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Q4 $y^2 p - xyq = x(z-2y)$

Soln Subsidiary eqn

$(p - xyq) = x(z-2y)$

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

Taking first two fraction

$$\frac{dx}{y^2} = \frac{-dy}{xy}$$

$$xdx - ydy = 0$$

$$\frac{x^2}{2} - \frac{y^2}{2} = c$$

$$\boxed{x^2 - y^2 = c}$$

Taking last two fraction

$$\frac{dy}{xy} = \frac{dz}{\cancel{x}(z-2y)}$$

$$(z-2y)dy = ydz$$

$$ydz + zdy - 2ydy = 0$$

$$d(yz) - 2ydy = 0$$

$$yz - y^2 = c_2$$

$f(x^2+y^2, yz-y^2)$

Q5 $(x^2-y^2-z^2)p + 2xyq = 2xz$

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

last 2 fractions

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\log y = \log z + \log c_1$$

$$\boxed{\frac{y}{z} = c_1}$$

using x, y, z multiplier

$$\frac{x\,dx + y\,dy + z\,dz}{x(x^2-y^2-z^2) + 2xy^2 + 2xz^2}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 + xy^2 + xz^2}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{x(x^2+y^2+z^2)}$$

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$$\frac{dy}{2\cancel{x}y} = \frac{xdx + ydy + zdx}{\cancel{x}(x^2+y^2+z^2)}$$

$$\log y = \log(x^2+y^2+z^2) + \log c_2$$

$$\boxed{c_2 = \frac{y}{x^2+y^2+z^2}}$$

$$F\left(\frac{y}{z}, \frac{y}{x^2+y^2+z^2}\right) = 0$$

Q6 $p - q = \log(x+y)$

$$dx = -dy = \frac{dz}{\log(x+y)}$$

$$dx = -dy$$

$$\boxed{x+y = c_1}$$

Taking Ist and last fraction

$$dx = \frac{dz}{\log(x+y)}$$

$$dx = \frac{dz}{\log(x+y)}$$

$\Rightarrow$ $(\log(x+y))\, dx = dz$

$\Rightarrow$ $(\log(c_1)) \cdot dx = dz$

$\log c_1\, x = \cancel{d}z + c_2$

$$\boxed{x \log(x+y) - z = c_2}$$

$$\boxed{f\left(x+y,\ x \log(x+y) - z\right) = 0}$$ Ans

Q7 $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$

each fracⁿ $= \dfrac{dx + dy + dz}{x^2 - yz + y^2 - zx + z^2 - xy}$

$= \dfrac{dx + dy + dz}{x^2 + y^2 + z^2 - xy - yz - zx}$

$= \dfrac{x\,dx + y\,dy + z\,dz}{(x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)}$

(i) = (ii)

$\dfrac{x\,dx + y\,dy + z\,dz}{(x+y+z)(\cancel{x^2 + y^2 + z^2 - xy - yz - zx})} = \dfrac{dx + dy + dz}{(\cancel{x^2 + y^2 + z^2 - xy - yz - zx})}$

$\dfrac{x\,dx + y\,dy + z\,dz}{(x+y+z)} = \dfrac{dx + dy + dz}{1}$

$x\,dx + y\,dy + z\,dz = (x+y+z)\,d(x+y+z)$

$x^2 + y^2 + z^2 = (x+y+z)^2 + c_1$

$$\boxed{c_1 = xy + yz + zx}$$

each fraction $= \dfrac{dx - dy}{(x^2-yz)-(y^2-zx)} = \dfrac{dy - dz}{(y^2-zx)-(z^2-xy)}$

$$\frac{d(x-y)}{(x-y)(\cancel{x+y+z})} = \frac{d(y-z)}{(y-z)(\cancel{x+y+z})}$$

$$\log(x-y) = \log(y-z) + \log c_2$$

$$\boxed{\frac{x-y}{y-z} = c_2}$$

$$\boxed{f\left(xy+yz+zx\ ,\ \frac{x-y}{y-z}\right) = 0}$$

Q8 $\quad x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)}$$

using $\dfrac{1}{x^2}, \dfrac{1}{y^2}, \dfrac{1}{z}$ as multiplier

each fraction $= \dfrac{\dfrac{dx}{x^2} + \dfrac{dy}{y^2} + \dfrac{dz}{z^2}}{\cancel{y}-\cancel{z}+\cancel{z}-\cancel{x}+\cancel{x}-\cancel{y}}$

$$\frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2} = 0$$

$$\boxed{\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = c_1}$$

$\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multiplier

each fraction = $\dfrac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{x(y-z) + y(z-x) + z(x-y) \longrightarrow 0}$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$$\log x + \log y + \log z = \log c_2$$

$$\boxed{c_2 = xyz}$$

$$\boxed{f\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z},\ xyz\right) = 0}$$

Q9 $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$

use x, y, z as multipliers

$$\frac{xdx + ydy + zdz}{x^2y^2 - x^2z^2 + y^2z^2 - x^2y^2 + z^2x^2 - z^2y^2 \longrightarrow 0}$$

$$xdx + ydy + zdz = 0$$

Integrate

$$\boxed{\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_1}$$

$$\boxed{x^2 + y^2 + z^2 = c_1}$$

Using $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multiplier

$$\frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{0}$$

$$\log x + \log y + \log z = \log c_2$$

$$\boxed{xyz = c_2}$$

$$\boxed{f(x^2+y^2+z^2,\ xyz) = 0}$$

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