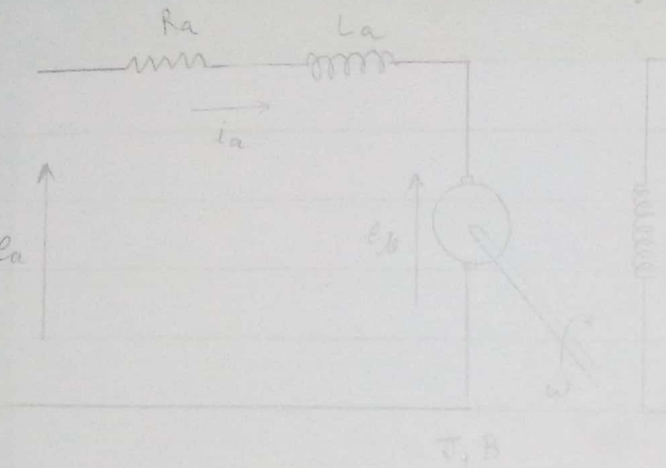
Schematic Digram



e_a : armature voltage (V)

i_a : armature current (amp.)

R_a : armature resistance (ohms)

L_a : armature inductance (H)

i_f : field current (amp.)

J : Moment of inertia of rotor including external loading (Nm/Rad/sec²)

B : Viscous friction coefficient

Equations

$T_m = K_T i_a$ where, T_m = motor torque (newton-m)

K_T = torque

$e_b = K_b \omega$ where, K_b : back emf constant ω: angular velocity

for armature circuit model.

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

for mechanical model.

$$J \frac{d\omega}{dt} + B\omega + T_L = T_M$$

T_M : Motor torque, T_L : Load torque

Taking laplace & rearranging

$$\frac{\omega(s)}{E_a(s)} = \frac{K_T}{(sL_a + R_a)(sJ + B) + K_T K_b}$$

as inductance of armature circuit is very small

$$\boxed{G_m(s) \triangleq \frac{\omega(s)}{E_a(s)} = \frac{K_T / R_a}{Js + B + \frac{K_T K_b}{R_a}} = \frac{K_m}{s\tau_m + 1}} \quad (1)$$

where $K_m = \dfrac{K_T}{R_a B + K_T K_b}$ i.e. motor gain constant

$\tau_m = \dfrac{R_a J}{R_a B + K_T K_b}$ i.e. motor time constant

Steady state armature current

$$I_a = \frac{E_a - E_b}{R_a} = \frac{E_a}{R_a} - \frac{K_b\, \omega}{R_a}$$

Steady state torque generated

$$T_m = K_T I_a = -\frac{K_T K_b}{R_a}\omega + \frac{K_T}{R_a} \cdot E_a \quad - (2)$$

As motor runs at constant speed

electrical power input $P_{in} = E_a \times I_a$ watts

Power lost in R_a = $R_a \times I_a \times I_a$

Power available in the armature, $P_{arm} = (E_a - I_a R_a) I_a$

$= E_b I_a$

Mechanical power,

$= K_b \times \omega \times I_a$

$P_m = T_m \times \omega = K_T I_a \omega$

Assuming 100% conversion of power from electrical input to mechanical output, i.e. $K_T = K_b$, (2) can be written as

$$T_m = -\frac{K_b^2}{R_a}\omega + \frac{K_b}{R_a} \cdot E_a$$

Also,

$$T_m = K_T I_a = K_b I_a = \frac{E_b}{\omega} I_a = \left(\frac{E_a - I_a R_a}{\omega}\right) \times I_a$$

Also B = negative of slope of torque speed eq^n (i.e. T_m [illegible] for steady state)

$$K_b = \frac{E_b}{\omega} = \frac{E_a - I_a R_a}{\omega}$$

For step Response

$$\omega(t) = E \cdot K_m \left(1 - e^{-t/\tau_m}\right), \quad \omega(t)\Big|_{t\to\infty} = \omega_{ss} = E \cdot K_m$$

$\Rightarrow K_m = N/E = \pi N / 30 E_a$ rad/sec/volt