ***on*** and the pair of switches S_3 and S_4 are turned ***off***. Just as in ***case 1***, the current takes time to become ***zero*** and diodes D_1 and D_2 conduct as long as its ***non-zero***. This condition is shown in **Figure 10b**.

The instantaneous current through the $R-L$ load is given by

$$i_L = \sum_{n=1,3,\ldots}^{\infty} \frac{4V_{in}}{n\pi\sqrt{R^2+(\omega L)^2}} \sin(n\omega t-\theta_n)$$

where (15)

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$$

The current and voltage waveforms for $R-L$ load are shown in **Figure 11**.In this figure the conduction is divided into 4 distinct zones. In *Zone I* the diode D_1 and D_2 conduct until i_L becomes zero. Once, i_L equals ***zero***, the switches S_1 and S_2 conduct and it is marked as *Zone II*. At time $t=T_2$, the diodes D_3 and D_4 conduct and this is marked as *Zone III* in **Figure 11**. Finally, in *Zone IV* the switches S_3 and S_4 conduct.

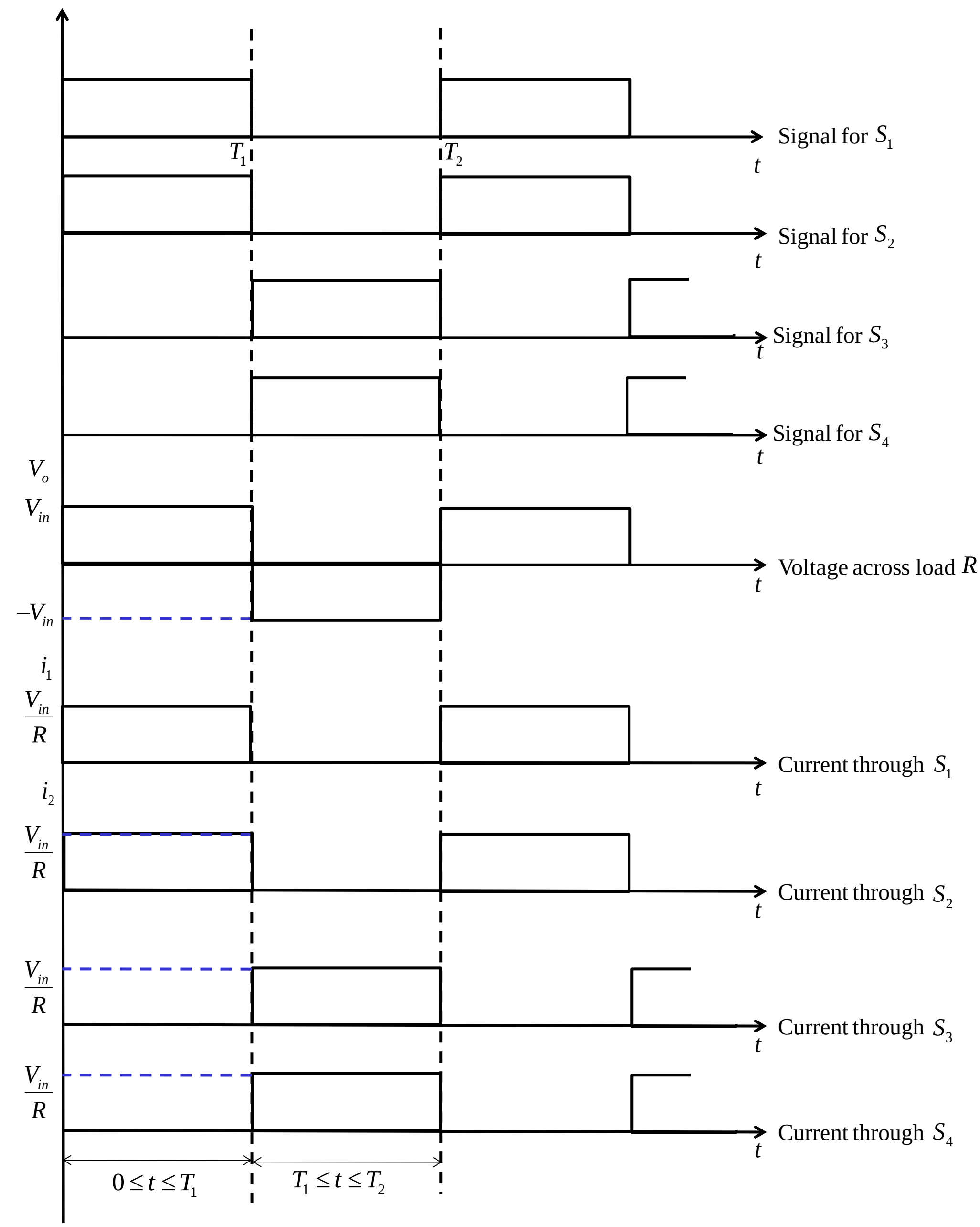

Figure 9: Instantaneous voltage and current waveforms for full bridge DC-AC inverter

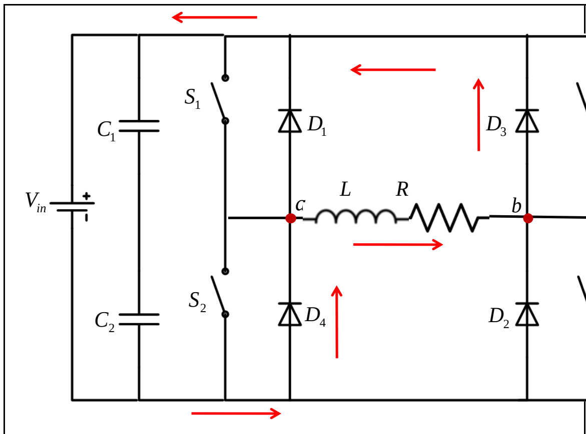	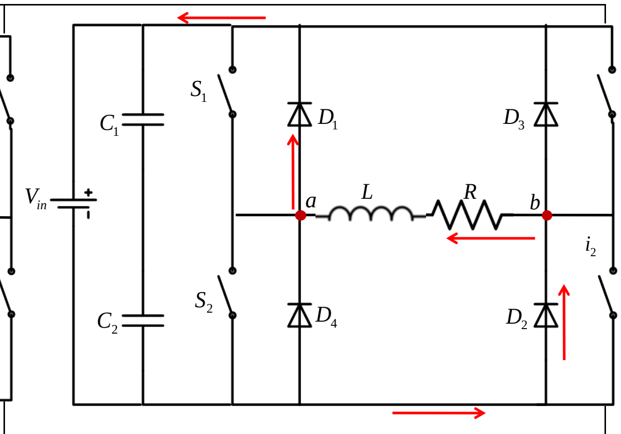
Figure 10a: Current flow in case 1	**Figure 10b: Current flow in case 2**

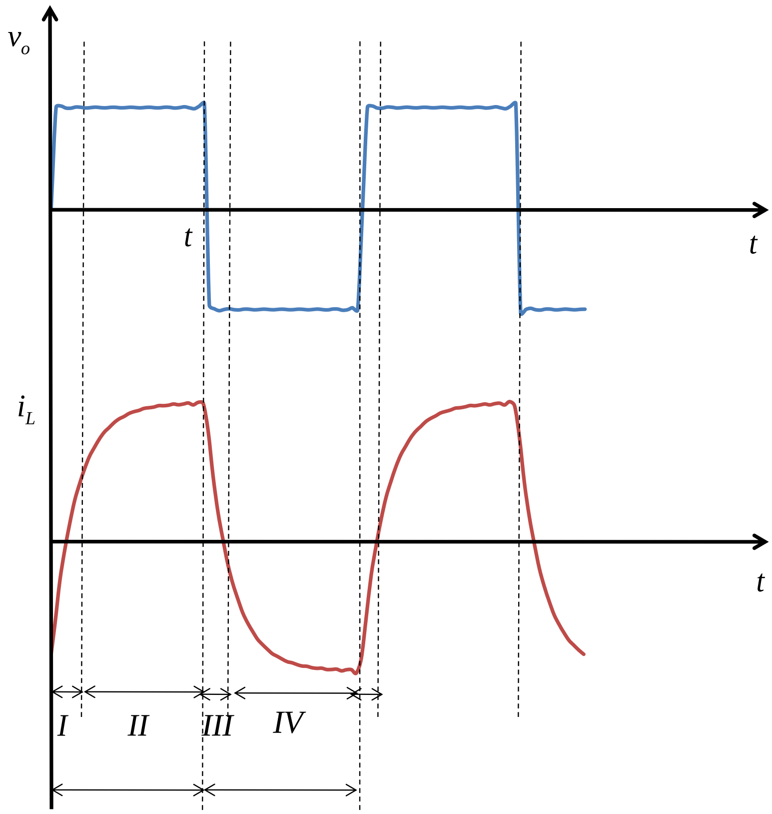

Figure 11: Voltage and current waveforms in case of *R-L* loads

References:

[1] M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

Suggested Reading:

[1] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*, 3rd edition, Pearson, 2004

[2] V. R. Moorthi, *Power Electronics: Devices, Circuits and Industrial Applications*, Oxford University Press, 2007

Lecture 15: Three Phase DC-AC Inverters

Three Phase DC-AC Inverters

Introduction

The topics covered in this chapter are as follows:

- Three phase DC-AC Converters
- 180-Degree Conduction with Star Connected Resistive Load
- 180-Degree Conduction with Star Connected *R-L* Load

Three Phase DC-AC Converters

Three phase inverters are normally used for high power applications. The advantages of a three phase inverter are:

- The frequency of the output voltage waveform depends on the switching rate of the switches and hence can be varied over a wide range.
- The direction of rotation of the motor can be reversed by changing the output phase sequence of the inverter.
- The ac output voltage can be controlled by varying the dc link voltage.

The general configuration of a three phase DC-AC inverter is shown in **Figure 1**. Two types of control signals can be applied to the switches:

- 180^{o} conduction
- 120^{o} conduction

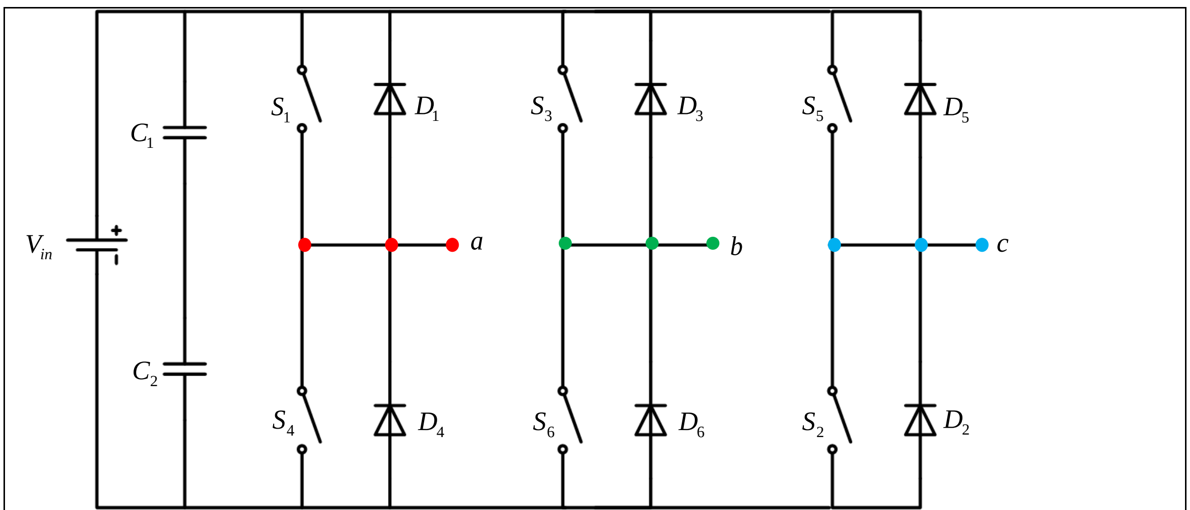

Figure 1: Configuration of a Three-Phase DC-AC Inverter

180-Degree Conduction with Star Connected Resistive Load

The configuration of the three phase inverter with star connected resistive load is shown in **Figure** 2. The following convention is followed:

- A current leaving a node point ***a***, ***b*** or ***c*** and entering the neutral point ***n*** is assumed to be positive.
- All the three resistances are equal, $R_a = R_b = R_c = R$.

In this mode of operation each switch conducts for 180^o. Hence, at any instant of time ***three switches*** remain ***on***. When S_1 is ***on***, the terminal ***a*** gets connected to the positive terminal of input DC source. Similarly, when S_4 is ***on***, terminal ***a*** gets connected to the negative terminal of input DC source. There are six possible modes of operation in a cycle and each mode is of 60^o duration and the explanation of each mode is as follows:

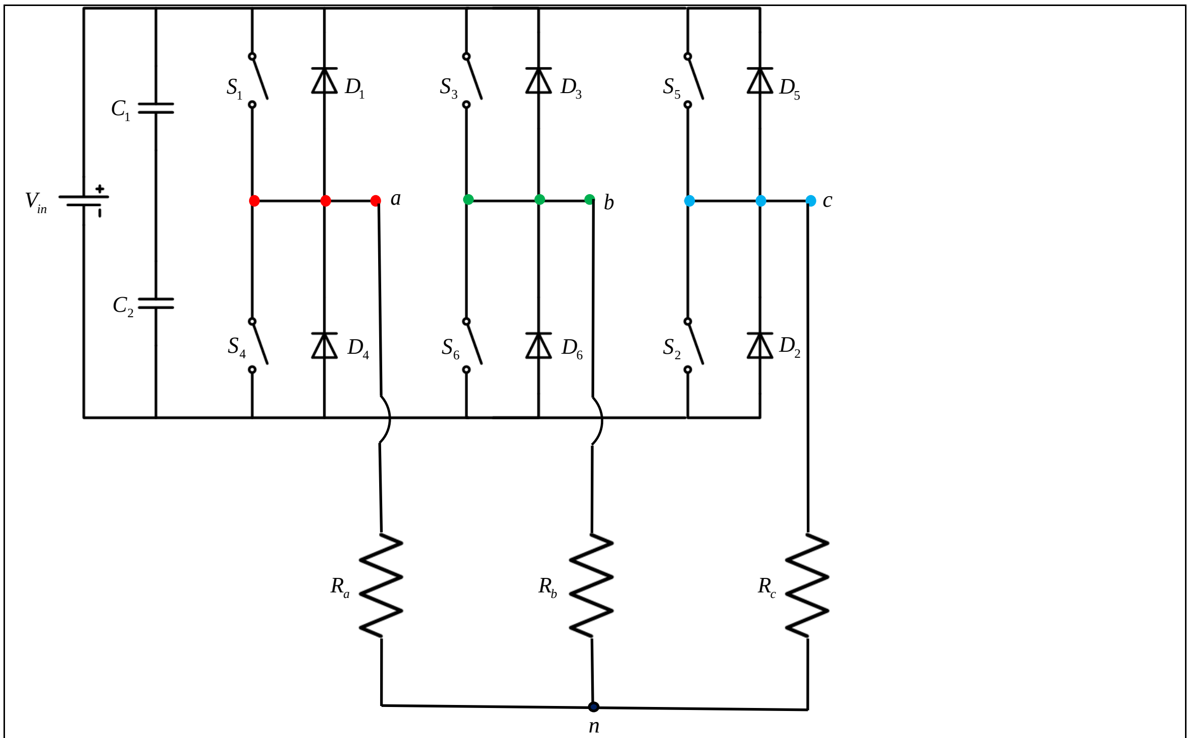

Figure 2: Three-Phase DC-AC Inverter with star connect resistive load

Mode 1**:** In this mode the switches S_5, S_6 and S_1 are turned ***on*** for time interval $0 \le \omega t \le \frac{\pi}{3}$. As a result of this the terminals ***a*** and ***c*** are connected to the positive terminal of the input DC source and the terminal ***b*** is connected to the negative terminal of the DC source. The current flow through R_a, R_b and R_c is shown in **Figure 3a** and the equivalent circuit is shown in **Figure 3b**. The equivalent resistance of the circuit shown in **Figure 3b** is

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2} \tag{1}$$

The current i delivered by the DC input source is

$$i = \frac{V_{in}}{R_{eq}} = \frac{2}{3}\frac{V_{in}}{R} \tag{2}$$

The currents i_a and i_b are

$$i_a = i_c = \frac{1}{3}\frac{V_{in}}{R} \tag{3}$$

Keeping the current convention in mind, the current i_b is

$$i_b = -i = -\frac{2}{3}\frac{V_{in}}{R} \tag{4}$$

Having determined the currents through each branch, the voltage across each branch is

$$v_{an} = v_{cn} = i_a R = \frac{V_{in}}{3};\; v_{bn} = i_b R = -\frac{2V_{in}}{3} \tag{5}$$

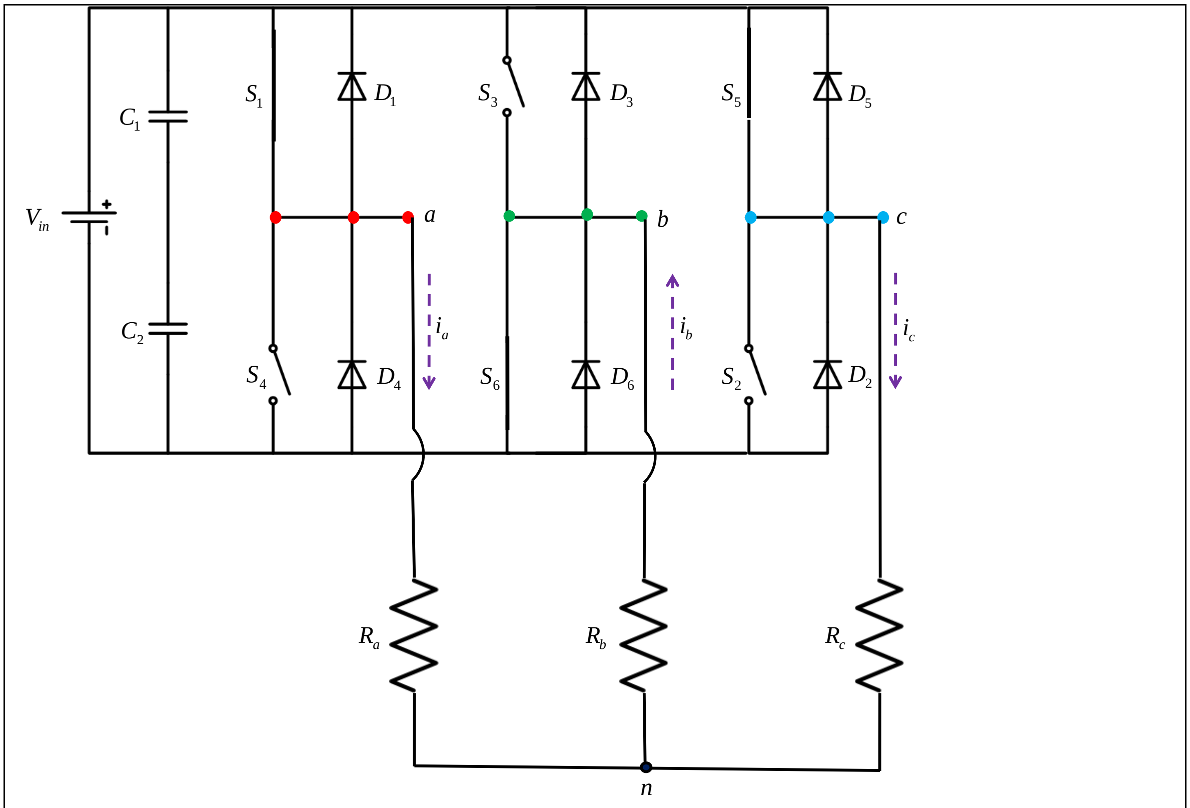

Figure 3a: Current through the load in *Mode 1*

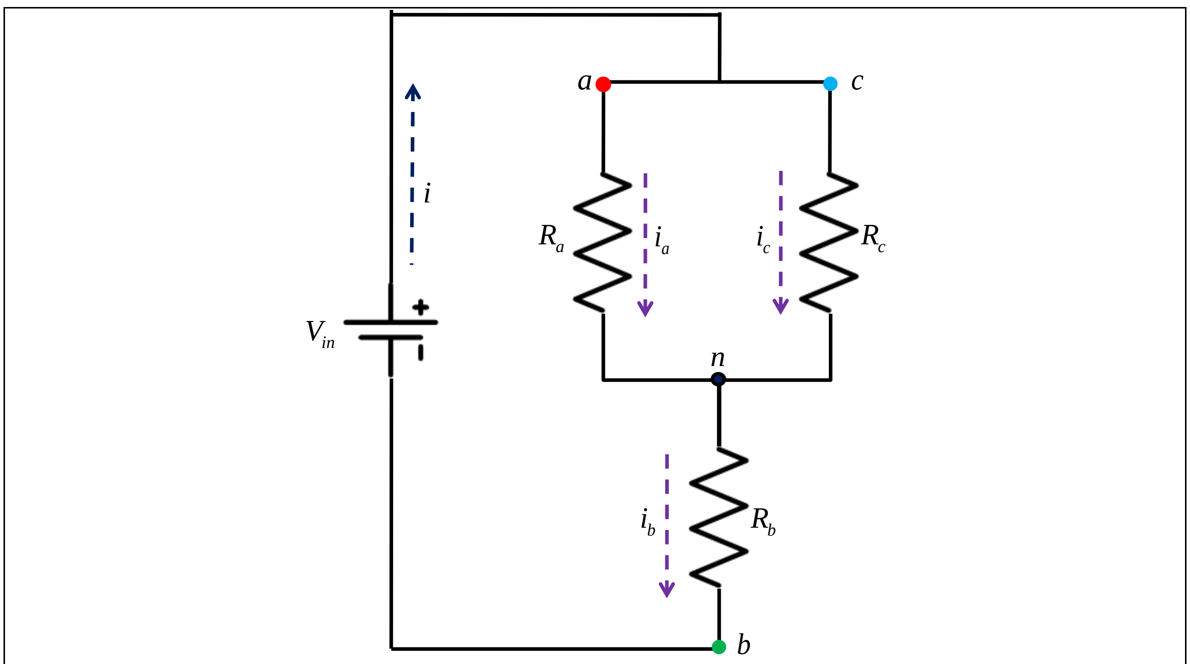

Figure 3b: Equivalent circuit in *Mode 1*

Mode 2*:** In this mode the switches S_6, S_1 and S_2 are turned ***on for time interval $\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$.

The current flow and the equivalent circuits are shown in **Figure 4a** and **Figure 4b** respectively. Following the reasoning given for ***mode 1***, the currents through each branch and the voltage drops are given by

$$i_b = i_c = \frac{1}{3}\frac{V_{in}}{R};\ i_a = -\frac{2}{3}\frac{V_{in}}{R} \tag{6}$$

$$v_{bn} = v_{cn} = \frac{V_{in}}{3};\ v_{an} = -\frac{2V_{in}}{3} \tag{7}$$

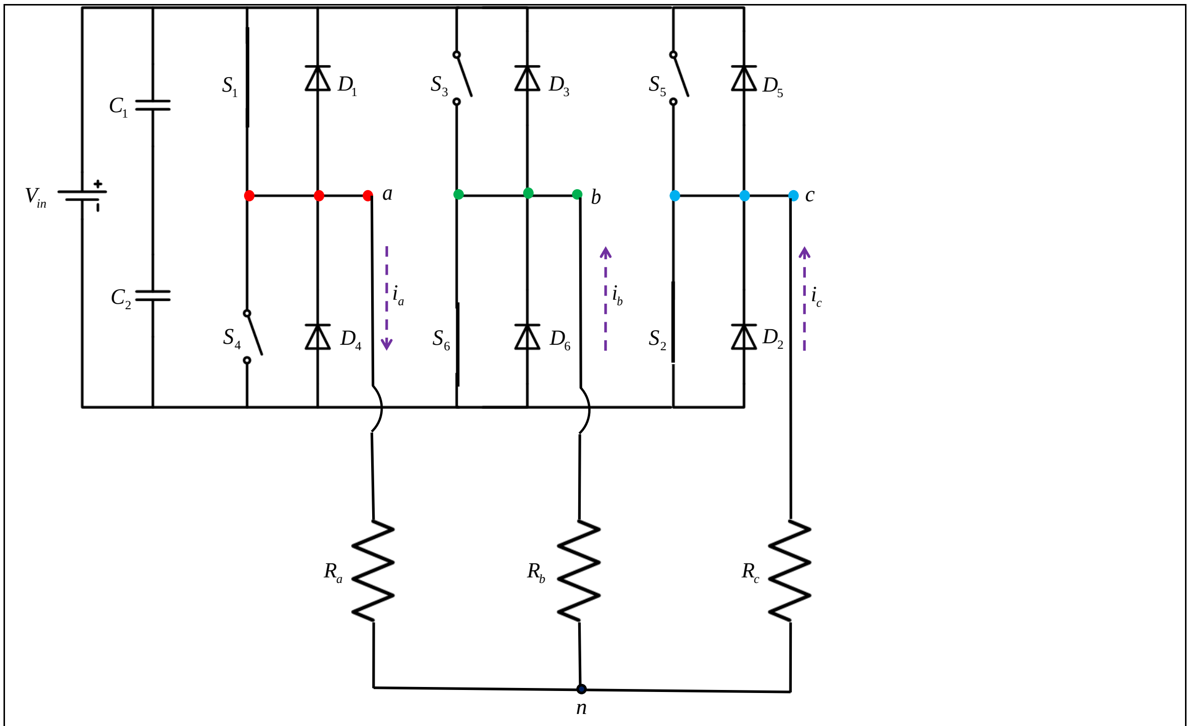

Figure 4a: Current through the load in *Mode 2*

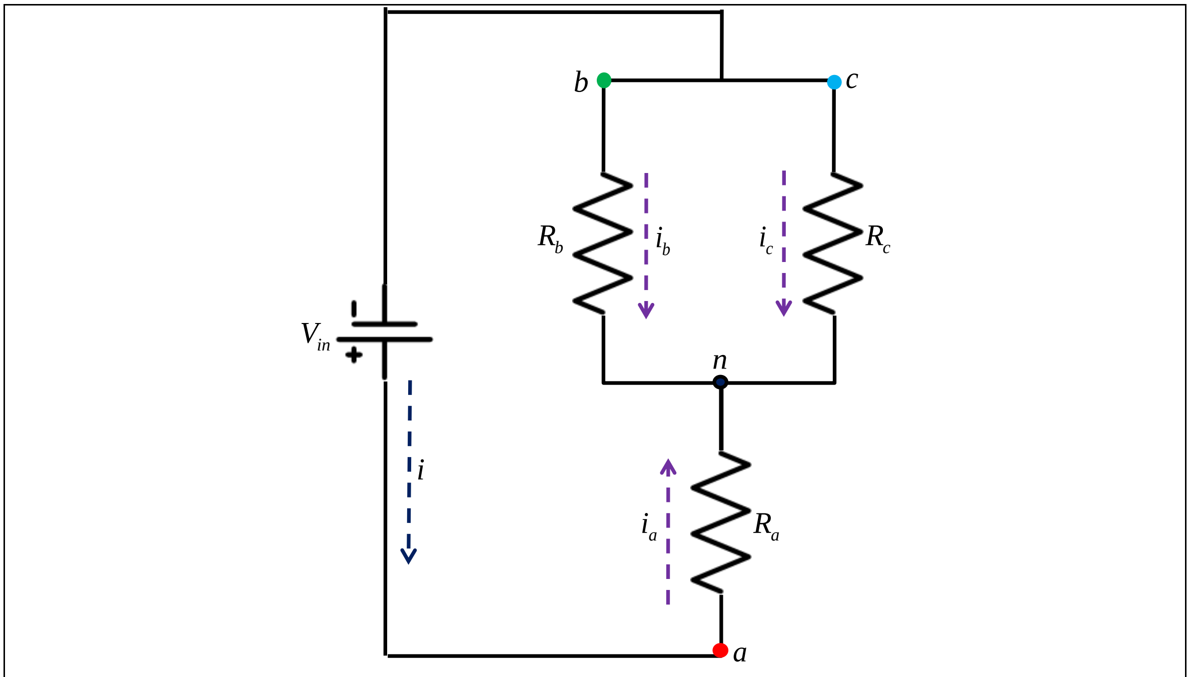

Figure 4b: Equivalent circuit in *Mode 2*

Mode 3*:** In this mode the switches S_1, S_2 and S_3 are ***on for $\frac{2\pi}{3} \le \omega t \le \pi$. The current flow and the equivalent circuits are shown in **Figure 5a** and **figure 5b** respectively. The magnitudes of currents and voltages are:

$$i_a = i_b = \frac{1}{3}\frac{V_{in}}{R};\ i_c = -\frac{2}{3}\frac{V_{in}}{R} \tag{8}$$

$$v_{an} = v_{bn} = \frac{V_{in}}{3};\ v_{cn} = -\frac{2V_{in}}{3} \tag{9}$$

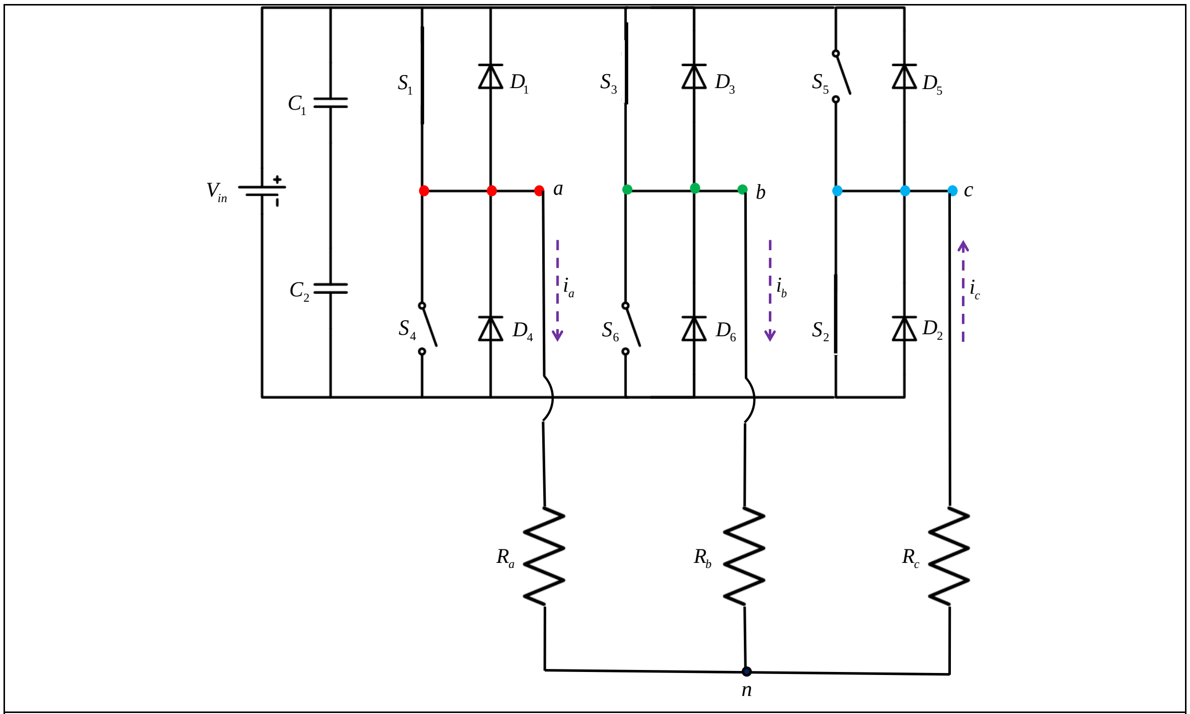

Figure 5a: Current through the load in *Mode 32*

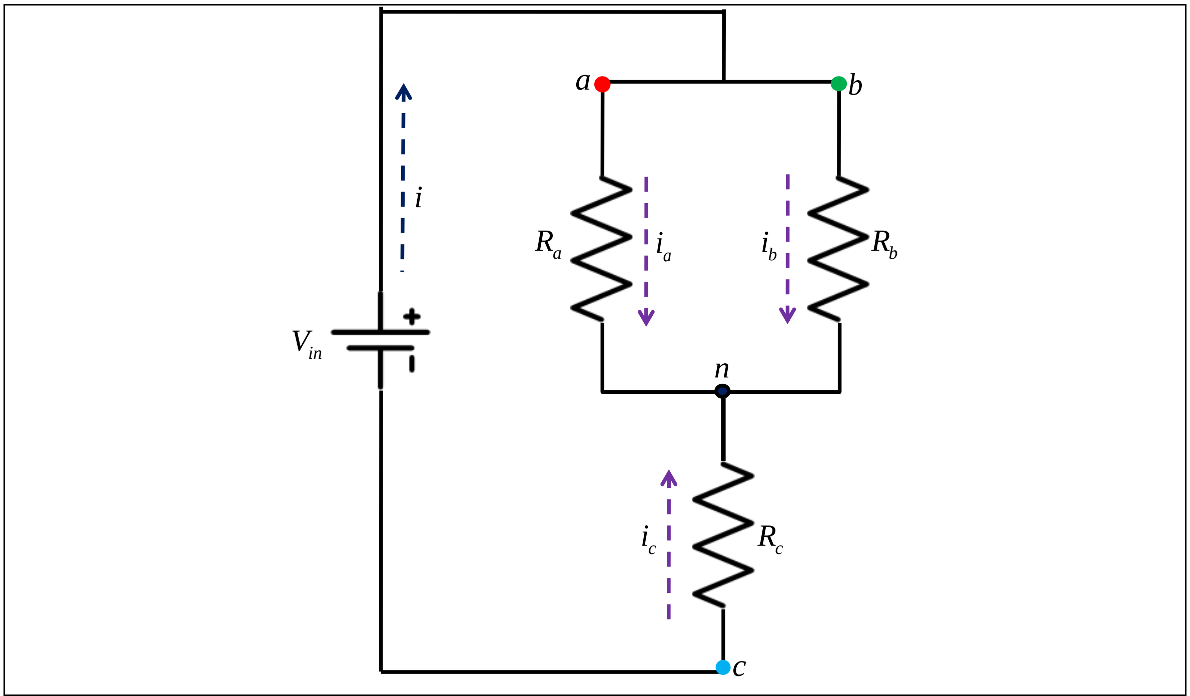

Figure 5b: Equivalent circuit in *Mode 3*

For ***modes 4, 5*** and ***6*** the equivalent circuits will be same as ***modes 1, 2*** and ***3*** respectively. The voltages and currents for each mode are:

$$\left.\begin{aligned} i_a = i_c = -\frac{1}{3}\frac{V_{in}}{R};\ i_b = \frac{2}{3}\frac{V_{in}}{R} \\ v_{an} = v_{cn} = -\frac{V_{in}}{3}; V_{bn} = \frac{2V_{in}}{3} \end{aligned}\right\} \text{for } \textbf{\textit{mode 4}} \tag{10}$$

$$\left.\begin{aligned} i_b = i_c = -\frac{1}{3}\frac{V_{in}}{R};\ i_a = \frac{2}{3}\frac{V_{in}}{R} \\ v_{bn} = v_{cn} = -\frac{V_{in}}{3}; V_{an} = \frac{2V_{in}}{3} \end{aligned}\right\} \text{for } \textbf{\textit{mode5}} \tag{11}$$

$$\left.\begin{aligned} i_a = i_b = -\frac{1}{3}\frac{V_{in}}{R};\ i_c = \frac{2}{3}\frac{V_{in}}{R} \\ v_{an} = v_{bn} = -\frac{V_{in}}{3}; V_{cn} = \frac{2V_{in}}{3} \end{aligned}\right\} \text{for } \textbf{\textit{mode 6}} \tag{12}$$

The plots of the phase voltages (v_{an}, v_{bn} and v_{cn}) and the currents (i_a, i_b and i_c) are shown in **Figure 6**. Having known the phase voltages, the line voltages can also be determined as:

$$\begin{aligned} v_{ab} &= v_{an} - v_{bn} \\ v_{bc} &= v_{bn} - v_{cn} \\ v_{ca} &= v_{cn} - v_{an} \end{aligned} \tag{13}$$

The plots of line voltages are also shown in **Figure 6** and the phase and line voltages can be expressed in terms of Fourier series as:

$$\begin{aligned} v_{an} &= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{in}}{3n\pi}\left[1+\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\right]\sin(n\omega t) \\ v_{bn} &= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{in}}{3n\pi}\left[1+\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\right]\sin\left(n\omega t-\frac{2n\pi}{3}\right) \\ v_{cn} &= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{in}}{3n\pi}\left[1+\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\right]\sin\left(n\omega t-\frac{4n\pi}{3}\right) \end{aligned} \tag{14}$$

$$\begin{aligned} v_{ab} = v_{an} - v_{bn} &= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{in}}{n\pi}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3}\sin\left(n\omega t+\frac{n\pi}{6}\right) \\ v_{bc} = v_{bn} - v_{cn} &= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{in}}{n\pi}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3}\sin\left(n\omega t-\frac{n\pi}{2}\right) \\ v_{ca} = v_{cn} - v_{an} &= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{in}}{n\pi}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3}\sin\left(n\omega t-\frac{7n\pi}{6}\right) \end{aligned} \tag{15}$$

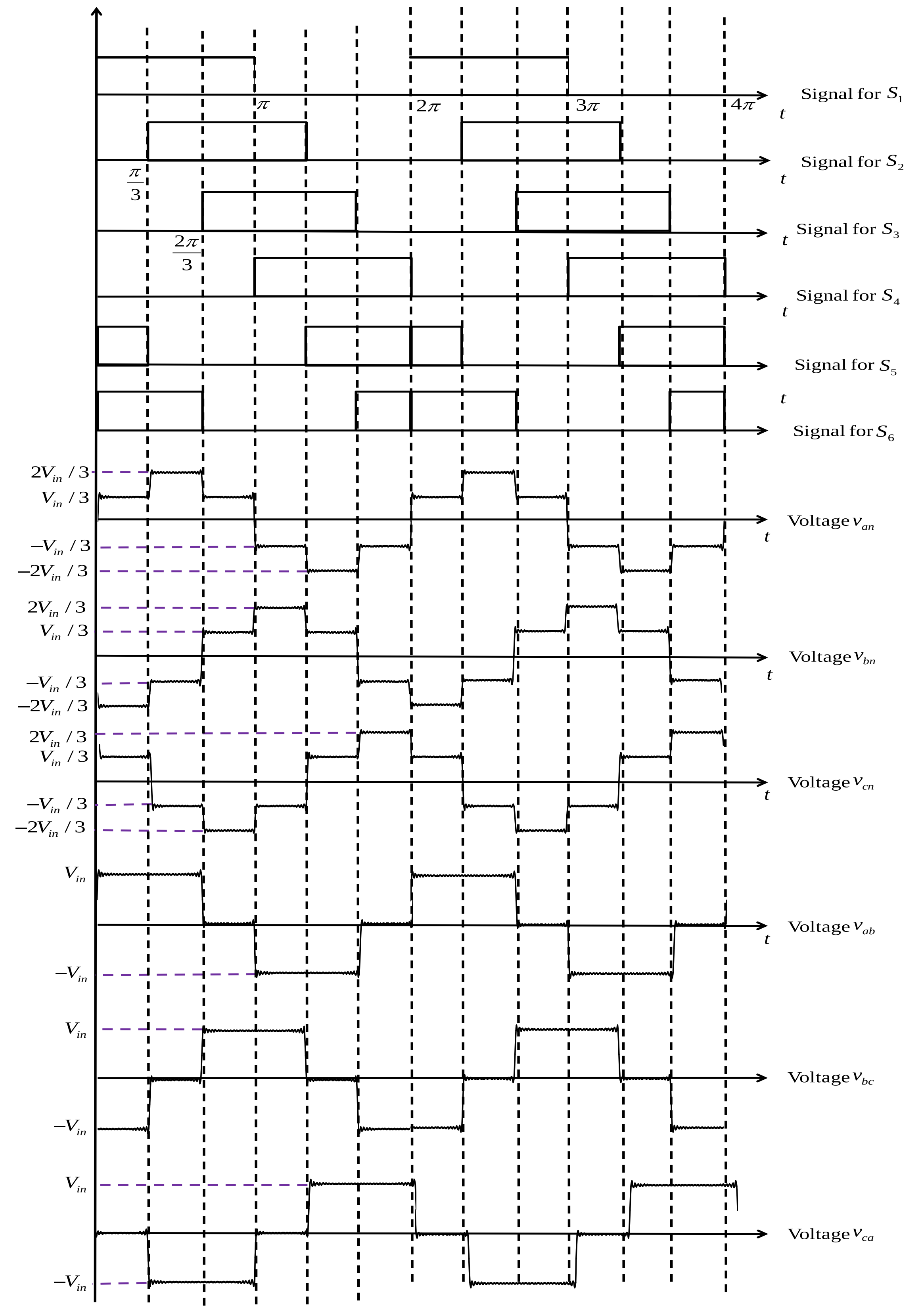

Figure 6: Voltage waveforms for Resistive load for 180°

180-Degree Conduction with Star Connected *R-L* Load

In ***mode 1*** the switches S_5, S_6 and S_1 are turned ***on***. The mode previous to ***mode1*** was ***mode 6*** and the in ***mode 6*** the switches S_4, S_5 and S_6 were ***on***. In the transition from ***mode 6*** to ***mode 1*** the switch S_4 is turned ***off*** and S_1 turned ***on*** and the current i_a changes its direction (***outgoing phase***). When the switch S_4 was ***on***, the direction of current was from point ***n*** to point ***a***, the circuit configuration is shown in **Figure 7a** and the equivalent circuit is shown in **Figure 7b**. When S_1 is turned ***on*** the direction of current should be from point ***a*** to point ***n***. However, due to the presence of inductance, the current cannot change its direction instantaneously and continues to flow in the previous direction through diode D_1 (**Figure 7c**) and the equivalent circuit of the configuration is shown in **Figure 7d**. Once $i_a = 0$, the diode D_1 ceases to conduct and the current starts flowing through S_1 as shown already in **Figure 3a** and **Figure 3b**. When ever one mode gets over and the next mode starts, the current of the outgoing phase cannot change its direction immediately due to presence of the inductance and hence completes its path through the freewheeling diode.

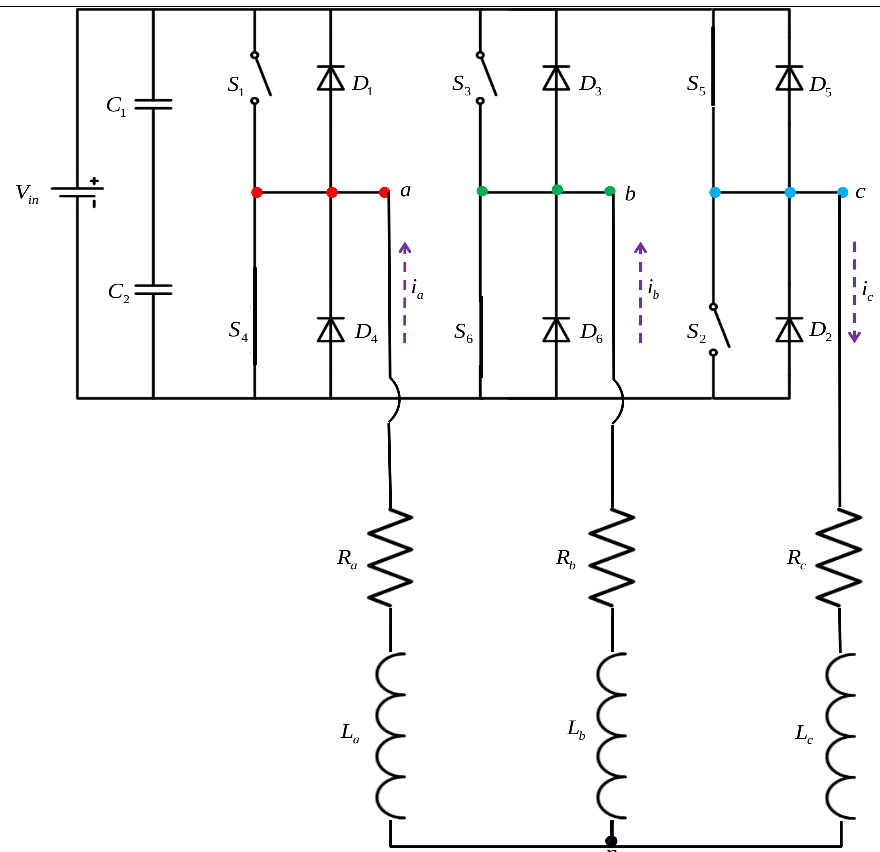

Figure 7a: Current through the load in *Mode 6*

The phase currents are determined as follows:

$$i_a = \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{\sqrt{R^2+(n\omega L)^2}} \frac{4V_{in}}{3n\pi}\left[1+\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\right]\sin\left(n\omega t-\theta_n\right)$$

$$i_b = \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{\sqrt{R^2+(n\omega L)^2}} \frac{4V_{in}}{3n\pi}\left[1+\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\right]\sin\left(n\omega t-\frac{2n\pi}{3}-\theta_n\right)$$

$$i_c = \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{\sqrt{R^2+(n\omega L)^2}} \frac{4V_{in}}{3n\pi}\left[1+\sin\frac{n\pi}{2}\sin\frac{n\pi}{6}\right]\sin\left(n\omega t-\frac{4n\pi}{3}-\theta_n\right) \quad (16)$$

where

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$$

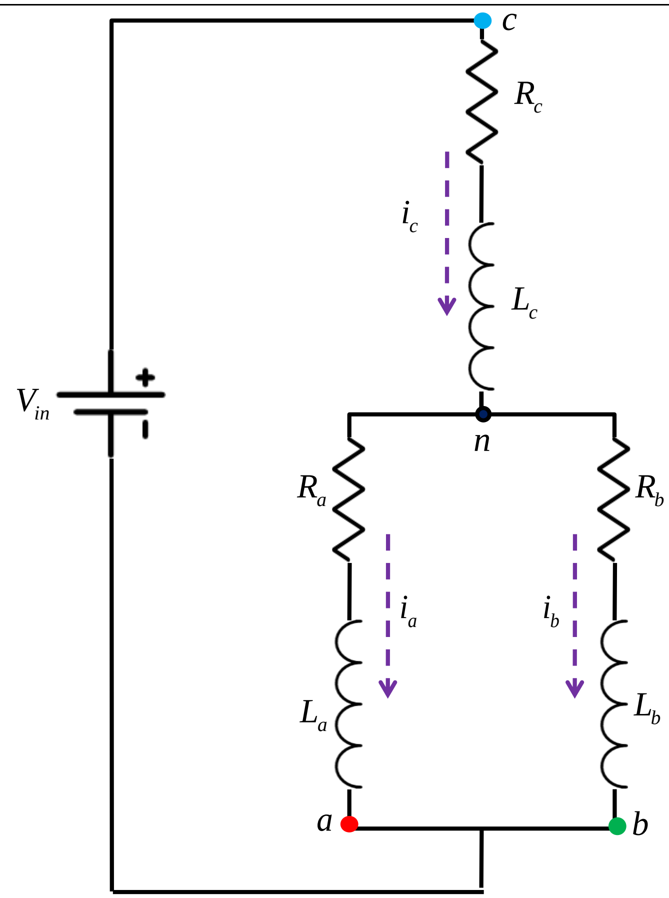

Figure 7b: Equivalent circuit for *Mode 6*

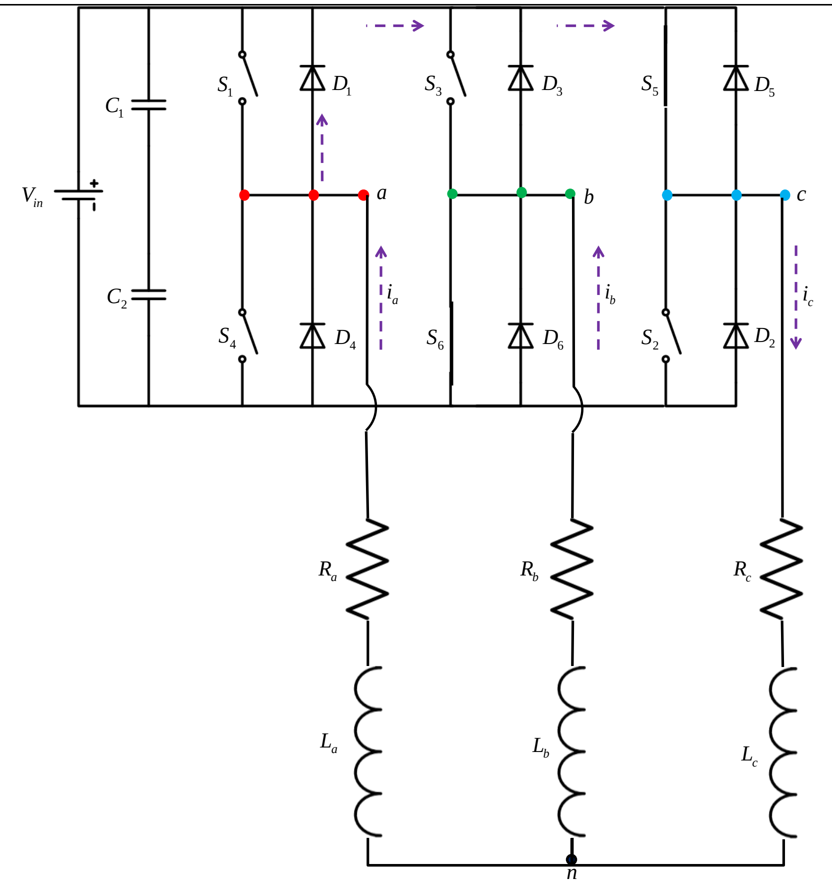

Figure 7d: Current through the load during transition from *Mode 6* to *Mode 1*

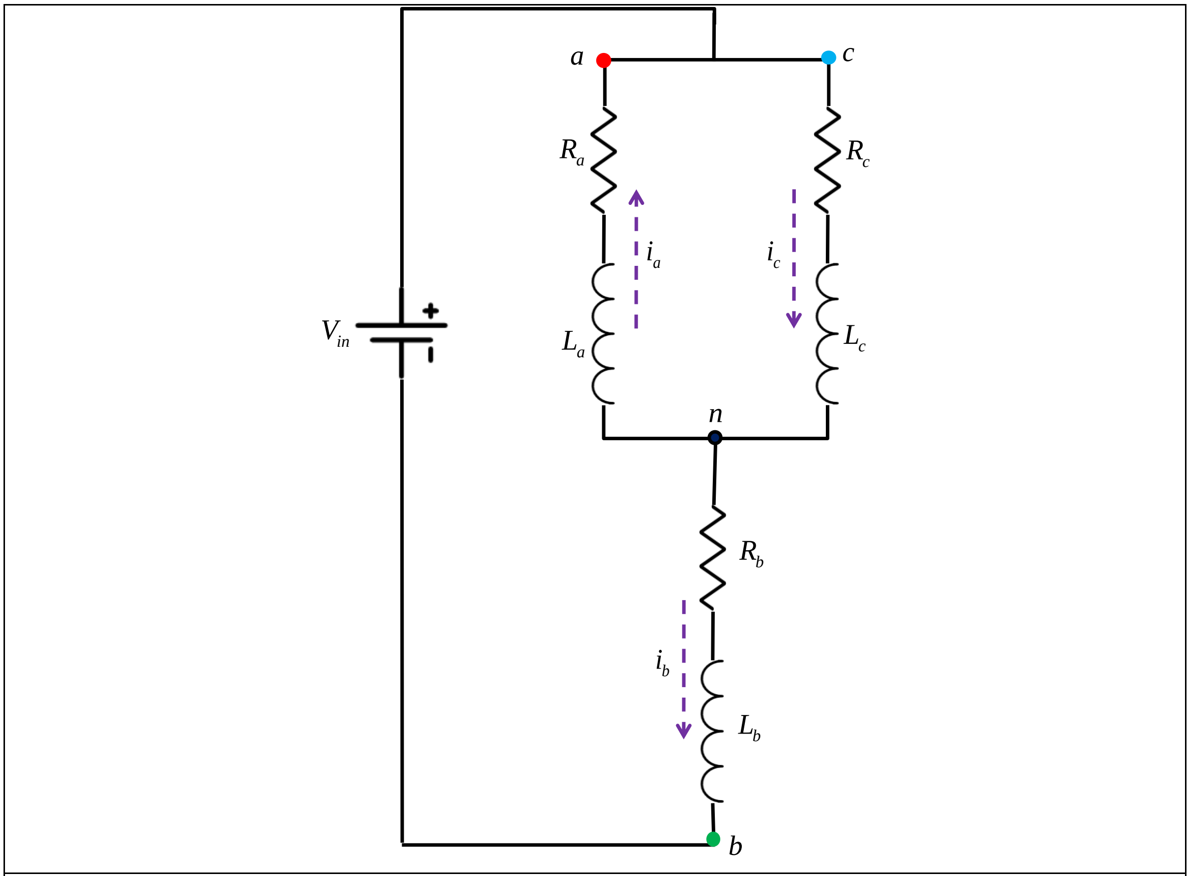

Figure 7e: Equivalent circuit during transition from *Mode 6* to *Mode 1*

Suggested Reading:

[1] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*, 3rd edition, Pearson, 2004

[2] V. R. Moorthi, *Power Electronics: Devices, Circuits and Industrial Applications*, Oxford University Press, 2007

Lecture 16: Voltage Control of DC-AC Inverters Using PWM

Voltage Control of DC-AC Inverters Using PWM

Introduction

The topics covered in this chapter are as follows:

- Need for PWM
- Single Pulse Width Modulation
- Sinusoidal Pulse Width Modulation
- Three Phase Sinusoidal Pulse Width Modulation

Need for PWM in Voltage Source Inverters

The electric motors used in EV applications are required to have large speed ranges as shown in **Figure 1**. Large speed ranges can be achieved by feeding the motor with voltages of different frequencies and also different voltage magnitudes. One of the most convenient voltage control technique to generate variable frequency and magnitude voltages is **P**ulse **W**idth **M**odulation (PWM).

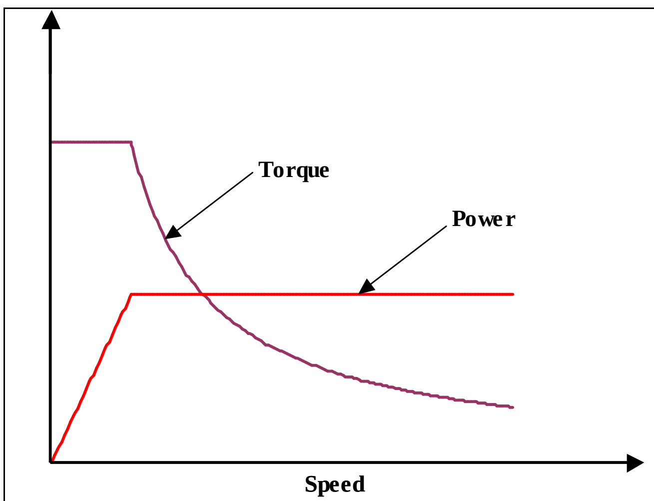

Figure 1: Ideal performance characteristics for a traction motor

The voltage control techniques for single phase inverters are:

- **Single Pulse Width Modulation**
- **Multiple Pulse Width Modulation**
- **Sinusoidal Pulse Width Modulation**
- Modified Sinusoidal Pulse Width Modulation
- Phase Displacement Control

Some of the important voltage control techniques for three phase inverters are:

- **Sinusoidal PWM**
- Space vector modulation

In this lecture, the techniques marked bold are discussed.

Voltage Control of Single Phase Inverter

The single phase DC-AC inverter considered in this section is shown in **Figure 2**.

Single Pulse Width Modulation

In this modulation only one pulse per half cycle exists and the width of the pulse is varied to control the inverter output voltage. The generation of the gating signals and the output voltage of single phase full-bridge inverters are shown in **Figure 3**. The gating signals are generated by comparing a rectangular reference signal of amplitude A_r with a triangular carrier wave of amplitude A_c. ***The frequency of the reference signal determines the fundamental frequency of the output voltage***. The ratio of A_r to A_c is the control variable and defined as the amplitude ***modulation index*** or ***modulation index*** and is given by

$$M = \frac{A_r}{A_c} \tag{1}$$

The output voltage shown in **Figure 3** can be expressed as

$$v_o = \sum_{n=1,3,5,...}^{\infty} \frac{4V_{in}}{n\pi} \sin\frac{n\pi}{2} \sin\frac{n\delta}{2} \sin n\omega t \tag{2}$$

And the rms value of the output voltage is

$$V_{o,rms} = \left[\frac{2}{2\pi} \int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} V_{in}^2 d(\omega t) \right]^{1/2} = V_{in}\sqrt{\frac{\delta}{\pi}} \tag{3}$$

The relation between δ and modulation index M is:

$$M = \frac{\delta}{\pi} = \frac{A_r}{A_c} \tag{4}$$

Using **equation 4**, the rms voltage can be expressed as

$$V_{o,rms} = V_{in}\sqrt{M} \tag{5}$$

The load current in case of resistive load is

$$i_L = \sum_{n=1,3,5,...}^{\infty} \frac{4V_{in}}{n\pi R} \sin\frac{n\pi}{2} \sin\frac{n\delta}{2} \sin n\omega t \tag{6}$$

For ***R-L*** load, the load current is given by

$$i_L = \sum_{n=1,3,5,...}^{\infty} \frac{4V_{in}}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin\frac{n\pi}{2} \sin\frac{n\delta}{2} \sin(n\omega t - \theta_n)$$

where (7)

$$\theta_n = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The currents for both ***R*** and ***R-L*** loads are also shown in **Figure 3**.

By varying A_r from 0 to A_c, the pulse width δ can be modified from 0^o to 180^o and the rms voltage $V_{o,rms}$ from 0 to V_{in}. The harmonic content for different harmonics for different modulation indices is shown in **Figure 4**.

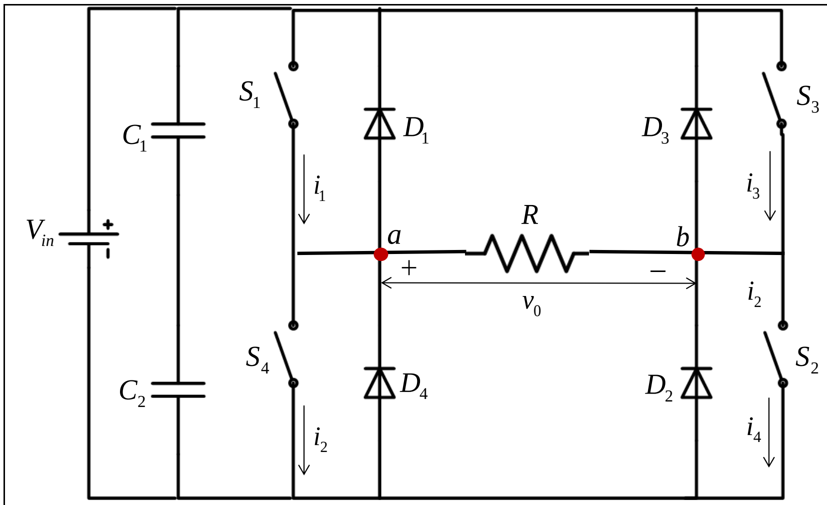

Figure 2: Configuration of the single phase inverter

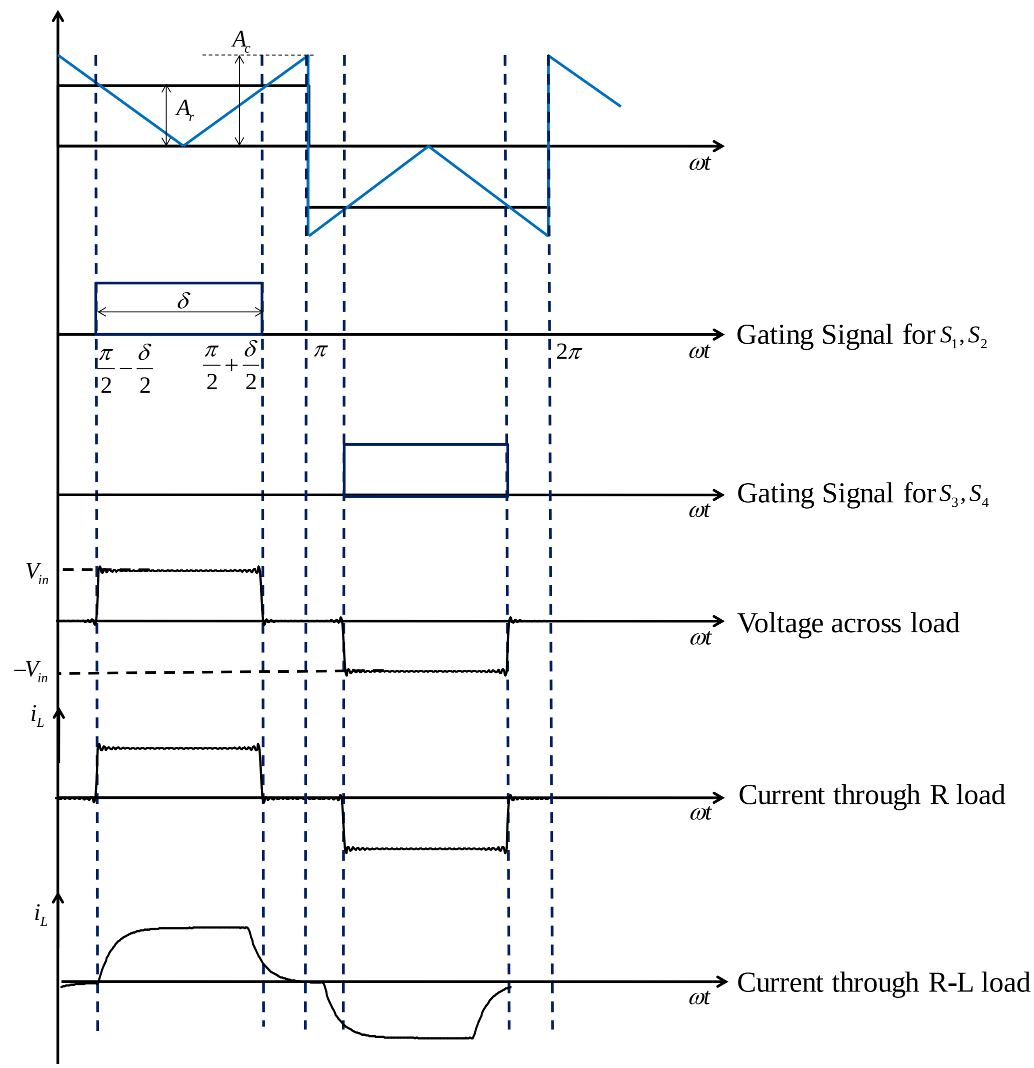

Figure 3: Currents and voltages in single phase DC-AC inverter for single PWM

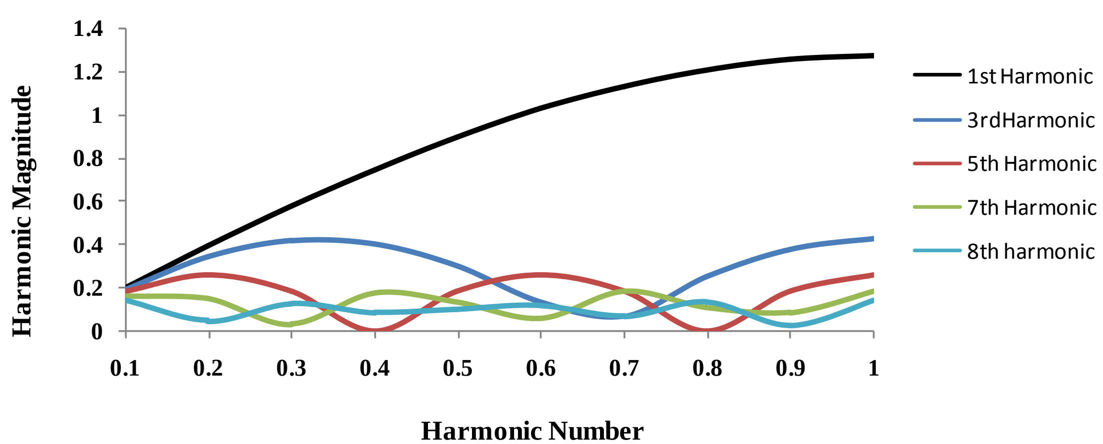

Figure 4: Harmonic content for single pulse width modulation

Single Pulse Width Modulation

The harmonic content in the voltage v_o can be reduced by using several pulses in each half cycle. The generation of the gating signal is done by comparing a reference signal with a triangular carrier waveform (**Figure 5**). The generated gate signals are shown in **Figure 5**. The frequency of the reference signal f_r and the carrier signal f_c determine the number of pulses per half cycle (n_p) as

$$n_p = \frac{f_c}{2f_r} = \frac{m_f}{2} \tag{8}$$

where

$m_f = \frac{f_c}{f_r}$ is frequency modulation ration

The instantaneous output voltage (v_o) and the current for resistive and inductive loads are shown in **Figure 5**. The output voltage in terms of Fourier series is given by

$$v_o = \sum_{n=1,3,5,\ldots}^{\infty} B_n \sin(n\omega t)$$

where

$$B_n = \sum_{m=1}^{2n_p} \frac{4V_{in}}{n\pi} \sin\frac{n\pi}{2} \sin\frac{n\delta}{2} \sin\left(n\omega t - n\alpha_m + \frac{(2n_p - 1)}{2} n\delta\right)$$

where

n_p is number of pulses in the half cycle

M is modulation index

δ is width of each pulse

α is the angle of left most pulse

$$\delta = \frac{M}{n_p} \times 180$$

$$\alpha = \frac{180(1-M)}{2n_p}$$

$$\alpha_m = (2m-1)\alpha + (m-1)\delta \tag{9}$$

The magnitude of the currents for ***R-L*** load are given by

$$i_L = \sum_{n=1,3,5,...}^{\infty} A_n \sin(n\omega t)$$

where

$$A_n = \sum_{m=1}^{2n_p} \frac{4V_{in}}{n\pi} \frac{1}{Z_n} \sin\frac{n\pi}{2} \sin\frac{n\delta}{2} \sin\left(n\omega t - n\alpha_m + \frac{(2n_p - 1)}{2} n\delta - \theta_n \right) \qquad (10)$$

where

$$Z_n = \sqrt{R^2 + (n\omega L)^2}$$

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$$

The rms value of the output voltage is

$$V_{o,rms} = \left[\frac{2n_p}{2\pi} \int_{\left(\frac{\pi}{2}-\delta\right)/2}^{\left(\frac{\pi}{2}+\delta\right)/2} V_{in}^2 d(\omega t) \right]^{1/2} = V_{in}\sqrt{\frac{n_p \delta}{\pi}} \qquad (11)$$

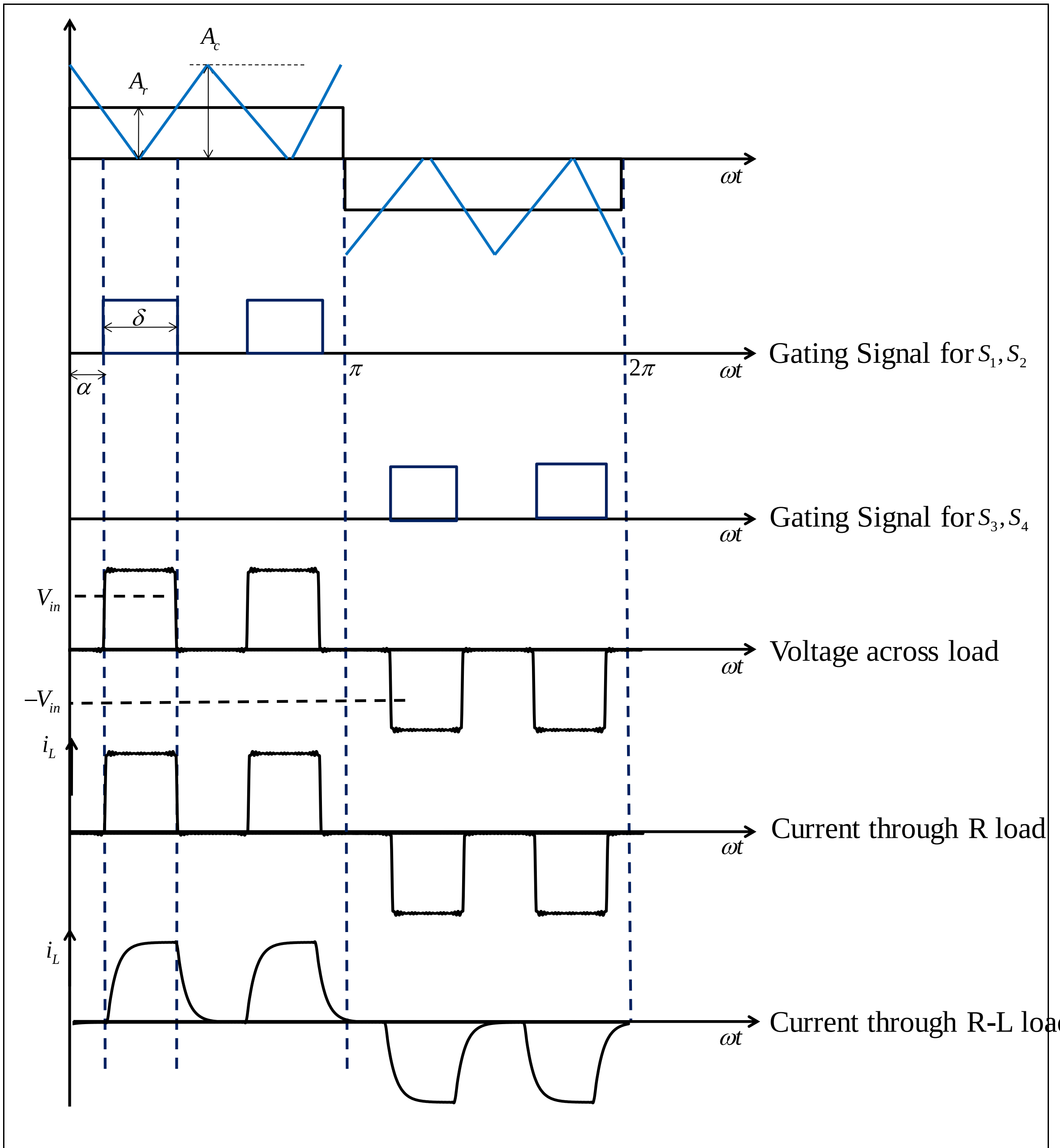

Figure 5: Currents and voltages in single phase DC-AC inverter for Multi-PWM

Sinusoidal Pulse Width Modulation

In sinusoidal PWM, also called ***sine-PWM***, the resulting pulse widths are varied throughout the half cycle in such a way that they are proportional to the instantaneous value of the reference sine wave at the centre of the pulses. The distance between the centres of the pulses is kept constant as in multi-PWM. Voltage control is achieved by varying the widths of all pulses without disturbing the sinusoidal relationship. The generation of the gating signals for sinusoidal PWM and the output voltage and currents is shown in **Figure 6**.

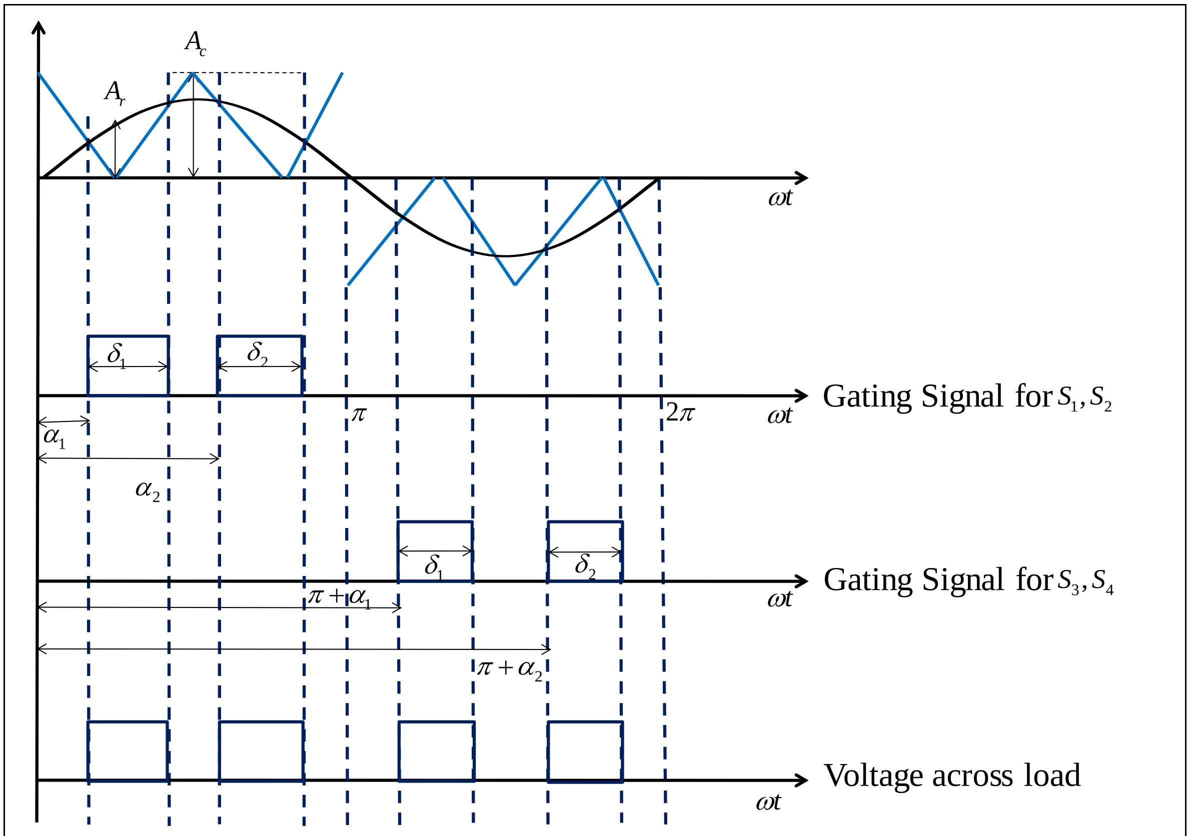

Figure 6: Gating Signal in Sinusoidal PWM

The output voltage in case of sinusoidal PWM can be expressed as

$$v_o = \sum_{n=1,3,5...}^{\infty} \frac{4V_{in}}{n\pi} \sum_{k=1,2,...}^{n_p} \sin\frac{n\delta_k}{2}\left[\cos\left(n\omega t - \frac{n\delta_k}{2} - n\alpha_k\right)\right] \tag{12}$$

where

n_p is the number of pulses in the half cycle

δ_k is the width of the *kth* pulse

α_k is the starting angle of the *kth* pulse

The width of the ***k^th^ pulse*** (δ_k) is approximately given by

$$\delta_k = \left(\frac{\pi}{n_p}\right) m_a \sin(\alpha_k) \tag{13}$$

where

$m_a = \frac{A_r}{A_c}$ is the modulation index

The value of the starting angle of the ***k^th^*** pulse (α_k) is given by numerically solving the following equation

$$m_a \sin(\alpha_k) = -\frac{m_f}{\pi}\alpha_k + (2k-1) \tag{14}$$

where

$m_f = 2n_p$

The angles θ and α for a sine PWM with 6 pulses per half cycle are calculated using **equations 13** and **14** and listed in **Table 1**. The waveforms of the voltage and current are shown in **Figure 7.**

The r.m.s value of the output voltage is

$$v_o = V_{in}\sqrt{\sum_{m=1}^{2n_p}\frac{\delta_m}{\pi}} \tag{15}$$

The load for an ***R-L*** load is given by

$$i_L = \sum_{n=1,3,5...}^{\infty}\frac{4V_{in}}{n\pi Z_n}\sum_{k=1,2,...}^{n_p}\sin\frac{n\delta_k}{2}\left[\cos\left(n\omega t - \frac{n\delta_k}{2} - n\alpha_k - \theta_n\right)\right] \tag{16}$$

where

$Z_n = \sqrt{R^2 + (n\omega L)^2}$

$\theta_n = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Table 1: The starting angle and pulse width for Sine PWM with 6 pulses per half cycle

Pulse Number	**Starting angle α [^{o}]**	**Pulse Width δ [^{o}]**
1	12.98	4.04
2	39.30	11.40
3	66.73	16.54
4	96.05	17.90
5	127.90	14.20
6	162.26	5.49

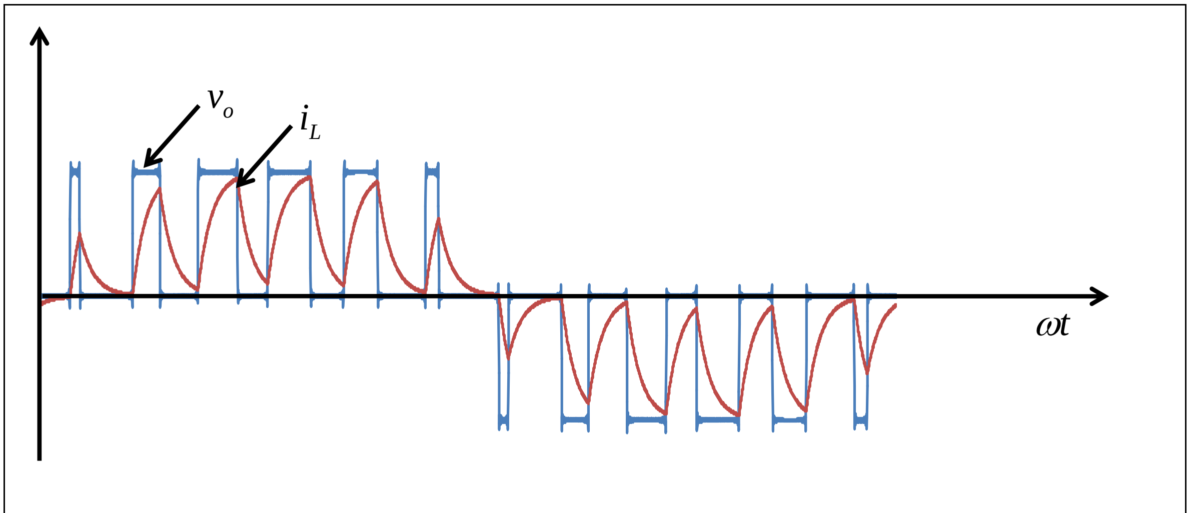

Figure 7: Voltage and Current waveforms for Sinusoidal PWM with 6 pulses per half cycle

Voltage Control of Three Phase DC-AC Inverter using Sinusoidal PWM

The generation of gating signals for a three phase DC-AC inverter with sine PWM are shown in **Figure 8**. There are three sinusoidal reference waves (v_{ra}, v_{rb}, v_{rc}) each shifted by 120^o. A triangular carrier wave is compared with the reference signals to produce the gating signals. Comparing the carrier signal v_{cr} with the reference phases v_{ra}, v_{rb}, v_{rc} produces the signals for gates 1, 2 and 3 (g_1, g_2, g_3). The instantaneous line-to-line output voltage is

$$v_{ab} = V_{in}\left(g_1 - g_3\right) \tag{12}$$

The output voltage is generated by eliminating the condition that two switching devices in the same arm cannot conduct at the same time.

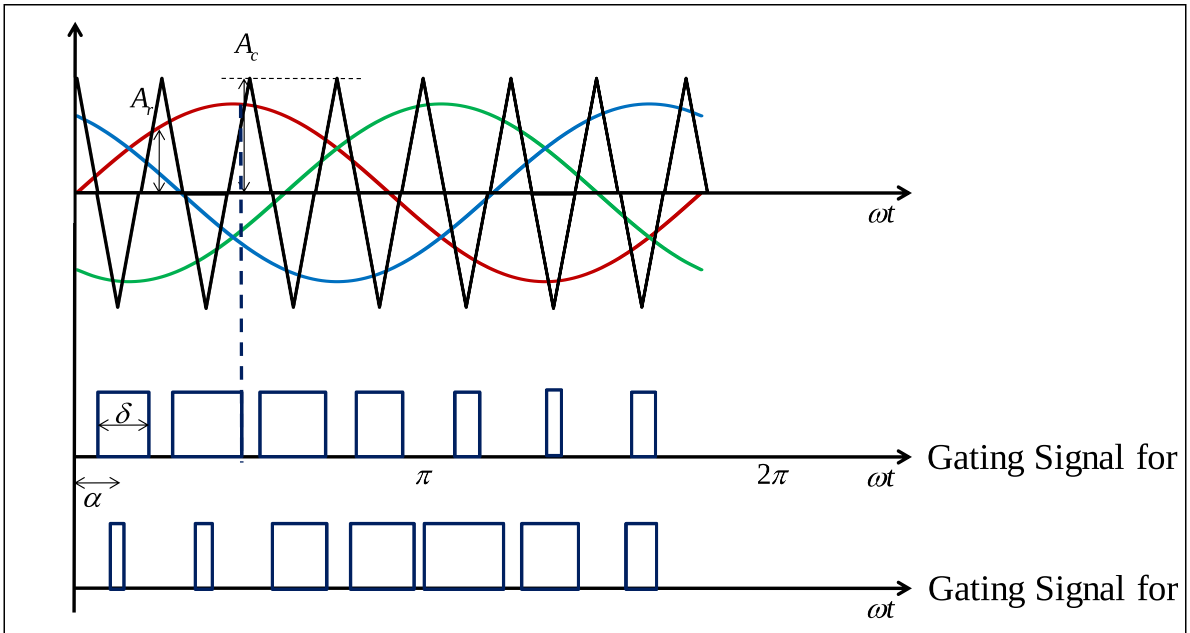

Figure 8: Voltage and Current waveforms for three phase Sinusoidal PWM

Suggested Reading:

[1] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*, 3rd edition, Pearson, 2004

[2] V. R. Moorthi, *Power Electronics: Devices, Circuits and Industrial Applications*, Oxford University Press, 2007