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+ Control System- Assignment-2:-

Root locus-

* For unity feedback system with transfer function.

$$G(s)H(s) = K/[s(s+3)(s+9)]$$

Plot root locus.

No of poles (p) = 3, zeros z=0

Loci = 3 (i.e. max(p,z))

No of asymptotes = p-z = 3

Ceteroid = $\frac{\Sigma \text{Real part of } P - \Sigma \text{real part of } z}{p-z}$

= -4

Angle of asymptotes

$\theta = [(2q+1)/(p-z)] * 180°$;

q = 0, 1, 2

$\theta = 60°, 180°, 300°$

char eq^n

$$1 + G(s)H(s)$$

$$1 + K/s(s+3)(s+9) = 0$$

also

$$\frac{dK}{ds} = 0 \Rightarrow s = -1.18; -22.8$$

Characteristics eq^n

$$s^3 + 12s^2 + 27s + K = 0$$

Routh Array

s^3	1	27
s^2	12	K
s	$324-K/12$	0
s^0	K	

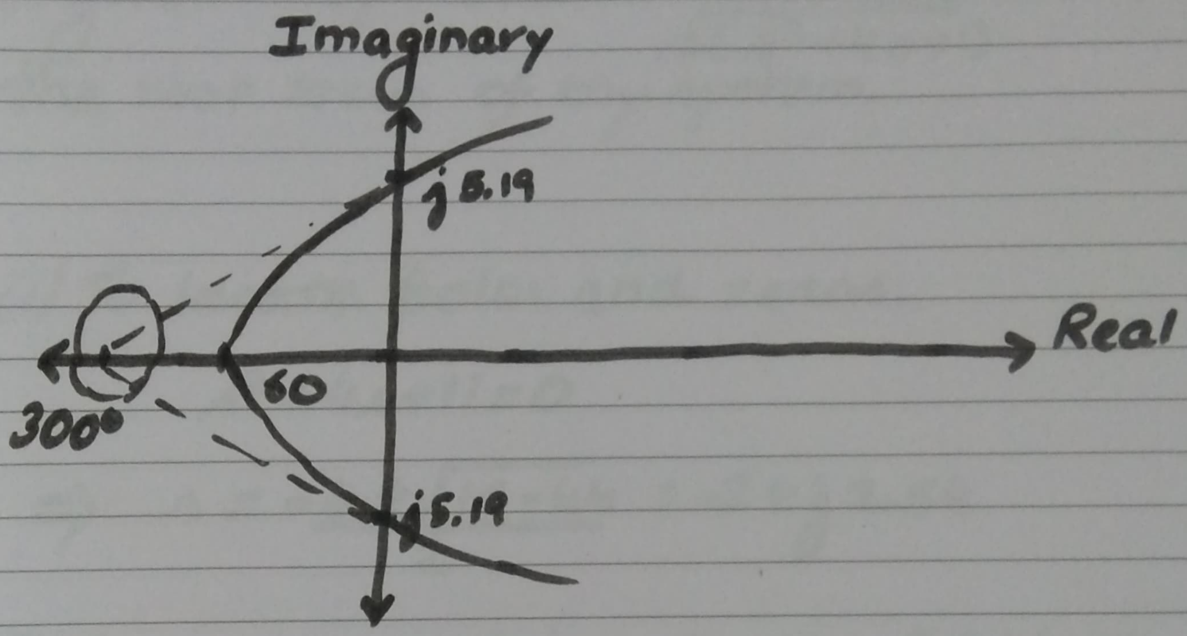
K>0, for stability

1.ecz $< K < 324$

$A(s) = 0$

$$12s^2 + K = 0$$

for $K = 324$ $s = \pm j5.19$



Q2.) The open loop transfer function of a unity feedback system is given by $G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$. Sketch the root locus of thy system.

(i) To locate poles and zeros.

$$s^2+4s+11=0$$

$$\Rightarrow s = \frac{-4 \pm \sqrt{16-44}}{2} = -2 + j2.64$$

$\therefore$ The poles are lying at $s=0, -2+j2.64, -2-j2.64$.

The zeroes are lying at $s=-9$ and infinity
Let us denote the poles as p_1, p_2, p_3, finite zero by z_1

$$\Rightarrow p_1 = 0$$
$$p_2 = -2+j2.64$$
$$p_3 = -2-j2.64$$
$$z_1 = -9$$

(ii) To find angles of asymptotes and centeroid
Angle of asymptotes

$$= \frac{\pm 180°(2q+1)}{n-m}$$

$q = 0, 1, 2, \ldots, n-m$
$n=3$ and $m=0$
$\therefore q = 0, 1, 2, 3.$

When $q=0$ Angles $= \pm \frac{180^\circ}{3} = \pm 60^\circ$

When $q=1$ Angles $= \pm 180 \times 3/2 = \pm 270 = \pm 90^\circ$

When $q=2$ Angles $= \pm \frac{180^\circ \times 5}{2} = \pm 450^\circ = \pm 90^\circ$

Centeroid $= \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$

$= (0-2+j2.64-2-j2.64-(-9))/2$

$= 2.5$

iii) Breakaway and breaking point.

It is concluded that there is no possibility of breakaway or breaking points.

iv) To find angles of departure

Let the angle of the vectors be θ_1, θ_2 & θ_3

$\theta_1 = 180^\circ - \tan^{-1}(2.64/2) = 127.1^\circ$

$\theta_2 = 90^\circ$

$\theta_3 = \tan^{-1}(2.64/7) = 20.7^\circ$

Angle of departure from the complex

$= 180^\circ - (\theta_1 + \theta_2) + \theta_3$

$= 180^\circ - (127.1^\circ + 90^\circ) + 20.7^\circ$

$= -16.4^\circ$

The angle of departure at the complex pole p_3 is -ive of the angle of departures at complex poles β_3

$\therefore$ Angle of departures at pole $p_3 = -(-16.4)$
$= 16.4^\circ$

(iv) To find the crossing pt of imaginary axis.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K(s+9)}{s(s^2+4s+11)}}{1+\frac{K(s+9)}{s(s^2+4s+11)}} = \frac{K(s+9)}{s(s^2+4s+11)+K(s+9)}$$

$\therefore$ putting $s = j\omega$

eq^n is

$$-j\omega^3 - 4\omega^2 + j11\omega + jK\omega + 9K = 0$$

on equation img part to 0

$$\omega^2 = 11+K; \quad K = 8.8 \quad -②$$

$$\omega = \pm\sqrt{19.8} = \pm 4.4$$

On equating real part to 0

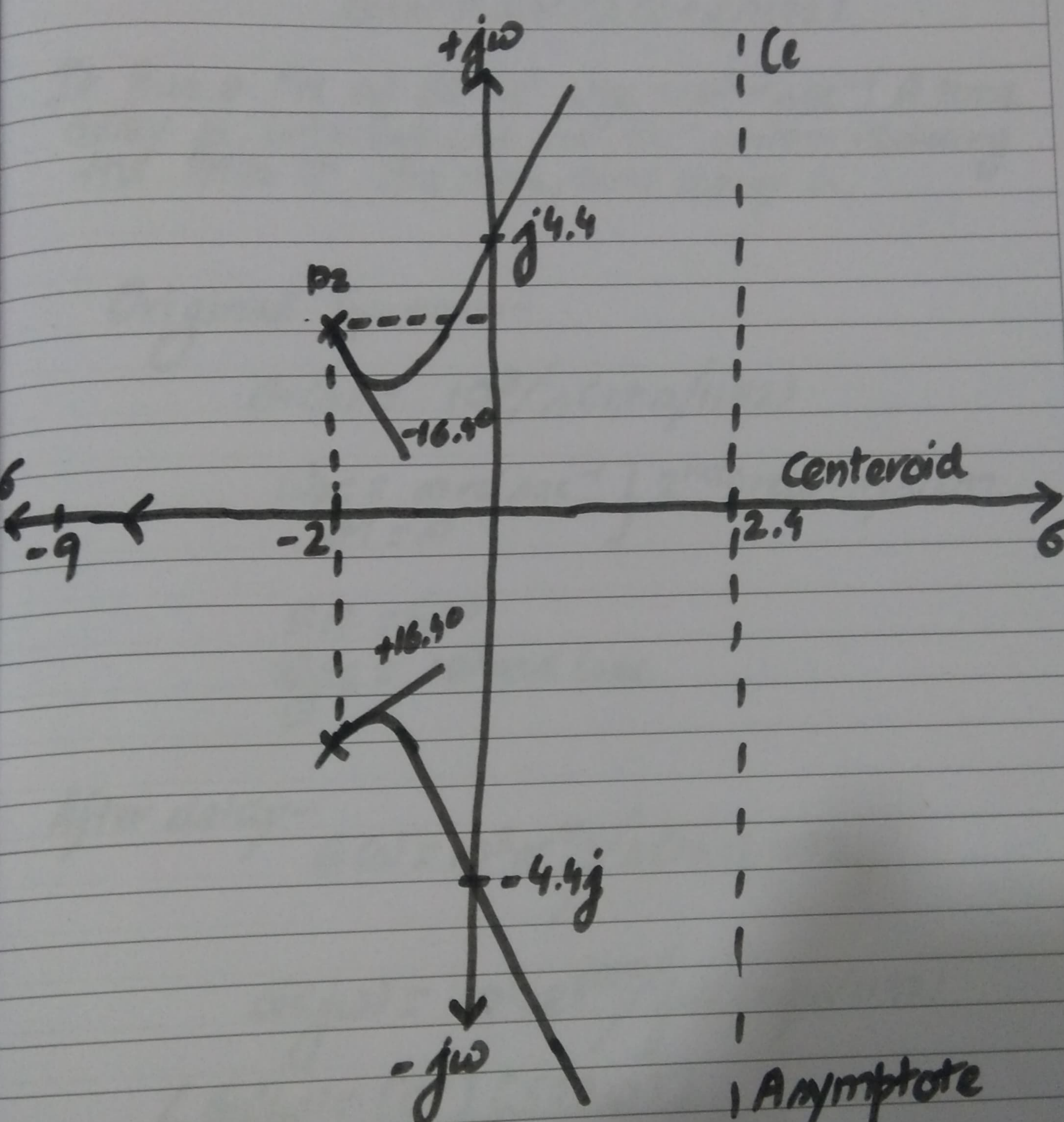
$$-4\omega^2 + 9K = 0$$

$$9K = 4(11+K) \quad -①$$

$$5K = 44$$

$$K = 8.8 \quad ②$$

The crossing pt. of root locous is $j \pm 4.4$.
The values of K at this crossing pt
$K = 6.8$



Q2) The OLTF of a ufb. system is

$$G(s) = 10^3/s(1+s/1192) \quad \text{(II order)}$$

It has a PM of 50° at $\omega_{gc} = 10^3 \, r\,sec^{-1}$! A time delay is introduced in the system reducing the PM to 40°. The max, time delay is.

Original System-

$$G(s) = 10^3/s(1+s/1192)$$

$$\left.\begin{aligned} \omega_{pc} &= \infty \, rd\,sec^{-1} \\ GM &= \infty \end{aligned}\right\} \text{2nd order system}$$

$$P.M. = 50°$$
$$\omega_{g.c} = 10^3 \, rad/sec$$

After delay-

$$G(s) = 10^3 e^{-Ts} / s(1+s)/1192$$

$$G(j\omega) = 10^3 . e^{j\omega t} / j\omega(1+j\omega/1192)$$

$$\angle G(j\omega) = \frac{[0°][57.3\omega T]}{[90°][\tan^{-1}\omega/1192]}$$

$$\Rightarrow 57.3\omega T - 90° = \tan^{-1}(\omega/1192) \quad -(1)$$

Now,
as per ques, $PM_{new} = 40^\circ$

$$PM = 180 + \phi$$
$$40 = 180 + \phi \Rightarrow \phi = -140^\circ$$

$$\phi = \angle G(j\omega)\Big|_{\omega = \omega_{gc} = 10^3\, r/sec}$$ (remains same)

from ①

$$-140 = -90 - 57.3 \times 10^3 T - \tan^{-1}\left(\frac{1000}{1192}\right)$$

$$T = 0.174 \times 10^{-3}\ \text{secs.}$$