ENEL 693 Restructured Electricity Market

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Unit Commitment Project

Unit Commitment
Project220 marks

For the following system with the provided data, consider a 4-hour planning horizon with hourly demandsequal to 1, 0.9, 1.3, and 1.5 puMW, respectively. The reserve requirement is 10% of the demand at each time period. There is a single reliability area comprising the three nodes. Both generating units were online in the time period prior to the beginning of the planning horizon with power outputs equal to 0.7 and 0.6 puMW for generating units #1 and #2, respectively. The unit for B and Pmax is in PU. Line limits are 1PU.

Table 1: Generation Units Data

Parameter	**Unit 1**	**Unit 2**
Minimum Power Output (puMW)	0.15	0.2
Maximum Power Output (puMW)	1.5	1.5
Ramping-Down Limit (puMW/h)	0.2	0.3
Shut-Down Ramping Limit (puMW/h)	0.2	0.3
Ramping-Up Limit (puMW/h)	0.2	0.3
Start-Up Ramping Limit (puMW/h)	0.2	0.3
Fixed Cost ($)	3.0	1.0
Start-Up Cost ($)	5.0	2.0
Shut-Down Cost ($)	1.0	1.0
Variable Cost ($/puMW)	10	20

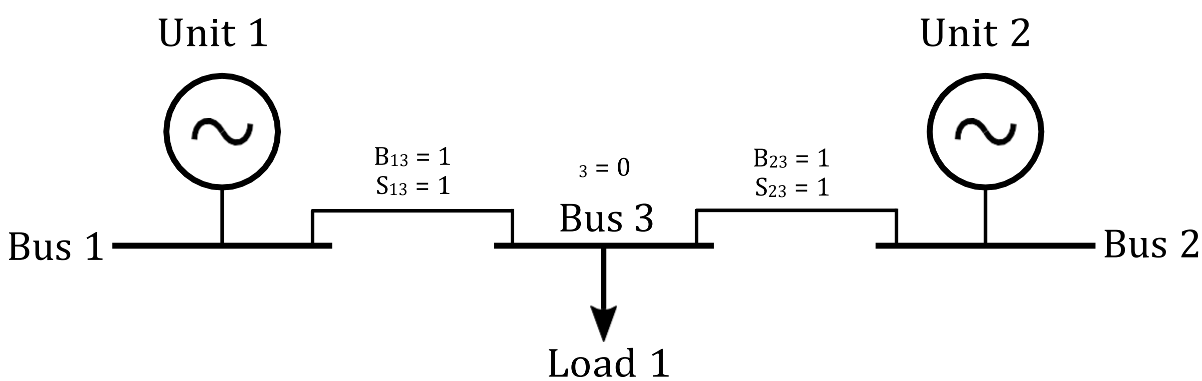


Figure 1: Case 2 - Network Constrained System

Report Tasks:

(a) Using a similar approach as the one practiced in class and detailed in the slides for Unit Commitment, identify one feasible solution for this unit commitment problem. You do not have to find all feasible solutions. Provide only one feasible solution and verify and justify why each individual constraint that is honored. Please verify each single constraint one by one and clearly refer to the equation number and verify feasibility of your solution. Also, calculate the associated total cost for your proposed feasible solution. Note that what you propose does not have to be optimal. (30 marks).

The following data is given in the problem.

Unit	P_{min} $(puMW)$	P_{max} $(puMW)$	Ramp Up limit $(puMW/h)$	Ramp Down Limit $(puMW/h)$	Start-up ramp up limit $(puMW/h)$	Shut-down ramp limit $(puMW/h)$	Fixed Cost $(\$)$	Variable cost $(\$/MWh)$	Initial Status
1	0.15	1.5	0.2	0.2	0.2	0.2	3.0	10	ON
2	0.2	1.5	0.3	0.3	0.3	0.3	1.0	20	ON

Table 1: Unit Data

Load scenario:

Hourly load demand is given as below for four different time period. 10% of demand is considered as reserve capacity.

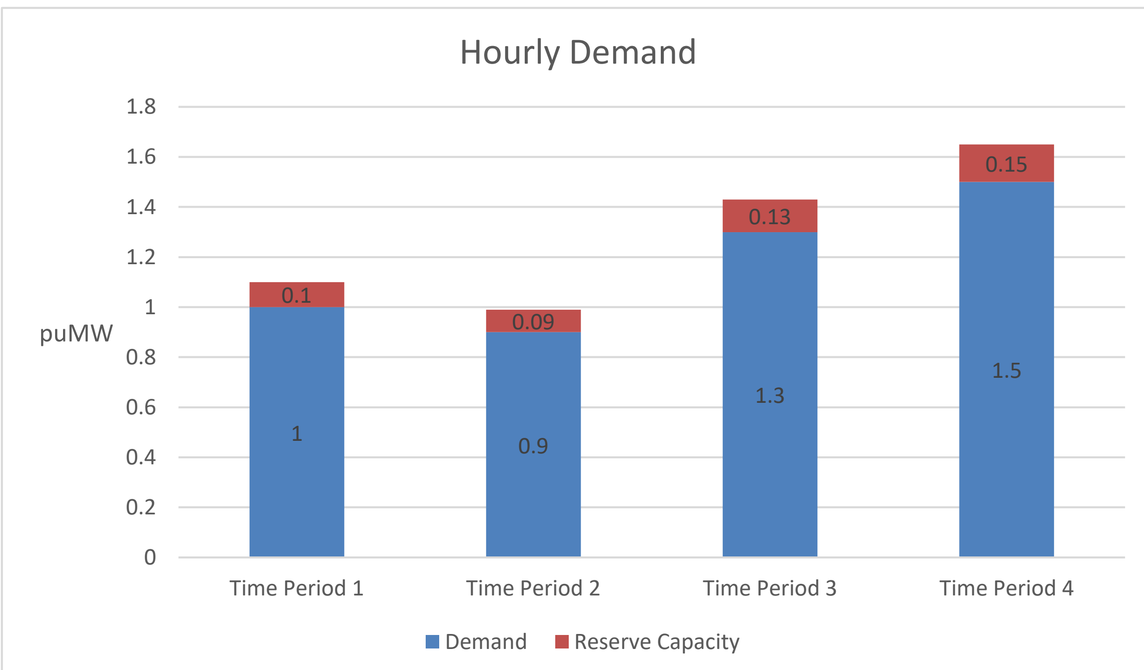


Figure 1: Load Scenario

The following possible feasible Unit states are shown below for different time period considering the reserve capacity. (Line constraints not considered yet).

Combinations		P_{min} $puMW$	P_{max} $puMW$
Unit-1	Unit-2		
ON	ON	0.35	3
ON	OFF	0.15	1.5
OFF	ON	0.2	1.5
OFF	OFF	0	0

Time Period-1	Time Period-2	Time Period-3	Time Period-4
$1 + 0.1\ puMW$	$0.9 + 0.09\ puMW$	$1.3 + 0.13\ puMW$	$1.5 + 0.15\ puMW$
●	●	●	●
●	●	●	○
●	●	●	○
○	○	○	○

Table 2: Possible feasible states

○ ---> Not possible solution

● ---> Possible solution

The following feasible combinations are shown below considering the required reserve capacity.

Combinations	
P_1	P_2
1	1
1	0
0	1
0	0

Initial State	Time Period-1	Time Period-2	Time Period-3	Time Period-4
	1 + 0.1	0.9 + 0.09	1.3 + 0.13	1.5 + 0.15
●	●	●	●	●
	●	●	●	
	●	●	●	

Table 3: Possible feasible combinations

From all the above possible cases, I have selected one feasible solution as shown below. In which both the units remain ON throughout all the time period. Initially P_1 and P_2 Unit are ON.

Combinations	
P_1	P_2
1	1
1	0
0	1
0	0

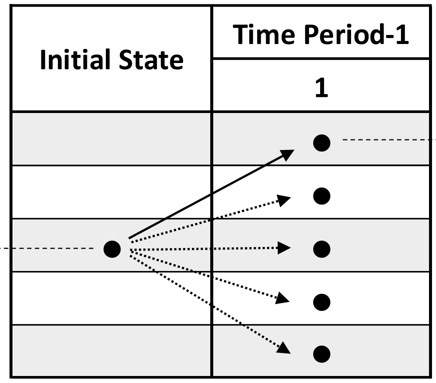
Initial State	Time Period-1	Time Period-2	Time Period-3	Time Period-4
	1 + 0.1	0.9 + 0.09	1.3 + 0.13	1.5 + 0.15
● →	● →	● →	● →	●

Table 4: Selected one feasible solution

<u>Initial State to Time Period-1:</u>

Initially P_1 and P_2 units are running with 0.7 $puMW$ and 0.6 $puMW$. There are many possible cases to mitigate the demand in time period-1; however, I have selected the case in which P_1 and P_2 units are running with 0.7 $puMW$ and 0.3 $puMW$ as shown in table below.

Initial state	
P_1	P_2
0.9	0.4
0.8	0.5
0.7	**0.6**
0.6	0.7
0.5	0.6



Time Period-1		
P_1	P_2	*Actual demand*
0.7	**0.3**	**1**
0.6	0.4	1
0.5	0.5	1
0.4	0.6	1
0.3	0.7	1

Table 5: Initial State-->Time Period-1

$\longrightarrow$ Selected Case

$\dashrightarrow$ Possible Cases

Ramp up Constraint:

- **Actual Ramp up value:** 0 $puMW/h$ and 0 $puMW/h$ for P_1 and P_2 units.
- **Given Ramp up limit:** 0.2 $puMW/h$ and 0.3 $puMW/h$ for P_1 and P_2 units.

Ramp down Constraint:

- **Actual Ramp down:** 0 $puMW/h$ and 0.3 $puMW/h$ for P_1 and P_2 units.
- **Given Ramp down limit:** 0.2 $puMW/h$ and 0.3 $puMW/h$ for P_1 and P_2 units.

Start-up Ramp up Constraint: In this case, our units remain ON. Thus, this constraint is not applicable.

Shut down Ramp down Constraint: In this case, our units remain ON. Thus, this constraint is not applicable.

Line Loading constraint:

- **Actual line loading:** $L_{13} = 0.7\ puMW$ and $L_{23} = 0.3\ puMW$.
- **Given line loading limit:** $L_{13} = 1.0\ puMW$ and $L_{23} = 1.0\ puMW$.

Reserve capacity:

$$\gg P_1^{max} u_{11} + P_2^{max} u_{21} \geq\ P_1^D +\ R_1^D$$
$$\gg 1.5 * 1 + 1.5 * 1 \geq\ 1.0 +\ 0.1$$
$$\gg 3.0 \geq\ 1.1$$

where, u_{ij} *is the binary variable which state that unit is either ONLINE or OFFLINE for* i^{th} *unit and in* j^{th} *time period.*

P_j^D *is the total system demand (in puMW) in* j^{th} *time period.*

R_j^D *is the required reserve capacity of total system demand (in puMW) in* j^{th} *time period.*

The reserve capacity constraint satisfied in our selected combination.

Fixed Cost:

Time period-1 Fixed cost $c_{g1}^F\ = Unit - 1\ Fixed\ cost\ +\ Unit - 2\ Fixed\ cost$

$$= 3.0\ \$ + 1.0\ \$$$
$$= 4..0\ \$$$

Variable Cost:

Time period-1 Variable cost $c_{g1}^{V} = Unit - 1\, Variable\ cost * p_{11} + Unit - 2\, Variable\ cost * p_{21}$

$$= 10 * 0.7 + 20 * 0.3$$
$$= 13\ \$/puMW$$

where, p_{ij} is actual generation of i^{th} unit in $puMW$ and in j^{th} time period.

Start-up Cost:

In this case, both the Units remain ON. So, no any start-up cost in time period-1.

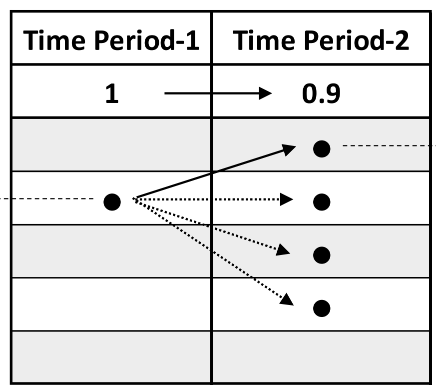
Shut down Cost:

In this case, both the Units remain ON. So, no any shut down cost in time period-1.

<u>Time Period-1 to Time Period-2:</u>

As discuss above, in time period 1, P_1 and P_2 units are running with 0.7 $puMW$ and 0.3 $puMW$. There are many possible cases to mitigate the demand in time period-2; however, I have selected the case in which P_1 and P_2 units are running with 0.7 $puMW$ and 0.2 $puMW$ as shown in table below.

Time Period-1	
P_1	P_2
0.8	0.2
0.7	**0.3**
0.6	0.4
0.5	0.5
0.4	0.6



Time Period-2		
P_1	P_2	*Actual demand*
0.7	**0.2**	**0.9**
0.6	0.3	0.9
0.5	0.4	0.9
0.4	0.5	0.9
0.3	0.6	0.9

Table 6: Time Period-1 --> Time Period-2

Ramp up Constraint:

- **Actual Ramp up value:** 0 $puMW/h$ and 0 $puMW/h$ for P_1 and P_2 units.
- **Given Ramp up limit:** 0.2 $puMW/h$ and 0.3 $puMW/h$ for P_1 and P_2 units.

Ramp down Constraint:

- **Actual Ramp down:** 0 $puMW/h$ and 0.1 $puMW/h$ for P_1 and P_2 units.
- **Given Ramp down limit:** 0.2 $puMW/h$ and 0.3 $puMW/h$ for P_1 and P_2 units.

Start-up Ramp up Constraint: In this case, our units remain ON. Thus, this constraint is not applicable.

Shut down Ramp down Constraint: In this case, our units remain ON. Thus, this constraint is not applicable.

Line Loading constraint:

- **Actual line loading:** $L_{13} = 0.7\ puMW$ and $L_{23} = 0.2\ puMW$.
- **Given line loading limit:** $L_{13} = 1.0\ puMW$ and $L_{23} = 1.0\ puMW$.

Reserve capacity:

$$\gg P_1^{max} u_{12} + P_2^{max} u_{22} \geq\ P_2^D +\ R_2^D$$
$$\gg 1.5 * 1 + 1.5 * 1 \geq\ 0.9 +\ 0.09$$
$$\gg 3.0 \geq\ 0.99$$

Here, the reserve capacity constraint satisfied for selected combination.

Fixed Cost:

Time period-2 Fixed cost $c_{g2}^F\ =\ Unit - 1\ Fixed\ cost\ +\ Unit - 2\ Fixed\ cost$

$$= 3.0\ \$ + 1.0\ \$$$
$$= 4..0\ \$$$

Variable Cost:

Time period-2 Variable cost $c_{g2}^V\ = Unit - 1\ Variable\ cost * p_{12} +\ Unit - 2\ Variable\ cost * p_{22}$

$$= 10 * 0.7 + 20 * 0.2$$
$$= 11\ \$/puMW$$

Start-up Cost:

In this case, both the Units remain ON. So, no any start-up cost in time period-2.

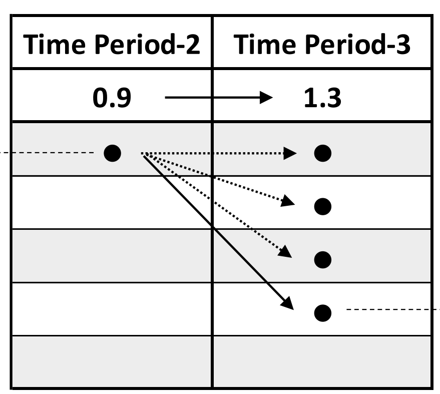
Shut down Cost:

In this case, both the Units remain ON. So, no any shut down cost in time period-2.

__Time Period-2 to Time Period-3:__

As discuss above, in time period 2, P_1 and P_2 units are running with 0.7 $puMW$ and 0.2 $puMW$. There are many possible cases to mitigate the demand in time period-3; however, I have selected the case in which P_1 and P_2 units are running with 0.9 $puMW$ and 0.4 $puMW$ as shown in table below.

Time Period-2	
P_1	P_2
0.7	**0.2**
0.6	0.3
0.5	0.4
0.4	0.5
0.3	0.6



Time Period-3		
P_1	P_2	*Actual demand*
0.6	0.7	1.3
0.7	0.6	1.3
0.8	0.5	1.3
0.9	**0.4**	**1.3**
1	0.3	1.3

Table 7: Time Period -2 --> Time Period-3

Ramp up Constraint:

- **Actual Ramp up value:** $0.2\ puMW/h$ and $0.2\ puMW/h$ for P_1 and P_2 units.
- **Given Ramp up limit:** $0.2\ puMW/h$ and $0.3\ puMW/h$ for P_1 and P_2 units.

Ramp down Constraint:

- **Actual Ramp down:** $0\ puMW/h$ and $0\ puMW/h$ for P_1 and P_2 units.
- **Given Ramp down limit:** $0.2\ puMW/h$ and $0.3\ puMW/h$ for P_1 and P_2 units.

Start-up Ramp up Constraint: In this case, our units remain ON. Thus, this constraint is not applicable.

Shut down Ramp down Constraint: In this case, our units remain ON. Thus, this constraint is not applicable.

Line Loading constraint:

- **Actual line loading:** $L_{13} = 0.9\ puMW$ and $L_{23} = 0.4\ puMW$.
- **Given line loading limit:** $L_{13} = 1.0\ puMW$ and $L_{23} = 1.0\ puMW$.

Reserve capacity:

$$\gg P_1^{max} u_{13} + P_2^{max} u_{23} \geq P_3^D + R_3^D$$
$$\gg 1.5 * 1 + 1.5 * 1 \geq 1.3 + 0.13$$
$$\gg 3.0 \geq 1.43$$

Here, the reserve capacity constraint satisfied for selected combination.

Fixed Cost:

Time period-3 Fixed cost $c_{g3}^F = Unit - 1\ Fixed\ cost + Unit - 2\ Fixed\ cost$

$$= 3.0\ \$ + 1.0\ \$$$
$$= 4..0\ \$$$

Variable Cost:

Time period-3 Variable cost $c_{g3}^V = Unit - 1\ Variable\ cost * p_{13} + Unit - 2\ Variable\ cost * p_{23}$

$$= 10 * 0.9 + 20 * 0.4$$
$$= 17\ \$/puMW$$

Start-up Cost:

In this case, both the Units remain ON. So, no any start-up cost in time period-3.

Shut down Cost:

In this case, both the Units remain ON. So, no any shut down cost in time period-3.

Time Period-3 to Time Period-4:

As discuss above, in time period 3, P_1 and P_2 units are running with 0.9 $puMW$ and 0.4 $puMW$. There are many possible cases to mitigate the demand in time period-4; however, I have selected the case in which P_1 and P_2 units are running with 0.9 $puMW$ and 0.6 $puMW$ as shown in table below.

Time Period-3	
P_1	P_2
0.6	0.7
0.7	0.6
0.8	0.5
0.9	**0.4**
1	0.3

Time Period-3	Time Period-4
1.3 →	**1.5**
	●
	●
●	●
	●

Time Period-4		
P_1	P_2	*Actual demand*
0.6	0.9	1.5
0.7	0.8	1.5
0.8	0.7	1.5
0.9	**0.6**	**1.5**
1	0.5	1.5

Table 8: Time Period-3 --> Time Period-4

Ramp up Constraint:

- **Actual Ramp up value:** $0.0\ puMW/h$ and $0.2\ puMW/h$ for P_1 and P_2 units.
- **Given Ramp up limit:** $0.2\ puMW/h$ and $0.3\ puMW/h$ for P_1 and P_2 units.

Ramp down Constraint:

- **Actual Ramp down:** $0\ puMW/h$ and $0\ puMW/h$ for P_1 and P_2 units.
- **Given Ramp down limit:** $0.2\ puMW/h$ and $0.3\ puMW/h$ for P_1 and P_2 units.

Start-up Ramp up Constraint: In this case, our units remain ON. Thus, this constraint is not applicable.

Shut down Ramp down Constraint: In this case, our units remain ON. Thus, this constraint is not applicable.

Line Loading constraint:

- **Actual line loading:** $L_{13} = 0.9\ puMW$ and $L_{23} = 0.6\ puMW$.
- **Given line loading limit:** $L_{13} = 1.0\ puMW$ and $L_{23} = 1.0\ puMW$.

Reserve capacity:

$$\gg P_1^{max} u_{14} + P_2^{max} u_{24} \geq P_4^D + R_4^D$$
$$\gg 1.5 * 1 + 1.5 * 1 \geq 1.5 + 0.15$$
$$\gg 3.0 \geq 1.65$$

Here, the reserve capacity constraint satisfied for selected combination.

Fixed Cost:

Time period-4 Fixed cost $c_{g4}^F = Unit - 1\ Fixed\ cost + Unit - 2\ Fixed\ cost$

$$= 3.0\ \$ + 1.0\ \$$$
$$= 4..0\ \$$$

Variable Cost:

Time-period-4 Variable-cost $c_{g4}^{V} = Unit - 1\, Variable\ cost * p_{14} + Unit - 2\, Variable\ cost * p_{24}$

$$= 10 * 0.9 + 20 * 0.6$$
$$= 21\ \$/puMW$$

Start-up Cost:

In this case, both the Units remain ON. So, no any start-up cost in time period-4.

Shut down Cost:

In this case, both the Units remain ON. So, no any shut down cost in time period-4.

Total cost:

$$Total\ cost = c_{g1}^{F} + c_{g2}^{F} + c_{g3}^{F} + c_{g4}^{F} + c_{g1}^{V} + c_{g2}^{V} + c_{g3}^{V} + c_{g4}^{V}$$
$$= 4 + 4 + 4 + 4 + 13 + 11 + 17 + 21$$
$$= 78\ \$$$

Summary Table: -

Decision Variables	Time Period-1		Time Period-2		Time Period-3		Time Period-4	
Unit-1 Output (puMW)	p_{11}	0.7	p_{12}	0.7	p_{13}	0.9	p_{14}	0.9
Unit-2 Output (puMW)	p_{21}	0.3	p_{22}	0.2	p_{23}	0.4	p_{24}	0.6
Unit-1 running status binary variable	u_{11}	1	u_{12}	1	u_{13}	1	u_{14}	1
Unit-2 running status binary variable	u_{21}	1	u_{22}	1	u_{23}	1	u_{24}	1
Unit-1 start-up binary variable	y_{11}	0	y_{12}	0	y_{13}	0	y_{14}	0
Unit-2 start-up binary variable	y_{21}	0	y_{22}	0	y_{23}	0	y_{24}	0
Unit-1 shut down binary variable	z_{11}	0	z_{12}	0	z_{13}	0	z_{14}	0
Unit-2 shut down binary variable	z_{21}	0	z_{22}	0	z_{23}	0	z_{24}	0
Auxiliary Variables	**Time Period-1**		**Time Period-2**		**Time Period-3**		**Time Period-4**	
P13 line loading (puMW)	L_{11}	0.7	L_{12}	0.7	L_{13}	0.9	L_{14}	0.9
P23 line loading (puMW)	L_{21}	0.3	L_{22}	0.2	L_{23}	0.4	L_{24}	0.6
Parameters	**Time Period-1**		**Time Period-2**		**Time Period-3**		**Time Period-4**	
Demand (puMW)	P_1^D	1	P_2^D	0.9	P_3^D	1.3	P_4^D	1.5
Fixed Cost ($)	C_1^F	4	C_2^F	4	C_3^F	4	C_4^F	4
Variable cost ($/puMW)	C_1^V	13	C_2^V	11	C_3^V	17	C_4^V	21
Start-up cost ($)	C_1^{SU}	0	C_2^{SU}	0	C_3^{SU}	0	C_4^{SU}	0
Shut down cost ($)	C_1^{SD}	0	C_2^{SD}	0	C_3^{SD}	0	C_4^{SD}	0
Total cost ($)	C_1^{Total}	17	C_2^{Total}	15	C_3^{Total}	21	C_4^{Total}	25

Table 9: Summary Table

(b) Write a detailed, specific formulation for an optimal least-cost unit commitment problem for this system. Clearly name and specify your variables and parameters. Label all your equations and describe what each equation does. (30 marks)

Objective Function:

The total cost of producing electricity by thermal generating units can be expressed as:

$$\boldsymbol{Total\ cost\ c_{gt}} = c_{gt}^{F} + c_{gt}^{V} + c_{gt}^{SU} + c_{gt}^{SD},\ \$ \quad \forall g, \forall t$$

where:

c_{gt} *is the total cost of unit g in time period t,*

c_{gt}^{F} *is the fixed cost of unit g in time period t,*

c_{gt}^{V} *is the variable cost of unit g in time period t,*

c_{gt}^{SU} *is the start − up cost of unit g in time period t, and*

c_{gt}^{SD} *is the shut − down cost of unit g in time period t,*

The no-load or fixed cost can be computed as:

$$\boldsymbol{Fixed\ cost\ c_{gt}^{F}} = C_{g}^{F} * u_{gt},\ \$\ \forall g, \forall t,$$

$$= C_{1}^{F} * (u_{11} + u_{12} + u_{13} + u_{14}) + C_{2}^{F} * (u_{21} + u_{22} + u_{23} + u_{24})$$

u_{gt} *is a binary variable that is equal to* 1 *if generating unit g is online in time period t, and* 0 *otherwise.*

C_{g}^{F} *is the no load cost of generating unit g in* $\$$.

When a generating unit is producing an electricity, it has a variable cost:

$$\boldsymbol{Variable\ cost\ C_{gt}^{V}} = C_{g}^{V} * p_{gt},\ \$\ \forall g, \forall t,$$

$$= C_{1}^{V} * (p_{11} + p_{12} + p_{13} + p_{14}) + C_{2}^{V} * (p_{21} + p_{22} + p_{23} + p_{24})$$

p_{gt} *is the output power of generating unit g in puMW during time period t.*

C_{g}^{V} *is the variable cost of generating unit g in* $\$/puMW$.

When a generating unit is started up, its start-up cost is incurred:

$$\boldsymbol{Start - up\ cost\ c_{gt}^{SU}} = C_{g}^{SU} * y_{gt},\ \$\ \forall g, \forall t,$$

$$= C_{1}^{SU} * (y_{11} + y_{12} + y_{13} + y_{14}) + C_{2}^{SU} * (y_{21} + y_{22} + y_{23} + y_{24})$$

y_{gt} *is a binary variable that is equal to* 1 *if generating unit g is started up at the beginning of time period t, and* 0 *otherwise.*

C_{g}^{SU} *is the start up cost of generating unit g in* $\$$.

Similarly, when a generating unit is shutting down, its shut-down cost is incurred:

$$\boldsymbol{Shut - down\ cost\ c_{gt}^{SD}} = C_{g}^{SD} * z_{gt},\ \$\ \forall g, \forall t,$$

$$= C_{1}^{SD} * (z_{11} + z_{12} + z_{13} + z_{14}) + C_{2}^{SD} * (z_{21} + z_{22} + z_{23} + z_{24})$$

z_{gt} *is a binary variable that is equal to* 1 *if generating unit* g *is shutting down at the beginning of time period* t, *and* 0 *otherwise.*
C_g^{SD} *is the shut down cost of generating unit* g *in* \$.

Our **objective function** is to minimize the total cost for Unit 1 and Unit 2.

$$
\begin{aligned}
\min_{u_{gt};p_{gt};y_{gt};z_{gt};} \\
&= C_1^F * (u_{11} + u_{12} + u_{13} + u_{14}) + C_2^F * (u_{21} + u_{22} + u_{23} + u_{24}) \\
&+ C_1^V * (p_{11} + p_{12} + p_{13} + p_{14}) + C_2^V * (p_{21} + p_{22} + p_{23} + p_{24}) \\
&+ C_1^{SU} * (y_{11} + y_{12} + y_{13} + y_{14}) + C_2^{SU} * (y_{21} + y_{22} + y_{23} + y_{24}) \\
&+ C_1^{SD} * (z_{11} + z_{12} + z_{13} + z_{14}) + C_2^{SD} * (z_{21} + z_{22} + z_{23} + z_{24}) \ \$
\end{aligned}
$$

$$
\begin{aligned}
\min_{u_{gt};p_{gt};y_{gt};z_{gt};} \\
&= 3 * (u_{11} + u_{12} + u_{13} + u_{14}) + 1 * (u_{21} + u_{22} + u_{23} + u_{24}) \\
&+ 10 * (p_{11} + p_{12} + p_{13} + p_{14}) + 20 * (p_{21} + p_{22} + p_{23} + p_{24}) \\
&+ 5 * (y_{11} + y_{12} + y_{13} + y_{14}) + 2 * (y_{21} + y_{22} + y_{23} + y_{24}) \\
&+ 1 * (z_{11} + z_{12} + z_{13} + z_{14}) + 1 * (z_{21} + z_{22} + z_{23} + z_{24}) \ \$
\end{aligned}
$$

<u>Logical Conditions:</u>

Any thermal generating unit that is on-line can be shut down but not started up. Analogously, any generating unit that is off-line, can be started up but not shut-down. This can be expressed as:

$$y_{gt} - z_{gt} = u_{gt} - u_{g,t-1}, \quad \forall g, \forall t, \tag{1a}$$

$$y_{gt} + z_{gt} \leq 1, \quad \forall g, \forall t, \tag{1b}$$

$$u_{gt}, y_{gt}, z_{gt} \in \{0,1\}, \quad \forall g, \forall t, \tag{1c}$$

Note that constraint $(1a)$ include binary variables in both time periods t and $t-1$. Thus, it is necessary to rewrite these constraints for the first time period of the planning horizon as:

$$y_{g1} - z_{g1} = u_{g1} - U_{g0}, \quad \forall g,$$

Where U_{g0} is equal to 1 if generating unit g is online in the time period prior to the beginning of the planning horizon, and 0 otherwise.

So, we can write the Logical constraints for Unit 1 and Unit 2 as below…

<u>Logical conditions-1:</u>

	$y_{11} - z_{11} = u_{11} - 1$	Time period-1
Unit-1	$y_{12} - z_{12} = u_{12} - u_{11}$	Time period-2
	$y_{13} - z_{13} = u_{13} - u_{12}$	Time period-3
	$y_{14} - z_{14} = u_{14} - u_{13}$	Time period-4

	$y_{21} - z_{21} = u_{21} - 1$	Time period-1
Unit-2	$y_{22} - z_{22} = u_{22} - u_{21}$	Time period-2
	$y_{23} - z_{23} = u_{23} - u_{22}$	Time period-3
	$y_{24} - z_{24} = u_{24} - u_{23}$	Time period-4

Logical conditions-2:

	$y_{11} + z_{11} \leq 1$	Time period-1
Unit-1	$y_{12} + z_{12} \leq 1$	Time period-2
	$y_{13} + z_{13} \leq 1$	Time period-3
	$y_{14} + z_{14} \leq 1$	Time period-4

	$y_{21} + z_{21} \leq 1$	Time period-1
Unit-2	$y_{22} + z_{22} \leq 1$	Time period-2
	$y_{23} + z_{23} \leq 1$	Time period-3
	$y_{24} + z_{24} \leq 1$	Time period-4

Logical conditions-3:

$$u_{11}, u_{12}, u_{13}, u_{14}, y_{11}, y_{12}, y_{13}, y_{14}, z_{11}, z_{12}, z_{13}, z_{14} \in \{0\,, 1\}$$

$$u_{21}, u_{22}, u_{23}, u_{24}, y_{21}, y_{22}, y_{23}, y_{24}, z_{21}, z_{22}, z_{23}, z_{24} \in \{0\,, 1\}$$

Power Bounds:

Thermal generating units cannot operate below a minimum power output and above a maximum power output (capacity):

$$P_g^{min}\, u_{gt} \leq p_{gt} \leq P_g^{max}\, u_{gt}, \forall g, \forall t,$$

Where:

P_g^{min} is the minimum power output of generating unit g in $puMW$ and

P_g^{max} is the capacity of generating unit g in $puMW$.

- If unit g is on-line during time period t ($u_{gt} = 1$), it indicates that unit can operate between P_g^{min} and P_g^{max}.
- If generating unit g is off-line during time period t ($u_{gt} = 0$), the constraints impose $0 \leq p_{gt} \leq 0$, i.e., $p_{gt} = 0$.

So, we can write the power bound limits for the Unit-1 and Unit-2 as below…

$P_g^{min} * u_{11} \leq p_{11} \leq P_g^{max} * u_{11}$	Time Period-1
$P_g^{min} * u_{12} \leq p_{12} \leq P_g^{max} * u_{12}$	Time Period-2
$P_g^{min} * u_{13} \leq p_{13} \leq P_g^{max} * u_{13}$	Time Period-3
$P_g^{min} * u_{14} \leq p_{14} \leq P_g^{max} * u_{14}$	Time Period-4

	$0.15 * u_{11} \leq p_{11} \leq 1.5 * u_{11}$	Time Period-1
Unit-1	$0.15 * u_{12} \leq p_{12} \leq 1.5 * u_{12}$	Time Period-2
	$0.15 * u_{13} \leq p_{13} \leq 1.5 * u_{13}$	Time Period-3
	$0.15 * u_{14} \leq p_{14} \leq 1.5 * u_{14}$	Time Period-4

$P_g^{min} * u_{21} \leq p_{21} \leq P_g^{max} * u_{21}$	Time Period-1
$P_g^{min} * u_{22} \leq p_{22} \leq P_g^{max} * u_{22}$	Time Period-2
$P_g^{min} * u_{23} \leq p_{23} \leq P_g^{max} * u_{23}$	Time Period-3
$P_g^{min} * u_{24} \leq p_{24} \leq P_g^{max} * u_{24}$	Time Period-4

	$0.2 * u_{21} \leq p_{21} \leq 1.5 * u_{21}$	Time Period-1
Unit-2	$0.2 * u_{22} \leq p_{22} \leq 1.5 * u_{22}$	Time Period-2
	$0.2 * u_{23} \leq p_{23} \leq 1.5 * u_{23}$	Time Period-3
	$0.2 * u_{24} \leq p_{24} \leq 1.5 * u_{24}$	Time Period-4

Ramp Up Limit:

From one time period to the next one, any thermal generating unit cannot increase its power output above a maximum level, is called the ramping-up limit.

Analogously, if the generating units starts up, its maximum power output in that time period is limited by the so-called start-up ramping limit.

This can be represented as:

$$p_{gt} - p_{g,t-1} \leq R_g^U u_{g,t-1} + R_g^{SU} y_{gt}, \forall g, \forall t,$$

Where:

R_g^U is the ramping-up limit of generating unit g in $puMW/h$ and

R_g^{SU} is the start-up ramping limit of generating unit g in $puMW/h$.

For the first time period of the planning horizon, these constraints become:

$$p_{g1} - P_{g0} \leq R_g^U U_{g0} + R_g^{SU} y_{g1}, \forall g,$$

Where P_{g0} is the power output of generating unit g just prior to the first period of the planning horizon.

So, we can write the ramp up limit constraints for Unit 1 and Unit 2 as below…

	$p_{11} - P_{10} \leq R_1^U U_{10} + R_1^{SU} y_{11}$	Time Period-1
Unit-1	$p_{12} - p_{11} \leq R_1^U u_{11} + R_1^{SU} y_{12}$	Time Period-2
	$p_{13} - p_{12} \leq R_1^U u_{12} + R_1^{SU} y_{13}$	Time Period-3
	$p_{14} - p_{13} \leq R_1^U u_{13} + R_1^{SU} y_{14}$	Time Period-4

	$p_{11} - 0.7 \le 0.2 + 0.2 * y_{11}$	Time Period-1
Unit-1	$p_{12} - p_{11} \le 0.2 * u_{11} + 0.2 * y_{12}$	Time Period-2
	$p_{13} - p_{12} \le 0.2 * u_{12} + 0.2 * y_{13}$	Time Period-3
	$p_{14} - p_{13} \le 0.2 * u_{13} + 0.2 * y_{14}$	Time Period-4

	$p_{21} - P_{20} \le R_2^U U_{20} + R_2^{SU} y_{21}$	Time Period-1
Unit-2	$p_{22} - p_{21} \le R_2^U u_{21} + R_2^{SU} y_{22}$	Time Period-2
	$p_{23} - p_{22} \le R_2^U u_{22} + R_2^{SU} y_{23}$	Time Period-3
	$p_{24} - p_{23} \le R_2^U u_{23} + R_2^{SU} y_{24}$	Time Period-4

	$p_{21} - 0.6 \le 0.3 + 0.3 * y_{21}$	Time Period-1
Unit-2	$p_{22} - p_{21} \le 0.3 * u_{21} + 0.3 * y_{22}$	Time Period-2
	$p_{23} - p_{22} \le 0.3 * u_{22} + 0.3 * y_{23}$	Time Period-3
	$p_{24} - p_{23} \le 0.3 * u_{23} + 0.3 * y_{24}$	Time Period-4

Ramp Down Limit:

Similarly, any thermal generating unit cannot decrease its power output above a limit, which is called the ramping-down limit.

$$p_{g,t-1} - p_{gt} \le R_g^D u_{gt} + R_g^{SD} z_{gt}, \forall g, \forall t$$

Where:

R_g^D is the ramping-down limit of generating unit g in $puMW$ and

R_g^{SD} is the shut-down ramping limit of generating unit g in $puMW$.

For the first time period of the planning horizon, these constraints become:

$$P_{g0} - p_{g1} \le R_g^D u_{g1} + R_g^{SD} z_{g1}, \forall g.$$

So, we can write the ramp down limit constraints for Unit 1 and Unit 2 as below…

	$P_{10} - p_{11} \le R_1^D u_{11} + R_1^{SD} z_{11}$	Time Period-1
Unit-1	$p_{11} - p_{12} \le R_1^D u_{12} + R_1^{SD} z_{12}$	Time Period-2
	$p_{12} - p_{13} \le R_1^D u_{13} + R_1^{SD} z_{13}$	Time Period-3
	$p_{13} - p_{14} \le R_1^D u_{14} + R_1^{SD} z_{14}$	Time Period-4

	$0.7 - p_{11} \le 0.2 * u_{11} + 0.2 * z_{11}$	Time Period-1
Unit-1	$p_{11} - p_{12} \le 0.2 * u_{12} + 0.2 * z_{12}$	Time Period-2
	$p_{12} - p_{13} \le 0.2 * u_{13} + 0.2 * z_{13}$	Time Period-3
	$p_{13} - p_{14} \le 0.2 * u_{14} + 0.2 * z_{14}$	Time Period-4

Unit-2

$$P_{20} - p_{21} \le R_2^D u_{21} + R_2^{SD} z_{21} \qquad \text{Time Period-1}$$

$$p_{21} - p_{22} \le R_2^D u_{22} + R_2^{SD} z_{22} \qquad \text{Time Period-2}$$

$$p_{22} - p_{23} \le R_2^D u_{23} + R_2^{SD} z_{23} \qquad \text{Time Period-3}$$

$$p_{23} - p_{24} \le R_2^D u_{24} + R_2^{SD} z_{24} \qquad \text{Time Period-4}$$

Unit-2

$$0.6 - p_{21} \le 0.3 * u_{21} + 0.3 * z_{21} \qquad \text{Time Period-1}$$

$$p_{21} - p_{22} \le 0.3 * u_{22} + 0.3 * z_{22} \qquad \text{Time Period-2}$$

$$p_{22} - p_{23} \le 0.3 * u_{23} + 0.3 * z_{23} \qquad \text{Time Period-3}$$

$$p_{23} - p_{24} \le 0.3 * u_{24} + 0.3 * z_{24} \qquad \text{Time Period-4}$$

Power Balance:

The available generating units are used to satisfy the demand at each time period:

$$\sum_g p_{gt} = P_t^D, \qquad \forall t,$$

Where P_t^D is the demand$(in\ puMW)$ in time period t.

Active power balance per node:

$$\sum_{g \in \Omega_n^G} p_{gt} - \sum_{j \in \Omega_n^D} D_{jt} = \sum_{m \in \Lambda_n} B_{nm} * (\delta_{nt} - \delta_{mt}), \qquad \forall n, \forall t$$

D_j is the active power load demand j,

Ω_n^G is the set of generating units located at node n in the time period t, and

Ω_n^D is the set of demands located at node n in the time period t.

For node-1:

$$p_{11} = B_{13} * (\delta_{11} - \delta_{31}) \qquad \text{Time period 1}$$

$$p_{12} = B_{13} * (\delta_{12} - \delta_{32}) \qquad \text{Time period 2}$$

$$p_{13} = B_{13} * (\delta_{13} - \delta_{33}) \qquad \text{Time period 3}$$

$$p_{14} = B_{13} * (\delta_{14} - \delta_{34}) \qquad \text{Time period 4}$$

For node-2:

$$p_{21} = B_{23} * (\delta_{21} - \delta_{31}) \qquad \text{Time period 1}$$

$$p_{22} = B_{23} * (\delta_{22} - \delta_{32}) \qquad \text{Time period 2}$$

$$p_{23} = B_{23} * (\delta_{23} - \delta_{33}) \qquad \text{Time period 3}$$

$$p_{24} = B_{23} * (\delta_{24} - \delta_{34}) \qquad \text{Time period 4}$$

For node-3:

$$D_1 = 1 = B_{13} * (\delta_{11} - \delta_{31}) + B_{23} * (\delta_{21} - \delta_{31}) \qquad \text{Time period 1}$$

$$D_2 = 0.9 = B_{13} * (\delta_{12} - \delta_{32}) + B_{23} * (\delta_{22} - \delta_{32}) \qquad \text{Time period 2}$$

$$D_3 = 1.3 = B_{13} * (\delta_{13} - \delta_{33}) + B_{23} * (\delta_{23} - \delta_{33}) \qquad \text{Time period 3}$$

$$D_4 = 1.5 = B_{13} * (\delta_{14} - \delta_{34}) + B_{23} * (\delta_{24} - \delta_{34}) \qquad \text{Time period 4}$$

Where $\delta_{31} = \delta_{32} = \delta_{33} = \delta_{34} = 0$. (Node 3 given as reference node)

Transmission line loading capacity:

Any line should be operated below its transmission capacity limit:

$$-P_{nm}^{max} \leq B_{nm} * (\delta_n - \delta_m) \leq P_{nm}^{max}, \ \forall n, \forall m \in \Lambda_n,$$

Where P_{nm}^{max} is the transmission capacity of transmission line $n - m \ in \ puMW$.

Transmission line loading capacity for $Line_{1-3}$ can be written as…

$$-1 \leq B_{13} * (\delta_{11} - \delta_{31}) \leq 1 \qquad \text{Time period-1}$$

$$-1 \leq B_{13} * (\delta_{12} - \delta_{32}) \leq 1 \qquad \text{Time period-2}$$

$$-1 \leq B_{13} * (\delta_{13} - \delta_{33}) \leq 1 \qquad \text{Time period-3}$$

$$-1 \leq B_{13} * (\delta_{14} - \delta_{34}) \leq 1 \qquad \text{Time period-4}$$

Transmission line loading capacity for $Line_{2-3}$ can be written as…

$$-1 \leq B_{23} * (\delta_{21} - \delta_{31}) \leq 1 \qquad \text{Time period-1}$$

$$-1 \leq B_{23} * (\delta_{22} - \delta_{32}) \leq 1 \qquad \text{Time period-2}$$

$$-1 \leq B_{23} * (\delta_{23} - \delta_{33}) \leq 1 \qquad \text{Time period-3}$$

$$-1 \leq B_{23} * (\delta_{24} - \delta_{34}) \leq 1 \qquad \text{Time period-4}$$

Reserve capacity:

Finally, for security reasons, the total output power available on-line should be larger than the actual demand by a pre-specified amount:

$$\sum_g P_g^{max} u_{gt} \geq P_t^D + R_t^D, \qquad \forall t,$$

Where R_t^D is the amount of required reserve (capacity available over the demand) in time period t.

Here, in our Unit commitment problem, it is given to keep 10% margin of total demand on total generation.

So, the reserve capacity constraints for Unit 1 and Unit 2 can be written as below…

$$P_1^{max} u_{11} + P_2^{max} u_{21} \geq P_1^D + R_1^D = P_1^D + 10\% * P_1^D \qquad \text{Time Period-1}$$

$$P_1^{max} u_{12} + P_2^{max} u_{22} \geq P_2^D + R_2^D = P_2^D + 10\% * P_2^D \qquad \text{Time Period-2}$$

$$P_1^{max} u_{13} + P_2^{max} u_{23} \geq P_3^D + R_3^D = P_3^D + 10\% * P_3^D \qquad \text{Time Period-3}$$

$$P_1^{max} u_{14} + P_2^{max} u_{24} \geq P_4^D + R_4^D = P_4^D + 10\% * P_4^D \qquad \text{Time Period-4}$$

$$1.5 * u_{11} + 1.5 * u_{21} \geq 1 + 0.1 \qquad \text{Time Period-1}$$

$$1.5 * u_{12} + 1.5 * u_{22} \geq 0.9 + 0.09 \qquad \text{Time Period-2}$$

$$1.5 * u_{13} + 1.5 * u_{23} \geq 1.3 + 0.13 \qquad \text{Time Period-3}$$

$$1.5 * u_{14} + 1.5 * u_{24} \geq 1.5 + 0.15 \qquad \text{Time Period-4}$$

(c) Implement the formulation in a computer simulation code and find the optimal solution. No generic code. The code must be specific to the specific equations that you developed for this network in the previous part. Use comments to associate each line in your code to the formulation. List the optimal values of all decision variables and all auxiliary variables in two separate well-organized/labelled tables. Do not forget the units. (50 marks)

```
#Pyomo objects exist within the pyomo.environ namespace
#Every Pyomo model starts with this; it tells Python to Load the Pyomo Modeling Environment
from pyomo.environ import *
from pyomo.opt import SolverFactory

#Create an instance of a Concrete model
m = ConcreteModel("Question-(c)")

#Defining desicion variables and limit their values.

# p1[1,j] is power output limit of unit 1 in j time period.
# p1[1,0] is the initial power output of unit 1 = 0.7 pu.
m.p10 = Var(bounds=(0.7,0.7))
m.p11 = Var(bounds=(0.0,1.5))
m.p12 = Var(bounds=(0.0,1.5))
m.p13 = Var(bounds=(0.0,1.5))
m.p14 = Var(bounds=(0.0,1.5))

# p2[1,j] is power output limit of unit 2 in j time period.
# p2[2,0] is the initial power output of unit 2 = 0.6 pu.
m.p20 = Var(bounds=(0.6,0.6))
m.p21 = Var(bounds=(0.0,1.5))
m.p22 = Var(bounds=(0.0,1.5))
m.p23 = Var(bounds=(0.0,1.5))
m.p24 = Var(bounds=(0.0,1.5))

# u[i,j] is a binary variable that indicate that unit i is either online or offline in j time period.
# u10 = 1 indicate that unit 1 is online before the start of the time period.
m.u10= Var(bounds=(1,1))
m.u11 = Var(within=Binary)
m.u12 = Var(within=Binary)
m.u13 = Var(within=Binary)
m.u14 = Var(within=Binary)
m.u20= Var(bounds=(1,1))
m.u21 = Var(within=Binary)
m.u22 = Var(within=Binary)
m.u23 = Var(within=Binary)
m.u24 = Var(within=Binary)

# y[i,j] is a binary variable that indicate that unit i is started up or not in j time period.
m.y11 = Var(within=Binary)
m.y12 = Var(within=Binary)
m.y13 = Var(within=Binary)
m.y14 = Var(within=Binary)

m.y21 = Var(within=Binary)
m.y22 = Var(within=Binary)
m.y23 = Var(within=Binary)
m.y24 = Var(within=Binary)

# z[i,j] is a binary variable that indicate that unit i is shut down or not in j time period.
m.z11 = Var(within=Binary)
m.z12 = Var(within=Binary)
m.z13 = Var(within=Binary)
m.z14 = Var(within=Binary)

m.z21 = Var(within=Binary)
m.z22 = Var(within=Binary)
m.z23 = Var(within=Binary)
m.z24 = Var(within=Binary)
```

```
# Define voltage angle for node 1 and node 2. For node 3, voltage angle is given as 0 reference value.
m.δ11 = Var(bounds=(-3.14,3.14))
m.δ12 = Var(bounds=(-3.14,3.14))
m.δ13 = Var(bounds=(-3.14,3.14))
m.δ14 = Var(bounds=(-3.14,3.14))

m.δ21 = Var(bounds=(-3.14,3.14))
m.δ22 = Var(bounds=(-3.14,3.14))
m.δ23 = Var(bounds=(-3.14,3.14))
m.δ24 = Var(bounds=(-3.14,3.14))
```

```
# Define B[i,j] susceptance of transmission line between node i and j.
m.B13 = Var(bounds=(1,1))
m.B23 = Var(bounds=(1,1))

# D is for load demand at node 2 for 4 different time period frame.
m.D1 = Var(bounds=(1,1))
m.D2 = Var(bounds=(0.9,0.9))
m.D3 = Var(bounds=(1.3,1.3))
m.D4 = Var(bounds=(1.5,1.5))

# Defining an objective function to minimize the total cost for Unit 1 and Unit 2.
m.objective = Objective(expr = 3*(m.u11 + m.u12 + m.u13 + m.u14)\
                           +1*(m.u21 + m.u22 + m.u23 + m.u24)\
                           +10*(m.p11 + m.p12 + m.p13 + m.p14)\
                           +20*(m.p21 + m.p22 + m.p23 + m.p24)\
                           +5*(m.y11 + m.y12 + m.y13 + m.y14)\
                           +2*(m.y21 + m.y22 + m.y23 + m.y24)\
                           +(m.z11 + m.z12 + m.z13 + m.z14)\
                           +(m.z21 + m.z22 + m.z23 + m.z24))

### List of all constraints###

#Below statement indicate the following LOGICAL constraints for Unit-1
m.constraint1 = Constraint(expr = m.y11 - m.z11 == m.u11 - m.u10)   # 1. y11 - z11 = u11 - U10     --->Time period 1
m.constraint2 = Constraint(expr = m.y12 - m.z12 == m.u12 - m.u11)   # 2. y12 - z12 = u12 - u11     --->Time period 2
m.constraint3 = Constraint(expr = m.y13 - m.z13 == m.u13 - m.u12)   # 3. y13 - z13 = u13 - u12     --->Time period 3
m.constraint4 = Constraint(expr = m.y14 - m.z14 == m.u14 - m.u13)   # 4. y14 - z14 = u14 - u13     --->Time period 4

#Below statement indicate the following LOGICAL constraints for Unit-2
m.constraint5 = Constraint(expr = m.y21 - m.z21 == m.u21 - m.u20)   # 5. y21 - z21 = u21 - U20     --->Time period 1
m.constraint6 = Constraint(expr = m.y22 - m.z22 == m.u22 - m.u21)   # 6. y22 - z22 = u22 - u21     --->Time period 2
m.constraint7 = Constraint(expr = m.y23 - m.z23 == m.u23 - m.u22)   # 7. y23 - z23 = u23 - u22     --->Time period 3
m.constraint8 = Constraint(expr = m.y24 - m.z24 == m.u24 - m.u23)   # 8. y24 - z24 = u24 - u23     --->Time period 4
```

```
#Below statement indicate the following LOGICAL constraints for Unit-1
m.constraint9 = Constraint(expr = m.y11 + m.z11 <= 1)    # 09. y11 + z11 <= 1     --->Time period 1
m.constraint10 = Constraint(expr = m.y12 + m.z12 <= 1)   # 10. y12 + z12 <= 1     --->Time period 2
m.constraint11 = Constraint(expr = m.y13 + m.z13 <= 1)   # 11. y13 + z13 <= 1     --->Time period 3
m.constraint12 = Constraint(expr = m.y14 + m.z14 <= 1)   # 12. y14 + z14 <= 1     --->Time period 4

#Below statement indicate the following LOGICAL constraints for Unit-2
m.constraint13 = Constraint(expr = m.y21 + m.z21 <= 1)   # 13. y21 + z21 <= 1     --->Time period 1
m.constraint14 = Constraint(expr = m.y22 + m.z22 <= 1)   # 14. y22 + z22 <= 1     --->Time period 2
m.constraint15 = Constraint(expr = m.y23 + m.z23 <= 1)   # 15. y23 + z23 <= 1     --->Time period 3
m.constraint16 = Constraint(expr = m.y24 + m.z24 <= 1)   # 16. y24 + z24 <= 1     --->Time period 4

#Below statements indicate the following POWER BOUND LIMITS for unit-1
# 17. 0.15*u11 <= p11 <= 1.5*u11    --->Time period 1
# 18. 0.15*u12 <= p12 <= 1.5*u12    --->Time period 2
# 19. 0.15*u13 <= p13 <= 1.5*u13    --->Time period 3
# 20. 0.15*u14 <= p14 <= 1.5*u14    --->Time period 4
m.constraint17_0 = Constraint(expr = 0.15*m.u11 <= m.p11)
m.constraint18_0 = Constraint(expr = 0.15*m.u12 <= m.p12)
m.constraint19_0 = Constraint(expr = 0.15*m.u13 <= m.p13)
m.constraint20_0 = Constraint(expr = 0.15*m.u14 <= m.p14)
m.constraint17_1 = Constraint(expr = 1.5*m.u11 >= m.p11)
m.constraint18_1 = Constraint(expr = 1.5*m.u12 >= m.p12)
m.constraint19_1 = Constraint(expr = 1.5*m.u13 >= m.p13)
m.constraint20_1 = Constraint(expr = 1.5*m.u14 >= m.p14)
```

```
#Below statements indicate the following POWER BOUND LIMITS for unit-2
# 21. 0.15*u11 <= p11 <= 1.5*u11    --->Time period 1
# 22. 0.15*u12 <= p12 <= 1.5*u12    --->Time period 2
# 23. 0.15*u13 <= p13 <= 1.5*u13    --->Time period 3
# 24. 0.15*u14 <= p14 <= 1.5*u14    --->Time period 4
m.constraint21_0 = Constraint(expr = 0.2*m.u21 <= m.p21)
m.constraint22_0 = Constraint(expr = 0.2*m.u22 <= m.p22)
m.constraint23_0 = Constraint(expr = 0.2*m.u23 <= m.p23)
m.constraint24_0 = Constraint(expr = 0.2*m.u24 <= m.p24)
m.constraint21_1 = Constraint(expr = 1.5*m.u21 >= m.p21)
m.constraint22_1 = Constraint(expr = 1.5*m.u22 >= m.p22)
m.constraint23_1 = Constraint(expr = 1.5*m.u23 >= m.p23)
m.constraint24_1 = Constraint(expr = 1.5*m.u24 >= m.p24)

#Below statements indicates the following RAMPING UP LIMITs for Unit-1
m.constraint25=Constraint(expr=m.p11-m.p10 <= 0.2*m.u10 + 0.2*m.y11) # 25. p11 - 0.7 <= 0.2 + 0.2*y11      --->Time period 1
m.constraint26=Constraint(expr=m.p12-m.p11 <= 0.2*m.u11 + 0.2*m.y12) # 26. p12 - p11 <= 0.2*u11 + 0.2*y12  --->Time period 2
m.constraint27=Constraint(expr=m.p13-m.p12 <= 0.2*m.u12 + 0.2*m.y13) # 27. p13 - p12 <= 0.2*u12 + 0.2*y13  --->Time period 3
m.constraint28=Constraint(expr=m.p14-m.p13 <= 0.2*m.u13 + 0.2*m.y14) # 28. p14 - p13 <= 0.2*u13 + 0.2*y14  --->Time period 4

#Below statements indicates the following RAMPING UP LIMITs for Unit-2
m.constraint29=Constraint(expr=m.p21-m.p20 <= 0.3*m.u20 + 0.3*m.y21) # 29. p21 - 0.6 <= 0.3 + 0.3*y21       --->Time period 1
m.constraint30=Constraint(expr=m.p22-m.p21 <= 0.3*m.u21 + 0.3*m.y22) # 30. p22 - p21 <= 0.3*u21 + 0.3*y22   --->Time period 2
m.constraint31=Constraint(expr=m.p23-m.p22 <= 0.3*m.u22 + 0.3*m.y23) # 31. p23 - p22 <= 0.3*u22 + 0.3*y23   --->Time period 3
m.constraint32=Constraint(expr=m.p24-m.p23 <= 0.3*m.u23 + 0.3*m.y24) # 32. p24 - p23 <= 0.3*u23 + 0.3*y24   --->Time period 4

#Below statements indicates the following RAMP DOWN LIMITs for Unit-1
m.constraint33=Constraint(expr=m.p10-m.p11 <= 0.2*m.u11 + 0.2*m.z11) # 33. 0.7 - p11 <= 0.2*u11 + 0.2*z11   --->Time period 1
m.constraint34=Constraint(expr=m.p11-m.p12 <= 0.2*m.u12 + 0.2*m.z12) # 34. p11 - p12 <= 0.2*u12 + 0.2*z12   --->Time period 2
m.constraint35=Constraint(expr=m.p12-m.p13 <= 0.2*m.u13 + 0.2*m.z13) # 35. p12 - p13 <= 0.2*u13 + 0.2*y13   --->Time period 3
m.constraint36=Constraint(expr=m.p13-m.p14 <= 0.2*m.u14 + 0.2*m.z14) # 36. p13 - p14 <= 0.2*u14 + 0.2*y14   --->Time period 4

#Below statements indicates the following RAMP DOWN LIMITs for Unit-2
m.constraint37=Constraint(expr=m.p20 - m.p21 <= 0.3*m.u21 + 0.3*m.z21) # 37. 0.6 - p21 <= 0.3*u21 + 0.3*z21   --->Time period 1
m.constraint38=Constraint(expr=m.p21 - m.p22 <= 0.3*m.u22 + 0.3*m.z22) # 38. p21 - p22 <= 0.3*u22 + 0.3*z22   --->Time period 2
m.constraint39=Constraint(expr=m.p22 - m.p23 <= 0.3*m.u23 + 0.3*m.z23) # 39. p22 - p23 <= 0.3*u23 + 0.3*z23   --->Time period 3
m.constraint40=Constraint(expr=m.p23 - m.p24 <= 0.3*m.u24 + 0.3*m.z24) # 40. p23 - p24 <= 0.3*u24 + 0.3*z24   --->Time period 4
```

```
#Below statements indicates the following ACTIVE POWER BALANCE AT NODE 1. Here δ3 = 0 for all time period.
m.constraint41 = Constraint(expr = m.p11 == m.B13*m.δ11) # 41. p11 = B13*(δ11 - δ31)    --->Time period 1
m.constraint42 = Constraint(expr = m.p12 == m.B13*m.δ12) # 42. p12 = B13*(δ12 - δ32)    --->Time period 2
m.constraint43 = Constraint(expr = m.p13 == m.B13*m.δ13) # 43. p13 = B13*(δ13 - δ33)    --->Time period 3
m.constraint44 = Constraint(expr = m.p14 == m.B13*m.δ14) # 44. p14 = B13*(δ14 - δ34)    --->Time period 4

#Below statements indicates the following ACTIVE POWER BALANCE AT NODE 2. Here δ3 = 0 for all time period.
m.constraint45 = Constraint(expr = m.p21 == m.B23*m.δ21) # 45. p21 = B23*(δ21 - δ31)    --->Time period 1
m.constraint46 = Constraint(expr = m.p22 == m.B23*m.δ22) # 46. p22 = B23*(δ22 - δ32)    --->Time period 2
m.constraint47 = Constraint(expr = m.p23 == m.B23*m.δ23) # 47. p23 = B23*(δ23 - δ33)    --->Time period 3
m.constraint48 = Constraint(expr = m.p24 == m.B23*m.δ24) # 48. p24 = B23*(δ24 - δ34)    --->Time period 4

#Below statements indicates the following TRANSMISSION LINE LOADING CAPACITY constraints for line 1-3.
# 49. -1 <= B13*(δ11-δ31) <= 1    --->Time period 1
# 50. -1 <= B13*(δ12-δ32) <= 1    --->Time period 2
# 51. -1 <= B13*(δ13-δ33) <= 1    --->Time period 3
# 52. -1 <= B13*(δ14-δ34) <= 1    --->Time period 4
m.constraint49_0 = Constraint(expr = -1 <= m.B13*m.δ11)
m.constraint50_0 = Constraint(expr = -1 <= m.B13*m.δ12)
m.constraint51_0 = Constraint(expr = -1 <= m.B13*m.δ13)
m.constraint52_0 = Constraint(expr = -1 <= m.B13*m.δ14)
m.constraint49_1 = Constraint(expr = 1 >= m.B13*m.δ11)
m.constraint50_1 = Constraint(expr = 1 >= m.B13*m.δ12)
m.constraint51_1 = Constraint(expr = 1 >= m.B13*m.δ13)
m.constraint52_1 = Constraint(expr = 1 >= m.B13*m.δ14)
```

```
#Below statements indicate the following TRANSMISSION LINE LOADING CAPACITY constraints for line 2-3.
# 53. -1 <= B23*(δ21-δ31) <= 1    --->Time period 1
# 54. -1 <= B23*(δ22-δ32) <= 1    --->Time period 2
# 55. -1 <= B23*(δ23-δ33) <= 1    --->Time period 3
# 56. -1 <= B23*(δ24-δ34) <= 1    --->Time period 4
m.constraint53_0 = Constraint(expr = -1 <= m.B23*m.δ21)
m.constraint54_0 = Constraint(expr = -1 <= m.B23*m.δ22)
m.constraint55_0 = Constraint(expr = -1 <= m.B23*m.δ23)
m.constraint56_0 = Constraint(expr = -1 <= m.B23*m.δ24)
m.constraint53_1 = Constraint(expr = 1 >= m.B23*m.δ21)
m.constraint54_1 = Constraint(expr = 1 >= m.B23*m.δ22)
m.constraint55_1 = Constraint(expr = 1 >= m.B23*m.δ23)
m.constraint56_1 = Constraint(expr = 1 >= m.B23*m.δ24)

#Below statement indicates the following Reserve capacity constraints.
m.constraint57 = Constraint(expr = 1.5*m.u11 + 1.5*m.u21 >= 1.1*m.D1) # 57. 1.5*u11 + 1.5*u21 >= 1 + 0.1      --->Time period 1
m.constraint58 = Constraint(expr = 1.5*m.u12 + 1.5*m.u22 >= 1.1*m.D2) # 58. 1.5*u12 + 1.5*u22 >= 0.9 + 0.09   --->Time period 2
m.constraint59 = Constraint(expr = 1.5*m.u13 + 1.5*m.u23 >= 1.1*m.D3) # 59. 1.5*u13 + 1.5*u23 >= 1.3 + 0.13   --->Time period 3
m.constraint60 = Constraint(expr = 1.5*m.u14 + 1.5*m.u24 >= 1.1*m.D4) # 60. 1.5*u14 + 1.5*u24 >= 1.5 + 0.15   --->Time period 4

#Below statement indicates the following Reserve capacity constraints.
m.constraint61 = Constraint(expr = m.D1 == m.B13*m.δ11 + m.B23*m.δ21)  # 61. D1 = B13*δ11 + B23*δ21   --->Time period 1
m.constraint62 = Constraint(expr = m.D2 == m.B13*m.δ12 + m.B23*m.δ22)  # 62. D2 = B13*δ12 + B23*δ22   --->Time period 2
m.constraint63 = Constraint(expr = m.D3 == m.B13*m.δ13 + m.B23*m.δ23)  # 63. D3 = B13*δ13 + B23*δ23   --->Time period 3
m.constraint64 = Constraint(expr = m.D4 == m.B13*m.δ14 + m.B23*m.δ24)  # 64. D4 = B13*δ14 + B23*δ24   --->Time period 4

#Solving models
opt = SolverFactory('gurobi')
opt.solve(m)

#Display the result
m.display()
```

```
Model 'Question-(c)'

  Variables:
    p10 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.7 :   0.7 :   0.7 : False : False :  Reals
    p11 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.7000000000000008 :   1.5 : False : False :  Reals
    p12 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.9 :   1.5 : False : False :  Reals
    p13 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   1.0 :   1.5 : False : False :  Reals
    p14 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   1.0 :   1.5 : False : False :  Reals
    p20 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.6 :   0.6 :   0.6 : False : False :  Reals
    p21 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.2999999999999991 :   1.5 : False : False :  Reals
    p22 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.0 :   1.5 : False : False :  Reals
    p23 : Size=1, Index=None
        Key  : Lower : Value               : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.30000000000000004 :   1.5 : False : False :  Reals
    p24 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.5 :   1.5 : False : False :  Reals
```

```
u10 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     1 :   1.0 :     1 : False : False :  Reals
u11 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
u12 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
u13 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
u14 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
u20 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     1 :   1.0 :     1 : False : False :  Reals
u21 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
u22 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
u23 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
u24 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
y11 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
y12 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
y13 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
y14 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
y21 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
y22 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
y23 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
y24 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
z11 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
z12 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
```

```
z13 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
z14 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
z21 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
z22 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   1.0 :     1 : False : False : Binary
z23 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
z24 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     1 : False : False : Binary
δ11 : Size=1, Index=None
    Key  : Lower : Value              : Upper : Fixed : Stale : Domain
    None : -3.14 : 0.7000000000000008 :  3.14 : False : False :  Reals
δ12 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None : -3.14 :   0.9 :  3.14 : False : False :  Reals
δ13 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None : -3.14 :   1.0 :  3.14 : False : False :  Reals
δ14 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None : -3.14 :   1.0 :  3.14 : False : False :  Reals
δ21 : Size=1, Index=None
    Key  : Lower : Value              : Upper : Fixed : Stale : Domain
    None : -3.14 : 0.2999999999999991 :  3.14 : False : False :  Reals
δ22 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None : -3.14 :   0.0 :  3.14 : False : False :  Reals
δ23 : Size=1, Index=None
    Key  : Lower : Value               : Upper : Fixed : Stale : Domain
    None : -3.14 : 0.30000000000000004 :  3.14 : False : False :  Reals
δ24 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None : -3.14 :   0.5 :  3.14 : False : False :  Reals
B13 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     1 :   1.0 :     1 : False : False :  Reals
B23 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     1 :   1.0 :     1 : False : False :  Reals
D1 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     1 :   1.0 :     1 : False : False :  Reals
D2 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :   0.9 :   0.9 :   0.9 : False : False :  Reals
D3 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :   1.3 :   1.3 :   1.3 : False : False :  Reals
D4 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :   1.5 :   1.5 :   1.5 : False : False :  Reals
```

```
Objectives:
  objective : Size=1, Index=None, Active=True
      Key  : Active : Value
      None :   True :  76.0

Constraints:
  constraint1 : Size=1
      Key  : Lower : Body : Upper
      None :   0.0 :  0.0 :   0.0
  constraint2 : Size=1
      Key  : Lower : Body : Upper
      None :   0.0 :  0.0 :   0.0
  constraint3 : Size=1
      Key  : Lower : Body : Upper
      None :   0.0 :  0.0 :   0.0
  constraint4 : Size=1
      Key  : Lower : Body : Upper
      None :   0.0 :  0.0 :   0.0
  constraint5 : Size=1
      Key  : Lower : Body : Upper
      None :   0.0 :  0.0 :   0.0
  constraint6 : Size=1
      Key  : Lower : Body : Upper
      None :   0.0 :  0.0 :   0.0
  constraint7 : Size=1
      Key  : Lower : Body : Upper
      None :   0.0 :  0.0 :   0.0
  constraint8 : Size=1
      Key  : Lower : Body : Upper
      None :   0.0 :  0.0 :   0.0
  constraint9 : Size=1
      Key  : Lower : Body : Upper
      None :  None :  0.0 :   1.0
  constraint10 : Size=1
      Key  : Lower : Body : Upper
      None :  None :  0.0 :   1.0
  constraint11 : Size=1
      Key  : Lower : Body : Upper
      None :  None :  0.0 :   1.0
  constraint12 : Size=1
      Key  : Lower : Body : Upper
      None :  None :  0.0 :   1.0
  constraint13 : Size=1
      Key  : Lower : Body : Upper
      None :  None :  0.0 :   1.0
  constraint14 : Size=1
      Key  : Lower : Body : Upper
      None :  None :  1.0 :   1.0
  constraint15 : Size=1
      Key  : Lower : Body : Upper
      None :  None :  1.0 :   1.0
  constraint16 : Size=1
      Key  : Lower : Body : Upper
      None :  None :  0.0 :   1.0
  constraint17_0 : Size=1
      Key  : Lower : Body                : Upper
      None :  None : -0.5500000000000008 :   0.0
  constraint18_0 : Size=1
      Key  : Lower : Body  : Upper
      None :  None : -0.75 :   0.0
```

```
constraint19_0 : Size=1
    Key  : Lower : Body  : Upper
    None :  None : -0.85 :   0.0
constraint20_0 : Size=1
    Key  : Lower : Body  : Upper
    None :  None : -0.85 :   0.0
constraint17_1 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -0.7999999999999992 :   0.0
constraint18_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -0.6 :   0.0
constraint19_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -0.5 :   0.0
constraint20_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -0.5 :   0.0
constraint21_0 : Size=1
    Key  : Lower : Body                 : Upper
    None :  None : -0.09999999999999909 :   0.0
constraint22_0 : Size=1
    Key  : Lower : Body : Upper
    None :  None :  0.0 :   0.0
constraint23_0 : Size=1
    Key  : Lower : Body                 : Upper
    None :  None : -0.10000000000000003 :   0.0
constraint24_0 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -0.3 :   0.0
constraint21_1 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -1.2000000000000008 :   0.0
constraint22_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None :  0.0 :   0.0
constraint23_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -1.2 :   0.0
constraint24_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -1.0 :   0.0
constraint25 : Size=1
    Key  : Lower : Body                 : Upper
    None :  None : -0.19999999999999912 :   0.0
constraint26 : Size=1
    Key  : Lower : Body                   : Upper
    None :  None : -8.326672684688674e-16 :   0.0
constraint27 : Size=1
    Key  : Lower : Body                 : Upper
    None :  None : -0.10000000000000003 :   0.0
constraint28 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -0.2 :   0.0
constraint29 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -0.6000000000000009 :   0.0
constraint30 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -0.5999999999999991 :   0.0
```

```
constraint31 : Size=1
    Key  : Lower : Body                  : Upper
    None :  None : 5.551115123125783e-17 :   0.0
constraint32 : Size=1
    Key  : Lower : Body                 : Upper
    None :  None : -0.10000000000000003 :   0.0
constraint33 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -0.2000000000000009 :   0.0
constraint34 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -0.3999999999999992 :   0.0
constraint35 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -0.3 :   0.0
constraint36 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -0.2 :   0.0
constraint37 : Size=1
    Key  : Lower : Body                  : Upper
    None :  None : 8.881784197001252e-16 :   0.0
constraint38 : Size=1
    Key  : Lower : Body                   : Upper
    None :  None : -8.881784197001252e-16 :   0.0
constraint39 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -0.6000000000000001 :   0.0
constraint40 : Size=1
    Key  : Lower : Body                 : Upper
    None :  None : -0.49999999999999994 :   0.0
constraint41 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint42 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint43 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint44 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint45 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint46 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint47 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint48 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint49_0 : Size=1
    Key  : Lower : Body               : Upper
    None :  -1.0 : 0.7000000000000008 :  None
constraint50_0 : Size=1
    Key  : Lower : Body : Upper
    None :  -1.0 :  0.9 :  None
constraint51_0 : Size=1
    Key  : Lower : Body : Upper
    None :  -1.0 :  1.0 :  None
```

```
constraint52_0 : Size=1
    Key  : Lower : Body : Upper
    None :  -1.0 :  1.0 :  None
constraint49_1 : Size=1
    Key  : Lower : Body               : Upper
    None :  None : 0.7000000000000008 :   1.0
constraint50_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None :  0.9 :   1.0
constraint51_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None :  1.0 :   1.0
constraint52_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None :  1.0 :   1.0
constraint53_0 : Size=1
    Key  : Lower : Body               : Upper
    None :  -1.0 : 0.2999999999999991 :  None
constraint54_0 : Size=1
    Key  : Lower : Body : Upper
    None :  -1.0 :  0.0 :  None
constraint55_0 : Size=1
    Key  : Lower : Body                : Upper
    None :  -1.0 : 0.30000000000000004 :  None
constraint56_0 : Size=1
    Key  : Lower : Body : Upper
    None :  -1.0 :  0.5 :  None
constraint53_1 : Size=1
    Key  : Lower : Body               : Upper
    None :  None : 0.2999999999999991 :   1.0
constraint54_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None :  0.0 :   1.0
constraint55_1 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : 0.30000000000000004 :   1.0
constraint56_1 : Size=1
    Key  : Lower : Body : Upper
    None :  None :  0.5 :   1.0
constraint57 : Size=1
    Key  : Lower : Body : Upper
    None :  None : -1.9 :   0.0
constraint58 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -0.5099999999999999 :   0.0
constraint59 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -1.5699999999999998 :   0.0
constraint60 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -1.3499999999999999 :   0.0
constraint61 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint62 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint63 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
constraint64 : Size=1
    Key  : Lower : Body : Upper
    None :   0.0 :  0.0 :   0.0
```

Output Summary Table:

Decision Variables	Time Period-1		Time Period-2		Time Period-3		Time Period-4	
Unit-1 Output (puMW)	p_{11}	0.7	p_{12}	0.9	p_{13}	1	p_{14}	1
Unit-2 Output puMW)	p_{21}	0.3	p_{22}	0	p_{23}	0.3	p_{24}	0.5
Unit-1 running status binary variable	u_{11}	1	u_{12}	1	u_{13}	1	u_{14}	1
Unit-2 running status binary variable	u_{21}	1	u_{22}	0	u_{23}	1	u_{24}	1
Unit-1 start-up binary variable	y_{11}	0	y_{12}	0	y_{13}	0	y_{14}	0
Unit-2 start-up binary variable	y_{21}	0	y_{22}	0	y_{23}	1	y_{24}	0
Unit-1 shut down binary variable	z_{11}	0	z_{12}	0	z_{13}	0	z_{14}	0
Unit-2 shut down binary variable	z_{21}	0	z_{22}	1	z_{23}	0	z_{24}	0
Auxiliary Variables	**Time Period-1**		**Time Period-2**		**Time Period-3**		**Time Period-4**	
δ for node-1	δ_{11}	0.7	δ_{12}	0.9	δ_{13}	1	δ_{14}	1
δ for node-2	δ_{21}	0.3	δ_{22}	0	δ_{23}	0.3	δ_{24}	0.5
P13 line loading (puMW)	L_{11}	0.7	L_{12}	0.9	L_{13}	1	L_{14}	1
P23 line loading (puMW)	L_{21}	0.3	L_{22}	0	L_{23}	0.3	L_{24}	0.5
Parameter	**Time Period-1**		**Time Period-2**		**Time Period-3**		**Time Period-4**	
Demand (puMW)	P_1^D	1	P_2^D	0.9	P_3^D	1.3	P_4^D	1.5
Fixed Cost ($)	C_1^F	4	C_2^F	3	C_3^F	4	C_4^F	4
Variable cost ($/puMW)	C_1^V	13	C_2^V	9	C_3^V	16	C_4^V	20
Start-up cost ($)	C_1^{SU}	0	C_2^{SU}	0	C_3^{SU}	2	C_4^{SU}	0
Shut down cost ($)	C_1^{SD}	0	C_2^{SD}	1	C_3^{SD}	0	C_4^{SD}	0
Total cost ($)	76							

Table 10: Output Summary

(d) Suggest one investigation that you could be facing in your role as a power systems operation, planning or investment engineer that must use an UC model like this to make an operation, planning or investment decision. Describe the hypothetical situation and the key question that must be answered. Then propose an experiment using your model that can help you answer the key question. Then set up your problem with proper data from real-world to run your proposed experiment. Then run your simulations, and based on the findings, provide a recommendation or answer to the key question. Your investigation must involve renewable energy sources (wind or solar), and must use real-life daily historical load and renewable energy patterns with an hourly resolution. You may normalize the data and use in a way that supports your experiment. Typical examples of such investigations include, but not limited to, adding renewable sources to this network and the potential economic or technical challenges, investment or decommissioning decisions as more renewable resources are added to the system, transmission line investments need in the future as the load grows and renewable resources are added, potential wind/solar energy curtailment volumes and the associated economic impact on renewable investors etc. This is an open-ended question and there is no single answer. You must clearly justify your suggestions and approach. Make sure to cite the references that you use in your research for answering this question. Use the IEEE citation format. (100 marks)

Introduction: High wind penetration of renewable generation such as wind has posed great challenges to the power system operators in grid management and generation scheduling. Unlike other conventional and controllable generation sources, wind power and solar power is unpredictable and intermittent. The impact of large amounts of renewable sources has complicated implications to UC and ED.

Problem Formulation: A hybrid network that consists of two thermal units, Solar-park and a wind power generator is interconnected as shown below. Due to some technical problem, the variable cost of the Unit-2 has been increased to $20\$/puMW$ from $15\$/puMW$. Thus, it is forecasted to take planned shut-down of thermal Unit-2 to fix the technical problem for 2 hours without any compromising in total cost on 19^{th} March, 2022 at given time period frame (noon time) when mostly solar is available. On 17^{th} March, as an Engineer, I was asked to calculate the Unit commitment/Economic dispatch problem to forecast either Unit-2 can be shut down or not, considering all the technical constraints? If no, why? What measures we should consider while forecasting with renewable energy sources? Both the thermal units will run at $1.2\,puMW$ and $0.8\,puMW$ respectively before the time period frame. Specific time slot is allotted for maintenance as technical expert team is coming to look after the maintenance in that period. Geographic location is assumed from the Alberta province.

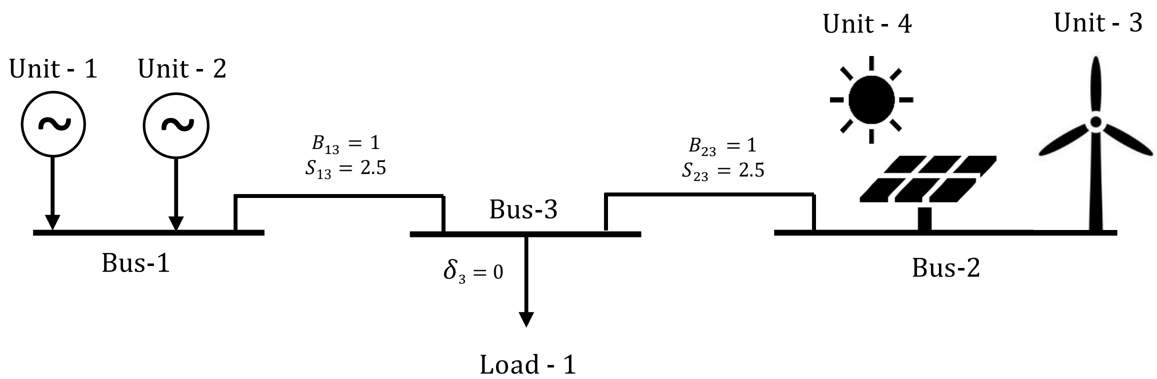


Figure 2: Power System Network

Given Data:

Parameter	Unit 1	Unit 2	Unit 3	Unit 4
Minimum Power Output (puMW)	0.2	0.3	0.0	0.0
Maximum Power Output (puMW)	1.4	1.4	2.2	0.8
Ramping – Down Limit (puMW/h)	0.2	0.3	NA	NA
Shut – Down Ramping Limit (puMW/h)	0.2	0.3	NA	NA
Ramping – Up Limit (puMW/h)	0.2	0.3	NA	NA
Start – Up Ramping Limit (puMW/h)	0.2	0.3	NA	NA
Fixed Cost ($)	3	1	0	0
Start – Up Cost ($)	5	2	0	0
Shut – Down Cost ($)	1	1	0	0
Variable Cost ($/puMW)	10	20	0	0

Table 11: Given Data [2] [9]

In our power system network, renewable energy poses great challenges in our network due to their unpredictability. In our investigation, wind forecast, solar forecast and system demand forecast taken from the Alberta electric system operator. For the simplicity, we have manipulated the real-life daily data into the per unit system for the simplicity. The availability of another thermal unit is 100% throughout all the time period [3].

- **Load forecast:** I have considered the time-span (Hourly) between 09:00 AM to 3:00 PM for the calculation. I have considered Alberta's total system demand and manipulated(rescale) it into the pu limits to formulate the problematic scenario, although, keeping the same realistic system demand variation. [1]

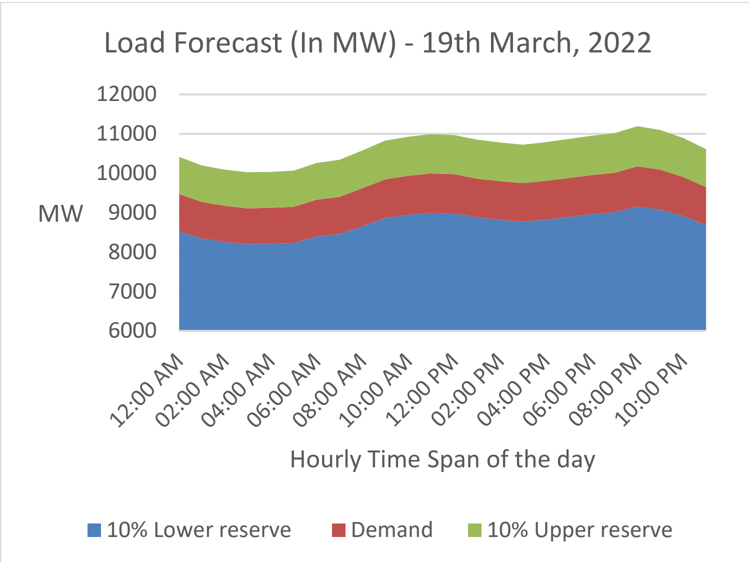


Figure – 3

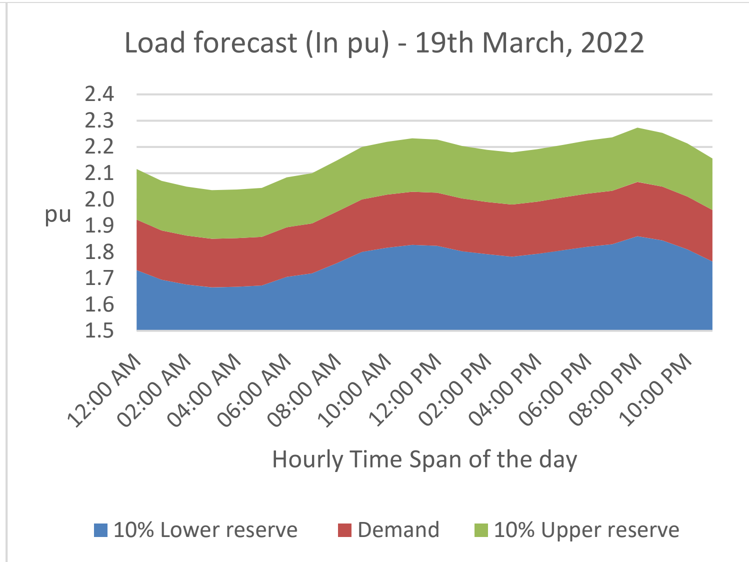


Figure - 4

- **Wind forecast:** I have considered the span (Hourly) between 09:00 AM to 3:00 PM for the calculation. I have taken Alberta's total wind power generation and manipulated it considering the wind farm maximum capacity ($2.2\ puMW$), although, keeping the realistic wind variation [4].

$$0 \leq P_{wind} \leq 2.2\ puMW$$

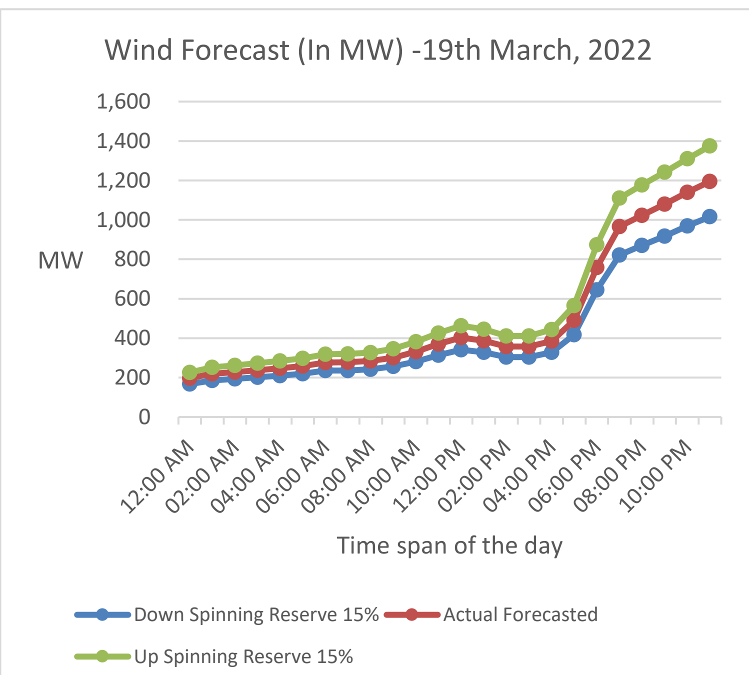


Figure – 5

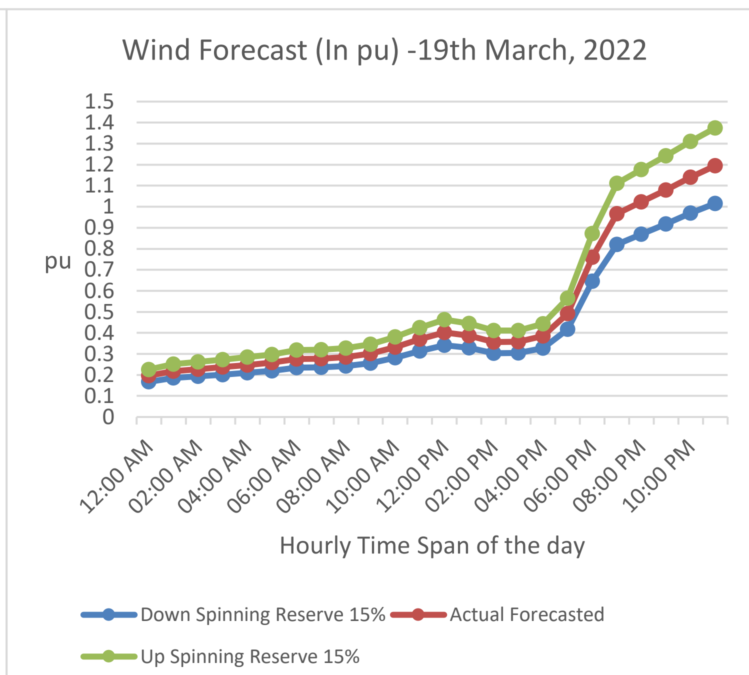


Figure - 6

- **Solar forecast:** We have considered the span (Hourly) between 09:00 AM to 3:00 PM for the calculation. I have taken Alberta's solar generation and manipulated the data into pu considering the wind farm maximum capacity ($0.8\ puMW$) although, keeping the realistic wind variation [4].

$$0 \leq P_{solar} \leq 0.8\ puMW$$

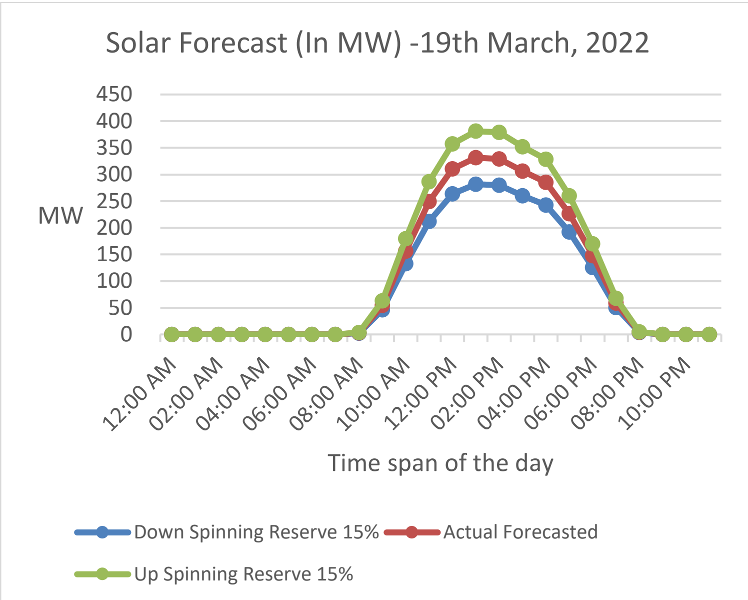


Figure – 7

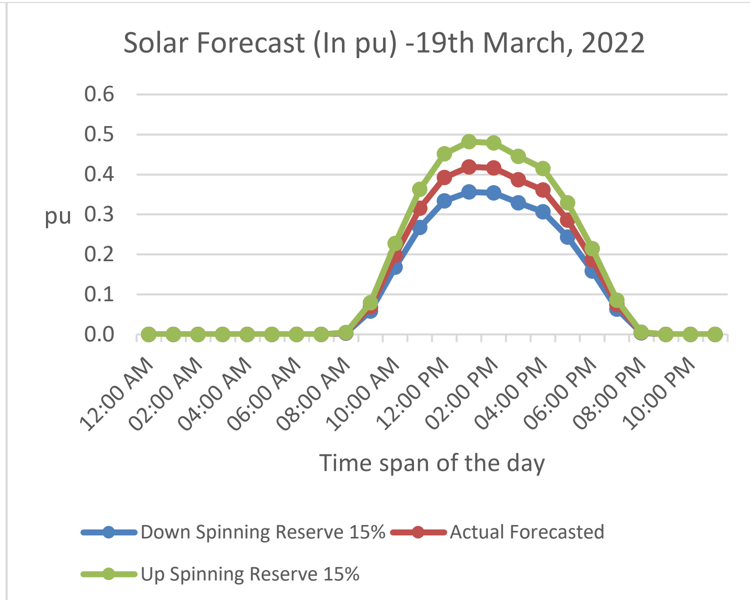


Figure – 8

Time-Period	**Wind Generation** $(In\ puMW)$	**Solar Generation** $(In\ puMW)$	**Total system Demand** $(In\ puMW)$
1	0.30	0.07	2.00
2	0.33	0.20	2.02
3	0.37	0.32	2.03
4	0.40	0.39	2.03
5	0.39	0.42	2.00
6	0.36	0.42	1.99

Table 12: Renewable energy source generation data

Following constraints have been taken into the consideration.

- No load shedding allowed in given time period.
- No wind generation curtailments allowed as it is the cheapest source of the supply.
- No solar generation curtailments allowed as it is the cheapest source of the supply.
- Logical constraint that online Unit cannot be started and vice-versa.
  - $y_{gt} - z_{gt} = u_{gt} - u_{g,t-1}, \quad \forall g, \forall t,$

    where, if $t = 1, \ y_{g1} - z_{g1} = u_{g1} - U_{g0}, \quad \forall g$
  - $y_{gt} + z_{gt} \leq 1, \quad \forall g, \forall t,$
  - $u_{gt}, y_{gt}, z_{gt} \in \{0,1\}, \quad \forall g, \forall t,$

Where, u_{gt} is a binary variable that is equal to 1 if generating unit g is online in time period t, and 0 otherwise.

Where, z_{gt} is a binary variable that is equal to 1 if generating unit g is shut down in time period t, and 0 otherwise.

Where, y_{gt} is a binary variable that is equal to 1 if generating unit g is started in time period t, and 0 otherwise.

- Power Bound limit of thermal generators.
  - $P_g^{min}\, u_{gt} \leq p_{gt} \leq P_g^{max}\, u_{gt}, \forall g, \forall t,$

Where, P_g^{min} is the minimum power output of generating unit g and

Where, P_g^{max} is the maximum capacity of generating unit g.

Where, p_{gt} is the power output of generating unit g in time period t.

- Ramp-up/Ramp-down and Unit start-up/shut-down ramp limit of thermal power stations.
  - $p_{gt} - p_{g,t-1} \leq R_g^U\, u_{g,t-1} + R_g^{SU}\, y_{gt}, \forall g, \forall t,$

    where, if t = 1, $p_{g1} - P_{g0} \leq R_g^U\, U_{g0} + R_g^{SU}\, y_{g1}, \forall g,$
  - $p_{g,t-1} - p_{gt} \leq R_g^D\, u_{gt} + R_g^{SD}\, z_{gt}, \forall g, \forall t$

    where, if = 1, $P_{g0} - p_{g1} \leq R_g^D\, u_{g1} + R_g^{SD}\, z_{g1}, \forall g.$

Where, R_g^U is the ramping-up limit of generating unit g and

Where, R_g^{SU} is the start-up ramping limit of generating unit g.

Where, R_g^D is the ramping-down limit of generating unit g and

Where, R_g^{SD} is the shut-down ramping limit of generating unit g.

- Transmission line loading capacity.
  - $-P_{nm}^{max} \leq B_{nm} * (\delta_n - \delta_m) \leq P_{nm}^{max}, \ \forall n, \forall m \in \wedge_n,$
- Power balance at each node.
  - $\sum_{g \in \Omega_n^G} p_{gt} - \sum_{j \in \Omega_n^D} D_{jt} = \sum_{m \in \wedge_n} B_{nm} * (\delta_{nt} - \delta_{mt}), \ \forall n, \forall t$

Where P_t^D is the demand in time period t.

- 10% spinning UP/DOWN reserve requirement without considering wind/solar power generation (In case system demand increase or decrease unexpected).
  - $\sum_g P_g^{max} u_{gt} + P_t^{wind} + P_t^{solar} \geq P_t^D + R_{t,UP}^D, \ \forall t,$
  - $\sum_g P_g^{min} u_{gt} + P_t^{wind} + P_t^{solar} \leq P_t^D - R_{t,DOWN}^D, \ \forall t,$

Where $R_{t,UP}^D = 10\%\, P_t^D$ is the amount of additional spinning up reserve requirement (capacity available over the total system demand) in time period t.

Where $R_{t,DOWN}^D = 10\%\, P_t^D$ is the amount of spinning down reserve requirement (margin to reduce the generation if system demand goes down unexpected) in time period t.

Where P_t^{wind} is the Power output of wind generator at time period t.

Where P_t^{solar} is the Power output of solar generator at time period t.

- Considering the Economic dispatch, our objective function would be.

$$Total\ cost\ c_{gt} = c_{gt}^F + c_{gt}^V + c_{gt}^{SU} + c_{gt}^{SD},\ \$ \quad \forall g, \forall t$$

$where:$

$c_{gt}\ is\ the\ total\ cost\ of\ unit\ g\ in\ time\ period\ t,$

$c_{gt}^F\ is\ the\ fixed\ cost\ of\ unit\ g\ in\ time\ period\ t,$

$c_{gt}^V\ is\ the\ variable\ cost\ of\ unit\ g\ in\ time\ period\ t,$

$c_{gt}^{SU}\ is\ the\ start-up\ cost\ of\ unit\ g\ in\ time\ period\ t, and$

c_{gt}^{SD} *is the shut − down cost of unit g in time period t*

I have simulated the problem same in question number 2 & 3 and got the result as shown below.

Output table of scenario-1												
Decision Variables	**Time Period-1**		**Time Period-2**		**Time Period-3**		**Time Period-4**		**Time Period-5**		**Time Period-6**	
Unit-1 Output ($puMW$)	p_{11}	1.129	p_{12}	1.188	p_{13}	1.045	p_{14}	1.231	p_{15}	1.197	p_{16}	1.217
Unit-2 Output ($puMW$)	p_{21}	0.499	p_{22}	0.299	p_{23}	0.299	p_{24}	0.0	p_{25}	0.0	p_{26}	0.0
Unit-3 Output ($puMW$)	p_{31}	0.301	p_{32}	0.331	p_{33}	0.369	p_{34}	0.402	p_{35}	0.386	p_{36}	0.356
Unit-4 Output ($puMW$)	p_{41}	0.069	p_{42}	0.197	p_{43}	0.315	p_{44}	0.392	p_{45}	0.419	p_{46}	0.416
Unit-1 running status binary variable	u_{11}	1	u_{12}	1	u_{13}	1	u_{14}	1	u_{15}	1	u_{16}	1
Unit-2 running status binary variable	u_{21}	1	u_{22}	1	u_{23}	1	u_{24}	0	u_{25}	0	u_{26}	0
Unit-1 start-up binary variable	y_{11}	0	y_{12}	0	y_{13}	0	y_{14}	0	y_{15}	0	y_{16}	0
Unit-2 start-up binary variable	y_{21}	0	y_{22}	0	y_{23}	0	y_{24}	0	y_{25}	0	y_{26}	0
Unit-1 shut down binary variable	z_{11}	0	z_{12}	0	z_{13}	0	z_{14}	0	z_{15}	0	z_{16}	0
Unit-2 shut down binary variable	z_{21}	0	z_{22}	0	z_{23}	0	z_{24}	1	z_{25}	0	z_{26}	0
Auxiliary Variables	**Time Period-1**		**Time Period-2**		**Time Period-3**		**Time Period-4**		**Time Period-5**		**Time Period-6**	
P13 line loading ($puMW$)	L_{11}	1.62	L_{12}	1.48	L_{13}	1.34	L_{14}	1.23	L_{15}	1.19	L_{16}	1.21
P23 line loading ($puMW$)	L_{21}	0.37	L_{22}	0.52	L_{23}	0.68	L_{24}	0.79	L_{25}	0.80	L_{26}	0.77
Demand ($puMW$)	P_1^D	1.99	P_2^D	2.01	P_1^D	2.02	P_1^D	2.02	P_1^D	2.00	P_6^D	1.99
Unit- 1 Fixed Cost ($)	C_1^F	3	C_1^F	3	C_1^F	3	C_1^F	3	C_1^F	3	C_1^F	3
Unit- 2 Fixed Cost ($)	C_2^F	1	C_2^F	1	C_2^F	1	C_2^F	0	C_2^F	0	C_2^F	0
Unit-1 Variable cost ($\$/puMW$)	C_1^V	11.2	C_1^V	11.8	C_1^V	10.45	C_1^V	12.31	C_1^V	11.97	C_1^V	12.17
Unit-2 Variable cost ($\$/puMW$)	C_2^V	10	C_2^V	6	C_2^V	6	C_2^V	0	C_2^V	0	C_2^V	0
Unit-1 Start-up cost ($)	C_1^{SU}	0	C_1^{SU}	0	C_1^{SU}	0	C_1^{SU}	0	C_1^{SU}	0	C_1^{SU}	0
Unit-1 Shut down cost ($)	C_1^{SD}	0	C_1^{SD}	0	C_1^{SD}	0	C_1^{SD}	0	C_1^{SD}	0	C_1^{SD}	0
Unit-2 Start-up cost ($)	C_2^{SU}	0	C_2^{SU}	0	C_2^{SU}	0	C_2^{SU}	0	C_2^{SU}	0	C_2^{SU}	0
Unit-2 Shut down cost ($)	C_2^{SD}	0	C_2^{SD}	0	C_2^{SD}	0	C_2^{SD}	1	C_2^{SD}	0	C_2^{SD}	0
Total cost ($)	114.09 $											

Table 13: Output Table Case-1

Result: Output result indicate that the Unit-2 (thermal) can be shut down during the given time period for maintenance. Here, our power system working with renewable energy sources and we have considered spinning reserve requirement only for change in forecasted demand by 10% up or 10% down then the predicted; however, as wind power is unpredictable and intermittent, we must consider the Up/Down spinning reserve requirement to the total system demand considering the wind power generators [5]. In addition, the forecasted solar power generation might be not accurate. In order to keep the margin at online thermal generators to compensate the unpredicted change in wind and solar power generation, we must consider up/down spinning reserve requirement considering the wind and solar energy sources.

Now, I run the above problem considering the addition constraints as shown below [5] [6].

- 15% spinning UP/DOWN reserve requirement considering wind power generation (In case wind generation increase or decrease unexpected).
- 15% spinning UP/DOWN reserve requirement considering solar power generation (In case solar generation increase or decrease unexpected).
  - $\sum_g P_g^{max} u_{gt} + P_t^{wind} + P_t^{solar} \geq P_t^D + R_{t,UP}^D + R_{t,UP}^{wind} + R_{t,UP}^{solar}, \ \forall t,$
  - $\sum_g P_g^{min} u_{gt} + P_t^{wind} + P_t^{solar} \leq P_t^D - R_{t,DOWN}^D - R_{t,DOWN}^{wind} - R_{t,DOWN}^{wind} \ \ \forall t,$

Where $R_{t,UP}^D = 10\% \, P_t^D$ is the amount of additional spinning up reserve requirement (capacity available over the total system demand) in time period t.

Where $R_{t,DOWN}^D = 10\% \, P_t^D$ is the amount of spinning down reserve requirement (margin to reduce the generation if system demand goes down unexpected) in time period t.

Where P_t^{wind} is the Power output of wind generator at time period t.

Where P_t^{solar} is the Power output of solar generator at time period t.

Where $R_{t,UP}^{wind} = 15\% P_t^{wind}$ is the amount of additional up reserve requirement considering the wind power generation in time period t. (In the case when wind generation suddenly reduce)

Where $R_{t,DOWN}^{wind} = 15\% P_t^{wind}$ is the amount of down reserve requirement considering the wind power generation in time period t. (In the case when wind generation suddenly increase)

Where $R_{t,UP}^{solar} = 15\% P_t^{solar}$ is the amount of additional up reserve requirement considering the solar power generation in time period t. (In the case when solar generation suddenly reduce)

Where $R_{t,DOWN}^{solar} = 15\% P_t^{solar}$ is the amount of down reserve requirement considering the solar power generation in time period t. (In the case when solar generation suddenly increase)

I have simulated the problem same in question number 2 & 3 and got the result as shown below.

Output table of scenario-2												
Decision Variables	**Time Period-1**		**Time Period-2**		**Time Period-3**		**Time Period-4**		**Time Period-5**		**Time Period-6**	
Unit-1 Output ($puMW$)	p_{11}	1.129	p_{12}	1.188	p_{13}	1.045	p_{14}	0.931	p_{15}	0.897	p_{16}	0.917
Unit-2 Output ($puMW$)	p_{21}	0.499	p_{22}	0.299	p_{23}	0.299	p_{24}	0.299	p_{25}	0.299	p_{26}	0.299
Unit-3 Output ($puMW$)	p_{31}	0.301	p_{32}	0.331	p_{33}	0.369	p_{34}	0.402	p_{35}	0.386	p_{36}	0.356
Unit-4 Output ($puMW$)	p_{41}	0.069	p_{42}	0.197	p_{43}	0.315	p_{44}	0.392	p_{45}	0.419	p_{46}	0.416
Unit-1 running status binary variable	u_{11}	1	u_{12}	1	u_{13}	1	u_{14}	1	u_{15}	1	u_{16}	1
Unit-2 running status binary variable	u_{21}	1	u_{22}	1	u_{23}	1	u_{24}	1	u_{25}	1	u_{26}	1
Unit-1 start-up binary variable	y_{11}	0	y_{12}	0	y_{13}	0	y_{14}	0	y_{15}	0	y_{16}	0
Unit-2 start-up binary variable	y_{21}	0	y_{22}	0	y_{23}	0	y_{24}	0	y_{25}	0	y_{26}	0
Unit-1 shut down binary variable	z_{11}	0	z_{12}	0	z_{13}	0	z_{14}	0	z_{15}	0	z_{16}	0
Unit-2 shut down binary variable	z_{21}	0	z_{22}	0	z_{23}	0	z_{24}	0	z_{25}	0	z_{26}	0
Auxiliary Variables	**Time Period-1**		**Time Period-2**		**Time Period-3**		**Time Period-4**		**Time Period-5**		**Time Period-6**	
P13 line loading ($puMW$)	L_{11}	1.62	L_{12}	1.48	L_{13}	1.34	L_{14}	1.23	L_{15}	1.19	L_{16}	1.21
P23 line loading ($puMW$)	L_{21}	0.37	L_{22}	0.52	L_{23}	0.68	L_{24}	0.79	L_{25}	0.80	L_{26}	0.77
Demand ($puMW$)	P_1^D	1.99	P_2^D	2.01	P_1^D	2.02	P_1^D	2.02	P_1^D	2.00	P_6^D	1.99
Unit- 1 Fixed Cost ($)	C_1^F	3	C_1^F	3	C_1^F	3	C_1^F	3	C_1^F	3	C_1^F	3
Unit- 2 Fixed Cost ($)	C_2^F	1	C_2^F	1	C_2^F	1	C_2^F	1	C_2^F	1	C_2^F	1
Unit-1 Variable cost ($\$/puMW$)	C_1^V	11.29	C_1^V	11.88	C_1^V	10.45	C_1^V	9.31	C_1^V	8.97	C_1^V	9.17
Unit-2 Variable cost ($\$/puMW$)	C_2^V	10	C_2^V	6	C_2^V	6	C_2^V	6	C_2^V	6	C_2^V	6
Unit-1 Start-up cost ($)	C_1^{SU}	0	C_1^{SU}	0	C_1^{SU}	0	C_1^{SU}	0	C_1^{SU}	0	C_1^{SU}	0
Unit-1 Shut down cost ($)	C_1^{SD}	0	C_1^{SD}	0	C_1^{SD}	0	C_1^{SD}	0	C_1^{SD}	0	C_1^{SD}	0
Unit-2 Start-up cost ($)	C_2^{SU}	0	C_2^{SU}	0	C_2^{SU}	0	C_2^{SU}	0	C_2^{SU}	0	C_2^{SU}	0
Unit-2 Shut down cost ($)	C_2^{SD}	0	C_2^{SD}	0	C_2^{SD}	0	C_2^{SD}	0	C_2^{SD}	0	C_2^{SD}	0
Total cost	125.095 $											

Table 14: Output Table Case-2

Key solution: Output result indicate that if we simulate the UC/ED problem in the given hybrid isolated power system considering all the constraints and spinning reserve, it is not possible to take shutdown of Unit-2 on given time period considering the spinning reserve requirement for renewable energy sources. If conventional units were present in place of renewable sources, we can forecast the energy output accurately and no need to keep spinning reserve.

Furthermore, the reliability of the system may be hampered in case of unpredicted changes in wind and solar power because the available ramping capability of on-line units in the system may not be sufficient to accommodate this change. Thus, while working with renewable energy sources, we must run/choose the conventional units in parallel which can ramp up and ramp down easily to accommodate the changes in renewable sources i.e., hydro and gas plant, although, in our defined problem, fluctuation in system demand, wind generation and solar generation is less. [5] [7]

The Most common problems in hybrid power system:

- Wind and solar forecasting errors bring great uncertainty to the system operations, since the real time wind power and solar output may be very different from what is forecasted.
- The reliability of the system may be impeded in case of unforeseen decreases in wind and solar power because the available ramping capability of on-line units in the system may not be sufficient to mitigate this change.
- Also, a large upward ramp in wind power may be unfavorable in a system in which sufficient downward Impact of Wind Power Forecasting on Unit Commitment and Dispatch reserves from other resources are not present. In this case, wind power may have to be curtailed, which leads to a waste of available resources.
- The same rationale applies to the wind power supply surplus that may happen at night, when the wind is usually the strongest but the system load is low.

Recommendations:

- In hybrid power system, to mitigate the forecasting errors in renewable sources, we must keep spinning reserve margin to other conventional units. In the case of sudden change in renewable output, we would be able to ramp up or ramp down the other conventional units' generation to mitigate the change in renewable energy source output.
- To avoid the renewable energy curtailments, other sources having sufficient large ramp up/down limit must be present to dispatch the reserves.
- In the case of surplus power at night when wind is usually the strongest, wind generation may also have to be spilled to maintain normal operation of other slow-start units, such as coal and nuclear, because of the physical and economic constraints of those units.

Consequently, several areas for improving unit commitment and dispatch have been proposed to address the uncertainty and variability of wind and solar power. Some researchers focus on revising the current security-constrained unit commitment (SCUC) formulation. Others aim at novel UC methods. [5]

Economic result: Looking towards economical point of view, if we shut down the thermal unit, we can save $= 125.095\ \$ - 114.09\$ = 11.005\$$ in given time period frame. In addition, unit-2 variable cost would be reduced to $15\$/puMW$.

Conclusion: Although the operating cost of renewable is very cheap and there is no greenhouse gas emission, renewable energy is unpredictable and intermittent. Integration of wind and solar power plants into the existing isolated system presents challenges to power system planners and operators. Because of the intermittency and unpredictability of wind and solar power generation, additional physical constraints such as high ramp up and ramp down due to uncertainty and economic operation constraints must be taken into consideration to reach a compromise between system security and total production cost. To reiterate, having capability of high ramp up and ramp down Units and addition spinning reserve required in our power system to compensate the sudden unexpected change in renewable energy output. [7]

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