UNIVERSITY OF CALGARY

ENEL 693 Restructured Electricity Market

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Economic Dispatch Project

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Deadline: January 31
140 marks

The purpose of this assignment is to explore an economic dispatch problem and understand the modelling, the solution, and the results when a commercial-level optimization solver is employed. The required optimization tool for this course is Pyomo in Python (http://www.pyomo.org).

For a simple 3-unit system the following data are provided:

Unit	a	b	P_G^{min}	P_G^{max}	d	e	f
	(\$/MWh)	(\$/$(MWh)^2$)	(MW)	(MW)	tonCO_2/h	tonCO_2/(MWh)	tonCO_2/(MWh)^2
1	20	0.05	20	340	150	0.2	0.001
2	25	0.10	20	300	150	0.2	0.001
3	40	0.20	30	250	40	0.1	0.0005

Table 1: Unit Generator Data for the ELD problem

The cost functions for the units are provided as follows:

$$C_j(P_{Gj}) = a_j P_{Gj} + \frac{1}{2} b_j P_{Gj}^2 \qquad \$/h \tag{1}$$

The emission functions for the units are provided as follows:

$$E_j(P_{Gj}) = d_j + e_j P_{Gj} + f_j P_{Gj}^2 \qquad tonCO_2/h \tag{2}$$

The power demand at a particular planning interval is 500 MW. The region is considering a carbon tax of \$50/tonCO_2. All three units are online and synchronized.

Project Tasks:

(a) Manually formulate the optimal economic dispatch with clear objective and all constraints for minimizing only the fuel cost. Write down the full form and not the general/concise form of the optimization problem (i.e., no summation terms nor "for all" ∀ expressions etc.) (10 marks)

- **Following calculations are only for fuel cost consideration.**

Cost function of Unit 1 $C_1(P_{G_1}) = a_1 P_{G_1} + \frac{1}{2} b_1 P_{G_1}^2$

$$= 20P_{G_1} + \frac{1}{2} * 0.05 * P_{G_1}^2$$

$$= 20P_{G_1} + 0.025 * P_{G_1}^2 \quad \$/h$$

Cost function of Unit 2 $C_2(P_{G_2}) = a_2 P_{G_2} + \frac{1}{2} b_2 P_{G_2}^2$

$$= 25P_{G_2} + \frac{1}{2} * 0.10 * P_{G_2}^2$$

$$= 25P_{G_2} + 0.05 * P_{G_2}^2 \quad \$/h$$

Cost function of Unit 3 $C_3(P_{G_3}) = a_3 P_{G_3} + \frac{1}{2} b_3 P_{G_3}^2$

$$= 40P_{G_3} + \frac{1}{2} * 0.20 * P_{G_3}^2$$

$$= 40P_{G_3} + 0.10 * P_{G_3}^2 \quad \$/h$$

Objective Function

Total Cost function

$$C_i(P_{G_i}) = 20P_{G_1} + \frac{1}{2} * 0.05 * P_{G_1}^2 + 25P_{G_2} + \frac{1}{2} * 0.10 * P_{G_2}^2 + 40P_{G_3} + \frac{1}{2} * 0.20 * P_{G_3}^2$$

$$= 20P_{G_1} + 0.025P_{G_1}^2 + 25P_{G_2} + 0.05P_{G_2}^2 + 40P_{G_3} + 0.10P_{G_3}^2 \quad \$/h$$

Constraints

1. $P_G^{min} \le P_{G_1} \le P_G^{max} \quad \rightarrow \quad 20 \le P_{G_1} \le 340$
2. $P_G^{min} \le P_{G_2} \le P_G^{max} \quad \rightarrow \quad 20 \le P_{G_2} \le 300$
3. $P_G^{min} \le P_{G_3} \le P_G^{max} \quad \rightarrow \quad 30 \le P_{G_3} \le 250$
4. $P_{G_1} + P_{G_2} + P_{G_3} = 500$

Inequality constraints

$\Longrightarrow \quad g_i(P_i) \leq 0$

1. $P_{G_1}^{min} - P_{G_1} \leq 0 \quad \rightarrow \quad 20 - P_{G_1} \leq 0$
2. $P_{G_2}^{min} - P_{G_2} \leq 0 \quad \rightarrow \quad 20 - P_{G_2} \leq 0$
3. $P_{G_3}^{min} - P_{G_3} \leq 0 \quad \rightarrow \quad 30 - P_{G_3} \leq 0$

$\Longrightarrow \quad h_i(P_i) \leq 0$

1. $\mathrm{P}_{\mathrm{G}_1} - \mathrm{P}_{\mathrm{G}_1}^{\max} \leq 0 \quad \rightarrow \quad \mathrm{P}_{\mathrm{G}_1} - 340 \leq 0$
2. $\mathrm{P}_{\mathrm{G}_2} - \mathrm{P}_{\mathrm{G}_2}^{\max} \leq 0 \quad \rightarrow \quad \mathrm{P}_{\mathrm{G}_2} - 300 \leq 0$
3. $\mathrm{P}_{\mathrm{G}_3} - \mathrm{P}_{\mathrm{G}_3}^{\max} \leq 0 \quad \rightarrow \quad \mathrm{P}_{\mathrm{G}_3} - 250 \leq 0$

Equality constraint

$\Longrightarrow \quad \emptyset_i(P_i) = 0$

1. $P.D. - \sum_{i=1}^{3} P_i = 0 \quad \rightarrow \quad 500 - P_{G_1} - P_{G_2} - P_{G_3} = 0$

(b) Manually derive the optimality condition equations based on KKT method showing all steps. (10 marks)

LaGrange Equation

$$L(P_G, \lambda, \mu, \gamma) = \sum_{i=1}^{3} C_i\left(P_{G_i}\right) + \lambda \emptyset_i(P_i) + \sum_{i=1}^{3} \mu_i g_i(P_i) + \sum_{i=1}^{3} \gamma_i h_i(P_i)$$

$$\begin{aligned} L(P_G, \lambda, \mu, \gamma) = {} & C_1\left(P_{G_1}\right) + C_2\left(P_{G_2}\right) + C_3\left(P_{G_3}\right) + \lambda\left[P.D. - P_{G_1} - P_{G_2} - P_{G_3}\right] \\ & + \mu_1\left[P_{G_1}^{min} - P_{G_1}\right] + \mu_2\left[P_{G_2}^{min} - P_{G_2}\right] + \mu_3\left[P_{G_3}^{min} - P_{G_3}\right] + \gamma_1\left[\mathrm{P}_{\mathrm{G}_1} - \mathrm{P}_{\mathrm{G}_1}^{\max}\right] \\ & + \gamma_2\left[\mathrm{P}_{\mathrm{G}_2} - \mathrm{P}_{\mathrm{G}_2}^{\max}\right] + \gamma_3\left[\mathrm{P}_{\mathrm{G}_3} - \mathrm{P}_{\mathrm{G}_3}^{\max}\right] \end{aligned}$$

$$\begin{aligned} L(P_G, \lambda, \mu, \gamma) = {} & 20P_{G_1} + 0.025P_{G_1}^2 + 25P_{G_2} + 0.05P_{G_2}^2 + 40P_{G_3} + 0.10P_{G_3}^2 \\ & + \lambda\left[500 - P_{G_1} - P_{G_2} - P_{G_3}\right] + \mu_1\left[20 - P_{G_1}\right] + \mu_2\left[20 - P_{G_2}\right] \\ & + \mu_3\left[30 - P_{G_3}\right] + \gamma_1\left[\mathrm{P}_{\mathrm{G}_1} - 340\right] + \gamma_2\left[\mathrm{P}_{\mathrm{G}_2} - 300\right] + \gamma_3\left[\mathrm{P}_{\mathrm{G}_3} - 250\right] \end{aligned}$$

Where λ, μ_i and γ_i are LaGrange multipliers

Karush-Kuhn Tucker's (KKT, or KT) Conditions of Optimality:

KKT conditions

1. $\frac{\partial L}{\partial P_{G_i}} = \frac{\partial C_i}{\partial P_{G_i}} - \lambda - \mu_i - \gamma_i \ = \ 0;$
   a. $\frac{\partial L}{\partial P_{G_1}} = 20 + 0.05\, P_{G_1} - \lambda - \mu_1 + \gamma_1 = 0$
   b. $\frac{\partial L}{\partial P_{G_2}} = 25 + 0.1\, P_{G_2} - \lambda - \mu_2 + \gamma_2 = 0$
   c. $\frac{\partial L}{\partial P_{G_3}} = 40 + 0.20\, P_{G_3} - \lambda - \mu_3 + \gamma_3 = 0$
2. $\frac{\partial L}{\partial \lambda} = 0;$
   a. $\frac{\partial L}{\partial \lambda} = \ 500 - \ P_{G_1} - \ P_{G_2} - \ P_{G_3} = 0$
3. Complimentary slackness conditions

$\{\mu_i(P_{G_i}^{min} \ - \ P_{G_i}) = 0; \qquad\qquad \forall\ i = 1,2,3$

Either $\mu_i = 0,\ P_{G_i}^{min} \ - \ P_{G_i} \ \neq 0 \ \ OR \ \ \mu_i \neq 0,\ P_{G_i}^{min} \ - \ P_{G_i} = 0$

a. $\mu_1 \ = \ 0,\ P_{G_1}^{min} \ - \ P_{G_1} \ \neq \ 0$ OR $\mu_1 \ \neq \ 0,\ P_{G_1}^{min} \ - \ P_{G_1} \ = \ 0$

$\mu_1 \ = \ 0,\ 20 \ - \ P_{G_1} \ \neq \ 0$ OR $\mu_1 \ \neq \ 0,\ 20 \ - \ P_{G_1} \ = \ 0$

b. $\mu_2 \ = \ 0,\ P_{G_2}^{min} \ - \ P_{G_2} \ \neq \ 0$ OR $\mu_2 \ \neq \ 0,\ P_{G_2}^{min} \ - \ P_{G_2} \ = \ 0$

$\mu_2 \ = \ 0,\ 20 \ - \ P_{G_2} \ \neq \ 0$ OR $\mu_2 \ \neq \ 0,\ 20 \ - \ P_{G_2} \ = \ 0$

c. $\mu_3 \ = \ 0,\ P_{G_3}^{min} \ - \ P_{G_3} \ \neq \ 0$ OR $\mu_3 \ \neq \ 0,\ P_{G_3}^{min} \ - \ P_{G_3} \ = \ 0$

$\mu_3 \ = \ 0,\ 30 \ - \ P_{G_3} \ \neq \ 0$ OR $\mu_3 \ \neq \ 0,\ 30 \ - \ P_{G_3} \ = \ 0$

$\{\gamma_i(P_{G_i} - \ P_{G_i}^{max}) = 0; \qquad\qquad \forall\ i = 1,2,3$

Either $\gamma_i = 0,\ P_{G_i} - \ P_{G_i}^{max} \ \neq 0 \ \ OR \ \ \gamma_i \neq 0,\ P_{G_i} - \ P_{G_i}^{max} = 0$

a. $\gamma_1 \ = \ 0,\ P_{G_1} - \ P_{G_1}^{max} \ \neq \ 0$ OR $\gamma_1 \ \neq \ 0,\ P_{G_1} - \ P_{G_1}^{max} \ = \ 0$

$\gamma_1 \ = \ 0,\ P_{G_1} - \ 340 \ \neq \ 0$ OR $\gamma_1 \ \neq \ 0,\ P_{G_1} - \ 340 \ = \ 0$

b. $\gamma_2 \ = \ 0,\ P_{G_2} - \ P_{G_2}^{max} \ \neq \ 0$ OR $\gamma_2 \ \neq \ 0,\ P_{G_2} - \ P_{G_2}^{max} \ = \ 0$

$\gamma_2 \ = \ 0,\ P_{G_2} - \ 300 \ \neq \ 0$ OR $\gamma_2 \ \neq 0,\ P_{G_2} - \ 300 \ = \ 0$

c. $\gamma_3 \ = \ 0,\ P_{G_3} - \ P_{G_3}^{max} \ \neq \ 0$ OR $\gamma_3 \ \neq \ 0,\ P_{G_3} - \ P_{G_3}^{max} \ = \ 0$

$\gamma_3 \ = \ 0,\ P_{G_3} - \ 250 \ \neq \ 0$ OR $\gamma_3 \ \neq \ 0,\ P_{G_3} - \ 250 \ = \ 0$

4. $\mu_i \geq 0$, $\gamma_i \geq 0$

   i.e. $\mu_1, \mu_2, \mu_3 \geq 0$, $\gamma_1, \gamma_2, \gamma_3 \geq 0$

(c) Manually solve the equations you formed in part (b) and find the optimal power generation schedule that would minimize the total cost of electricity generation (without considering the emissions). Also, find the system marginal cost, unit incremental costs, and the LaGrange multipliers associated with every binding or non-binding constraint. Show your work. (20 marks)

Step -1: Assumse that P_i is within limits

If all generators operate within the limits, as mentioned earlier in *complimentary slackness condition*, $\mu_i = 0$ and $\gamma_i = 0$. Thus, our equations will reduce to:

$$\frac{\partial L}{\partial P_{G_1}} = 20 + 0.05\, P_{G_1} - \lambda = 0 \qquad \text{... (1)}$$

$$\frac{\partial L}{\partial P_{G_2}} = 25 + 0.1\, P_{G_2} - \lambda = 0 \qquad \text{... (2)}$$

$$\frac{\partial L}{\partial P_{G_3}} = 40 + 0.20\, P_{G_3} - \lambda = 0 \qquad \text{... (3)}$$

$$\frac{\partial L}{\partial \lambda} = 500 - P_{G_1} - P_{G_2} - P_{G_3} = 0 \qquad \text{... (4)}$$

We have 4 equations with 4 unknown variables ($P_{G_1}, P_{G_2}, P_{G_3}, \lambda$). Thus, by solving equation 1,2,3 and 4, we get the feasible solution as below.

$P_{G_1} = 371.42\, MW$ $\quad$ $P_{G_2} = 135.71\, MW$ $\quad$ $P_{G_3} = -7.14\, MW$ $\quad$ $\lambda = 38.57\ \$/MWh$

Here, our solution does not satisfy the *inequality constraints* for $P_{G_1} = 371.42\, MW$ which crosses the upper limit of Unit 1 and $P_{G_3} = -7.14\, MW$ which is below the lower limit of Unit 3.

Step- 2: Assume that $P_{G_1} = P_{G_1}^{max}$, all other $P_i (P_2$ and $P_3)$ within limits

Here, we have bounded our Unit 1 generation to upper limit $P_{G_1} = P_{G_1}^{max} = 340\, MW$

As we know the *complementary slackness condition*

1. $\cancel{\gamma_1 = 0,\ P_{G_1} - P_{G_1}^{max} \neq 0}$ OR $\gamma_1 \neq 0,\ P_{G_1} - P_{G_1}^{max} = 0$

   $\cancel{\gamma_1 = 0,\ P_{G_1} - 340 \neq 0}$ OR $\gamma_1 \neq 0,\ P_{G_1} - 340 = 0$

Thus, $\gamma_1 \neq 0$

So, our equation would change to…

$$\frac{\partial L}{\partial \gamma_1} = P_{G_1} - 340 = 0 \qquad \text{... (1.1)}$$

$$\frac{\partial L}{\partial P_{G_2}} = 25 + 0.1\, P_{G_2} - \lambda = 0 \qquad \text{... (2.1)}$$

$$\frac{\partial L}{\partial P_{G_3}} = 40 + 0.20\, P_{G_3} - \lambda = 0 \qquad \text{... (3.1)}$$

$$\frac{\partial L}{\partial \lambda} = 500 - P_{G_1} - P_{G_2} - P_{G_3} = 0 \qquad \text{... (4.1)}$$

We have 4 equations with 4 unknown variables $(P_{G_1}, P_{G_2}, P_{G_3}, \lambda)$. Thus, by solving equation 1.1, 2.1, 3.1 and 4.1, we get the feasible solution as below.

$P_{G_1} = 340\ MW$ $\quad$ $P_{G_2} = 156.66\ MW$ $\quad$ $P_{G_3} = 3.33\ MW$ $\quad$ $\lambda = 40.67\ \$/MWh$

Here, our solution does not satisfy the *inequality constraints* for $P_{G_3} = 3.33\ MW$ which is below the lower limit of Unit 3.

Step- 3: Assumption that $P_{G_3} = P_{G_3}^{min}$, all other P_i within limits

Here, we have bounded our Unit 3 generation to lower limit $P_{G_3} = P_{G_3}^{min} = 30\ MW$ due to *inequality constraint*.

As we know the *complementary slackness condition*

1. $\cancel{\mu_3 = 0,\ P_{G_3}^{min} - P_{G_3} \neq 0}$ OR $\mu_3 \neq 0,\ P_{G_3}^{min} - P_{G_3} = 0$

   $\cancel{\mu_3 = 0,\ 30 - P_{G_3} \neq 0}$ OR $\mu_3 \neq 0,\ 30 - P_{G_3} = 0$

Thus, $\mu_3 \neq 0$

So, our equation would change to…

$$\frac{\partial L}{\partial \gamma_1} = P_{G_1} - 340 \qquad \text{... (1.2)}$$

$$\frac{\partial L}{\partial P_{G_2}} = 25 + 0.1\, P_{G_2} - \lambda = 0 \qquad \text{... (2.2)}$$

$$\frac{\partial L}{\partial \mu_3} = 30 - P_{G_3} = 0 \qquad \text{... (3.2)}$$

$$\frac{\partial L}{\partial \lambda} = 500 - P_{G_1} - P_{G_2} - P_{G_3} = 0 \qquad \text{... (4.2)}$$

We have 4 equations with 4 unknown variables $(P_{G_1}, P_{G_2}, P_{G_3}, \lambda)$. Thus, by solving equation 1.2, 2.2, 3.2 and 4.2, we get the feasible solution as below.

$\boldsymbol{P_{G_1} = 340\ MW}$ $\quad$ $\boldsymbol{P_{G_2} = 130\ MW}$ $\quad$ $\boldsymbol{P_{G_3} = 30\ MW}$ $\quad$ $\boldsymbol{\lambda = 38\ \$/MWh}$

From the above optimal power generation schedule, we get the total minimized cost…

$$C_i(P_{G_i}) = 20P_{G_1} + \frac{1}{2} * 0.05 * P_{G_1}^2 + 25P_{G_2} + \frac{1}{2} * 0.10 * P_{G_2}^2 + 40P_{G_3} + \frac{1}{2} * 0.20 * P_{G_3}^2$$
$$= 20P_{G_1} + 0.025P_{G_1}^2 + 25P_{G_2} + 0.05P_{G_2}^2 + 40P_{G_3} + 0.10P_{G_3}^2 \quad \$/h$$
$$= 20 * 340 + 0.025 * (340)^2 + 25 * 130 + 0.05 * (130)^2 + 40 * 30 + 0.10 * (30)^2$$
$$= 15075 \quad \$/h$$

<u>System marginal cost</u> = The multiplier λ appears in the Lagrange function $L(P_G, \lambda, \mu, \gamma)$ associated with the demand-supply balance. It denotes the sensitivity of the objective function (cost, in this case) to a change in system demand. Simply stated, λ denotes the change in the system cost for a 1 MW change in the system demand. λ is commonly known as the system marginal cost.

System marginal cost $\boldsymbol{\lambda} = \mathbf{38}\ \$/\boldsymbol{MWh}$

<u>Unit incremental cost</u> =

Unit 1

Here, we have bounded our Unit 1 generation to upper limit $P_{G_1} = P_{G_1}^{max} = 340\ MW$

Thus, $\gamma_1 \neq 0$

Our Unit 1 generation is not bounded by lower limit i.e., $\mu_1 = 0,\ P_{G_1}^{min} - P_{G_1} \neq 0$

$$\frac{\partial L}{\partial P_{G_1}} = 20 + 0.05\, P_{G_1} - \lambda + \gamma_1 = 0$$
$$\Rightarrow 20 + 0.05 * 340 - 38 + \gamma_1 = 0$$
$$\Rightarrow \boldsymbol{\gamma_1} = +\mathbf{1}\$/\boldsymbol{MWh}$$

Thus, Unit 1 incremental cost $= \frac{\partial L}{\partial P_{G_1}} = \frac{\partial C_1(P_{G_1})}{\partial P_{G_1}} - \lambda + \gamma_1 = 0$

$$\Rightarrow \frac{\partial C_1(P_{G_1})}{\partial P_{G_1}} - \lambda + \gamma_1 = 0$$
$$\Rightarrow \frac{\partial C_1(P_{G_1})}{\partial P_{G_1}} = \lambda - \gamma_1$$
$$\Rightarrow \frac{\partial C_1(P_{G_1})}{\partial P_{G_1}} = 38 - 1$$
$$\Rightarrow \boldsymbol{\frac{\partial C_1(P_{G_1})}{\partial P_{G_1}}} = +\mathbf{37}\ \$/\boldsymbol{MWh}$$

Lagrange multiplier $\boldsymbol{\gamma_1} = +\mathbf{1}\ \$/\boldsymbol{MWh}$

Lagrange multiplier $\boldsymbol{\mu_1} = \mathbf{0}\ \$/\boldsymbol{MWh}$

Unit 2

Here, our unit 2 is not bounded between the upper and lower limit

Thus, $\mu_2 = 0,\ P_{G_2}^{min} - P_{G_2} \neq 0$ and $\gamma_2 = 0,\ P_{G_2} - P_{G_2}^{max} \neq 0$

$\mu_2 = 0,\ 20 - P_{G_2} \neq 0$ $\gamma_2 = 0,\ P_{G_2} - 300 \neq 0$

Thus, Unit 2 incremental cost $= \frac{\partial L}{\partial P_{G_2}} = \frac{\partial C_2(P_{G_2})}{\partial P_{G_2}} - \lambda - \mu_2 + \gamma_2 = 0$

$$\Rightarrow \frac{\partial L}{\partial P_{G_2}} = \frac{\partial C_2(P_{G_2})}{\partial P_{G_2}} - \lambda = 0$$

$$\Rightarrow \frac{\partial C_2(P_{G_2})}{\partial P_{G_2}} = \boldsymbol{\lambda} = \mathbf{+38\$/MWh}$$

Lagrange multiplier $\boldsymbol{\gamma_2} = \mathbf{0}\ \$/\boldsymbol{MWh}$

Lagrange multiplier $\boldsymbol{\mu_2} = \mathbf{0}\ \$/\boldsymbol{MWh}$

Unit 3

Here, we have bounded our Unit 3 generation to lower limit $P_{G_3} = P_{G_3}^{min} = 30\ MW$

Thus, $\mu_3 \neq 0$

Our Unit 3 generation is not binding by upper limit i.e., $\gamma_3 = 0,\ P_{G_3} - P_{G_3}^{max} \neq 0$

$$\frac{\partial L}{\partial P_{G_3}} = 40 + 0.20\,P_{G_3} - \lambda - \mu_3 + \gamma_3 = 0$$

$$\Rightarrow 40 + 0.20 * 30 - 38 - \mu_3 = 0$$

$$\Rightarrow \mu_3 = +8\$/MWh$$

Thus, Unit 3 incremental cost $= \frac{\partial L}{\partial P_{G_3}} = \frac{\partial C_3(P_{G_3})}{\partial P_{G_3}} - \lambda - \mu_3 = 0$

$$\Rightarrow \frac{\partial C_3(P_{G_3})}{\partial P_{G_3}} = \lambda + \mu_3$$

$$\Rightarrow \frac{\partial C_3(P_{G_3})}{\partial P_{G_3}} = 38 + 8$$

$$\Rightarrow \frac{\partial C_3(\boldsymbol{P_{G_3}})}{\partial \boldsymbol{P_{G_3}}} = +\mathbf{46}\ \$/\boldsymbol{MWh}$$

Lagrange multiplier $\boldsymbol{\gamma_3} = \mathbf{0}\ \$/\boldsymbol{MWh}$

Lagrange multiplier $\boldsymbol{\mu_3} = \mathbf{8}\ \$/\boldsymbol{MWh}$

(d) Using the results of part (c), and without re-solving the problem, answer the following questions:

1) How much would have been the added cost if the demand were to be increased to 501 MW? (2marks)

Answer: - If Total Demand increase to $501\,MW$ instead of $500\,MW$, the additional cost would be equal to the system marginal cost which is $\boldsymbol{\lambda = +38\,\$/MWh}$**.**

2) How much would have been the cost reduction for a 500 MW demand if the upper limit of Unit1 were 341 MW? (2marks)

Answer: - If we change the upper limit of Unit 1 to $341\,MW$ instead of $340\,MW$, the Unit incremental cost of Unit 1 would be $= \lambda - \gamma_1 = +37\,\$/MWh$. Thus, cost reduction for a 500 MW demand would be the difference between system marginal cost and Unit incremental cost of Unit 1 which is equal to $38\$ - 37\$ = 1\$/h$ reduction.

3) How much would have been the cost reduction for a 500 MW demand if the lower limit of Unit 3 were 29 MW? (2marks)

Answer: - If we change the lower limit of Unit 3 to $29\,MW$ instead of $30\,MW$, the Unit incremental cost of Unit 1 would be $= \lambda + \mu_3 = +46\,\$/MWh$. Thus, cost reduction for a 500 MW demand would be the difference between Unit incremental cost and system marginal cost of Unit 3 which is equal to $+46\$ - 38\$ = 8\$$ reduction.

(e) Implement the problem of part (a) in a computer optimization code in Pyomo, solve it and clearly compare the computer solution with your manually obtained results in part (c). The expressions in the code must be analogous to the ones in part (a) (i.e., using the specific parameters with no general functions or expressions) Note: you need to solve an optimization problem as opposed to solving the set of linear equations of part (b). Make sure you leave clear comments for each line of your code so we can easily understand it. (20 marks)

<u>Computer optimization code in Pyomo</u>

```
#Pyomo objects exist within the pyomo.environ namespace
#Every Pyomo model starts with this; it tells Python to load the Pyomo Modeling Environment
from pyomo.environ import *

#Create an instance of a Concrete model
model = ConcreteModel("Economic Load dispatch considering only fuel cost")

#Define desicion variables and limit their values to only real number.
model.PG1 = Var(within = Reals)
model.PG2 = Var(within = Reals)
model.PG3 = Var(within = Reals)

#Define an objective function to minimize the total cost.
model.objective = Objective(expr = 20*model.PG1 + 25*model.PG2 + 40*model.PG3 + 0.025*(model.PG1)**2
                            + 0.05*(model.PG2)**2 + 0.10*(model.PG3)**2 )
#If “sense” is omitted in above objective function, Pyomo assumes minimization objective function.

#Define an equality constraint
model.constraint1 = Constraint(expr = 500 - model.PG1 - model.PG2 - model.PG3 == 0)

#Define an inequality constraints
model.constraint2 = Constraint(expr = model.PG1 - 340 <= 0)      # PG1 - 340 <= 0
model.constraint3 = Constraint(expr = -model.PG1 + 20 <= 0)      # -PG1 + 20 <= 0
model.constraint4 = Constraint(expr = model.PG2 - 300 <= 0)      # PG2 - 300 <= 0
model.constraint5 = Constraint(expr = -model.PG2 + 20 <= 0)      # -PG2 + 20 <= 0
model.constraint6 = Constraint(expr = model.PG3 - 250 <= 0)      # PG3 -250 <= 0
model.constraint7 = Constraint(expr = -model.PG3 + 30 <= 0)      # -PG3 + 30 <= 0

#Solving models
opt = SolverFactory("gurobi")
results = opt.solve(model)

#Display the result
model.display()
```

Output from the Pyomo code

```
Model Economic Load dispatch considering only fuel cost

  Variables:
    PG1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 339.99999999984965 :  None : False : False :  Reals
    PG2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 130.00000000015007 :  None : False : False :  Reals
    PG3 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 30.000000000000277 :  None : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 15075.000000000155

  Constraints:
    constraint1 : Size=1
        Key  : Lower : Body                  : Upper
        None :   0.0 : 7.105427357601002e-15 :   0.0
    constraint2 : Size=1
        Key  : Lower : Body                   : Upper
        None :  None : -1.503508428868372e-10 :   0.0
    constraint3 : Size=1
        Key  : Lower : Body                : Upper
        None :  None : -319.99999999984965 :   0.0
    constraint4 : Size=1
        Key  : Lower : Body                : Upper
        None :  None : -169.99999999984993 :   0.0
    constraint5 : Size=1
        Key  : Lower : Body                : Upper
        None :  None : -110.00000000015007 :   0.0
    constraint6 : Size=1
        Key  : Lower : Body                : Upper
        None :  None : -219.99999999999972 :   0.0
    constraint7 : Size=1
        Key  : Lower : Body                    : Upper
        None :  None : -2.7711166694643907e-13 :   0.0
```

Computer optimization code in Pyomo result	**Manual calculation result**
$\lambda = 38\,\$/MWh$	$\lambda = 38\,\$/MWh$
$P_{G_1} = 340\ MW$	$P_{G_1} = 340\ MW$
$P_{G_2} = 130\ MW$	$P_{G_2} = 130\ MW$
$P_{G_3} = 30\ MW$	$P_{G_3} = 30\ MW$
Total minimized cost = $15075\,\$/h$	Total minimized cost = $15075\,\$/h$
LaGrange Multipliers: $\mu_1 = 0\ \$/MWh$	LaGrange Multipliers: $\mu_1 = 0\ \$/MWh$
LaGrange Multipliers: $\gamma_1 = +1\,\$/MWh$	LaGrange Multipliers: $\gamma_1 = +1\ \$/MWh$
LaGrange Multipliers: $\mu_2 = 0\ \$/MWh$	LaGrange Multipliers: $\mu_2 = 0\ \$/MWh$
LaGrange Multipliers: $\gamma_2 = 0\ \$/MWh$	LaGrange Multipliers: $\gamma_2 = 0\ \$/MWh$
LaGrange Multipliers: $\mu_3 = +8\ \$/MWh$	LaGrange Multipliers: $\mu_3 = +8\ \$/MWh$
LaGrange Multipliers: $\gamma_3 = 0\ \$/MWh$	LaGrange Multipliers: $\gamma_3 = 0\ \$/MWh$
Unit 1 Incremental cost = $\lambda - \gamma_1 = +37\,\$/MWh$	Unit 1 Incremental cost = $\lambda - \gamma_1 = +37\,\$/MWh$
Unit 2 Incremental cost = $\lambda = +38\,\$/MWh$	Unit 2 Incremental cost = $\lambda = +38\,\$/MWh$
Unit 3 Incremental cost = $\lambda + \mu_3 = 46\,\$/MWh$	Unit 3 Incremental cost = $\lambda + \mu_3 = +46\,\$/MWh$

- By using the optimization code, we get the value of P_{G_1}, P_{G_2} *and* P_{G_3}. By putting these values in *equation number 2.2*, we get the value of $\lambda = 38\ \$/MWh$
- By putting the value of P_{G_1}, P_{G_2} *and* P_{G_3} *in part C*, we get the values of μ_1, μ_2, μ_3, $\gamma_1, \gamma_2, \gamma_3$ and Unit incremental cost of each unit.

Answer: From the above results, we can conclude that we get the same result in both the cases i.e., In manual calculation and optimization code result.

(f) Using your optimization code, verify and analytically demonstrate the values you found for part (d)- 1,2,3. (4 marks)

1) How much would have been the added cost if the demand were to be increased to 501 MW?

   Answer: - If Total Demand increase to $501\ MW$ instead of $500\ MW$, the additional cost would be equal to the system marginal cost which is $\boldsymbol{\lambda = +38\ \$/MWh}$.

<table>
<tr><th>Output of code with 500 MW demand</th><th>Output of code with 501 MW demand</th></tr>
<tr><td>

```
Model Economic Load dispatch considering only fuel cost

  Variables:
    PG1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 339.99999999984965 :  None : False : False :  Reals
    PG2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 130.00000000015007 :  None : False : False :  Reals
    PG3 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 30.000000000000277 :  None : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 15075.000000000155
```

</td><td>

```
Model Economic Load dispatch considering only fuel cost

  Variables:
    PG1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 339.99999999989365 :  None : False : False :  Reals
    PG2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 131.00000000010618 :  None : False : False :  Reals
    PG3 : Size=1, Index=None
        Key  : Lower : Value            : Upper : Fixed : Stale : Domain
        None :  None : 30.0000000000002 :  None : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 15113.05000000012
```

</td></tr>
</table>

By using the optimization code using Pyomo for the above problem, we get the total cost $= 15113\ \$/h$

Added in total cost is $\boldsymbol{= 15113\ \$/h - 15075\ \$/h = 38\ \$/h}$ which is the same as above.

2) How much would have been the cost reduction for a 500 MW demand if the upper limit of Unit1 were 341 MW? (2marks)

   Answer: - If we change the upper limit of Unit 1 to $341\ MW$ instead of $340\ MW$, the Unit incremental cost of Unit 1 would be $= \lambda - \gamma_1 = +37\ \$/MWh$. Thus, cost reduction for a 500 MW demand would be the difference between system marginal cost and Unit incremental cost of Unit 1 which is equal to $38\$ - 37\$ = 1\$/h$ reduction.

Output of code with Unit 1 limit kept at 340 MW

```
Model Economic Load dispatch considering only fuel cost

  Variables:
    PG1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 339.99999999984965 :  None : False : False :  Reals
    PG2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 130.00000000015007 :  None : False : False :  Reals
    PG3 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 30.000000000000277 :  None : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 15075.000000000155
```

Output of code with Unit 1 limit kept at 341 MW

```
Model Economic Load dispatch considering only fuel cost

  Variables:
    PG1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 340.99999999927314 :  None : False : False :  Reals
    PG2 : Size=1, Index=None
        Key  : Lower : Value             : Upper : Fixed : Stale : Domain
        None :  None : 129.0000000007264 :  None : False : False :  Reals
    PG3 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 30.000000000000448 :  None : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 15074.075000000621
```

By using the optimization code using Pyomo for the above problem, we get the total cost $=$ $15074\ \$/h$

Reduction in total cost is $= \mathbf{15075\ \$/h - 15074\ \$/h = 1\ \$/h}$ which is the same as above.

3) How much would have been the cost reduction for a 500 MW demand if the lower limit of Unit 3 were 29 MW? (2marks)

Answer: - If we change the lower limit of Unit 3 to $29\ MW$ instead of $30\ MW$, the Unit incremental cost of Unit 1 would be $= \lambda + \mu_3 = +46\ \$/MWh$. Thus, cost reduction for a 500 MW demand would be the difference between Unit incremental cost and system marginal cost of Unit 3 which is equal to $+46\$/\mathrm{h} - 38\$/\mathrm{h} = 8\$/h$ reduction.

Output of code with Unit 3 lower limit kept at 30 MW

```
Model Economic Load dispatch considering only fuel cost

  Variables:
    PG1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 339.99999999984965 :  None : False : False :  Reals
    PG2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 130.00000000015007 :  None : False : False :  Reals
    PG3 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 30.000000000000277 :  None : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 15075.000000000155
```

Output of code with Unit 3 lower limit kept at 29 MW

```
Model Economic Load dispatch considering only fuel cost

  Variables:
    PG1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 339.99999999988796 :  None : False : False :  Reals
    PG2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 131.00000000011184 :  None : False : False :  Reals
    PG3 : Size=1, Index=None
        Key  : Lower : Value             : Upper : Fixed : Stale : Domain
        None :  None : 29.00000000000022 :  None : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 15067.150000000125
```

By using the optimization code using Pyomo for the above problem, we get the total cost $=$ $15067\ \$/h$

Reduction in total cost is $= \mathbf{15075\ \$/h - 15067\ \$/h = +8\ \$/h}$ which is the same as above.

(g) Repeat part (a) by considering total costs, i.e., both fuel and emission costs. (10 marks)

- **Following calculations are for both fuel and emission cost consideration.**

<u>Cost function considering fuel in $\$/h$</u>

Cost function of Unit 1 $C_1(P_{G_1}) = a_1 P_{G_1} + \frac{1}{2} b_1 P_{G_1}^2$

$$= 20P_{G_1} + \frac{1}{2} * 0.05 * P_{G_1}^2 \ \$/h$$

Cost function of Unit 2 $C_2(P_{G_2}) = a_2 P_{G_2} + \frac{1}{2} b_2 P_{G_2}^2$

$$= 25P_{G_2} + \frac{1}{2} * 0.10 * P_{G_2}^2 \ \$/h$$

Cost function of Unit 3 $C_3(P_{G_3}) = a_3 P_{G_3} + \frac{1}{2} b_3 P_{G_3}^2$

$$= 40P_{G_3} + \frac{1}{2} * 0.20 * P_{G_3}^2 \ \underline{\$/h}$$

<u>Emission function for Unit in $tonCO_2/h$</u>

Emission function of Unit 1 $E_1(P_{G_1}) = d_1 + e_1 P_{G_1} + f_1 P_{G_1}^2 \ tonCO_2/h$

$$= 150 + 0.2P_{G_1} + 0.001 * P_{G_1}^2$$

Emission function of Unit 2 $E_2(P_{G_2}) = d_2 + e_2 P_{G_2} + f_2 P_{G_2}^2 \ tonCO_2/h$

$$= 150 + 0.2P_{G_2} + 0.001 * P_{G_2}^2$$

Emission function of Unit 3 $E_3(P_{G_3}) = d_3 + e_3 P_{G_3} + f_3 P_{G_3}^2 \ tonCO_2/h$

$$= 40 + 0.1P_{G_3} + 0.0005 * P_{G_3}^2$$

<u>Converting the Emission function into the Cost function considering carbon tax($\$/tonCO_2$) for CO_2 in $\$/h$</u>

Cost function of Unit 1 $C_1'(P_{G_1}) = 50 * [d_1 + e_1 P_{G_1} + f_1 P_{G_1}^2] \ \$/h$

$$= 50 * [150 + 0.2P_{G_1} + 0.001 * P_{G_1}^2]$$

$$= 7500 + 10P_{G_1} + 0.05 * P_{G_1}^2 \ \$/h$$

Cost function of Unit 2 $C_2'(P_{G_2}) = 50 * [d_2 + e_2 P_{G_2} + f_2 P_{G_2}^2] \ \$/h$

$$= 50 * [150 + 0.2P_{G_2} + 0.001 * P_{G_2}^2]$$

$$= 7500 + 10P_{G_2} + 0.05 * P_{G_2}^2 \ \$/h$$

Cost function of Unit 3 $C_3'(P_{G_3}) = 50 * [d_3 + e_3 P_{G_3} + f_3 P_{G_3}^2] \ \$/h$

$$= 50 * \left[40 + 0.1P_{G_3} + 0.0005 * P_{G_3}^2\right]$$
$$= 2000 + 5P_{G_3} + 0.025 * P_{G_3}^2 \ \$/h$$

Objective Function

Total Cost function

$$C_i\left(P_{G_i}\right) = 20P_{G_1} + \frac{1}{2} * 0.05 * P_{G_1}^2 + 25P_{G_2} + \frac{1}{2} * 0.10 * P_{G_2}^2 + 40P_{G_3} + \frac{1}{2} * 0.20 * P_{G_3}^2 + 7500 + 10P_{G_1} + 0.05 * P_{G_1}^2 + 7500 + 10P_{G_2} + 0.05 * P_{G_2}^2 + 2000 + 5P_{G_3} + 0.025 * P_{G_3}^2$$
$$= 17000 + 30P_{G_1} + 0.075P_{G_1}^2 + 35P_{G_2} + 0.1P_{G_2}^2 + 45P_{G_3} + 0.125P_{G_3}^2 \ \$/h$$

Constraints

1. $P_G^{min} \le P_{G_1} \le P_G^{max} \quad \rightarrow \quad 20 \le P_{G_1} \le 340$
2. $P_G^{min} \le P_{G_2} \le P_G^{max} \quad \rightarrow \quad 20 \le P_{G_2} \le 300$
3. $P_G^{min} \le P_{G_3} \le P_G^{max} \quad \rightarrow \quad 30 \le P_{G_3} \le 250$
4. $P_{G_1} + P_{G_2} + P_{G_3} = 500$

Inequality constraints

$=\gg \quad g_i(P_i) \le 0$

1. $P_{G_1}^{min} - P_{G_1} \le 0 \quad \rightarrow \quad 20 - P_{G_1} \le 0$
2. $P_{G_2}^{min} - P_{G_2} \le 0 \quad \rightarrow \quad 20 - P_{G_2} \le 0$
3. $P_{G_3}^{min} - P_{G_3} \le 0 \quad \rightarrow \quad 30 - P_{G_3} \le 0$

$=\gg \quad h_i(P_i) \le 0$

1. $\mathrm{P_{G_1}} - \mathrm{P_{G_1}^{max}} \le 0 \quad \rightarrow \quad \mathrm{P_{G_1}} - 340 \le 0$
2. $\mathrm{P_{G_2}} - \mathrm{P_{G_2}^{max}} \le 0 \quad \rightarrow \quad \mathrm{P_{G_2}} - 300 \le 0$
3. $\mathrm{P_{G_3}} - \mathrm{P_{G_3}^{max}} \le 0 \quad \rightarrow \quad \mathrm{P_{G_3}} - 250 \le 0$

Equality constraint

$=\gg \quad \emptyset_i(P_i) = 0$

1. $P.D. - \sum_{i=1}^{3} P_i = 0 \quad \rightarrow \quad 500 - P_{G_1} - P_{G_2} - P_{G_3} = 0$

(h) Add additional lines to your original computer code and implement the problem of part (g), solve itand clearly compare the solution (dispatch, marginal costs, system total emissions etc.) with those you found in part (e) when you only considered fuel costs, and identify what has changed, why and how.(10 marks)

Computer optimization code in Pyomo

```
#Pyomo objects exist within the pyomo.environ namespace
#Every Pyomo model starts with this; it tells Python to load the Pyomo Modeling Environment
from pyomo.environ import *

#Create an instance of a Concrete model
model = ConcreteModel("Economic Load dispatch considering fuel cost as well as emission cost")

#Define desicion variables and limit their values to only real number.
model.PG1 = Var(within = Reals)
model.PG2 = Var(within = Reals)
model.PG3 = Var(within = Reals)

#Define an objective function to minimize the total cost.
model.objective = Objective(expr = 20*model.PG1 + 25*model.PG2 + 40*model.PG3 + 0.025*(model.PG1)**2
                            + 0.05*(model.PG2)**2 + 0.10*(model.PG3)**2 + 10*model.PG1 + 10*model.PG2
                            + 5*model.PG3 + 0.05*(model.PG1)**2 + 0.05*(model.PG2)**2 + 0.025*(model.PG3)**2
                            + 17000 )
#If “sense” is omitted in above objective function, Pyomo assumes minimization objective function.

#Define an equality constraint
model.constraint1 = Constraint(expr = 500 - model.PG1 - model.PG2 - model.PG3 == 0)

#Define an inequality constraints
model.constraint2 = Constraint(expr = model.PG1 - 340 <= 0)       # P1 - 340 <= 0
model.constraint3 = Constraint(expr = -model.PG1 + 20 <= 0)       # -P1 + 20 <= 0
model.constraint4 = Constraint(expr = model.PG2 - 300 <= 0)       # P2 - 300 <= 0
model.constraint5 = Constraint(expr = -model.PG2 + 20 <= 0)       # -P2 + 20 <= 0
model.constraint6 = Constraint(expr = model.PG3 - 250 <= 0)       # P3 -250 <= 0
model.constraint7 = Constraint(expr = -model.PG3 + 30 <= 0)       # -P3 + 30 <= 0

#Solving models
opt = SolverFactory("gurobi")
results = opt.solve(model)

#Display the result
model.display()
```

Output from the Pyomo code

```
Model Economic Load dispatch considering fuel cost as well as emission cost

  Variables:
    PG1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 248.93617021260417 :  None : False : False :  Reals
    PG2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  None : 161.70212765952695 :  None : False : False :  Reals
    PG3 : Size=1, Index=None
        Key  : Lower : Value             : Upper : Fixed : Stale : Domain
        None :  None : 89.36170212786885 :  None : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 42409.574468085106

  Constraints:
    constraint1 : Size=1
        Key  : Lower : Body                  : Upper
        None :   0.0 : 2.842170943040401e-14 :   0.0
    constraint2 : Size=1
        Key  : Lower : Body              : Upper
        None :  None : -91.06382978739583 :   0.0
    constraint3 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : -228.93617021260417 :   0.0
    constraint4 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : -138.29787234047305 :   0.0
    constraint5 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : -141.70212765952695 :   0.0
    constraint6 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : -160.63829787213115 :   0.0
    constraint7 : Size=1
        Key  : Lower : Body              : Upper
        None :  None : -59.36170212786885 :   0.0
```

By using the optimization code, we get the value of $P_{G_1}, P_{G_2}\ and\ P_{G_3}$. None of the Unit generation output is bounded by their upper or lower limits, thus by *complementary slackness condition*, we can say that $\mu_1, \mu_2, \mu_3,\ \gamma_1, \gamma_2, \gamma_3 = 0$ By putting these values in below equation, we get the value of $\lambda = 67.339\ \$/MWh$

System marginal cost

$$\Rightarrow L(P_G, \lambda, \mu, \gamma) = 17000 + 30P_{G_1} + 0.075P_{G_1}^2 + 35P_{G_2} + 0.1P_{G_2}^2 + 45P_{G_3} + 0.125P_{G_3}^2$$
$$+ \lambda\left[500 - P_{G_1} - P_{G_2} - P_{G_3}\right] + \mu_1\left[20 - P_{G_1}\right] + \mu_2\left[20 - P_{G_2}\right]$$

$$+ \mu_3\left[30 - P_{G_3}\right] + \gamma_1\left[\mathrm{P_{G_1}} - 340\right] + \gamma_2\left[\mathrm{P_{G_2}} - 300\right] + \gamma_3\left[\mathrm{P_{G_3}} - 250\right]$$

$$\Rightarrow \frac{\partial L}{\partial P_{G_1}} = \frac{\partial C}{\partial P_{G_1}} - \lambda - \mu_1 - \gamma_1 = 0;$$

$$\Rightarrow \frac{\partial L}{\partial P_{G_1}} = 30 + 0.15\, P_{G_1} - \lambda = 0$$

$$\Rightarrow \boldsymbol{\lambda = 67.339\ \$/MWh}$$

Result-1 with considering only fuel cost	**Result-2 with considering fuel as well as Emission cost**
$\lambda = +38\ \$/MWh$	$\lambda = +67.339\ \$/MWh$
$P_{G_1} = +340\ MW$	$P_{G_1} = +248.93\ MW$
$P_{G_2} = +130\ MW$	$P_{G_2} = +161.70\ MW$
$P_{G_3} = +30\ MW$	$P_{G_3} = +89.36\ MW$
Total minimized cost = $+15075\ \$/h$	Total minimized cost = $+42409.57\ \$/h$
LaGrange Multipliers: $\mu_1 = 0\ \$/MWh$	LaGrange Multipliers: $\mu_1 = 0\ \$/MWh$
LaGrange Multipliers: $\gamma_1 = +1\ \$/MWh$	LaGrange Multipliers: $\gamma_1 = 0\ \$/MWh$
LaGrange Multipliers: $\mu_2 = 0\ \$/MWh$	LaGrange Multipliers: $\mu_2 = 0\ \$/MWh$
LaGrange Multipliers: $\gamma_2 = 0\ \$/MWh$	LaGrange Multipliers: $\gamma_2 = 0\ \$/MWh$
LaGrange Multipliers: $\mu_3 = +8\ \$/MWh$	LaGrange Multipliers: $\mu_3 = 0\ \$/MWh$
LaGrange Multipliers: $\gamma_3 = 0\ \$/MWh$	LaGrange Multipliers: $\gamma_3 = 0\ \$/MWh$
Unit 1 Incremental cost = $\lambda - \gamma_1 = +37\ \$/MWh$	Unit 1 Incremental cost = $\lambda = +67.339\ \$/MWh$
Unit 2 Incremental cost = $\lambda = +38\ \$/MWh$	Unit 2 Incremental cost = $\lambda = +67.339\ \$/MWh$
Unit 3 Incremental cost = $\lambda + \mu_3 = +46\ \$/MWh$	Unit 3 Incremental cost = $\lambda = +67.339\ \$/MWh$

Conclusion: Here, in result-2, we get the optimized generator schedule different as compare to result-1.

- For P_{G_1}, the Unit 1 is not working at its maximum upper limit because the Emission function for Unit 1 says that it produces more $tonCO_2/h$ as compare to other Units which results in high carbon tax ($\$/tonCO_2$) cost. Thus, to reduce the cumulative cost, output of Unit 1 is kept lower compare to previous case.
- For P_{G_2}, the Unit 2 is not scheduled as its previous value. If we run Unit 2 at higher output, we find that we get lower total cost for Unit 2 in result-2 as oppose to the total cost for Unit 2 in result-1. To reiterate, we can say that in Unit 2 case, the cost of Emission is more dominant over the cost of fuel which is comparatively higher. So ultimately, we get minimum total cost by increasing Unit 2 output.
- For P_{G_3}, the Unit 3 is not working at its minimum lower limit because the Emission function for Unit 3 says that it produces less $tonCO_2/h$ as compare to other Units which results in low carbon tax ($\$/tonCO_2$) cost. So, to reduce the cumulative cost, output of Unit 3 is kept higher compare

to previous case.

- The significance of λ says that the cost needed to generate one more next MW. Here, the high value of λ in result-2 says that the cost needed to generate one more next MW considering the fuel cost as well as Emission cost which results in high cost for one MW output.
- Here in result-2, out result for $P_{G_1}, P_{G_2}, P_{G_3}$ says that none of the units is running at their upper or lower limit. So, by *complimentary slackness conditions* as described in (b), we can say that $\mu_1, \mu_2, \mu_3,\ \gamma_1, \gamma_2, \gamma_3 = 0$.
- Unit incremental cost for all three Units in result-2 is same as none of the Unit hits the Upper maximum or lower minimum limit. So, the added cost to generate the next one MW is same as system marginal cost λ regardless of which Unit is selected to generate next MW. So, for all three Units, each unit incremental cost is same as the system marginal cost.

(i) Determine the annual emission and economic saving/cost to the society should the carbon policy move forward. (5 marks)

Answer:

		Fuel Cost ($/year)	Emission cost + Fuel Cost ($/year)	Emission cost ($/year)	Emission (tonCO2/year)
P1	340	13,20,57,000	38,16,90,720	24,96,38,100	49,92,762
P2	130				
P3	30				
P1	248.94	14,23,57,358	37,15,07,833	22,91,50,474	45,83,009
P2	161.7				
P3	89.36				
Total saving/Loss over the year		1,03,00,358 Loss	1,01,82,887 Profit	2,04,87,626 Profit	4,09,753 Profit

Annual Emission saving $= \mathbf{4,09,753}\ \boldsymbol{tonCO_2}\ /\ \boldsymbol{year}$

Annual Emission cost saving $= \mathbf{2,04,87,626}\ \$\ /\ \boldsymbol{year}$

Annual Economic cost saving $= \mathbf{1,01,82,887}\ \$\ /\ \boldsymbol{year}$

To sum up by looking above figures, policy should be moved forward by everyone. Society would gain an economic cost saving of around $= \mathbf{10.18}\ \boldsymbol{million}\ \$$ by implementing the carbon tax over the year period.

(j) Would you support this policy? Explain your position- this could be a hypothetical position and could be irrelevant to your personal view- no judgment. Back your argument with the numbers you found here as well as other economic and social factors. Make sure to locate current federal and provincial carbon policies as well as international climate commitments in Canada and make them as part. Cite your references using an IEEE reference style format. (25 marks)

Would you support this policy?

From the above results in question number (i), when we consider the carbon tax, we end up spending around $= 10.3\ million\ \$\ /\ year$ more on fuel, however; on the other hand, we save around $= 20.4\ million\ \$\ /\ year$ on carbon emission tax. Considering the environment impacts at global level, carbon emission would be lessened by $4{,}09{,}753\ tonCO_2$ over the year. After considering the above numbers, **I would definitely support the carbon policy.**

Economic factors on Pricing Pollution

This section looks at the economic effects of the federal carbon pricing system under a scenario in which it is put in place in the nine jurisdictions that are not currently pricing carbon pollution in Canada. As noted above, this is a hypothetical scenario that we do not expect to occur; however, it provides an illustration of the potential effects of pricing carbon pollution in Canada [1].

Carbon pricing will make a significant contribution towards meeting Canada's greenhouse gas reduction target. A price on carbon could cut carbon pollution across Canada by 80 to 90 million tonnes in 2022, once all provinces and territories have systems that meet the federal standard. This is equivalent to taking 23-26 million cars off the road for a year or shutting down 20-23 coal-fired power plants for a year.Footnote2 Without this contribution, more costly regulatory interventions would be needed to meet our target [8].

GDP growth would remain strong with pan-Canadian carbon pricing. Applying the federal carbon pricing system to the nine provinces and territories that are not pricing carbon pollution today would not be expected to have any significant impact on national economic growth rates in the context of a more than $2 trillion economy. It is also likely to stimulate innovation, investments in clean technology and benefit long-term growth opportunities, although these benefits are not included in the modelling analysis [8].

Impacts on GDP

The federal carbon pricing system is not expected to have any significant impact on national economic growth rates. Between 2018 and 2022, the application of the federal carbon pollution pricing system in the nine jurisdictions that do not currently have their own regimes in place is estimated to impact average annual real GDP growth rates for Canada by less than one tenth of one percentage point. The difference in GDP in 2022 would amount to about $2 billion, or 0.1% of GDP. The average annual outlook for real GDP growth over this period would be 1.8%, either with or without federal carbon pricing. The model used to develop this estimate accounts for changes to provincial and territorial production and consumption patterns, interjurisdictional trade across Canada, and international imports and exports as a result of carbon pricing. This finding is consistent with previous analyses by governments and external experts [2].

Social factors on Pricing Pollution

Carbon pricing is a critical element of Canada's clean growth and climate plan and a major part of meeting our national target. However, it was never intended to be the only policy measure in the plan to reduce greenhouse gas emissions. Relying exclusively on a carbon price to achieve Canada's emission targets would require a very high price. Instead, Canada's climate plan includes complementary policies and investments that work in concert with carbon pollution pricing to reduce emissions across the economy. As Canada's Eco fiscal Commission has pointed out, complementary measures are particularly important to target emissions that cannot be covered by carbon pricing and to boost the price signal in certain markets [2].

These complementary measures will help make carbon pricing more effective, create incentives for innovation and clean growth, and will reduce the cost to Canadians. For example, energy efficiency standards, building code improvements, zero-emission vehicle policies, and investments in public transit all help Canadians use less energy and make cleaner choices that can save them money, including by reducing their exposure to carbon prices. Investments in clean technology and innovation help accelerate development of the next generation of technologies and ideas that will further improve efficiencies and lower emissions in the future [2].

The federal government has committed to return all direct revenues from the federal carbon pollution pricing system to the jurisdiction of origin. Around the world, governments use carbon pricing revenues for various purposes, including reducing business or individual taxes, helping businesses and households invest in energy efficiency, building transit and other infrastructure, and offsetting costs incurred by low-income households or other vulnerable groups. Canada's Eco fiscal Commission recommends a portfolio of approaches in order to address fairness and competitiveness concerns [2].

Governments can also use revenues to ensure that low-income households are protected. For example, in Alberta about 60 percent of households are eligible for a full or partial rebate on the province's carbon levy. A household with two adults and two children would be eligible to receive a rebate of up to $540 per year in 2018, more than the estimated cost of the levy for this size of household [2].

The federal government's commitment to ensuring all revenue from pricing pollution remains within the jurisdiction enables provinces and territories to best design the system for their economy and households [2].

Current federal and provincial carbon policies

Canada's approach is flexible: any province or territory can design its own pricing system tailored to local needs, or can choose the federal pricing system. The federal government sets minimum national stringency standards (the federal 'benchmark'), that all systems must meet to ensure they are comparable and effective in reducing greenhouse gas emissions. If a province or territory decides not to price pollution, or proposes a system that does not meet these standards, the federal system is put in place. This ensures consistency and fairness for all Canadians [3].

The federal pricing system has two parts: a regulatory charge on fossil fuels like gasoline and natural gas, known as the fuel charge, and a performance-based system for industries, known as the Output-Based Pricing System. The federal fuel charge applies in Ontario, Manitoba, Yukon, Alberta, Saskatchewan and Nunavut [3].

The federal Output-Based Pricing System applies in Ontario (until December 31, 2021), Manitoba, Prince Edward Island, Yukon, Nunavut, and partially in Saskatchewan. The Government of Canada has confirmed that the carbon pollution pricing systems in Quebec, Nova Scotia, Newfoundland and Labrador, the Northwest Territories, and British Columbia continue to meet the federal benchmark stringency requirements, and as of 2021, New Brunswick has a carbon pollution pricing system that also meets the benchmark requirements. Provincial systems in place in Prince Edward Island, Alberta, and Saskatchewan meet the federal benchmark stringency requirements for the emission sources they cover. The federal backstop applies in these provinces to emission sources not covered by the provincial systems. In Prince Edward Island, the federal OBPS applies alongside the provincial fuel charge. In Alberta and Saskatchewan, provincial output-based performance standards systems apply alongside the federal fuel charge. As of January 1, 2022, Ontario's provincial output-based pricing system will apply alongside the federal fuel charge [3]. The four provinces with carbon pricing systems in place – British Columbia, Alberta, Ontario and Quebec – cover 80 percent of Canada's population and were also the top four performers in GDP growth across the country in 2017 [2].

International climate commitments

On December 12, 2015, Canada and 194 other countries reached the Paris Agreement, an ambitious and balanced agreement to fight climate change. This new Agreement will strengthen the effort to limit the global average temperature rise to well below 2°C and pursue efforts to limit the increase to 1.5°C [5].

- Coal-fired electricity is responsible for 20 per cent of global greenhouse emissions [6].
- Moving away from coal will improve overall public health by creating cleaner, more breathable air. A recent analysis found that more than 800,000 people around the world die each year from the pollution generated by burning coal [6].
- Canada's electricity generation mix is already one of the cleanest in the world. But by phasing out coal-fired electricity, Canada will cut carbon pollution by nearly 13 million tonnes in 2030, representing a significant step toward reaching Canada's national target of reducing emissions by 40 to 45 per cent below 2005 levels by 2030 [6].
- Canada is the fourth largest producer and third largest exporter of oil in the world. The oil and gas sector are the largest contributor to Canada's greenhouse gas emissions, accounting for about 25 per cent of total emissions [6].
- In 2020, Canada released its strengthened climate plan, A Healthy Environment and a Healthy Economy, to accelerate emissions reductions and build a stronger, cleaner, more resilient and inclusive economy, putting in place some of the actions Canada will take to reach to net-zero emissions by 2050 [6].
- The Climate Investment Funds Accelerated Coal Transition Investment Program will support both public sector utilities and private sector operators with the relevant toolkit for transition, as appropriate and consistent with national priorities and Nationally Determined Contributions [6].
- The World Bank Energy Sector Management Assistance Program provides support to low- and middle-income countries to encourage low carbon development through a wide range of sustainable energy solutions [6].

Conclusion

Pricing carbon reduces pollution at the lowest cost to businesses and consumers while driving innovative ways to enhance energy efficiency and cleaner energy sources. This analysis shows that the adoption of carbon pricing at the federal standard throughout Canada will reduce greenhouse gas pollution by 80 to 90 million tonnes in 2022 while our economy continues to grow. The clean growth and innovation spurred by pricing pollution will help position Canada for success in the economy of the 21st century [2].

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