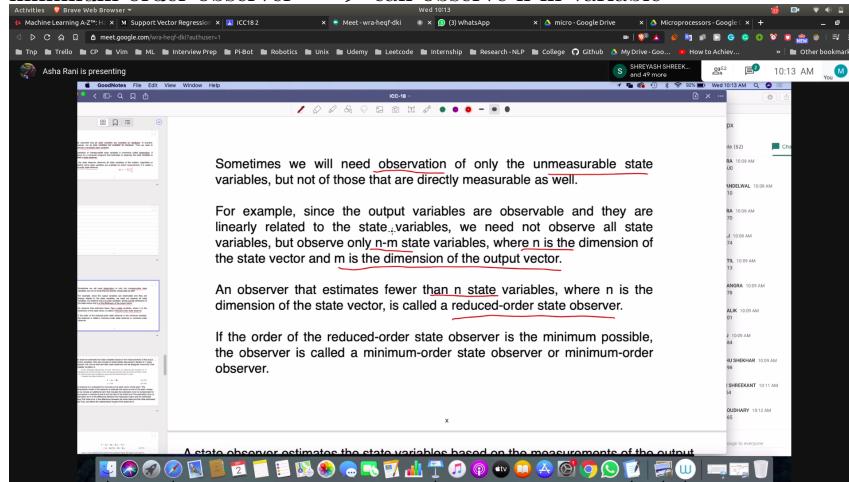


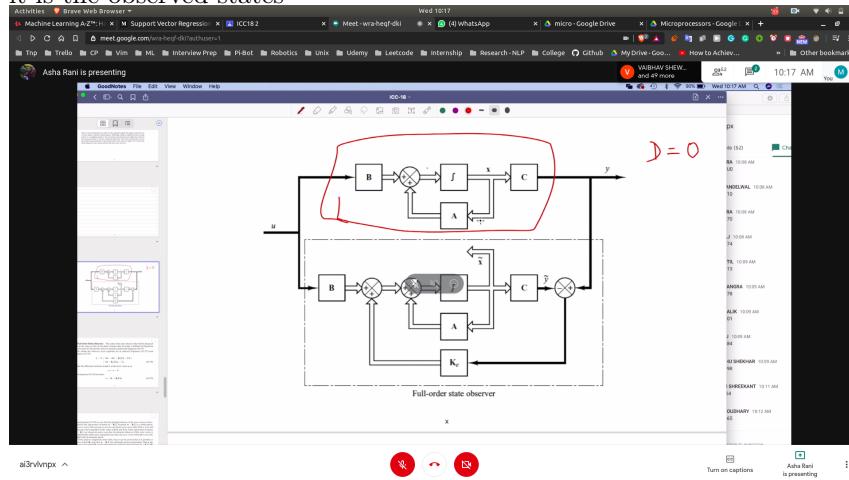
## Observer

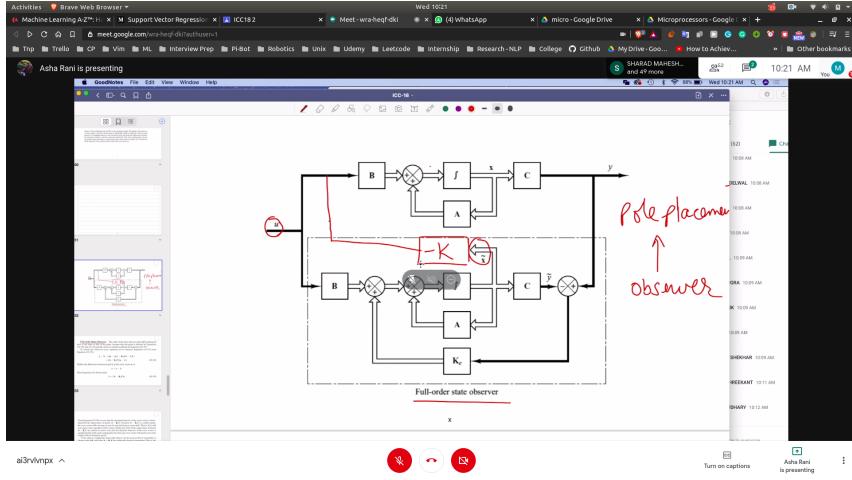
- So we have to observe the unmeasurable values
- we do not need to observe the state
- reduced order observer ==> can observe n-1 variables
- minimum order observer ==> can observe n-m variable



## Notation

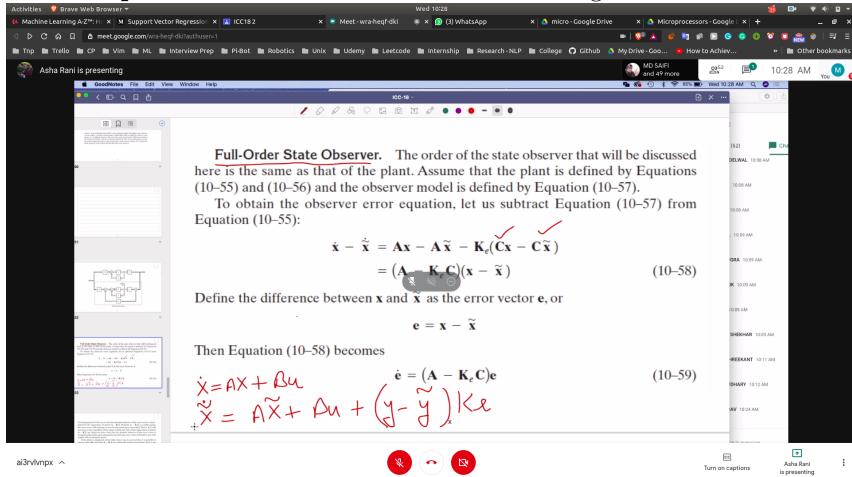
- $x\sim$  is the full state observer
- it is the observed states



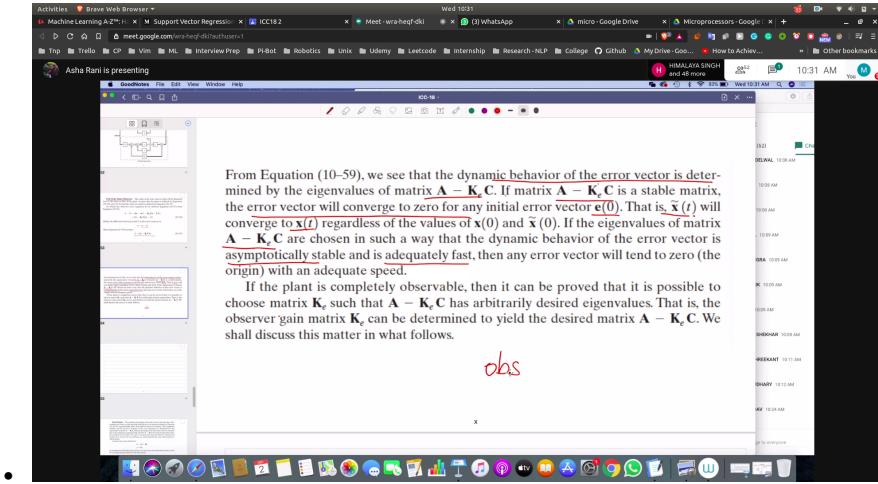


## Full Order State Observer

- here also we have to observe the  $K_e$  matrix
- now when we make  $y \sim = y$  then only we will get the correct  $x$  observer
- to do that we have to get the value fo  $K_e$
- So main problem is to estimate  $K_e \Rightarrow$  to get the correct estimate**

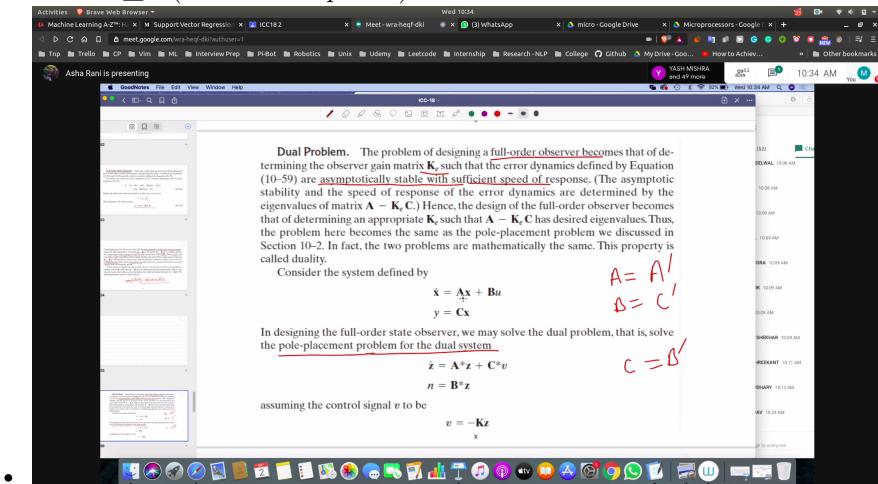


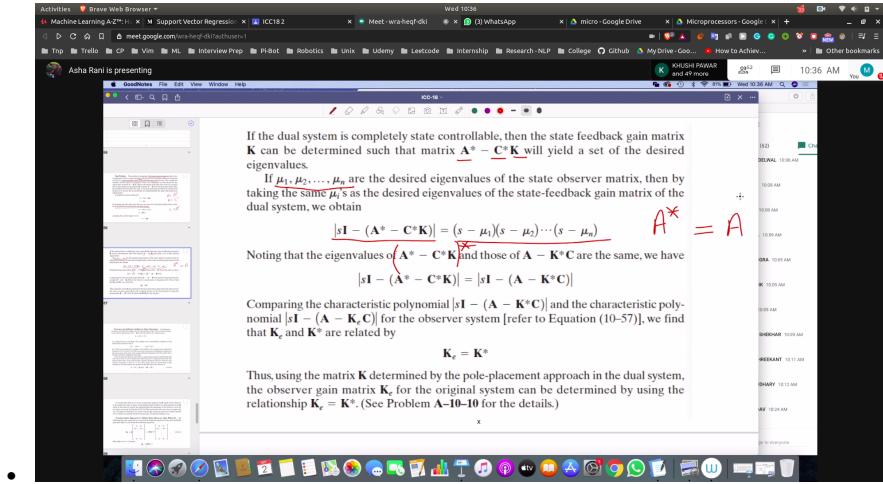
- now  $e$  here will be zero , when  $A - K_e C = 0$
- to do that we have to place correct eigen values of  $A - K_e C$



## Dual Problem

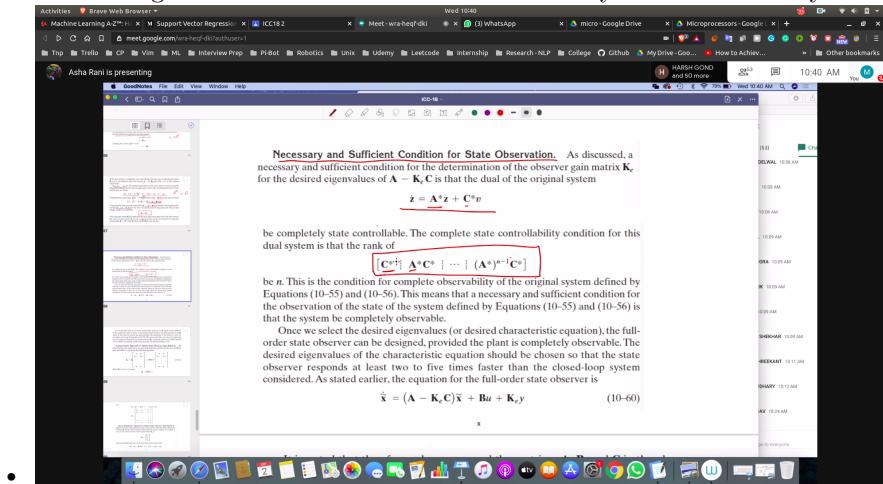
- We treat the system as finding the value of  $K$  for dual system
- basically in dual we take the transpose of the system
- $K_e = K^T$  ( in the dual problem)





## Condition for state observation

- so we design the dual and convert controllability into observability



## Transformation method for finding $K_e$

It is noted that thus far we have assumed the matrices  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  in the observer to be exactly the same as those of the physical plant. If there are discrepancies in  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  in the observer and in the physical plant, the dynamics of the observer error are no longer governed by Equation (10-59). This means that the error may not approach zero as expected. Therefore, we need to choose  $\mathbf{K}_e$  so that the observer is stable and the error remains acceptably small in the presence of small modeling errors.

**Transformation Approach to Obtain State Observer Gain Matrix  $\mathbf{K}_e$ .** By following the same approach as we used in deriving the equation for the state feedback gain matrix  $\mathbf{K}$ , we can obtain the following equation:

$$\mathbf{K}_e = \mathbf{Q} \begin{bmatrix} \alpha_n \\ \alpha_{n-1} - \alpha_{n-1} \\ \vdots \\ \alpha_1 - \alpha_1 \end{bmatrix} = (\mathbf{W}\mathbf{N}^*)^{-1} \begin{bmatrix} \alpha_n - \alpha_n \\ \alpha_{n-1} - \alpha_{n-1} \\ \vdots \\ \alpha_1 - \alpha_1 \end{bmatrix} \quad (10-61)$$

where  $\mathbf{K}_e$  is an  $n \times 1$  matrix,

$$\mathbf{Q} = (\mathbf{W}\mathbf{N}^*)^{-1} \xrightarrow{x} \text{controllable} \quad T = MW \xrightarrow{x} \text{observable}$$

## Direct Substitution Method for finding $K_e$

and

$$\mathbf{N} = [\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^* \mid \cdots \mid (\mathbf{A}^*)^{n-1}\mathbf{C}^*]$$

$$\mathbf{W} = \begin{bmatrix} \alpha_n & 1 & 0 & \dots & 0 \\ \alpha_{n-1} & \alpha_{n-2} & \cdots & \alpha_1 & 1 \\ \alpha_{n-2} & \alpha_{n-3} & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

**Direct-Substitution Approach to Obtain State Observer Gain Matrix  $\mathbf{K}_e$ .** Similar to the case of pole placement, if the system is of low order, then direct substitution of matrix  $\mathbf{K}_e$  into the desired characteristic polynomial may be simpler. For example, if  $\mathbf{x}$  is a 3-vector, then write the observer gain matrix  $\mathbf{K}_e$  as

$$\mathbf{K}_e = \begin{bmatrix} k_{e1} \\ k_{e2} \\ k_{e3} \end{bmatrix}$$

Substitute this  $\mathbf{K}_e$  matrix into the desired characteristic polynomial:

$$[s\mathbf{I} - (\mathbf{A} - \mathbf{K}_e\mathbf{C})] = [s - \mu_1][s - \mu_2][s - \mu_3]$$

By equating the coefficients of the like powers of  $s$  on both sides of this last equation,