Optimal Power Flow Project
220 marks

Consider the following system:

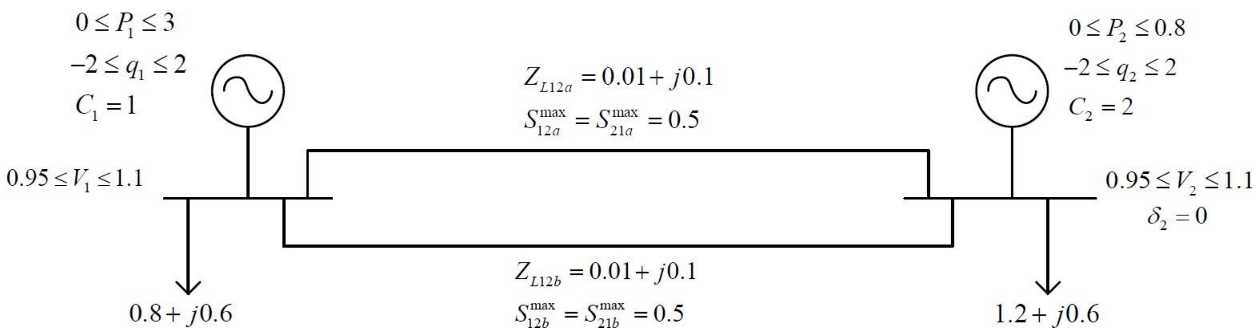


Figure 1: Two Node System

Report Tasks:

(a) Write the corresponding fully detailed formulation for ACOPF. Specify equations to the network in Figure 1 - no generic equations. Clearly name and specify your variables and parameters. Label all your equations and describe what each equation does. (20 marks)

Here, a power network with 2 nodes and two transmission lines are given as shown in figure above. Both the lines have same admittance. Thus,

Using the series impedance of transmission lines, we first compute the series admittances in per unit and radians as follows:

$$Y = \frac{1}{Z} = \frac{1}{0.01 + 0.1j} = Y_{\text{L12}}\angle\theta_{\text{L12}} = Y_{\text{L21}}\angle\theta_{\text{L21}} = 9.95\angle - 1.47^{r} = 0.99 - 9.9j = 9.95\angle - 1.47^{r}$$

We consider the following data,

1. Node 1 includes a generating unit with lower and upper bounds of active power generation equal to 0 and 3 puW, respectively, and of reactive power generation equal to -2 $puvar$ and 2 $puvar$, respectively.
2. Node 2 includes a generating unit with lower and upper bounds of active power generation equal to 0 and 0.8 puW, respectively, and of reactive power generation equal to -2 $puvar$ and 2 $puvar$, respectively.
3. The lower and upper limits of voltage magnitudes for two nodes are 0.95 puV and 1.10 puV, respectively.
4. The marginal production costs of generating units at nodes 1 and 2 are $\$1/puWh$ and $\$2/puWh$, respectively.
5. The line loading capacity of each transmission line between 1–2 (in both directions) is $S_{12a}^{max} = S_{21a}^{max} = S_{12b}^{max} = S_{21b}^{max} = 0.5\ puVA$.

The OPF problem formulation is provided below.

On the one hand, the objective$(in\ \$/h)$ is derived from the given incremental cost of each generator:

$\min_{P_1;P_2;Q_1;Q_2;v_1;v_2;\delta_1;\delta_2;}$

$$P_1 + 2\,P_2$$

Here, our objective function above is linear.

On the other hand, the constraints of the OPF problem are provided and briefly described below.

Active & Reactive Power Limits

Any generating unit g that is online can produce active power above and below a lower and an upper limit, respectively, i.e.:

$$P_g^{min} \le P_g \le P_g^{max}$$

Similarly, for reactive power limit for generator g;

$$Q_g^{min} \le Q_g \le Q_g^{max}$$

where:

- P_g^{min} $(in\ puW)$ and Q_g^{min} $(in\ puvar)$ are the minimum active and reactive power output limit of generating unit g respectively and
- P_g^{max} $(in\ puW)$ and Q_g^{max} $(in\ puvar)$ are the maximum active and reactive power output limit of generating unit g respectively.

Active power limits for generator 1 and 2 are$(in\ puW)$:

Constraint 1: $0 \le P_1 \le 3$

Constraint 2: $0 \le P_2 \le 0.8$

Reactive power limits for generator 1 and 2 are$(in\ puvar)$:

Constraint 3: $0 \le Q_1 \le 0.6$

Constraint 4: $0 \le Q_2 \le 0.6$

Voltage Magnitude Limits

The voltage magnitude of any node i throughout the network is bounded above and below. Therefore:

$$v_i^{min} \le v_i \le v_i^{max}$$

where v_i^{min} and v_i^{max} are, respectively, the lower and upper limits of the voltage magnitude at node i.

Voltage magnitude limits for node 1 and node 2 are $(in\ puV)$:

Constraint 5: $0.95 \le v_1 \le 1.1$

Constraint 6: $0.95 \le v_2 \le 1.1$

Voltage Angle Limits

Finally, the voltage angles should be within given bounds, i.e.:

$$\delta_{min} \le \delta_i \le \delta_{max}$$

Voltage angle limits for generator 1 is($in\ radian$):

Constraint 7: $$-\pi \le \delta_1 \le \pi$$

Voltage angle limits for generator 2 is given in question (reference bus) ($in\ radian$):

Constraint 8: $$\delta_2 = 0^r$$

Active and Reactive Power balance equation at node

The active power $P_{ik}(\cdot)$ flowing from node i to node k through transmission line ik.

$$P_g - P_L = P_{ik}(\cdot) = v_i^2 Y_{\mathrm{L}ik} \cos(\theta_{\mathrm{L}ik}) - v_i v_k Y_{\mathrm{L}ik} \cos(\delta_i - \delta_k - \theta_{\mathrm{L}ik}) + \frac{1}{2} v_i^2 Y_{\mathrm{S}ik} \cos(\theta_{\mathrm{S}ik})$$

Similarly, the reactive power $Q_{ik}(\cdot)$ flowing from node i to node k through transmission line ik.

$$Q_g - Q_L = Q_{ik}(\cdot) = -v_i^2 Y_{\mathrm{L}ik} \sin(\theta_{\mathrm{L}ik}) - v_i v_k Y_{\mathrm{L}ik} \sin(\delta_i - \delta_k - \theta_{\mathrm{L}ik}) - \frac{1}{2} v_i^2 Y_{\mathrm{S}ik} \sin(\theta_{\mathrm{S}ik})$$

Here, the shunt admittance is zero.

where:

- $\overline{v_i} = v_i \angle \delta_i^r$ and $\overline{v_k} = v_k \angle \delta_k^r$ are the voltages (magnitudes and angles) in per-unit at nodes i and k, respectively,
- P_g and P_L are the total net active power generation ($in\ puW$) and total active power load demand($in\ puW$) respectively at given node i.
- Q_g and Q_L are the total net reactive power generation ($in\ puvar$) and total reactive power load demand($in\ puvar$) respectively at given node i.
- $\bar{Y}_{\mathrm{L}ik} = Y_{\mathrm{L}ik} \angle \theta_{Lik}^r$ is the per-unit admittance of transmission line ik, and
- $\bar{Y}_{\mathrm{S}ik} = Y_{\mathrm{S}ik} \angle \theta_{Sik}^r$ is the per-unit shunt admittance of transmission line ik.
  Here, the shunt admittance is zero.

The active power and reactive power balance constraints at nodes 1 and 2 are respectively shown below. We are having two transmission lines between node 1 and node 2 with same characteristics.

$$P_1 - 0.8 = 2P_{12} = 2 * [v_1^2 Y_{\mathrm{L}12} \cos(\theta_{12}^r) - v_1 v_2 Y_{\mathrm{L}12} \cos(\delta_1 - \delta_2 - \theta_{12}^r)]\ puW$$

Constraint 9: $$P_1 - 0.8 = 2P_{12} = 2 * [9.95\, v_1^2 \cos(-1.47^r) - 9.95\, v_1 v_2 \cos(\delta_1 - \delta_2 + 1.47^r)]\ puW$$

$$Q_1 - 0.6 = 2Q_{12} = 2 * [-Y_{12} v_1^2 \sin(\theta_{12}^r) - v_1 v_2 Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12}^r)]\ puvar$$

Constraint 10: $$Q_1 - 0.6 = 2Q_{12} = 2 * [-9.95\, v_1^2 \sin(-1.47^r) - 9.95\, v_1 v_2 \sin(\delta_1 - \delta_2 + 1.47^r)]\ puvar$$

$$P_2 - 1.2 = 2P_{21} = 2 * [v_2^2 Y_{21} \cos(\theta_{21}^r) - v_1 v_2 Y_{21} \cos(\delta_2 - \delta_1 - \theta_{21}^r)]\ puW$$

Constraint 11: $$P_2 - 1.2 = 2P_{21} = 2 * [9.95\, v_1^2 \cos(-1.47^r) - 9.95\, v_1 v_2 \cos(\delta_2 - \delta_1 + 1.47^r)]\ puW$$

$$Q_2 - 0.6 = 2Q_{21} = 2 * [-v_2^2 Y_{21} \sin(\theta_{21}^r) - v_1 v_2 Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}^r)]\ puvar$$

Constraint 12: $$Q_2 - 0.6 = 2Q_{21} = 2 * [-9.95\, v_2^2 \sin(-1.47^r) - 9.95\, v_1 v_2 \sin(\delta_2 - \delta_1 + 1.47^r)]\ puvar$$

Transmission Line Limits

The apparent power (magnitude of the complex power $\bar{S}_{ik}(\cdot)$) flowing from node i to node k through transmission line ik is $(in\ puVA)$

$$S_{ik}(\cdot) = +\sqrt{\left(P_{ik}(\cdot)\right)^2 + \left(Q_{ik}(\cdot)\right)^2}$$

This apparent power is limited at each transmission line and thus:

$$+\sqrt{\left(P_{ik}(\cdot)\right)^2 + \left(Q_{ik}(\cdot)\right)^2} \leq S_{ik}^{max}$$

Where S_{ik}^{max} is the maximum apparent power through transmission line ik.

The capacity constraints of transmission lines in directions 1–2 and 2–1$(in\ puVA)$;

$$\left[\left(P_{12}(\cdot)\right)^2 + \left(Q_{12}(\cdot)\right)^2\right]^{\frac{1}{2}} \leq S_{12}^{max}$$

Constraint 13:

$$\left[\left(\frac{(P_1 - 0.8)}{2}\right)^2 + \left(\frac{Q_1 - 0.6}{2}\right)^2\right]^{\frac{1}{2}} \leq 0.5\ (puVA)$$

$$\left[\left(P_{21}(\cdot)\right)^2 + \left(Q_{21}(\cdot)\right)^2\right]^{\frac{1}{2}} \leq S_{21}^{max}$$

Constraint 14:

$$\left[\left(\frac{(P_2 - 1.2)}{2}\right)^2 + \left(\frac{Q_2 - 0.6}{2}\right)^2\right]^{\frac{1}{2}} \leq 0.5\ (puVA)$$

For both lines, capacity constraint will remain same (in particular direction) as both lines are having identical characteristics.

(b) Implement the formulation in Pyomo and find the optimal solution. Again, no general coding, explicitly express all equations. The code must be specific to the specific equations that you developed for this network in the previous part- no generic code. List the optimal values of all decision variables in a well-organized/labelled table. Do not forget the units. (30 marks)

```
#Pyomo objects exist within the pyomo.environ namespace
#Every Pyomo model starts with this; it tells Python to load the Pyomo Modeling Environment
from pyomo.environ import *
import cmath
import math

#Create an instance of a Concrete model
m = ConcreteModel("Optimal Power Flow Project (Question-b)")

#Assign a value of transmission line admitance.
Admitance = complex((0.99),(-9.9));  #(0.99 - 9.9j = 9.95∠(-1.47)^r)
Y = abs(Admitance)                   # Y is admitance magnitude in pu
θ = cmath.phase(Admitance)           # θ is in radian

#Define Active power P1 and P2 and their bounded limit values(Parameters).
#Constraint 1:
m.P1 = Var(bounds=(0,3))             # 0 ≤ P1 ≤ 3  where P1 is in puW
#Constraint 2:
m.P2 = Var(bounds=(0,0.8))           # 0 ≤ P2 ≤ 0.8  where P2 is in puW


#Define Reactive power Q1 and Q2 and their bounded limit values(Parameters).
#Constraint 3:
m.Q1 = Var(bounds=(-2,2))            # -2 ≤ Q1 ≤ 2   where Q1 is in puvar
#Constraint 4:
m.Q2 = Var(bounds=(-2,2))            # -2 ≤ Q2 ≤ 2   where Q2 is in puvar


#Define Voltage magnitude V1 and V2 and their bounded limit values(Parameters).
#Constraint 5:
m.v1 = Var(bounds=(0.95,1.10))       # 0.95 ≤ v1 ≤ 1.10   where v1 is in puV
#Constraint 6:
m.v2 = Var(bounds=(0.95,1.10))       # 0.95 ≤ v1 ≤ 1.10   where V2 is in puV


#Define Voltage angle δ1 & δ2 and their bounded limit values(Parameters).
#Constraint 7:
m.δ1 = Var(bounds=(-math.pi,math.pi))      # -π ≤ δ1 ≤ π    where δ1 is in radian
#Constraint 8:
m.δ2 = Var(bounds=(0,0))                   # δ2 = 0     where δ2 is in radian
#Define an objective funtion for minimizing the cost.
#If "sense" is omitted in above objective function, Pyomo assumes minimization objective function.
#objective function = P1 + 2P2   where objective function is in $/h
m.objective = Objective(expr = m.P1 + 2*m.P2)


#Define a constraint for active(in puW) and reactive power(in puvar) balance at node 1
m.constraint9 = Constraint(expr = m.P1 - 0.8 == 2*((m.v1**2)*Y*cos(θ) - (m.v1)*(m.v2)*Y*cos(m.δ1 - m.δ2 - θ)))
m.constraint10 = Constraint(expr = m.Q1 - 0.6 == 2*(-(m.v1**2)*Y*sin(θ) - (m.v1)*(m.v2)*Y*sin(m.δ1 - m.δ2 - θ)))

#Define a constraint for power balance at node 2
m.constraint11 = Constraint(expr = m.P2 - 1.2 == 2*((m.v2**2)*Y*(cos(θ)) - (m.v1)*(m.v2)*Y*cos(m.δ2 - m.δ1 - θ)))
m.constraint12 = Constraint(expr = m.Q2 - 0.6 == 2*(-(m.v2**2)*Y*sin(θ) - (m.v1)*(m.v2)*Y*sin(m.δ2 - m.δ1 - θ)))

#Define a constraint for line loading capacity L(12a) and L(21a) respectively.
#Here, both lines are having identical characteristics.Thus, constraint defined only oncefor both lines.
m.constraint13 = Constraint(expr = (((m.P1 - 0.8)/2)**2 + ((m.Q1 - 0.6)/2)**2)**0.5 <= 0.5)
#Devide by 2 is for single line capacity
m.constraint14 = Constraint(expr = (((m.P2 - 1.2)/2)**2 + ((m.Q2 - 0.6)/2)**2)**0.5 <= 0.5)
#Devide by 2 is for single line capacity

opt = SolverFactory("Ipopt")
results = opt.solve(m)

m.display()
```

Output:

```
Model 'Optimal Power Flow Project (Question-b)'

  Variables:
    P1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :     0 : 1.8000000174530684 :     3 : False : False :  Reals
    P2 : Size=1, Index=None
        Key  : Lower : Value               : Upper : Fixed : Stale : Domain
        None :     0 : 0.20413262853228192 :   0.8 : False : False :  Reals
    Q1 : Size=1, Index=None
        Key  : Lower : Value             : Upper : Fixed : Stale : Domain
        None :    -2 : 0.600000062708908 :     2 : False : False :  Reals
    Q2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :    -2 : 0.6413263971445908 :     2 : False : False :  Reals
    v1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  0.95 : 1.0999998440651486 :   1.1 : False : False :  Reals
    v2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  0.95 : 1.0963967554399974 :   1.1 : False : False :  Reals
    δ1 : Size=1, Index=None
        Key  : Lower              : Value               : Upper             : Fixed : Stale : Domain
        None : -3.141592653589793 : 0.04147415898185401 : 3.141592653589793 : False : False :  Reals
    δ2 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :     0 :   0.0 :     0 : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 2.208265274517632

  Constraints:
    constraint9 : Size=1
        Key  : Lower : Body                  : Upper
        None :   0.0 : 8.415490526658687e-14 :   0.0
    constraint10 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : -2.2937207688755734e-13 :   0.0
    constraint11 : Size=1
        Key  : Lower : Body                  : Upper
        None :   0.0 : -9.237055564881302e-14 :   0.0
    constraint12 : Size=1
        Key  : Lower : Body                  : Upper
        None :   0.0 : 1.4221956945448255e-13 :   0.0
    constraint13 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : 0.5000000087265352 :   0.5
    constraint14 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : 0.4983622409089031 :   0.5
```

No	Parameters	Values	No	Parameters	Values
0	$Cost$	$2.2082\ \$/h$			
1	P_1	$1.800\ puW$	7	δ_1	$0.0414\ rad$
2	P_2	$0.204\ puW$	8	δ_2	$0.0000\ rad$
3	Q_1	$0.600\ puvar$	9	S_{12a}	$0.500\ puVA$
4	Q_2	$0.641\ puvar$	10	S_{21a}	$0.498\ puVA$
5	v_1	$1.099\ puV$	11	S_{12b}	$0.500\ puVA$
6	v_2	$1.096\ puV$	12	S_{21b}	$0.498\ puVA$

(c) Extend the above formulation to a corrective SCOPF and write the corresponding detail full formulation. Specific equations to this network - no generic equations. Clearly name and specify your variables and parameters. Label all your equations and describe what each equation does. Assume that there is a possible contingency where the capacity of Line 12a is reduced to 0. The ramp limits of generating units at nodes 1 and 2 are 0.8 puW and 0.3 puW, respectively. List the optimal values of all variables in a well-organized/labelled table. Do not forget the units. (30 marks)

A SCOPF problem embodying a corrective approach and considering a selected number of contingencies indexed by ω is provided below:

Here the objective function can be written as $(in\ \$/h)$:

$\min_{P_1;P_2;Q_1;Q_2;v_1;v_2;\delta_1;\delta_2;P_{1\omega};P_{2\omega};Q_{1\omega};Q_{2\omega};v_{1\omega};v_{2\omega};\delta_{1\omega};\delta_{2\omega};}$

$$P_1 + 2\,P_2$$

Subject to…

Pre-contingency constraints

Pre-contingency constraints are written below.

Active power limits for generator 1 and 2 are$(in\ puW)$:

Constraint 1: $$0 \le P_1 \le 3$$

Constraint 2: $$0 \le P_2 \le 0.8$$

Reactive power limits for generator 1 and 2 are$(in\ puvar)$:

Constraint 3: $$0 \le Q_1 \le 0.6$$

Constraint 4: $$0 \le Q_2 \le 0.6$$

Voltage magnitude limits for node 1 and node 2 are $(in\ puV)$:

Constraint 5: $$0.95 \le v_1 \le 1.1$$

Constraint 6: $$0.95 \le v_2 \le 1.1$$

Voltage angle limits for generator 1 is$(in\ radian)$:

Constraint 7: $$-\pi \le \delta_1 \le \pi$$

Voltage angle limits for generator 2 is given in question (reference bus) $(in\ radian)$:

Constraint 8: $$\delta_2 = 0^r$$

Constraint 9: Active Power balance equation at node 1

$$P_1 - 0.8 = 2P_{12} = 2 * [9.95\, v_1^2 \cos(-1.47^r) - 9.95\, v_1 v_2 \cos(\delta_1 - \delta_2 + 1.47^r)]\ puW$$

Constraint 10: Reactive Power balance equation at node 1

$$Q_1 - 0.6 = 2Q_{12} = 2 * [-9.95\, v_1^2 \sin(-1.47^r) - 9.95\, v_1 v_2 \sin(\delta_1 - \delta_2 + 1.47^r)]\ puvar$$

Constraint 11: Active Power balance equation at node 2

$$P_2 - 1.2 = 2P_{21} = 2 * [9.95\, v_1^2 \cos(-1.47^r) - 9.95\, v_1 v_2 \cos(\delta_2 - \delta_1 + 1.47^r)]\; puW$$

Constraint 12: Reactive Power balance equation at node 2

$$Q_2 - 0.6 = 2Q_{21} = 2 * [-\,9.95\, v_2^2 \sin(-1.47^r) - 9.95\, v_1 v_2 \sin(\delta_2 - \delta_1 + 1.47^r)]\; puvar$$

Constraint 13: Line loading capacity constraints for both the line in direction $1-2$. For both the lines, in specific direction, line loading constraint will be the same**.**

$$\left[(\frac{(P_1 - 0.8)}{2})^2 + \left(\frac{Q_1 - 0.6}{2}\right)^2\right]^{\frac{1}{2}} \leq 0.5\; (puVA)$$

Constraint 14: Line loading capacity constraints for both the line in direction $2-1$.

$$\left[(\frac{(P_2 - 1.2)}{2})^2 + \left(\frac{Q_2 - 0.6}{2}\right)^2\right]^{\frac{1}{2}} \leq 0.5\; (puVA)$$

<u>Post-contingency</u>

Post contingency constraints indexed by ω is provided below: Here, line loading capacity of one line reduce to zero.

Active power limits for generator 1 and 2 are$(in\ puW)$:

Constraint 15: $0 \leq P_{1\omega} \leq 3$

Constraint 16: $0 \leq P_{2\omega} \leq 0.8$

Reactive power limits for generator 1 and 2 are $(in\ puvar)$:

Constraint 17: $0 \leq Q_{1\omega} \leq 0.6$

Constraint 18: $0 \leq Q_{2\omega} \leq 0.6$

Voltage magnitude limits for node 1 and node 2 are $(in\ puV)$:

Constraint 19: $0.95 \leq v_{1\omega} \leq 1.1$

Constraint 20: $0.95 \leq v_{2\omega} \leq 1.1$

Voltage angle limits for generator 1 is $(in\ radian)$:

Constraint 21: $-\pi \leq \delta_{1\omega} \leq \pi$

Voltage angle limits for generator 2 is given in question (reference bus) $(in\ radian)$:

Constraint 22: $\delta_{2\omega} = 0^r$

Constraint 23: Active Power balance equation at node 1. Here, the line loading capacity of one line reduce to zero. Thus, the equations can be written as below.

$$P_{1\omega} - 0.8 = P_{12\omega} = [9.95\, v_{1\omega}^2 \cos(-1.47^r) - 9.95\, v_{1\omega} v_{2\omega} \cos(\delta_{1\omega} - \delta_{2\omega} + 1.47^r)]\; puW$$

Constraint 24: Reactive Power balance equation at node 1. Here, the line loading capacity of one line reduce to zero. Thus, the equations can be written as below.

$$Q_{1\omega} - 0.6 = Q_{12\omega} = [-\, 9.95\, v_{1\omega}^2 \sin(-1.47^r) - 9.95\, v_{1\omega} v_{2\omega} \sin(\delta_{1\omega} - \delta_{2\omega} + 1.47^r)]\; puvar$$

Constraint 25: Active Power balance equation at node 2. Here, the line loading capacity of one line reduce to zero. Thus, the equations can be written as below.

$$P_{2\omega} - 1.2 = P_{21\omega} = [9.95\, v_{1\omega}^2 \cos(-1.47^r) - 9.95\, v_{1\omega} v_{2\omega} \cos(\delta_{2\omega} - \delta_{1\omega} + 1.47^r)]\; puW$$

Constraint 26: Reactive Power balance equation at node 2. Here, the line loading capacity of one line reduce to zero. Thus, the equations can be written as below.

$$Q_{2\omega} - 0.6 = Q_{21\omega} = [-\, 9.95\, v_{2\omega}^2 \sin(-1.47^r) - 9.95\, v_{1\omega} v_{2\omega} \sin(\delta_{2\omega} - \delta_{1\omega} + 1.47^r)]\; puvar$$

Constraint 27: Line loading capacity constraints for a line in direction $1 - 2$. Here, we have only one line in post contingency case. Thus, the equation can be written as:

$$[((P_{1\omega} - 0.8))^2 + (Q_{1\omega} - 0.6)^2]^{\frac{1}{2}} \leq 0.5\ (puVA)$$

Constraint 28: Line loading capacity constraints for both the line in direction $2 - 1$. Here, we have only one line in post contingency case. Thus, the equation can be written as:

$$[((P_{2\omega} - 1.2))^2 + (Q_{2\omega} - 0.6)^2]^{\frac{1}{2}} \leq 0.5\ (puVA)$$

Ramp up and Ramp down limit constraints after the contingency for the generating unit 1 and unit 2. Where, $R_1 = 0.8\ puW$ and $R_2 = 0.3\ puW$ are the Ramp up and down limit for Unit 1 and Unit 2 respectively.

For generating Unit 1:

Constraint 29:

$$P_{1\omega} - P_1 \leq R_1$$
$$P_{1\omega} - P_1 \leq 0.8\ (puW)$$

Constraint 30:

$$P_1 - P_{1\omega} \leq R_1$$
$$P_1 - P_{1\omega} \leq 0.8\ (puW)$$

For generating Unit 2:

Constraint 31:

$$P_{2\omega} - P_2 \leq R_2$$
$$P_{2\omega} - P_2 \leq 0.3\ (puW)$$

Constraint 32:

$$P_2 - P_{2\omega} \leq R_2$$
$$P_2 - P_{2\omega} \leq 0.3\ (puW)$$

(d) Implement the formulated SCOPF in Pyomo and find the optimal solution. The code must be specific to the specific equations that you developed for this network in the previous part - no generic code. List the optimal values of all decision variables in a well-organized/labelled table. Do not forget the units. (30 marks)

```
#Pyomo objects exist within the pyomo.environ namespace
#Every Pyomo model starts with this; it tells Python to load the Pyomo Modeling Environment
from pyomo.environ import *
import cmath
import math

#Create an instance of a Concrete model
m = ConcreteModel("Optimal Power Flow Project with SCOPF (Question-d)")

#Assign a value of transmission line admitance.
Admitance = complex((0.99),(-9.9));  #(0.99 - 9.9j = 9.95∠(-1.47)^r)
Y = abs(Admitance)                   # Y is admitance magnitude in pu
θ = cmath.phase(Admitance)           # θ is in radian


#Define Active power P1 & P2 and their bounded limit values(Parameters).
#Constrain 1:
m.P1 = Var(bounds=(0,3))                # 0 ≤ P1 ≤ 3  where P1 is in puW
#Constrain 2:
m.P2 = Var(bounds=(0.00,0.8))           # 0 ≤ P2 ≤ 0.8  where P2 is in puW

#Define Reactive power Q1 & Q2 and their bounded limit values(Parameters).
#Constrain 3:
m.Q1 = Var(bounds=(-2,2))              # -2 ≤ Q1 ≤ 2   where Q1 is in puvar
#Constrain 4:
m.Q2 = Var(bounds=(-2,2))              # -2 ≤ Q2 ≤ 2   where Q2 is in puvar

#Define Voltage magnitude V1 & V2  and their bounded limit values(Parameters).
#Constrain 5:
m.v1 = Var(bounds=(0.95,1.10))         # 0.95 ≤ v1 ≤ 1.10   where v1 is in puV
#Constrain 6:
m.v2 = Var(bounds=(0.95,1.10))         # 0.95 ≤ v2 ≤ 1.10   where V2 is in puV

#Define Voltage angle δ1 & δ2 and their bounded limit values(Parameters).
#Constrain 7:
m.δ1 = Var(bounds=(-math.pi,math.pi))      # -π ≤ δ1 ≤ π    where δ1 is in radian
#Constrain 8:
m.δ2 = Var(bounds=(0,0))                   # δ2 = 0    where δ2 is in radian

#Define an objective funtion for minimizing the cost.
#If “sense” is omitted in above objective function, Pyomo assumes minimization objective function.
m.objective = Objective(expr = m.P1 + 2*m.P2)    # objective function = P1 + 2P2   where objective function is in $/h



#####Pre-contingency constraints#####

#Define a constraint for active and reactive power balance at node 1
m.constraint9 = Constraint(expr = m.P1 - 0.8 == 2*((m.v1**2)*Y*cos(θ) - (m.v1)*(m.v2)*Y*cos(m.δ1 - m.δ2 - θ)))
m.constraint10 = Constraint(expr = m.Q1 - 0.6 == 2*(-(m.v1**2)*Y*sin(θ) - (m.v1)*(m.v2)*Y*sin(m.δ1 - m.δ2 - θ)))

#Define a constraint for active and reactive power balance at node 2
m.constraint11 = Constraint(expr = m.P2 - 1.2 == 2*((m.v2**2)*Y*(cos(θ)) - (m.v1)*(m.v2)*Y*cos(m.δ2 - m.δ1 - θ)))
m.constraint12 = Constraint(expr = m.Q2 - 0.6 == 2*(-(m.v2**2)*Y*(sin(θ)) - (m.v1)*(m.v2)*Y*sin(m.δ2 - m.δ1 - θ)))

#Define a constraint for line loading capacity L(12a) and L(21a) respectively.
#Here, both lines are having identical characteristics. Thus, constraint defined only once for both the lines.
m.constraint13 = Constraint(expr = (((m.P1 - 0.8)/2)**2 + ((m.Q1 - 0.6)/2)**2)**0.5 <= 0.5)
m.constraint14 = Constraint(expr = (((m.P2 - 1.2)/2)**2 + ((m.Q2 - 0.6)/2)**2)**0.5 <= 0.5)


###Post-contingency constraints###
#In which, one line loading capacity reduce to zero.

#Define Active power P1ω and P2ω and their bounded limit values(Parameters).
#Constraint 15:
m.P1ω = Var(bounds=(0,3))            # 0 ≤ P1ω ≤ 3  where P1ω is in puW
#Constraint 16:
m.P2ω = Var(bounds=(0,0.8))          # 0 ≤ P2ω ≤ 0.8  where P2ω is in puW

#Define Reactive power Q1ω & Q2ω and their bounded limit values(Parameters).
#Constraint 17:
m.Q1ω = Var(bounds=(-2,2))           # -2 ≤ Q1ω ≤ 2   where Q1ω is in puvar
#Constraint 18:
m.Q2ω = Var(bounds=(-2,2))           # -2 ≤ Q2ω ≤ 2   where Q2ω is in puvar
```

```
#Define Voltage magnitude V1ω & V2ω and their bounded limit values(Parameters).
#Constraint 19:
m.v1ω = Var(bounds=(0.95,1.10))      # 0.95 ≤ v1ω ≤ 1.10   where v1ω is in puV
#Constraint 20:
m.v2ω = Var(bounds=(0.95,1.10))      # 0.95 ≤ v2ω ≤ 1.10   where v2ω is in puV

#Define Voltage angle δ1ω & δ2ω and their bounded limit values(Parameters).
#Constraint 21:
m.δ1ω = Var(bounds=(-math.pi,math.pi))     # -π ≤ δ1ω ≤ π    where δ1ω is in radian
#Constraint 22:
m.δ2ω = Var(bounds=(0,0))                  # δ2ω = 0    where δ2ω is in radian


#Define a constraint for active and reactive power balance at node 1
m.constraint23 = Constraint(expr = m.P1ω - 0.8 == ((m.v1ω**2)*Y*cos(θ) - (m.v1ω)*(m.v2ω)*Y*cos(m.δ1ω - m.δ2ω - θ)))
m.constraint24 = Constraint(expr = m.Q1ω - 0.6 == (-(m.v1ω**2)*Y*sin(θ) - (m.v1ω)*(m.v2ω)*Y*sin(m.δ1ω - m.δ2ω - θ)))

#Define a constraint for active and reactive power balance at node 2
m.constraint25 = Constraint(expr = m.P2ω - 1.2 == ((m.v2ω**2)*Y*(cos(θ)) - (m.v1ω)*(m.v2ω)*Y*cos(m.δ2ω - m.δ1ω - θ)))
m.constraint26 = Constraint(expr = m.Q2ω - 0.6 == (-(m.v2ω**2)*Y*sin(θ) - (m.v1ω)*(m.v2ω)*Y*sin(m.δ2ω - m.δ1ω - θ)))

#Define a constraint for line loading capacity L(12a) and L(21a) respectively
#Here, for one transmission line loading capacity reduce to zero. Thus, line loading constraint would be for only one line.
m.constraint27 = Constraint(expr = (((m.P1ω - 0.8))**2 + ((m.Q1ω - 0.6))**2)**0.5 <= 0.5)
m.constraint28 = Constraint(expr = (((m.P2ω - 1.2))**2 + ((m.Q2ω - 0.6))**2)**0.5 <= 0.5)


#Defining a ramp up and ramp down limit constraint after the contingency for the generating unit 1 and unit 2.
m.constraint29 = Constraint(expr = (m.P1ω - m.P1) <= 0.8)
m.constraint30 = Constraint(expr = (m.P1 - m.P1ω) <= 0.8)
m.constraint31 = Constraint(expr = (m.P2ω - m.P2) <= 0.3)
m.constraint32 = Constraint(expr = (m.P2 - m.P2ω) <= 0.3)

opt = SolverFactory("Ipopt")
results = opt.solve(m)

m.display()
```

```
Model 'Optimal Power Flow Project with SCOPF (Question-d)'

  Variables:
    P1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :     0 : 1.6005824376953959 :     3 : False : False :  Reals
    P2 : Size=1, Index=None
        Key  : Lower : Value             : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.402066309944618 :   0.8 : False : False :  Reals
    Q1 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :    -2 : 0.6000044835161145 :     2 : False : False :  Reals
    Q2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :    -2 : 0.6264829928840328 :     2 : False : False :  Reals
    v1 : Size=1, Index=None
        Key  : Lower : Value             : Upper : Fixed : Stale : Domain
        None :  0.95 : 1.099999494004758 :   1.1 : False : False :  Reals
    v2 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  0.95 : 1.0969637937088792 :   1.1 : False : False :  Reals
    δ1 : Size=1, Index=None
        Key  : Lower              : Value               : Upper             : Fixed : Stale : Domain
        None : -3.141592653589793 : 0.03318288761124434 : 3.141592653589793 : False : False :  Reals
    δ2 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :     0 :   0.0 :     0 : False : False :  Reals
    P1ω : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :     0 : 1.3000000074562599 :     3 : False : False :  Reals
    P2ω : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :     0 : 0.7020663174219103 :   0.8 : False : False :  Reals
    Q1ω : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :    -2 : 0.6000000630911073 :     2 : False : False :  Reals
    Q2ω : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :    -2 : 0.6206631856905932 :     2 : False : False :  Reals
    v1ω : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  0.95 : 1.0999993394037657 :   1.1 : False : False :  Reals
    v2ω : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :  0.95 : 1.0963962471053628 :   1.1 : False : False :  Reals
    δ1ω : Size=1, Index=None
        Key  : Lower              : Value               : Upper             : Fixed : Stale : Domain
        None : -3.141592653589793 : 0.04147419689177698 : 3.141592653589793 : False : False :  Reals
    δ2ω : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :     0 :   0.0 :     0 : False : False :  Reals

  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 2.4047150575846317

  Constraints:
    constraint9 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : -6.705747068735946e-14 :   0.0
    constraint10 : Size=1
        Key  : Lower : Body                    : Upper
        None :   0.0 : -3.7336800318144014e-13 :   0.0
    constraint11 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : -5.129230373768223e-14 :   0.0
    constraint12 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : -8.008038676621254e-13 :   0.0
    constraint13 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : 0.4002912188539752 :   0.5
```

```
constraint14 : Size=1
    Key  : Lower : Body               : Upper
    None :  None : 0.3991865236444907 :   0.5
constraint23 : Size=1
    Key  : Lower : Body                  : Upper
    None :   0.0 : 1.887379141862766e-15 :   0.0
constraint24 : Size=1
    Key  : Lower : Body                    : Upper
    None :   0.0 : -1.1102230246251565e-16 :   0.0
constraint25 : Size=1
    Key  : Lower : Body                   : Upper
    None :   0.0 : -1.887379141862766e-15 :   0.0
constraint26 : Size=1
    Key  : Lower : Body                  : Upper
    None :   0.0 : 6.661338147750939e-16 :   0.0
constraint27 : Size=1
    Key  : Lower : Body               : Upper
    None :  None : 0.5000000074562638 :   0.5
constraint28 : Size=1
    Key  : Lower : Body               : Upper
    None :  None : 0.4983622372217439 :   0.5
constraint29 : Size=1
    Key  : Lower : Body               : Upper
    None :  None : -0.300582430239136 :   0.8
constraint30 : Size=1
    Key  : Lower : Body              : Upper
    None :  None : 0.300582430239136 :   0.8
constraint31 : Size=1
    Key  : Lower : Body               : Upper
    None :  None : 0.3000000074772923 :   0.3
constraint32 : Size=1
    Key  : Lower : Body                : Upper
    None :  None : -0.3000000074772923 :   0.3
```

Output Result in Table:

	OPF		SCOPF-Pre-contingency		SCOPF-Post-contingency	
No	**Variables**	**Values**	**Variables**	**Values**	**Variables**	**Values**
1	$Cost$	2.20 $\$/h$	$Cost$	2.40 $\$/h$		
2	P_1	1.800 puW	P_1	1.600 puW	$P_{1\omega}$	1.300 puW
3	P_2	0.204 puW	P_2	0.402 puW	$P_{2\omega}$	0.702 puW
4	Q_1	0.600 $puvar$	Q_1	0.600 $puvar$	$Q_{1\omega}$	0.600 $puvar$
5	Q_2	0.641 $puvar$	Q_2	0.626 $puvar$	$Q_{2\omega}$	0.620 $puvar$
6	v_1	1.099 puV	v_1	1.099 puV	$v_{1\omega}$	1.099 puV
7	v_2	1.096 puV	v_2	1.096 puV	$v_{2\omega}$	1.096 puV
8	δ_1	0.0414 rad	δ_1	0.033 rad	$\delta_{1\omega}$	0.0414 rad
9	δ_2	0.0000 rad	δ_2	0.000 rad	$\delta_{2\omega}$	0.000 rad
10	S_{12a}	0.500 $puVA$	S_{12a}	0.400 $puVA$	$S_{12a\omega}$	0.500 $puVA$
11	S_{21a}	0.498 $puVA$	S_{21a}	0.399 $puVA$	$S_{21a\omega}$	0.498 $puVA$
12	S_{12b}	0.500 $puVA$	S_{12b}	0.400 $puVA$	$S_{12b\omega}$	0.00 $puVA$
13	S_{21b}	0.498 $puVA$	S_{21b}	0 .399 $puVA$	$S_{21b\omega}$	0.00 $puVA$

Conclusion: By calculating optimal power flow problem considering security constraints, the operating cost increases to 2.4 $\$/h$ from 2.2 $\$/h$. However, it creates system more reliable and secure.

(e) Let's assume that the thermal plant in node 2 is decommissioned, and a wind power developer is applying to the system operator to build and connect a wind farm with the maximum capacity of 2 puW with a fuel cost of zero $\$/puWh$ in that node. As a transmission engineer, you are asked to study how this change would impact the network congestion and if there is a need for a new transmission development or transmission reinforcement. Wind/load curtailments are not allowed. Use your security constrained model and plan and conduct a study and make a recommendation. This recommendation must be based on a probabilistic study of wind power variability and the associated transmission loading/impacts. Describe your study plan and justify every choice that you make in terms of data, model, parameters etc. Your plan must be based on a sample wind regime in southern Alberta, and your sample size must be large enough to make your recommendation reliable given the variability of wind power. This is an open-ended study and you need to use your engineering judgment backed by research to plan and conduct the study. You must report measures and arguments that would support your recommendation/choices. There is no one way of doing this study and you are encouraged to do a bit of research and back your methodology with solid papers/industry reports/guidelines etc. (100 marks).

Introduction to Alberta's wind power generation scenario

Since the first commercial wind farm was built in southern Alberta in 1993, the government of Alberta has continued to support the development of wind power [1]. Currently, the wind energy capacity in Alberta is 2269 MW [2] and the government expects to at least doubles the current wind power capacity by 2030 [1]. This motivates us to find an efficient model to forecast the future performance of the current wind energy grid and the potential generation of new wind farms in Alberta.

Here, we have used the quantitative and qualitative insights regarding wind energy production in a geographical region using publicly available data. Wind speed is the most relevant factor related to wind power. In particular, the wind power curves for different types of wind turbines have roughly the same shape [1].

Most of Alberta's electricity generation, comes from conventional generation. Wind power capacity constitutes 13.72% of Alberta's electricity generating resource [AESO]. Alberta's power system has 26 wind farms with a total capacity of 2,269 MW, which are mostly located in central and southern Alberta [AESO]. Information on Alberta's generation system can be found in the Current Supply Demand Report [2].

The data used in the analysis is based on Alberta power system. The period that is considered in this analysis is from "***January 2020 to December 2020***".

Wind generation scenario over the last 5 years

Below **TABLE NO-1** summarizes the annual statistics for wind generation. No new wind facilities were installed in 2020, so the installed capacity at the end of the year remained at 1,781 MW. This represented 11 percent of the total installed generation capacity in Alberta. Wind generation produced seven percent of total AIL in 2020 [3].

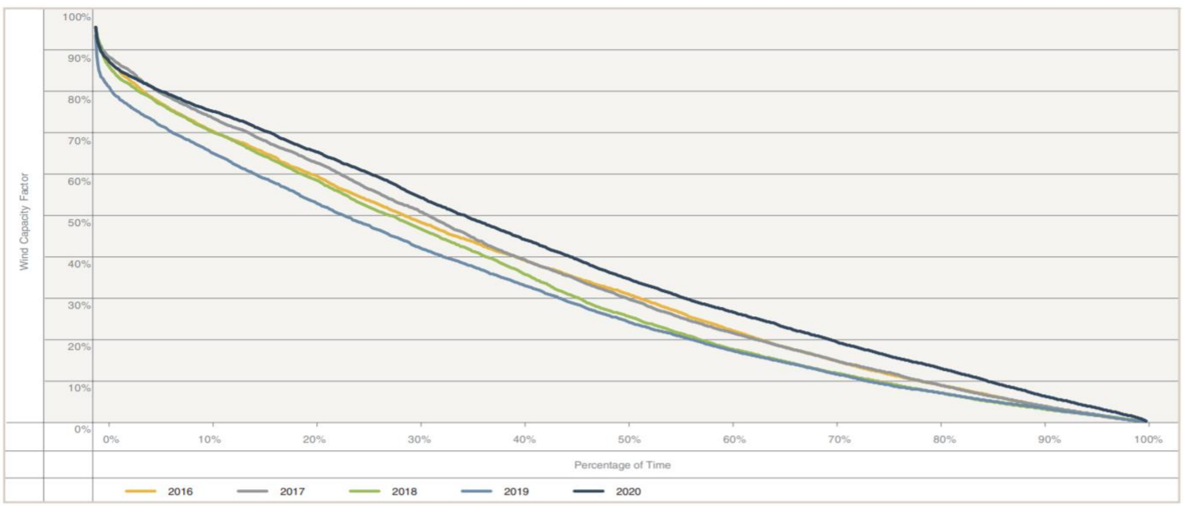
Year	2016	2017	2018	2019	2020
Installed wind capacity at year end (MW)	1,445	1,445	1,445	1,781	**1,781**
Total wind generation (GWh)	4,405	4,486	4,104	4,116	**6,079**
Wind generation as a percentage of total AIL	6%	5%	5%	5%	**7%**
Average hourly capacity factor	35%	35%	32%	30%	**39%**
Maximum hourly capacity factor	93%	96%	96%	94%	**96%**
Wind capacity factor during annual peak AIL	15%	6%	9%	0%	**8%**

Reference [4]

Table-1

Wind capacity factor over last five years

Below **FIGURE-1** illustrates annual duration curves for the hourly capacity factor for Alberta wind generation. Capacity factor represents the percentage of installed capacity used to generate electricity that was delivered to the grid. The duration curve represents the percentage of time that capacity factor of wind generation equals or exceeds a specific value [4].



Reference [4]

Figure-1

Alberta's wind power generation regional wise

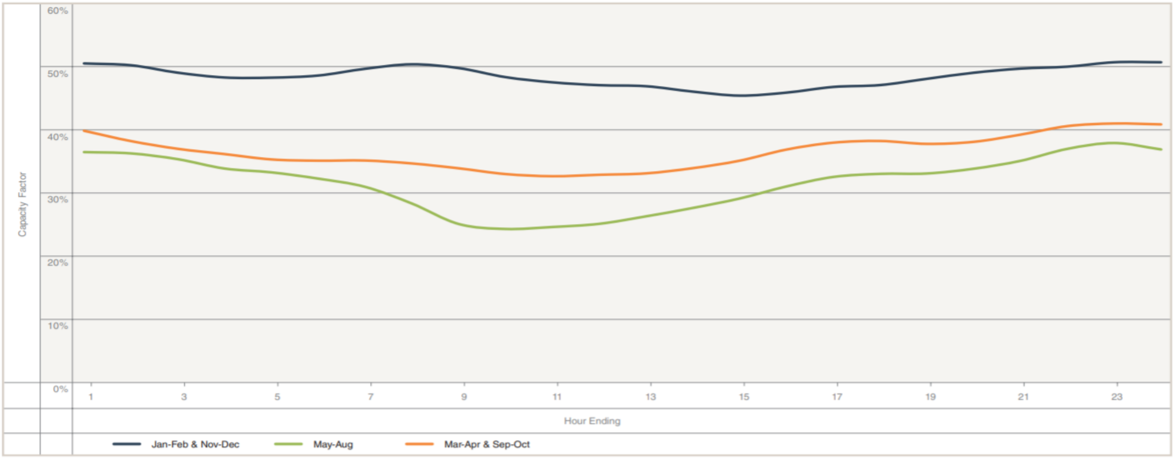
Below **TABLE-2'S** statistic illustrates the Installed wind capacity for two different regional. 80% wind farms are installed in the southern region [4].

Region	Central	South	Total
Installed wind capacity at year end (MW)	349	1432	**1781**
Total wind generation (GWh)	1183	4896	**6079**
Average wind capacity factor	39%	39%	**39%**
Achieved price ($/MWh)	$35.55	$33.14	**$33.61**

Reference [4]

Table-2

From the above data, we can conclude that the average 39% wind penetration was observed over the year period of 2020.



Reference [4]

Figure-2

Above **FIGURE-2** shows average hourly capacity factor of wind generation for different seasons of the year during 2020. It shows that wind generation is typically highest in the overnight hours and lowest during the day, with this phenomenon more pronounced in the summer than the winter. The capacity factor of wind is 15-20 % higher in the winter than the summer [4]. So, in conclusion, wind penetration changes observed over the different time of the day and different seasons.

As of today, existing wind power plant in Alberta

Asset	Maximum capacity (MW)	Asset	Maximum capacity (MW)
Ardenville Wind (ARD1)	68	McBride Lake Windfarm (AKE1)	73
BUL1 Bull Creek (BUL1)	13	Oldman 2 Wind Farm 1 (OWF1)	46
BUL2 Bull Creek (BUL2)	16	Rattlesnake Ridge Wind (RTL1)	130
Blackspring Ridge (BSR1)	300	Riverview (RIV1)	105
Blue Trail Wind (BTR1)	66	Soderglen Wind (GWW1)	71
Castle River #1 (CR1)	39	Summerview 1 (IEW1)	66
Castle Rock Ridge 2 (CRR2)	29	Summerview 2 (IEW2)	66
Castle Rock Wind Farm (CRR1)	77	Suncor Chin Chute (SCR3)	30
Cowley Ridge (CRE3)	20	Suncor Magrath (SCR2)	30
Enmax Taber (TAB1)	81	Whitla 1 (WHT1)	202
Ghost Pine (NEP1)	82	Whitla 2 (WHT2)	151
Halkirk Wind Power Facility (HAL1)	150	Windrise (WRW1)	207
Kettles Hill (KHW1)	63	Wintering Hills (SCR4)	88

Reference [2]

Trend.pdf

Table-3 [3]

The below data is taken from Alberta Electric System Operator (AESO) for 2020 year. The sample size is taken here for ***per hour over a period of one year*** [5]. AESO Wind generation data

Fuel Type	Season	Duration	Capacity (MW)	Availability (MW)	Utilized average wind generation per hour (MW/h)	Average Wind penetration (%)
Wind	Summer	Jan-2020 to Dec-2020	1783.83	1748.87	676.20	37.90
	Winter		1783.38	1750.31	691.83	38.79

Fuel Type	Season	Duration	Capacity (MW)	Availability (MW)	Max Wind penetration (%)	Min Wind penetration (%)
Wind	Summer	Jan-2020 to Dec-2020	1783.83	1748.877005	95.14	0
	Winter		1783.38	1750.311608	95.67	0

From the above data, our newly design Unit -2 would run against following wind penetration assumption.

- Effective power system operation should be achieved with annual average wind penetrations on Unit-2 with an average output of around 39% of its maximum capacity.
- From the above data, we can say that our power system should be run reliably when newly design generator-2 gives output during high and low wind penetration between 0% to 96%. (As seen in FIGURE-2, generator output may vary due to season changes)

New Objective Function

The fuel cost of Unit 2 is 0 $\$/puW$. Thus, our ***new objective*** function can be written as $(in\ \$/h)$:

$$\min_{P_1;P_2;Q_1;Q_2;v_1;v_2;\delta_1;\delta_2;P_{1\omega};P_{2\omega};Q_{1\omega};Q_{2\omega};v_{1\omega};v_{2\omega};\delta_{1\omega};\delta_{2\omega};}$$

$$Incremental\ Cost\ of\ P_1\ Unit * P_1$$

Base case: In the given problem, we are installing the wind power having a maximum capacity of $2\ puW$ at node 2 in place of existing plant. The output of newly designed wind generator can be varied between $0\ puW$ to $1.92\ puW$ over the entire year as discussed above due to change in wind penetrations.

Let's consider different cases for variability of wind power…

Case 1: If Unit 2 is running with annual average wind penetrations of 39%.

The average generation for Unit 2 $= 39\% * 2\ puW = 0.78\ puW$

In the QUESTION NUMBER (C), Our constraint number **2, 16, 31** and **32** change to…

Constraint 2: $P_2 = 0.78\ puW$

Constraint 16: $P_{2\omega} = 0.78\ puW$

Unit-2 generation out kept constant as wind power curtailments are not allowed.

Constraint 31:

$$P_{2\omega} - P_2 \leq R_2$$
$$P_{2\omega} - P_2 = 0.0$$

Constraint 32:

$$P_2 - P_{2\omega} \leq R_2$$
$$P_2 - P_{2\omega} = 0.0$$

Rest of the constraints will remain same as question number (c).

Case 2: If Unit 2 is running with 0% output either during lowest wind penetrations condition.

The average generation for Unit 2 $= 0\% * 2\ puW = 0.00\ puW$

In the **QUESTION NUMBER (C)**, Our constraint number **2, 16, 31** and **32** change to…

Constraint 2: $P_2 = 0.00\ puW$

Constraint 16: $P_{2\omega} = 0.00\ puW$

Unit-2 generation out kept constant as wind power curtailments are not allowed.

Constraint 31:

$$P_{2\omega} - P_2 \leq R_2$$
$$P_{2\omega} - P_2 = 0.0$$

Constraint 32:

$$P_2 - P_{2\omega} \leq R_2$$
$$P_2 - P_{2\omega} = 0.0$$

Rest of the constraints will remain same as question number (c).

Case 3: If Unit 2 is running with highest wind penetrations of 96%.

The average generation for Unit 2 $= 96\% * 2\ puW = 1.92\ puW$

In the **QUESTION NUMBER (C)**, Our constraint number **2, 16, 31** and **32** change to…

Constraint 2: $P_2 = 1.92\ puW$

Constraint 16: $P_{2\omega} = 1.92\ puW$

Unit-2 generation out kept constant as wind power curtailments are not allowed.

Constraint 31:

$$P_{2\omega} - P_2 \leq R_2$$
$$P_{2\omega} - P_2 = 0.0$$

Constraint 32:

$$P_2 - P_{2\omega} \leq R_2$$
$$P_2 - P_{2\omega} = 0.0$$

Rest of the constraints will remain same as question number (c).

From the above listed cases, we can conclude the results as shown in the table below.

Case 1 (Average wind penetration)			Case 2 (Zero wind penetration)			Case 3 (Maximum wind penetration)		
Constraint	**OPF Result**	**OPF Result SCOPF**	**Constraint**	**OPF Result**	**OPF Result SCOPF**	**Constraint**	**OPF Result**	**OPF Result SCOPF**
$0 \le P_1 \le 3$	$P_1 = 1.22\ puW$	$P_{1w} = 1.22\ puW$	$0 \le P_1 \le 3$	$P_1 = 2.00\ puW$	$P_{1w} = 2.00\ puW$	$0 \le P_1 \le 3$	$P_1 = 0.082\ puW$	$P_{1w} = 0.082\ puW$
$P_2 = 0.78\ puW$	$P_2 = 0.78\ puW$	$P_{2w} = 0.78\ puW$	$P_2 = 0.00\ puW$	$P_2 = 0.00\ puW$	$P_{2w} = 0.00\ puW$	$P_2 = 1.92\ puW$	$P_2 = 1.92\ puW$	$P_{2w} = 1.92\ puW$
$0 \le P_{1w} \le 3$	$S_{12a} = 0.21\ puVA$ $S_{12b} = 0.21\ puVA$	$S_{12aw} = 0.00\ puVA$ $S_{12bw} = 0.42\ puVA$	$0 \le P_{1w} \le 3$	$S_{12a} = 0.603\ puVA$ $S_{12b} = 0.603\ puVA$	$S_{12aw} = 0.00\ puVA$ $S_{12bw} = 1.21\ puVA$	$0 \le P_{1w} \le 3$	$S_{12a} = 0.358\ puVA$ $S_{12b} = 0.358\ puVA$	$S_{12a} = 0.00\ puVA$ $S_{12b} = 0.714\ puVA$
$P_{2w} = 0.78\ puW$	$S_{21a} = 0.21\ puVA$ $S_{21b} = 0.21\ puVA$	$S_{21aw} = 0.00\ puVA$ $S_{21bw} = 0.42\ puVA$	$P_{2w} = 0.00\ puW$	$S_{21a} = 0.600\ puVA$ $S_{21b} = 0.600\ puVA$	$S_{21aw} = 0.00\ puVA$ $S_{21bw} = 1.20\ puVA$	$P_{2w} = 1.92\ puW$	$S_{21a} = 0.36\ puVA$ $S_{21b} = 0.36\ puVA$	$S_{21a} = 0.00\ puVA$ $S_{21b} = 0.720\ puVA$
Cost	$1.22\ \$/h$		*Cost*	$2.00\ \$/h$		*Cost*	$0.082\ \$/h$	

<u>Conclusion from above analysis is summarized as follows:</u>

During the average wind penetration scenario (Case 1), the power system run reliably. However, during the low wind penetration (Case 2) condition, the transmission line becomes overload by 120% ($S = 0.603\ puVA$). In addition, during the high wind scenario, the power system runs healthy, although, creates a **reliability (N-1 contingency)** issue.

Let's try by implementing ***a new transmission line having the same capacity***, we get the solution as below…

<u>Implications of new transmission line analysis are summarized as follows:</u>

Case 1 (Average wind penetration)			Case 2 (Zero wind penetration)			Case 3 (Maximum wind penetration)		
Constraint	**OPF Result**	**OPF Result SCOPF**	**Constraint**	**OPF Result**	**OPF Result SCOPF**	**Constraint**	**OPF Result**	**OPF Result SCOPF**
$0 \le P_1 \le 3$	$P_1 = 1.22\ pu$	$P_{1w} = 1.22\ pu$	$0 \le P_1 \le 3$	$P_1 = 2.00\ puW$	$P_{1w} = 2.00\ puW$	$0 \le P_1 \le 3$	$P_1 = 0.0814\ puW$	$P_{1w} = 0.0814\ puW$
$P_2 = 0.78\ puW$	$P_2 = 0.78\ pu$	$P_{2w} = 0.78\ pu$	$P_2 = 0.00\ puW$	$P_2 = 0.00\ puW$	$P_{2w} = 0.00\ puW$	$P_2 = 1.92\ puW$	$P_2 = 1.92\ puW$	$P_{2w} = 1.92\ puW$
$0 \le P_{1w} \le 3$	$S_{12a} = 0.14\ puVA$ $S_{12b} = 0.14\ puVA$ $S_{12c} = 0.14\ puVA$	$S_{12aw} = 0.00\ puVA$ $S_{12bw} = 0.21\ puVA$ $S_{12cw} = 0.21\ puVA$	$0 \le P_{1w} \le 3$	$S_{12a} = 0.401\ puVA$ $S_{12b} = 0.401\ puVA$ $S_{12c} = 0.401\ puVA$	$S_{12aw} = 0.000\ puVA$ $S_{12bw} = 0.603\ puVA$ $S_{12cw} = 0.603\ puVA$	$0 \le P_{1w} \le 3$	$S_{12a} = 0.239\ puVA$ $S_{12b} = 0.239\ puVA$ $S_{12c} = 0.239\ puVA$	$S_{12aw} = 0.00\ puVA$ $S_{12bw} = 0.358\ puVA$ $S_{12cw} = 0.358\ puVA$
$P_{2w} = 0.78\ puW$	$S_{21a} = 0.14\ puVA$ $S_{21b} = 0.14\ puVA$ $S_{21c} = 0.14\ puVA$	$S_{21aw} = 0.00\ puVA$ $S_{21bw} = 0.21\ puVA$ $S_{21cw} = 0.21\ puVA$	$P_{2w} = 0.00\ pu$	$S_{21a} = 0.400\ puVA$ $S_{21b} = 0.400\ puVA$ $S_{21c} = 0.400\ puVA$	$S_{21aw} = 0.000\ puVA$ $S_{21bw} = 0.600\ puVA$ $S_{21cw} = 0.600\ puVA$	$P_{2w} = 1.92\ puW$	$S_{21a} = 0.24\ puVA$ $S_{21b} = 0.24\ puVA$ $S_{21c} = 0.24\ puVA$	$S_{21aw} = 0.00\ puVA$ $S_{21bw} = 0.36\ puVA$ $S_{21cw} = 0.36\ puVA$
Cost	$1.22\ \$/h$		*Cost*	$2.00\ \$/h$		*Cost*	$0.0814\ \$/h$	

<u>Conclusion from above analysis is summarized as follows:</u>

Still, by implementing the new transmission line having the same capacity, we get the stable power system without security constraint but unreliable and transmission congestion (N-1 contingency) issue still persist when there is low wind penetration (Case 2) or Unit is under maintenance.

Existing line loading capacity modification:

Let's try with existing transmission line modification by upgrading their transmission loading capacity.

$$S_{12a}^{new\_max} = S_{12b}^{new\_max} = S_{21a}^{new\_max} = S_{21b}^{new\_max} = 1.5\ puVA$$

By upgrading existing transmission line loading capacity, we get the following solution…

Case 1 (Average wind penetration)			Case 2 (Zero wind penetration)			Case 3 (High wind penetration)		
Constraint	**OPF Result**	**OPF Result SCOPF**	**Constraint**	**OPF Result**	**OPF Result SCOPF**	**Constraint**	**OPF Result**	**OPF Result SCOPF**
$0 \leq P_1 \leq 3$	$P_1 = 1.22\ puW$	$P_{1w} = 1.22\ puW$	$0 \leq P_1 \leq 3$	$P_1 = 2.006\ puW$	$P_{1w} = 2.006\ puW$	$0 \leq P_1 \leq 3$	$P_1 = 0.082\ puW$	$P_{1w} = 0.082\ puW$
$P_2 = 0.78\ puW$	$P_2 = 0.78\ puW$	$P_{2w} = 0.78\ puW$	$P_2 = 0.00\ puW$	$P_2 = 0.00\ puW$	$P_{2w} = 0.00\ puW$	$P_2 = 1.92\ puW$	$P_2 = 1.92\ puW$	$P_{2w} = 1.92\ puW$
$0 \leq P_{1w} \leq 3$	$S_{12a} = 0.21\ puVA$ $S_{12b} = 0.21\ puVA$	$S_{12aw} = 0.00\ puVA$ $S_{12bw} = 0.42\ puVA$	$0 \leq P_{1w} \leq 3$	$S_{12a} = 0.603\ puVA$ $S_{12b} = 0.603\ puVA$	$S_{12aw} = 0.00\ puVA$ $S_{12bw} = 1.214\ puVA$	$0 \leq P_{1w} \leq 3$	$S_{12a} = 0.359\ puVA$ $S_{12b} = 0.359\ puVA$	$S_{12aw} = 0.00\ puVA$ $S_{12bw} = 0.716\ puVA$
$P_{2w} = 0.78\ puW$	$S_{21a} = 0.21\ puVA$ $S_{21b} = 0.21\ puVA$	$S_{21aw} = 0.00\ puVA$ $S_{21bw} = 0.42\ puVA$	$P_{2w} = 0.00\ puW$	$S_{21a} = 0.60\ puVA$ $S_{21b} = 0.60\ puVA$	$S_{21aw} = 0.00\ puVA$ $S_{21bw} = 1.203\ puVA$	$P_{2w} = 1.92\ puW$	$S_{21a} = 0.36\ puVA$ $S_{21b} = 0.36\ puVA$	$S_{21aw} = 0.00\ puVA$ $S_{21bw} = 0.72\ puVA$
$Cost$	$1.22\ \$/h$		$Cost$	$2.006\ \$/h$		$Cost$	$0.082\ \$/h$	

Conclusion from above analysis is summarized as follows:

By upgrading the existing transmission line with new loading capacity to $1.5\ puVA$, we get the stable power system with considering security constraints.

Conclusion: By commissioning a new wind power plant at node 2 and upgrading the existing transmission line with new loading capacity equal to $1.5\ puVA$, the total operating cost is reduced from $2.4\ \$/h$ to $1.22\ \$/h$ considering the annual average wind power generation.

Assumptions:

1. Transmission line upgradation cost is not considered over the running cost of given power system to conclude the project. If we consider the line upgradation cost against the saving due to the wind power generator's zero fuel cost, conclusion will be depended on the ROI for a given period of time.
2. Variability of wind speed causes changes is active power output of WPP. This may lead to changes in reactive power output and consequently voltage at the Point of Connection [10]. Although, we have not consideration of wind variability effect (wind power plant under maintenance condition) on change in reactive power output, we can adopt reactive power control or voltage control method at local substation to mitigates the requirements [10].
3. We are upgrading the existing line loading capacity to $1.5\ pu$; however, in the above case when Unit 2 active power output is zero ***(Case 2)***, the maximum apparent power flow in line is $1.214\ pu$. Here, we have kept margin in transmission line loading capacity in order to meet the additional reactive power requirement at node 2 from node 1 when Unit 2 stop generating reactive power (Unit-2 may under maintenance condition scenario).

Note: In all the above questions, respective python code files have been attached with the project submission for your reference.

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