UNIVERSITY OF CALGARY

ENEL 693 Restructured Electricity Market

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Transmission Homework

Note: Please type your answers in the spaces provided. No handwriting. Also, please show the detail of your work. Please name the fine lastname_firstname.pdf and submit through D2L.

Consider a simple 3-zone power system with four generation centers and three load centres, as summarized in the following tables:

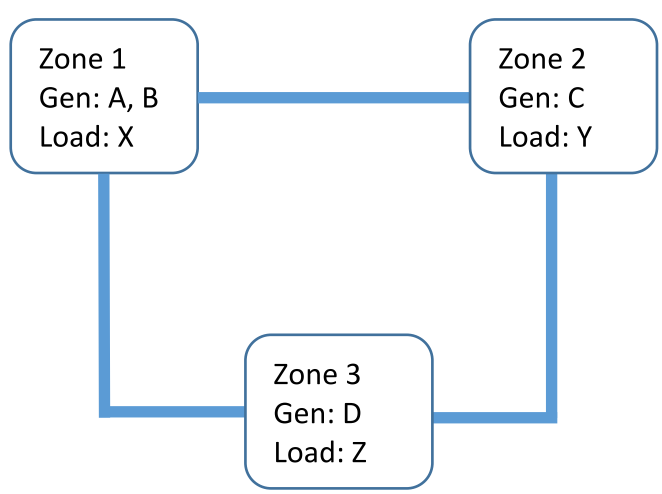
Generation center	MC ($/MWh)	Max Power (MW)	Min Power (MW)	Location
A	7.5	140	0	Zone 1
B	10.5	285	0	Zone 1
C	14	90	0	Zone 2
D	20	85	0	Zone 3

Load centre	Demand (MW)	Location
X	100	Zone 1
Y	100	Zone 2
Z	300	Zone 3

The three zones are connected to each other by three transmission corridors with the following characteristics:

Corridor connecting:	Reactance (x) in pu on a 100 MVA base	Max flow allowed (MW)
Zone 1 to Zone 2	0.2	300
Zone 1 to Zone 3	0.1	200
Zone 2 to Zone 3	0.2	300

A simple presentation of the system is shown below:



Without the use of a computer code, or a formal DC power flow calculation, and by only and only using the principles of superposition and current division, clearly answer the following questions.

a) **Determine the most economic dispatch when ignoring the line limits (1 marks).**

Cost function of each generator shown below.

- $C_p(A) = 7.5\ \$/MWh$
- $C_p(B) = 10.5\ \$/MWh$
- $C_p(C) = 14\ \$/MWh$
- $C_p(D) = 20\ \$/MWh$

Generator power limit constraints.

- $0 \leq P_1 \leq 140\ MW$
- $0 \leq P_2 \leq 285\ MW$
- $0 \leq P_3 \leq 90\ MW$
- $0 \leq P_4 \leq 85\ MW$

Our total system demand $= 500\ MW$.
The most economic dispatch solution without considering the line limit would be…

$\boldsymbol{P_1 = 140\ MW}$ $\boldsymbol{P_2 = 285\ MW}$ $\boldsymbol{P_3 = 75\ MW}$ $\boldsymbol{P_4 = 0\ MW}$

$$\text{Total Cost} = 140 * 7.5 + 285 * 10.5 + 75 * 14$$
$$= 5092.5\ \$/h$$

Here as shown in the *Figure-1*, cheapest generator generates the maximum power.

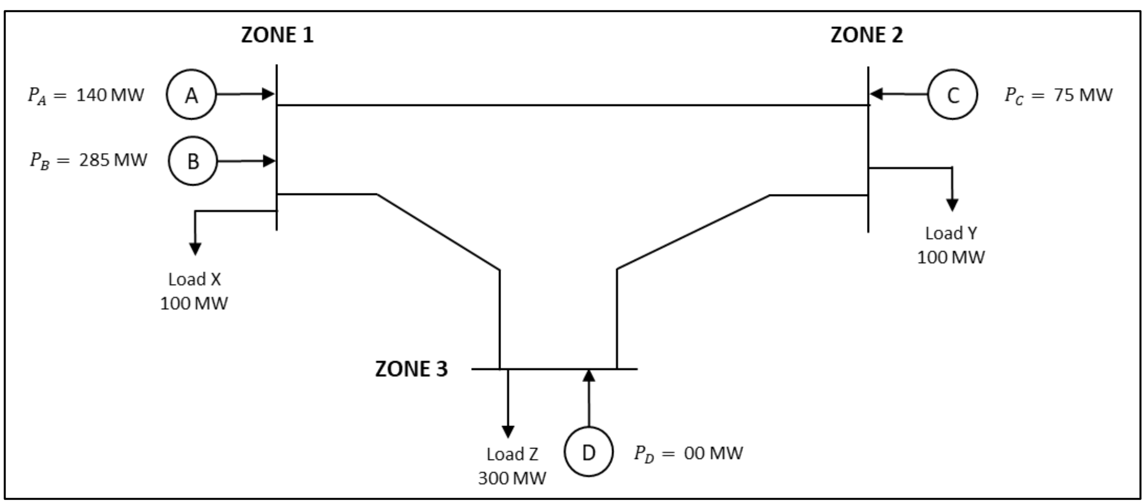


Figure-1: Economic dispatch solution ignoring the line limits

b) Determine the locational (zonal) marginal prices for part a (1 mark).

Here, cost of supplying additional MW of load at zone-1, zone-2 and zone-3 is set by the marginal price of Unit-C without considering the line loading limit. Here, Unit-D has higher marginal price compare to Unit-C and Unit-A and B are running at their maximum output capacity. So, addition one MW would be generated by Unit-C.

So, the marginal price of Unit-C is $\mathbf{14}\ \$/\boldsymbol{MWh}$ which is the locational marginal price for all node.

c) Determine the line flows for your dispatch and verify if it meets the flow limits (2 marks).

Figure-2 shows the Economic dispatch without considering the line limits.

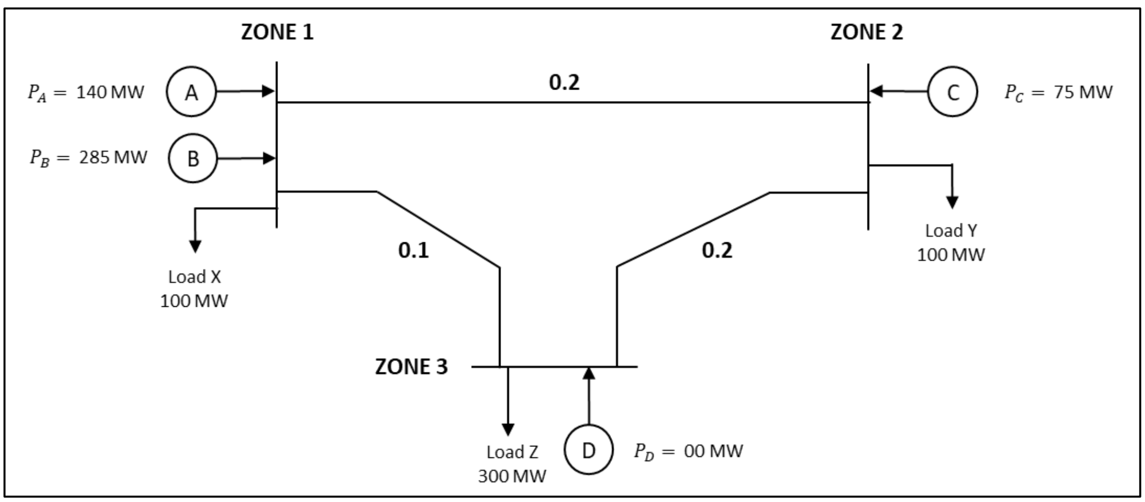


Figure-2: Economic dispatch ignoring the line limits

As shown in *Figure-3*, by using the super-position theorem, P_A and P_B units generate $400\ MW$ and dispatch a generation to Load X and Load Z. However, in *Figure-4*, by using super-position theorem, P_B and P_C units generate $100\ MW$ and dispatch a generation to Load Y.

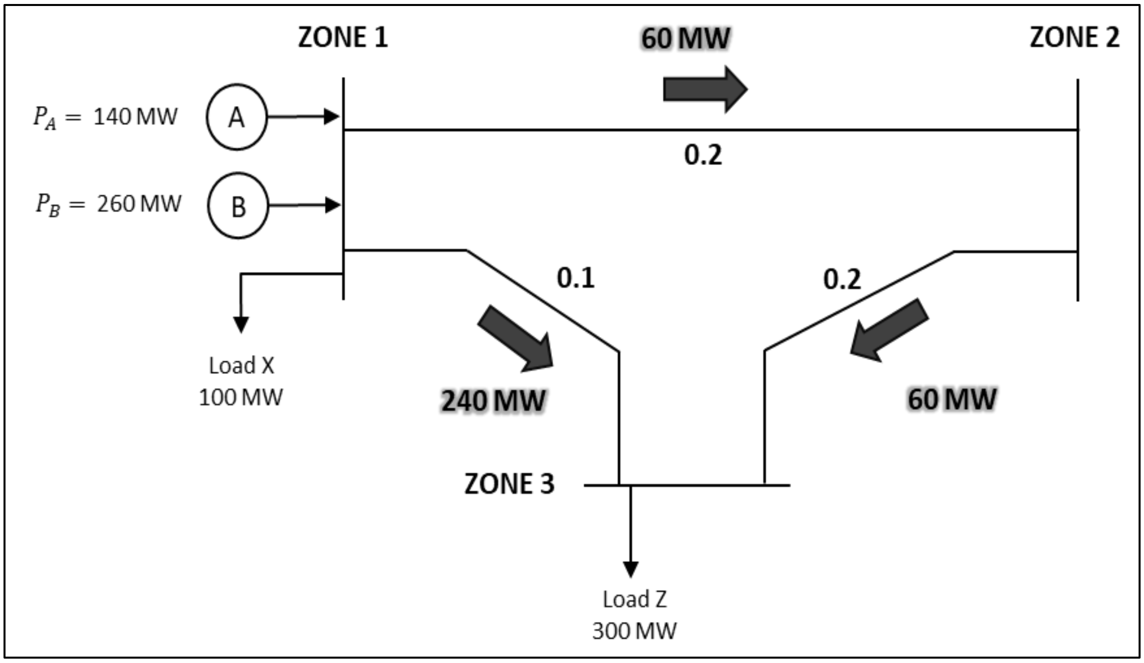


Figure-3: Case-1

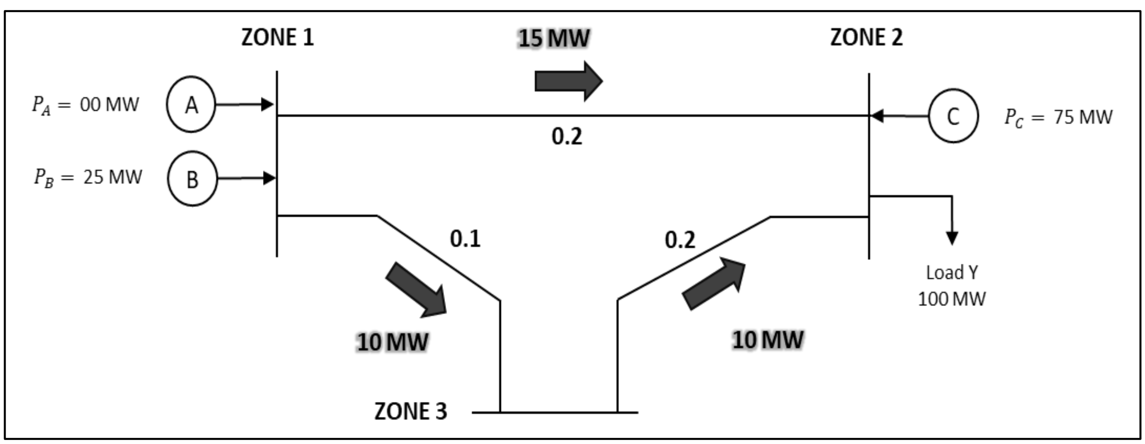


Figure-4: Case-2

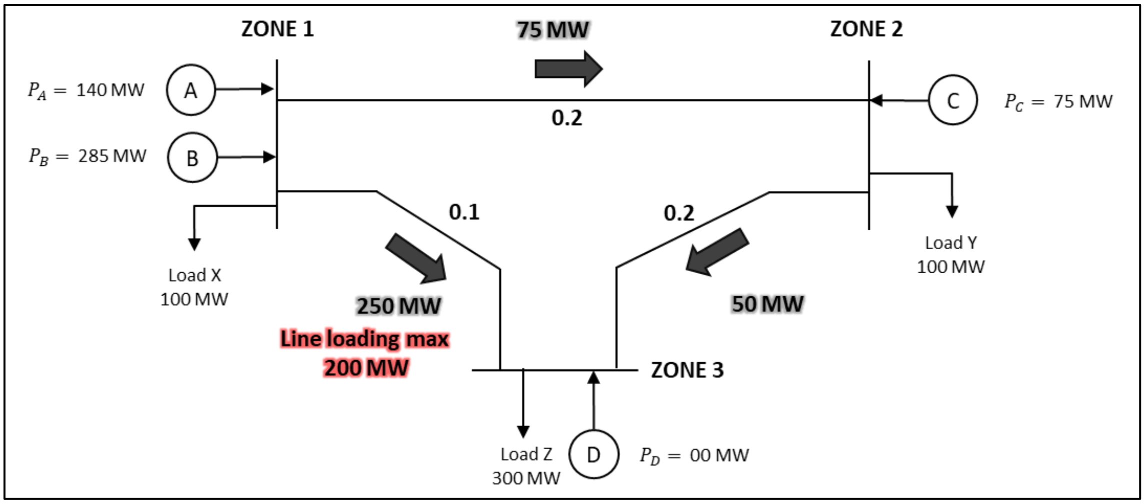


Figure 5: Case-1 + Case-2

Figure-5 shows the Economic dispatch without considering the line loading limit in which Line 1-3 overload by $50\ MW$. On the other hand, Line loading for Line 1-2 and Line 2-3 remain within limit.

d) If the flows violate the limits, determine the most economic dispatch to supply the load and yet meeting the line flow limits (6 marks). Similar to the example in the book/slides, you need to show the details of your solution- no need for additional graphs.

As shown in the *Figure-5*, the line 1-3 overload by ***50 MW***. Here, we will consider two cases, in which we will increase the generation of Unit-C and Unit-D respectively by reducing the generation of Unit-B as it is costlier than Unit-A.

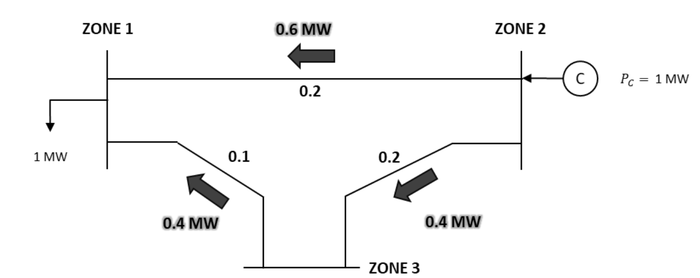


Figure 6: Addition of 1 MW at Unit-C

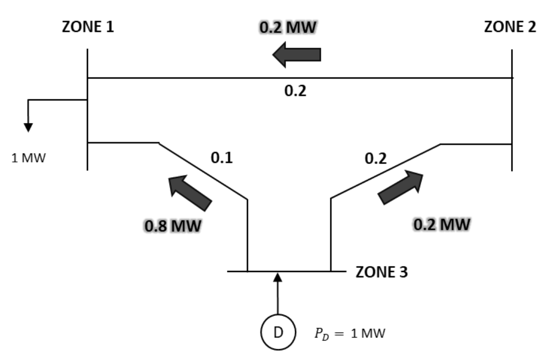


Figure 7: Addition of 1 MW at Unit-D

Above *Figure-6* and *Figure-7* shows the effect on transmission line loading due to 1 MW addition in output of Unit-C and Unit-D respectively.

Case-1: Let's add $62.5\ MW$ generation at Zone-3 by Unit-D in the most economical scenario as shown in *Figure-8* and *Figure-9.*

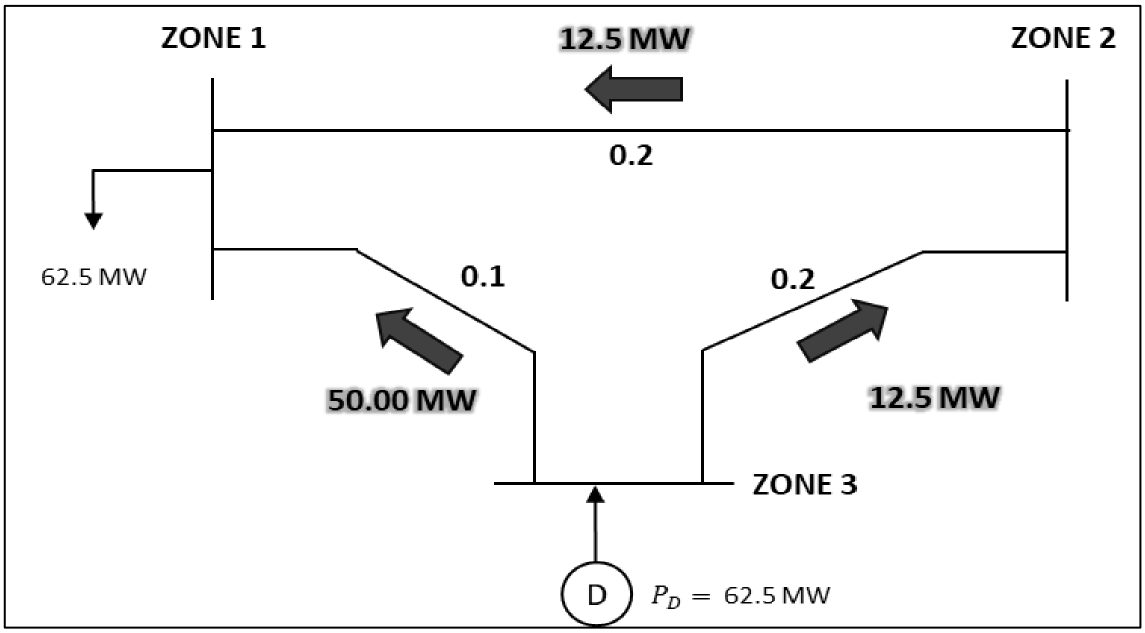


Figure 8: Adding generation at Zone-3 by Unit-D

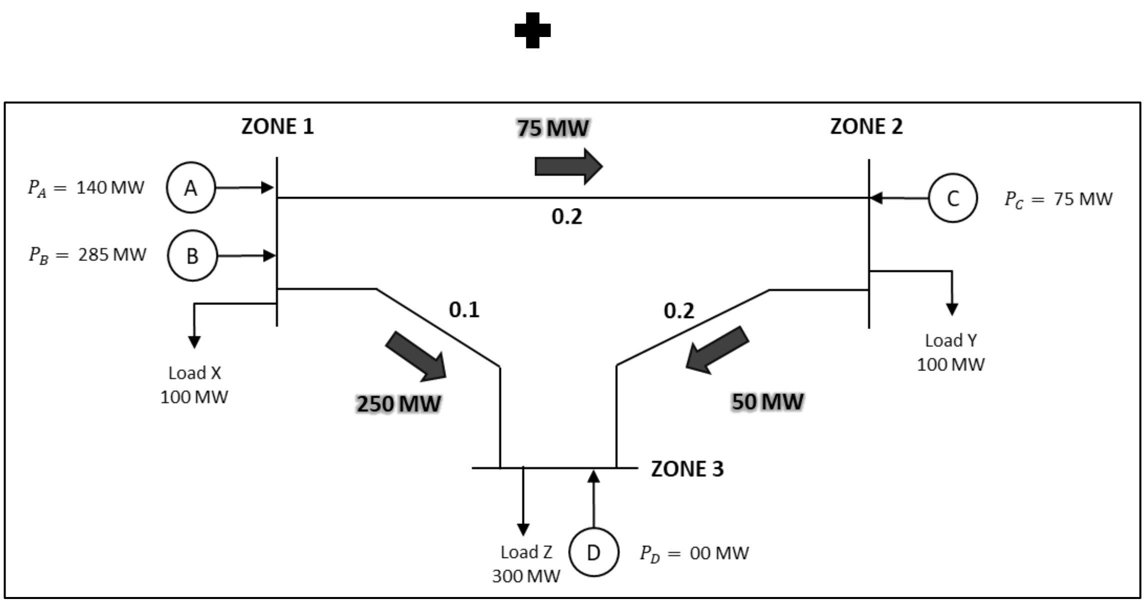


Figure 9: Most Economic scenario

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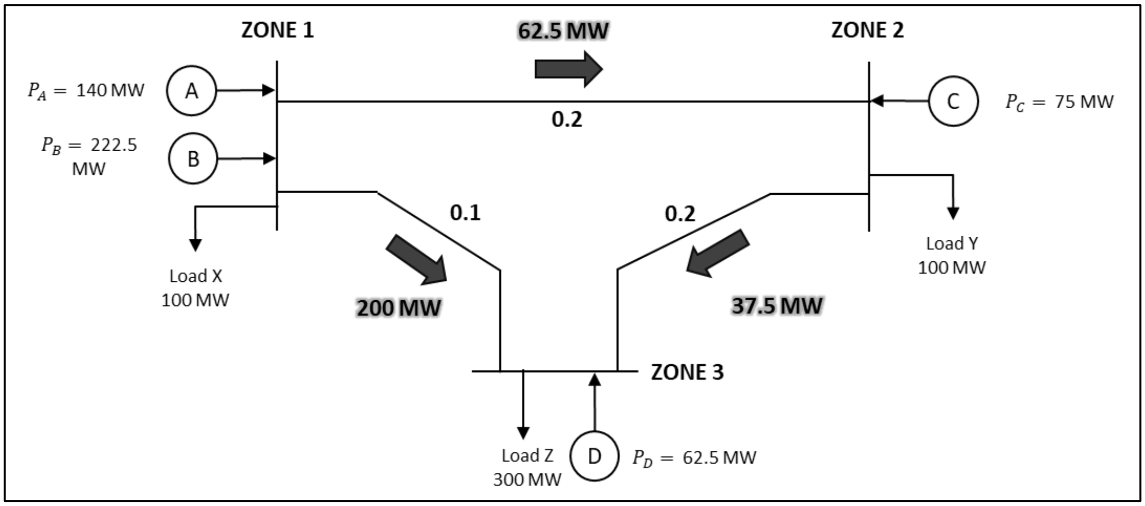

Figure 10 : Case-1 solution

Figure-10 gives the first solution which mitigate a line overloading constraint.

Unit Generation

- $P_A = 140\ MW$
- $P_B = 222.5\ MW$
- $P_C = 75\ MW$
- $P_D = 62.5\ MW$

Cost function

- $C_P(A) = 7.5\ \$/MWh$
- $C_P(B) = 10.5\ \$/MWh$
- $C_P(C) = 14\ \$/MWh$
- $C_P(D) = 20\ \$/MWh$

$$\text{Total cost of generation} = P_A * C_P(A) + P_B * C_P(B) + P_C * C_P(C) + P_D * C_P(D)$$

$$= 140 * 7.5 + 222.5 * 10.5 + 75 * 14 + 62.5 * 20$$

$$= 5686.25\ \$$$

Case-2: Let's add additional 15 MW (Maximum capacity) generation at Zone-2 by Unit-C as shown in *Figure-11* and by adding 17.5 MW generation at Zone-3 by Unit-D as shown in *Figure-12* in Most Economic case given in *Figure-13.*

We get the Output as shown in *Figure-14.*

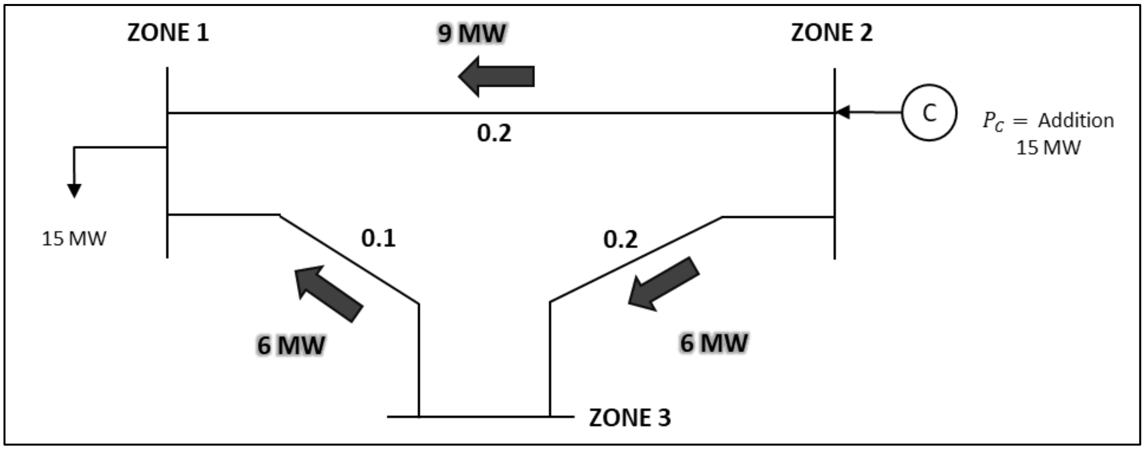

Figure 11: Additional 15 MW generation at Unit-C (Zone-2)

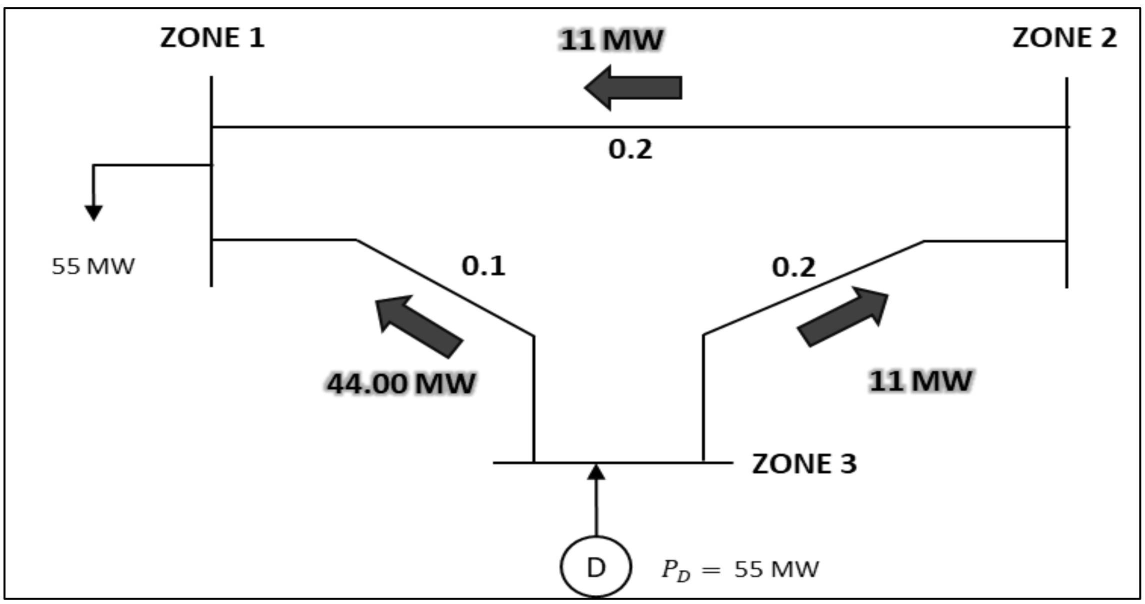

Figure 12: Adding generation of 55 MW by Unit-D (Zone-3)

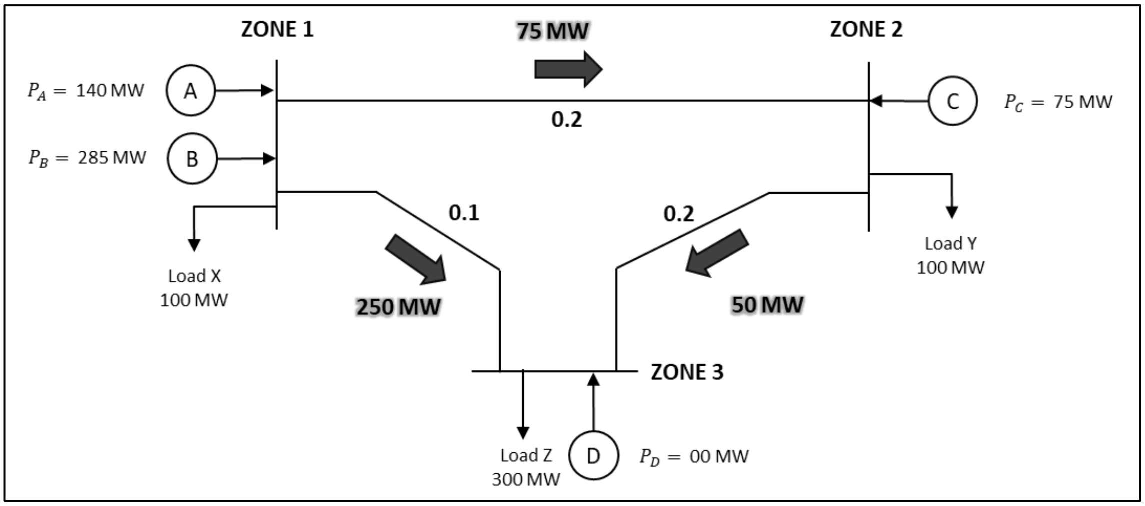

Figure 13: Most Economic scenario without considering line constraints

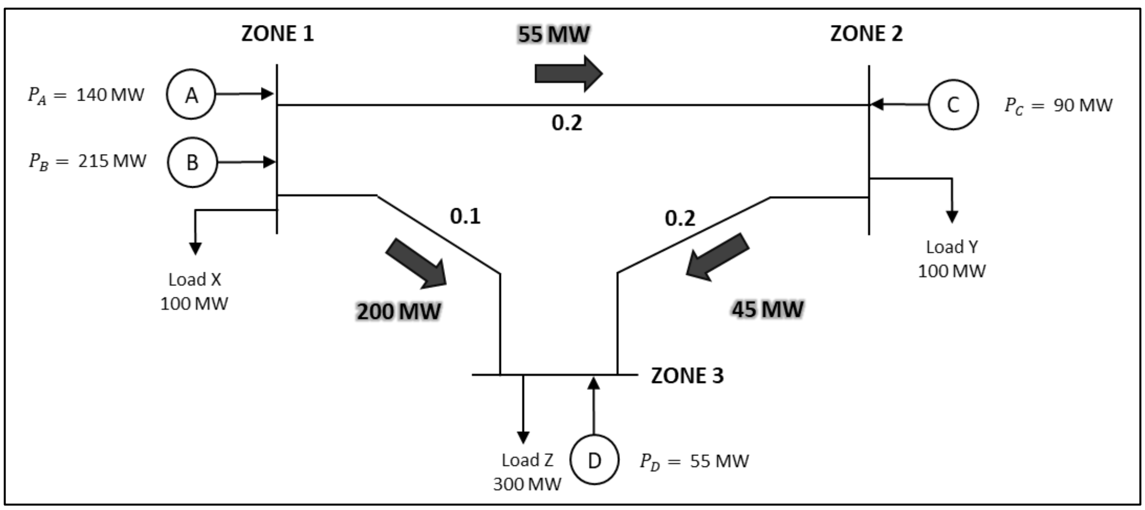

Figure 14: Case-2 solution

Figure-14 gives the second solution which mitigate a line overloading constraint.

Unit Generation

- $P_A = 140\ MW$
- $P_B = 215\ MW$
- $P_C = 90\ MW$
- $P_D = 55\ MW$

Cost function

- $C_P(A) = 7.5\ \$/MWH$
- $C_P(B) = 10.5\ \$/MWH$
- $C_P(C) = 14\ \$/MWH$
- $C_P(D) = 20\ \$/MWH$

$$\text{Total cost of generation} = P_A * C_P(A) + P_B * C_P(B) + P_C * C_P(C) + P_D * C_P(D)$$

$$= 140 * 7.5 + 215 * 10.5 + 90 * 14 + 55 * 20$$

$$= 5667.5 \$$$

Economical Dispatch: 5092.5 \$ from the question (a)
Case-1 Dispatch cost: 5686.25 \$
Case-2 Dispatch cost: 5667.5 \$

After calculating both cases, we get the most economic output in ***case-2*** as shown in *Figure-14* considering the transmission line loading limit.

So, our cost of security would be: $5667.5 - 5092.5 = 575$ \$

e) For your solution in part (d), determine zonal marginal prices, again only by using the principles of superposition and the definition of price, similar to the example in the slides/book (6 marks).

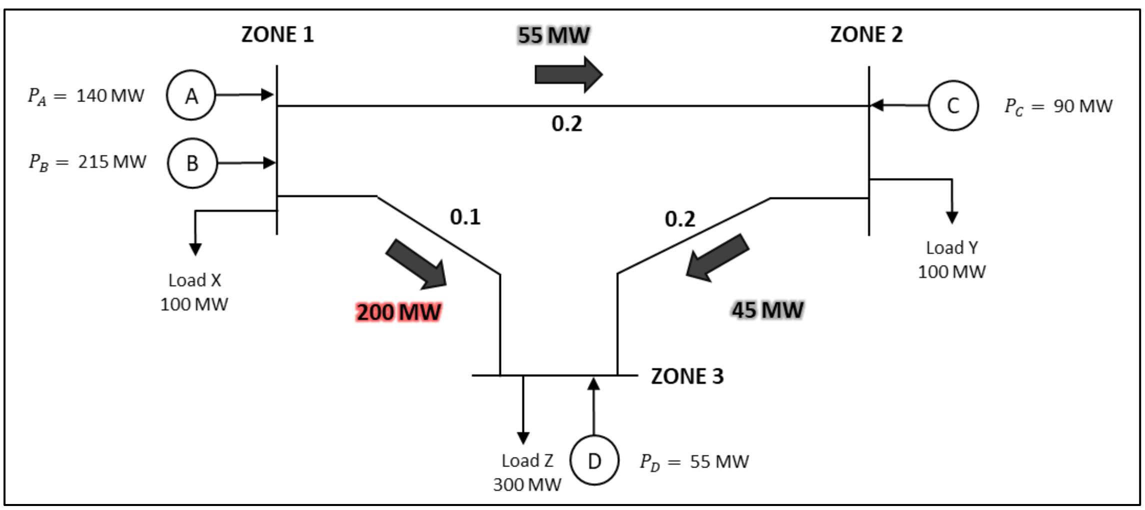

Figure 15: Economical dispatch considering the line limits

➢ <u>**Zonal marginal price at Zone-1:**</u>

At zone-1, Unit-A is running at maximum capacity and Unit-B is running at 215 MW. However, Unit-B is the cheapest generator among the Unit-C and Unit-D. So zonal marginal price at zone-1 $= \pi_1 = MC_B = 10.5\ \$/MWh$

➢ **Zonal marginal price at Zone-2:**

Here, Unit-C and Unit-A are running at its maximum capacity. Unit-B is cheaper than Unit-C and Unit-D.

Thus, adding 1 MW at Zone-1 by Unit-B would result in line 1-3 overloading as shown in *Figure-16* as Line 1-3 is already loaded at its maximum line loading.

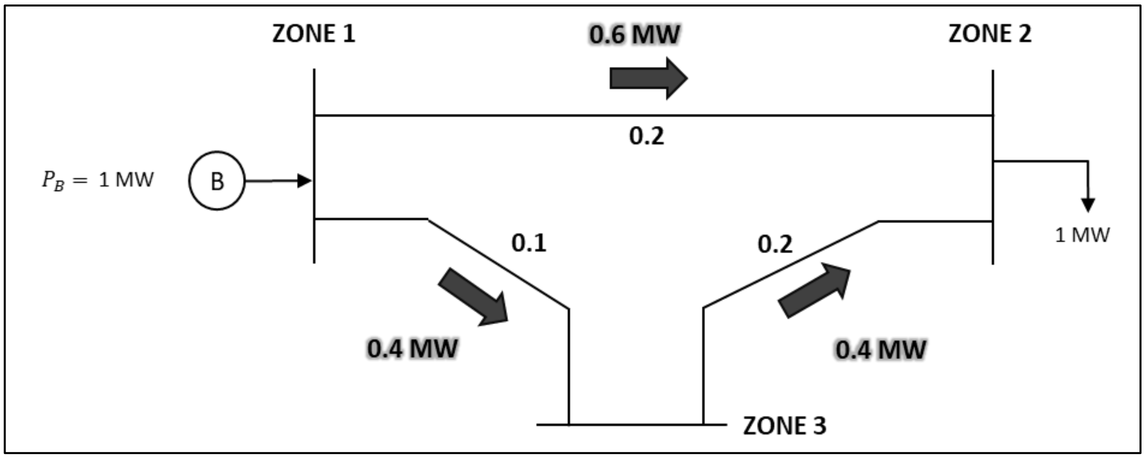

Figure 16: Adding 1 MW at Zone-1 by Unit-B

Now, by adding 1 MW at Zone-3 by Unit-D would result as shown in below *Figure-17.*

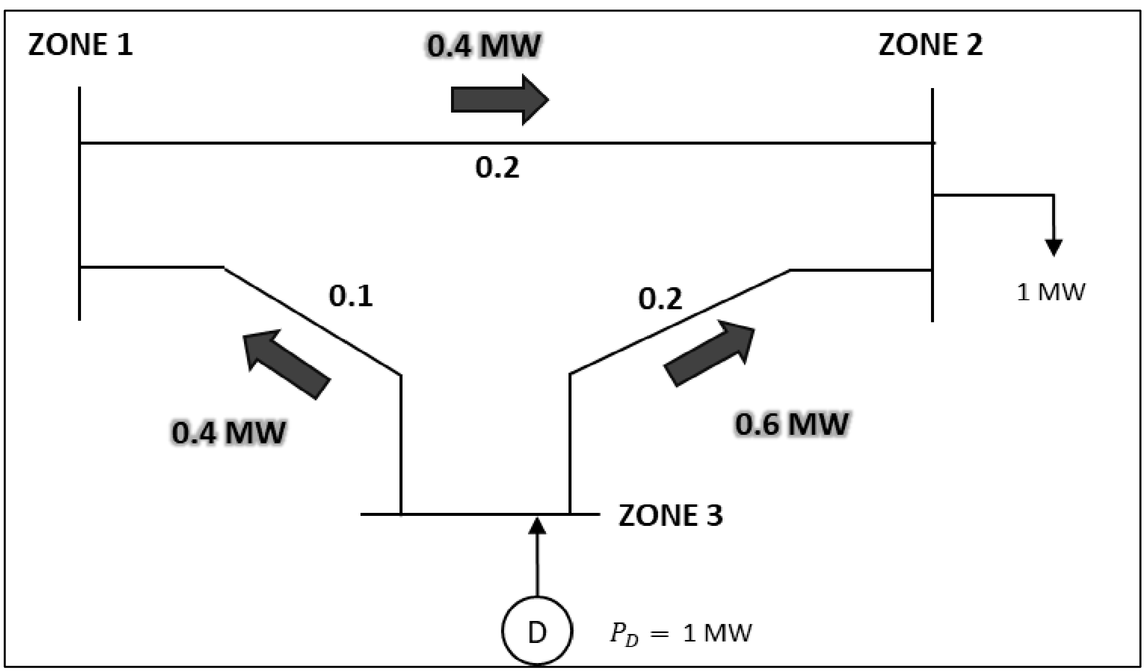

Figure 17: Adding 1 MW generation by Unit-D at Zone-3

So, to get economical zonal price, by adding 0.5 MW at Zone-1 by Unit-B and 0.5 MW at Zone-3 by Unit-D would not result in line overloading. Final result is shown in *Figure-18.*

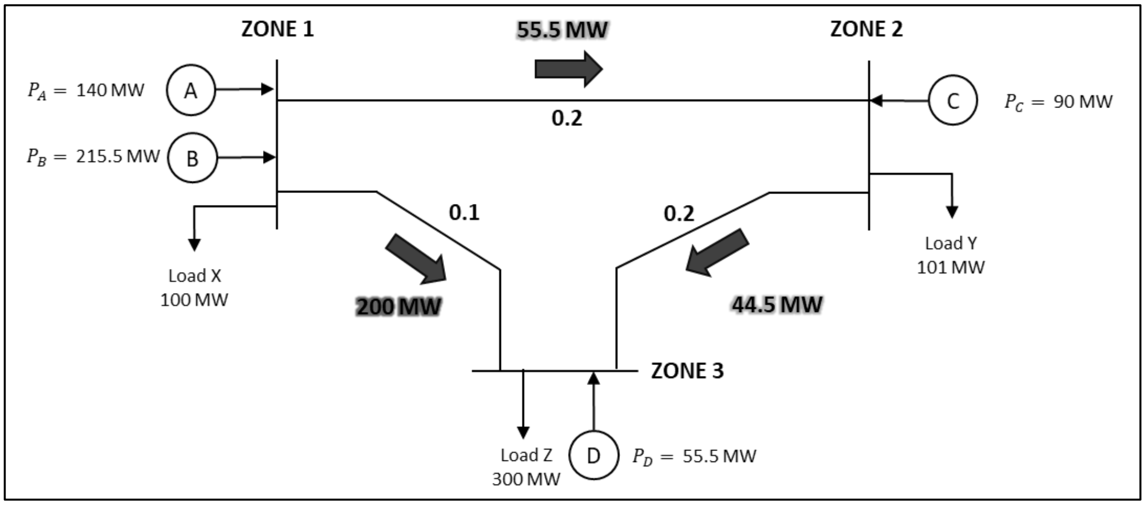

Figure 18: Adding 0.5 MW at zone-1 and zone-3

- Net change in Line 1-3 loading $= \ 0.5 * \Delta P_B - \ 0.5 * \Delta P_D = 0 \ MW$.
- Net change in generation $= \ \Delta P_B + \ \Delta P_D = 1MW$.

Where, $\Delta P_B = 0.5 \ MW \ and \ \Delta P_D = 0.5 \ MW$.

So, zonal marginal price at Zone-2 is $\pi_2 = 0.5 * MC_B + 0.5 * MC_D = 15.25 \ \$/MWh$.

- **<u>Zonal marginal price at Zone-3:</u>**

Here, Unit-C and Unit-A are running at its maximum capacity. Unit-B is cheaper than Unit-D.

Adding 1 MW at Zone-1 by Unit-B would result in line 1-3 overloading.

Thus, increasing 1 MW generation at Zone-3 by Unit-D.

Our zonal marginal price for Zone-3 will be…

$$Zonal \ marginal \ price \ at \ zone - 3 = \ \pi_3 = MC_C \ = 20 \ \$/MW.$$