CS-2
Lecture Notes

2018UIC3087
Manan.

* Condition for complete state controllability in the s-plane.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2.5 & -1.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$[B \,|\, AB] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

rank = 1

* Output controllability.

$$\dot{x} = Ax + Bu$$

$$y = Cx + Dy$$

It is output controllable iff $m \times (n+1)$ matrix

$$[CB \,|\, CAB \,|\, CA^2B \,|\, \dots \,|\, CA^{n-1}B \,|\, D]$$

is of rank M.

Stabilizibility

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$x(t) = e^{At} e(0) + \int_0^t e^{A(t-\tau)} Bu(\tau)\, d\tau$$

$$y(t) = Ce^{At} x(0) + C \int_0^t e^{A(t-\tau)} Bu(\tau)\, d\tau + Du$$

eg)
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

rank $\rightarrow [B \,\vdots\, AB] = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = 2$

completely state controllable.

~~rast~~ rank $\Rightarrow [CB \,\vdots\, CAB] = [0 \; 1] = 1$

complete output controllable

and rank of $[C^* | A^* C^*] = \begin{bmatrix} \cancel{1} 0 & 1 \\ 0 & 1 \end{bmatrix}$ is 2

$\therefore$ completely observable.

Principle of Quality

Let S_1 se,

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

and the dual system S_2 be

$$z = A^* z + CV$$
$$n = B^* n$$

(X^* - conjugate transpose of X).

The principle states that S_1 is completely state controllable iff S_2 is completely observable.