The equations of the string is $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

Let $y = X(x)T(t)$

X is a function of x and T is a function of T

$\frac{\partial y}{\partial t} = \frac{\partial (XT)}{\partial T} = XT'$ $\frac{\partial^2 y}{\partial x^2} = XT''$

$\frac{\partial y}{\partial x} = X''T$

$XT'' = c^2 X''T$

$\frac{1}{c^2}\frac{T''}{T} = \frac{X''}{X} = K = (-p^2)$

Using the seperation of variable we solve these 2 cases seperately

$\frac{1}{c^2}\frac{T''}{T} = -p^2$ $\frac{X'}{X} = -p^2$

$T'' = -c^2 p^2 T$ $X'' = -p^2 X$

$D' = \pm pci$ and $D = \pm pi$

$T = c_1 \cos cpt + c_2 \sin cpt$, $X = c_3 \cos px + c_4 \sin px$.

$y(0,t) = 0$, $y(l,t) = 0$ $\left(\frac{\partial u}{\partial t}\right)_{t=0}$ $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$

Applying boundaries condition in equation (∂^x)

$y(0t) = 0 = (c_1 \cos cpt + c_2 \cos cpt)\, c_3$

so $c_3 = 0$

Eq $\left(\frac{\partial u}{\partial t}\right) = \frac{n\pi c}{l}\left(c - c_1 \sin\frac{n\pi t}{l} + c_2 \cos\frac{n\pi t}{l}\right)\left(c_4 \sin\frac{n\pi x}{l}\right)$

$y(x,t) = \sum b_n \cos\frac{n\pi t}{l} \sin\frac{n\pi x}{l}$

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$$y(x,t) = \left[\frac{3y_0}{4} \cos \frac{\pi c t}{l} \sin \frac{n\pi x}{l} - \frac{y_0}{4} \cos \frac{3\pi c t}{l} \sin \frac{3\pi x}{l} \right]$$

Q2) Ans)

$$u(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos apt + c_4 \sin apt)$$

$$0 = c_1 (c_3 \cos apt + c_4 \sin apt)$$

$$u(x,t) = c_2 \sin px (c_3 \cos apt + c_4 \sin at)$$

on putting $x = \pi$ and $u = 0$

$$0 = c_2 \sin p\pi (c_3 \cos apt + c_4 \sin apt)$$

$$\sin \pi p = 0 \quad \text{or } p = n.$$

$$u(x,t) = c_2 \sin nx (c_3 \cos ant + c_4 \sin ant)$$

$$u(x,t) = \sin nx (b_1 \cos ant + b_2 \sin at)$$

Application of PDE

$$\frac{\partial u}{\partial t} = \sin nx (-ab_1 n \sin ant + ab_2 \cos at)$$

General solution is

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin nx \cos ant$$