Market Clearing Project
220 marks

Consider the following simple system (the base power is 1 MW):

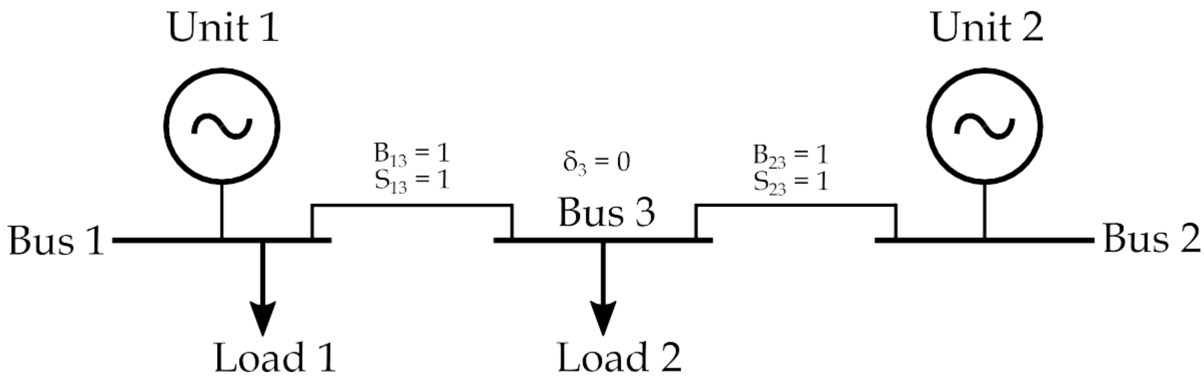

Figure 1: Case 2 - Network Constrained System

The generation offers are as follows:

Table 1: Generation Offers

Hour 1	Unit 1				Unit 2			
Block	1	2	3	4	1	2	3	4
Energy (puMWh)	0.8	0.6	0.4	0.2	0.5	0.5	0.5	0.5
Price ($/puMWh)	11.8	12.6	13.5	14.0	10.1	11.2	12.3	13.0
Hour 2	Unit 1				Unit 2			
Energy (puMWh)	0.8	0.6	0.4	0.2	0.5	0.5	0.5	0.5
Price ($/puMWh)	11.8	12.6	13.5	14.0	10.1	11.2	12.3	13.0

The demand bids are also as follows:

Table 2: Demand Bids

Hour 1	Demand 1				Demand 2			
Block	1	2	3	4	1	2	3	4
Energy (puMWh)	0.4	0.2	0.1	0.1	0.6	0.3	0.1	0.1
Price ($/puMWh)	16.2	14.6	12.1	10.0	16.1	14.2	12.5	11.0
Hour 2	Demand 1				Demand 2			
Energy (puMWh)	0.7	0.3	0.2	0.1	0.8	0.4	0.2	0.1
Price ($/puMWh)	16.2	14.6	12.1	10.0	16.1	14.2	12.5	11.0

Report Tasks:

(a) Write a detailed, specific formulation for an optimal social welfare maximization problem for this system. Ignore the network in this part. Clearly name and specify your variables and parameters. Label all your equations and describe what each equation does. (30 marks)

- **Producers** that own generating units and produce electrical energy.
- **Consumers** that consume electrical energy.

Here, in the question, two generating units given and two power consumers are given.

Consumer Surplus *is a measure of the happiness of the consumers as it is equal to the difference between the amount that the consumers are willing to pay (energy times bid price) and the actual consumers' payment (energy times market clearing price).*	***Producer Surplus*** *represents the happiness of the producers as it is equal to the difference between the actual producers' revenue (energy times market clearing price) and the revenue that producers are willing to accept (energy times offer price).*
Considering the market as whole, the ***social welfare*** *is a measure of happiness of both consumers and producers and equals consumer surplus plus producer surplus.*	

Formulation

In order to clear the market, the market operator seeks to maximize the social welfare, which is the sum of producer surplus and the consumer surplus.

This problem can be formulated as:

$$max_{p^D_{tdc},\forall t,\forall d,\forall c;p^G_{tgb},\forall t,\forall g,\forall b}$$

$$\sum_t \left[\sum_d \sum_{c\in\psi^D_d} \lambda^D_{tdc} p^D_{tdc} - \sum_g \sum_{b\in\psi^G_g} \lambda^G_{tgb} p^G_{tgb} \right] \quad (1a)$$

Subject to

$$0 \le p^D_{tdc} \le P^{Dmax}_{tdc}, \qquad \forall t, \forall d, \forall c \in \psi^D_d, \quad (1b)$$

$$0 \le p^G_{tgb} \le P^{Gmax}_{tgb}, \qquad \forall t, \forall g, \forall b \in \psi^G_g, \quad (1c)$$

$$\sum_g \sum_{b\in\psi^G_g} p^G_{tgb} - \sum_d \sum_{c\in\psi^D_d} p^D_{tdc} = 0, \qquad \forall t \quad (1d)$$

The objective function (1) represent the social welfare to be maximized by the market operator.

Constraints $(1b)$ are bid bounds on the consumption energy of all demands, constraints $(1c)$ are offer bounds on the production energy of all generating units, and constraint $(1d)$ represents the energy balance.

where,

The indexes of the market clearing auctions are:

b production blocks,

c consumption blocks,

d demands,

g generating units, and

t time periods.

Optimization variables:

p_{tdc}^{D} consumption block c bid by demand d at time period t $(in\ puMWh)$

p_{tgb}^{G} production block b offered by generating unit g at time period t $(in\ puMWh)$

δ_{tn} voltage angle at node n at time period t $(in\ purad)$

Constants:

B_{nm} susceptance of transmission line nm,

$P_{tdc}^{D max}$, size of consumption block c bid by demand d at time period t, $(in\ puMWh)$

$P_{tgb}^{G max}$, size of production block b offered by unit g at time period t, $(in\ puMWh)$

$P_{nm}^{l max}$, transmission capacity of line nm, $(in\ puMW)$

λ_{tdc}^{D} consumption bid price of block c of demand d at period t, and $(in\ \$/puMWh)$

λ_{tgb}^{G} production offer price of block b of unit g at period t $(in\ \$/puMWh)$.

Sets:

Λ_n set of nodes directly connected to node n

Ω_n^D set of demands located at node n

Ω_n^G set of generating units located at node n

ψ_g^G set of production blocks of generating unit g

ψ_d^D set of consumption blocks of demand d

We denote as λ^* the optimal value of the dual variable associated with the energy balance constraint.

This dual variable is the market clearing price, which is the price paid to power producers for providing electrical energy and the price paid by power consumers for consuming electrical energy.

The profit of each generating unit can be computed as follows:

$$\pi_g = \sum_{b \in \psi_g^G} (\lambda_t^* - C_{tgb}^G)\, p_{tgb}^{G*}, \qquad \forall g, \forall t \tag{1e}$$

This equation includes two terms, namely:

$\sum_{b \in \psi_g^G} \left(\lambda_t^*\, p_{tgb}^{G*}\right)$	The following term represents the revenues achieved by unit g in t period
$\sum_{b \in \psi_g^G} \left(C_{tgb}^G p_{tgb}^{G*}\right)$	The above term represents the costs incurred by unit g in t period

where, π_g profit of the producer that owns generating unit g$(in\ \$)$

C_{gb}^G cost of production block b of generating unit g $(in\ \$/puMWh)$

p_{gb}^{G*} optimal value of variable p_{gb}^G obtained for the given problem $(in\ puMWh)$

Formulating the given problem as below.

Offer	Hour 1								Hour 2							
	Unit 1				Unit 2				Unit 1				Unit 2			
Block	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4
Energy $(puMWh)$	0.8	0.6	0.4	0.2	0.5	0.5	0.5	0.5	0.8	0.6	0.4	0.2	0.5	0.5	0.5	0.5
Price $(\$/puMWh)$	11.8	12.6	13.5	14.0	10.1	11.2	12.3	13.0	11.8	12.6	13.5	14.0	10.1	11.2	12.3	13.0

Table 1: Generation offers for two-hour time period

Bid	Hour 1								Hour 2							
	Demand 1				Demand 2				Demand 1				Demand 2			
Block	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4
Energy $(puMWh)$	0.4	0.2	0.1	0.1	0.6	0.3	0.1	0.1	0.7	0.3	0.2	0.1	0.8	0.4	0.2	0.1
Price $(\$/puMWh)$	16.2	14.6	12.1	10.0	16.1	14.2	12.5	11.0	16.2	14.6	12.1	10.0	16.1	14.2	12.5	11.0

Table 2: Consumption Bids for two-hour time period

From the equation number $(1a)$, Our objective function to be maximized is given as:

$$max_{p^D_{tdc},\forall t,\forall d,\forall c;p^G_{tgb},\forall t,\forall g,\forall b}$$

$$\begin{aligned}
&= 16.2 * p^D_{111} + 14.6 * p^D_{112} + 12.1 * p^D_{113} + 10 * p^D_{114} \\
&+ 16.1 * p^D_{121} + 14.2 * p^D_{122} + 12.5 * p^D_{123} + 11 * p^D_{124} \\
&+ 16.2 * p^D_{211} + 14.6 * p^D_{212} + 12.1 * p^D_{213} + 10 * p^D_{214} \\
&+ 16.1 * p^D_{221} + 14.2 * p^D_{222} + 12.5 * p^D_{223} + 11 * p^D_{224} \\
&- (11.8 * p^G_{111} + 12.6 * p^G_{112} + 13.5 * p^G_{113} + 14 * p^G_{114}) \\
&- (10.1 * p^G_{121} + 11.2 * p^G_{122} + 12.3 * p^G_{123} + 13 * p^G_{124}) \\
&- (11.8 * p^G_{211} + 12.6 * p^G_{212} + 13.5 * p^G_{213} + 14 * p^G_{214}) \\
&- (10.1 * p^G_{221} + 11.2 * p^G_{222} + 12.3 * p^G_{223} + 13 * p^G_{224})\ \$
\end{aligned}$$

From the equation number $(1b)$, Consumption limits of consumption bids are given as $(in\ puMWh)$:

For the 1^{st} hour	For the 2^{nd} hour
$0 \le p^D_{111} \le 0.4,$	$0 \le p^D_{211} \le 0.7,$
$0 \le p^D_{112} \le 0.2,$	$0 \le p^D_{212} \le 0.3,$
$0 \le p^D_{113} \le 0.1,$	$0 \le p^D_{213} \le 0.2,$
$0 \le p^D_{114} \le 0.1,$	$0 \le p^D_{214} \le 0.1,$
$0 \le p^D_{121} \le 0.6,$	$0 \le p^D_{221} \le 0.8,$
$0 \le p^D_{122} \le 0.3,$	$0 \le p^D_{222} \le 0.4,$
$0 \le p^D_{123} \le 0.1,$	$0 \le p^D_{223} \le 0.2,$
$0 \le p^D_{124} \le 0.1,$	$0 \le p^D_{224} \le 0.1,$

From the equation number $(1c)$, Production limits of production offers($in\ puMWh$):

For the 1^{st} hour	For the 2^{nd} hour
$0 \le p^G_{111} \le 0.8,$	$0 \le p^G_{211} \le 0.8,$
$0 \le p^G_{112} \le 0.6,$	$0 \le p^G_{212} \le 0.6,$
$0 \le p^G_{113} \le 0.4,$	$0 \le p^G_{213} \le 0.4,$
$0 \le p^G_{114} \le 0.2,$	$0 \le p^G_{214} \le 0.2,$
$0 \le p^G_{121} \le 0.5,$	$0 \le p^G_{221} \le 0.5,$
$0 \le p^G_{122} \le 0.5,$	$0 \le p^G_{222} \le 0.5,$
$0 \le p^G_{123} \le 0.5,$	$0 \le p^G_{223} \le 0.5,$
$0 \le p^G_{124} \le 0.5,$	$0 \le p^G_{224} \le 0.5,$

From the equation number $(1d)$, Energy Balance equations are:

For the 1^{st} hour

$$\begin{aligned} &p^D_{111} + p^D_{112} + p^D_{113} + p^D_{114} + p^D_{121} + p^D_{122} + p^D_{123} + p^D_{124} \\ &- p^G_{111} - p^G_{112} - p^G_{113} - p^G_{114} - p^G_{121} - p^G_{122} - p^G_{123} - p^G_{124} = 0 \end{aligned}$$

For the 2^{nd} hour

$$\begin{aligned} &p^D_{211} + p^D_{212} + p^D_{213} + p^D_{214} + p^D_{221} + p^D_{222} + p^D_{223} + p^D_{224} \\ &- p^G_{211} - p^G_{212} - p^G_{213} - p^G_{214} - p^G_{221} - p^G_{222} - p^G_{223} - p^G_{224} = 0 \end{aligned}$$

(b) Implement the formulation in a computer simulation code and find the optimal solution. No generic code. The code must be specific to the specific equations that you developed for this network in the previous part. List the accepted bids and offers, and rejected bids and offers, and the market clearing prices for each of the intervals in separate well-organized/labelled tables. Do not forget the units. (70 marks)

```
#Pyomo objects exist within the pyomo.environ namespace
#Every Pyomo model starts with this; it tells Python to load the Pyomo Modeling Environment
from pyomo.environ import *
from pyomo.opt import SolverFactory

#Create an instance of a Concrete model
m = ConcreteModel("Question-(b)")

# In below equations, [i,j,k] indicates [time period, Generating unit number, block].

# Defining demand-1 for first hour
m.pd111 = Var(bounds=(0.0,0.4))    #--->Block 1
m.pd112 = Var(bounds=(0.0,0.2))    #--->Block 2
m.pd113 = Var(bounds=(0.0,0.1))    #--->Block 3
m.pd114 = Var(bounds=(0.0,0.1))    #--->Block 4

# Defining demand-2 for first hour
m.pd121 = Var(bounds=(0.0,0.6))    #--->Block 1
m.pd122 = Var(bounds=(0.0,0.3))    #--->Block 2
m.pd123 = Var(bounds=(0.0,0.1))    #--->Block 3
m.pd124 = Var(bounds=(0.0,0.1))    #--->Block 4

# Defining demand-1 for second hour
m.pd211 = Var(bounds=(0.0,0.7))    #--->Block 1
m.pd212 = Var(bounds=(0.0,0.3))    #--->Block 2
m.pd213 = Var(bounds=(0.0,0.2))    #--->Block 3
m.pd214 = Var(bounds=(0.0,0.1))    #--->Block 4

# Defining demand-2 for second hour
m.pd221 = Var(bounds=(0.0,0.8))    #--->Block 1
m.pd222 = Var(bounds=(0.0,0.4))    #--->Block 2
m.pd223 = Var(bounds=(0.0,0.2))    #--->Block 3
m.pd224 = Var(bounds=(0.0,0.1))    #--->Block 4

# Defining Unit-1 generation for first hour
m.pg111 = Var(bounds=(0.0,0.8))   #--->Block 1
m.pg112 = Var(bounds=(0.0,0.6))   #--->Block 2
m.pg113 = Var(bounds=(0.0,0.4))   #--->Block 3
m.pg114 = Var(bounds=(0.0,0.2))   #--->Block 4

# Defining Unit-2 generation for first hour
m.pg121 = Var(bounds=(0.0,0.5))   #--->Block 1
m.pg122 = Var(bounds=(0.0,0.5))   #--->Block 2
m.pg123 = Var(bounds=(0.0,0.5))   #--->Block 3
m.pg124 = Var(bounds=(0.0,0.5))   #--->Block 4

# Defining Unit-1 generation for second hour
m.pg211 = Var(bounds=(0.0,0.8))   #--->Block 1
m.pg212 = Var(bounds=(0.0,0.6))   #--->Block 2
m.pg213 = Var(bounds=(0.0,0.4))   #--->Block 3
m.pg214 = Var(bounds=(0.0,0.2))   #--->Block 4

# Defining Unit-2 generation for second hour
m.pg221 = Var(bounds=(0.0,0.5))   #--->Block 1
m.pg222 = Var(bounds=(0.0,0.5))   #--->Block 2
m.pg223 = Var(bounds=(0.0,0.5))   #--->Block 3
m.pg224 = Var(bounds=(0.0,0.5))   #--->Block 4
```

```
# Defining an objective function to maximize the social welfare.
m.objective = Objective(expr = 16.2*m.pd111 + 14.6*m.pd112 + 12.1*m.pd113 + 10*m.pd114\    #--->Time period-1
                        + 16.1*m.pd121 + 14.2*m.pd122 + 12.5*m.pd123 + 11*m.pd124\         #--->Time period-1
                        + 16.2*m.pd211 + 14.6*m.pd212 + 12.1*m.pd213 + 10*m.pd214\         #--->Time period-2
                        + 16.1*m.pd221 + 14.2*m.pd222 + 12.5*m.pd223 + 11*m.pd224\         #--->Time period-2
                        - (11.8*m.pg111 + 12.6*m.pg112 + 13.5*m.pg113 + 14*m.pg114)\       #--->Time period-1
                        - (10.1*m.pg121 + 11.2*m.pg122 + 12.3*m.pg123 + 13*m.pg124)\       #--->Time period-1
                        - (11.8*m.pg211 + 12.6*m.pg212 + 13.5*m.pg213 + 14*m.pg214)\       #--->Time period-2
                        - (10.1*m.pg221 + 11.2*m.pg222 + 12.3*m.pg223 + 13*m.pg224) , sense = maximize) #--->Time period-2

# Enegy balance equation for first hour.
m.constraint1 = Constraint(expr = m.pd111 + m.pd112 + m.pd113 + m.pd114 \
                        + m.pd121 + m.pd122 + m.pd123 + m.pd124\
                        - (m.pg111 + m.pg112 + m.pg113 + m.pg114)\
                        - (m.pg121 + m.pg122 + m.pg123 + m.pg124) == 0)

# Enegy balance equation for second hour.
m.constraint2 = Constraint(expr = m.pd211 + m.pd212 + m.pd213 + m.pd214\
                        + m.pd221 + m.pd222 + m.pd223 + m.pd224\
                        - (m.pg211 + m.pg212 + m.pg213 + m.pg214)\
                        - (m.pg221 + m.pg222 + m.pg223 + m.pg224) == 0)

# To extract the dual variable.
m.dual = Suffix(direction=Suffix.IMPORT)

#Solving models
opt = SolverFactory('gurobi')
opt.solve(m)

#Display the result
m.display()

#Display the extracted multipliers
m.dual.pprint()
```

```
Model 'Question-(b)'

  Variables:
    pd111 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.4 :   0.4 : False : False :  Reals
    pd112 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.2 :   0.2 : False : False :  Reals
    pd113 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.1 :   0.1 : False : False :  Reals
    pd114 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.0 :   0.1 : False : False :  Reals
    pd121 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.6 :   0.6 : False : False :  Reals
    pd122 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.3 :   0.3 : False : False :  Reals
    pd123 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.1 :   0.1 : False : False :  Reals
    pd124 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.0 :   0.1 : False : False :  Reals
    pd211 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.7 :   0.7 : False : False :  Reals
    pd212 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.3 :   0.3 : False : False :  Reals
    pd213 : Size=1, Index=None
        Key  : Lower : Value : Upper : Fixed : Stale : Domain
        None :   0.0 :   0.0 :   0.2 : False : False :  Reals
```

```
  pd214 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.1 : False : False :  Reals
  pd221 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.8 :   0.8 : False : False :  Reals
  pd222 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.4 :   0.4 : False : False :  Reals
  pd223 : Size=1, Index=None
      Key  : Lower : Value               : Upper : Fixed : Stale : Domain
      None :   0.0 : 0.10000000000000009 :   0.2 : False : False :  Reals
  pd224 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.1 : False : False :  Reals
  pg111 : Size=1, Index=None
      Key  : Lower : Value              : Upper : Fixed : Stale : Domain
      None :   0.0 : 0.7000000000000001 :   0.8 : False : False :  Reals
  pg112 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.6 : False : False :  Reals
  pg113 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.4 : False : False :  Reals
  pg114 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.2 : False : False :  Reals
  pg121 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.5 :   0.5 : False : False :  Reals

  pg122 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.5 :   0.5 : False : False :  Reals
  pg123 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.5 : False : False :  Reals
  pg124 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.5 : False : False :  Reals
  pg211 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.8 :   0.8 : False : False :  Reals
  pg212 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.6 : False : False :  Reals
  pg213 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.4 : False : False :  Reals
  pg214 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.2 : False : False :  Reals
  pg221 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.5 :   0.5 : False : False :  Reals
  pg222 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.5 :   0.5 : False : False :  Reals
  pg223 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.5 :   0.5 : False : False :  Reals

  pg224 : Size=1, Index=None
      Key  : Lower : Value : Upper : Fixed : Stale : Domain
      None :   0.0 :   0.0 :   0.5 : False : False :  Reals

Objectives:
  objective : Size=1, Index=None, Active=True
      Key  : Active : Value
      None :   True : 16.159999999999997
```

```
  Constraints:
    constraint1 : Size=1
        Key  : Lower : Body : Upper
        None :   0.0 :  0.0 :   0.0
    constraint2 : Size=1
        Key  : Lower : Body                  : Upper
        None :   0.0 : 2.220446049250313e-16 :   0.0
dual : Direction=Suffix.IMPORT, Datatype=Suffix.FLOAT
    Key         : Value
    constraint1 :  11.8
    constraint2 :  12.5
```

Accepted production offers of the generating units

Accepted Offer	**Hour 1**								**Hour 2**							
	Unit **1**				Unit **2**				Unit **1**				Unit **2**			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(puMWh)$	0.7				0.5	0.5			0.8				0.5	0.5	0.5	

Table 3: Accepted production offers of generation units

Accepted consumption bids of the demand

Accepted bids	**Hour 1**								**Hour 2**							
	Demand **1**				Demand **2**				Demand **1**				Demand **2**			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(puMWh)$	0.4	0.2	0.1		0.6	0.3	0.1		0.7	0.3			0.8	0.4	0.1	

Table 4: Accepted consumption bids of the demand

Rejected production offers of the generating units

Rejected Offer	**Hour 1**								**Hour 2**							
	Unit **1**				Unit **2**				Unit **1**				Unit **2**			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(puMWh)$	0.1	0.6	0.4	0.2			0.5	0.5		0.6	0.4	0.2				0.5

Table 5: Rejected production offers of the generating units

Rejected consumption bids of the demand

Rejected Bids	**Hour 1**								**Hour 2**							
	Demand **1**				Demand **2**				Demand **1**				Demand **2**			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(puMWh)$				0.1				0.1			0.2	0.1			0.1	0.1

Table 6: Rejected consumption bids of the demand

Market Clearing Price and Social Welfare

	Hour 1	**Hour 2**
Market Clearing Price	$11.8\ \$/puMWh$	$12.5\ \$/puMWh$
Social Welfare$(\$)$	*16.16 $*	

Table 7: Market Clearing Price and Social Welfare

We observe that, as expected, the accepted production offer blocks are those with the lowest offer prices, while the accepted bid blocks are those with the highest bid prices as shown in *Table 3-6.*

This solution provides an optimal value of the objective function, i.e., the social welfare, of $16.16.

The dual variable is known as the market clearing price and it is the price paid to producers for providing energy and the price paid by consumers for consuming energy.

The profit of each generating unit can be computed as follows:

Total profit of generating Unit-1

$$\pi_1 = (\lambda_1^* - C_{11}^G)\, p_{111}^G + (\lambda_1^* - C_{12}^G)\, p_{112}^G + (\lambda_1^* - C_{13}^G)\, p_{113}^G + (\lambda_1^* - C_{14}^G)\, p_{114}^G$$

$$+(\lambda_2^* - C_{11}^G)\, p_{211}^G + (\lambda_2^* - C_{12}^G)\, p_{212}^G + (\lambda_2^* - C_{13}^G)\, p_{213}^G + (\lambda_2^* - C_{14}^G)\, p_{214}^G$$

$$= (11.8 - 11.8) * 0.7 + (11.8 - 12.6) * 0.0 + (11.8 - 13.5) * 0.0 + (11.8 - 14.0) * 0.0$$

$$+ (12.5 - 11.8) * 0.8 + (12.5 - 12.6) * 0.0 + (12.5 - 13.5) * 0.0 + (12.5 - 14.0) * 0.0$$
$$= 0.56\ \$$$

Total profit of generating Unit-2

$$\pi_2 = (\lambda_1^* - C_{21}^G)\, p_{121}^G + (\lambda_1^* - C_{22}^G)\, p_{122}^G + (\lambda_1^* - C_{23}^G)\, p_{123}^G + (\lambda_1^* - C_{24}^G)\, p_{124}^G$$

$$+(\lambda_2^* - C_{21}^G)\, p_{221}^G + (\lambda_2^* - C_{22}^G)\, p_{222}^G + (\lambda_2^* - C_{23}^G)\, p_{223}^G + (\lambda_2^* - C_{24}^G)\, p_{224}^G$$

$$= (11.8 - 10.1) * 0.5 + (11.8 - 11.2) * 0.5 + (11.8 - 12.3) * 0.0 + (11.8 - 13.0) * 0.0$$

$$+ (12.5 - 10.1) * 0.5 + (12.5 - 11.2) * 0.5 + (12.5 - 12.3) * 0.5 + (12.5 - 13.0) * 0.0$$

$$= 0.85 + 0.3 + 1.2 + 0.65 + 0.1$$

$$= 3.1\ \$$$

Total Profit of generating Units

	Unit 1	Unit 2
Total profit	0.56 $	3.1 $

Table 8: Total Profit of generating Units

(c) Determine and present the additional specific equations that must be added to the formulation in part 1 in order to have an optimal DC-network-constrained multi-period market clearing model. (20 marks)

Transmission-Constrained Multi-Period Market Clearing Auction

The multi-period market clearing auction formulated in the previous section is extended here to consider also the constraints of the transmission network:

Transmission Network Constraints:

$$\sum_{g\in\Omega_n^G}\sum_{b\in\psi_g^G} p_{tgb}^G - \sum_{d\in\Omega_n^D}\sum_{c\in\psi_d^D} p_{tdc}^D = \sum_{m\in\Lambda_n} B_{nm}(\delta_{tn} - \delta_{tm}) \qquad : \lambda_{tn}, \forall t, \forall n, \qquad (1f)$$

$$-P_{nm}^{Lmax} \leq B_{nm}(\delta_{tn} - \delta_{tm}) \leq P_{nm}^{Lmax}, \qquad \forall t, \forall n, \forall m \in \Lambda_n, \qquad (1g)$$

$$\delta_{tn} = 0, \forall t, n{:}\, ref. \qquad (1h)$$

The differences between these two problems are constraints $(1f)$ that represent the energy balance per node and time period, constraints $(1g)$ that impose transmission capacity limits per transmission line and time period, and constraints $(1h)$ that fix to zero the voltage angle at the reference node for all time period.

Prices can be different at different nodes: we may use a cheap generating unit to supply a demand at a given node but this cheap generating unit may not be used to supply a demand at a different node due to transmission congestion.

Therefore, these dual variables λ_{tn} are usually known as locational marginal prices (LMPs) or spot prices.

Considering the network shown in figure, generating units 1 and 2 are located at Node-1 and Node-2 respectively. While, the demand-1 and 2 are located at Node-1 and Node-3 respectively.

The data of transmission network are given in the question.
I consider a base power of $1MW$ so that all the data of the previous example can be easily transformed into a per-unit system.
Using the above data, we formulate the required additional transmission-constrained multi-period auction as shown below.
Energy balance per node constraints:

For node-1

$$p_{111}^G + p_{112}^G + p_{113}^G + p_{114}^G = p_{111}^D + p_{112}^D + p_{113}^D + p_{114}^D + B_{13}(\delta_{11} - \delta_{13}) \ldots 1^{st}\ hour\ time\ period$$
$$p_{211}^G + p_{212}^G + p_{213}^G + p_{214}^G = p_{211}^D + p_{212}^D + p_{213}^D + p_{214}^D + B_{23}(\delta_{21} - \delta_{23}) \ldots 2^{st}\ hour\ time\ period$$

For node-2

$$p_{121}^G + p_{122}^G + p_{123}^G + p_{124}^G = B_{23}(\delta_{12} - \delta_{13}) \ldots 1^{st}\ hour\ time\ period$$
$$p_{221}^G + p_{222}^G + p_{223}^G + p_{224}^G = B_{23}(\delta_{22} - \delta_{23}) \ldots 2^{nd}\ hour\ time\ period$$

For node-3

$$p_{121}^D + p_{122}^D + p_{123}^D + p_{124}^D = B_{23}(\delta_{12} - \delta_{13}) + B_{13}(\delta_{11} - \delta_{13}) \ldots 1^{st}\ hour\ time\ period$$
$$p_{221}^D + p_{222}^D + p_{223}^D + p_{224}^D = B_{23}(\delta_{22} - \delta_{23}) + B_{13}(\delta_{21} - \delta_{23}) \ldots 2^{nd}\ hour\ time\ period$$

Transmission Line loading capacity:

Transmission Line L_{13}

$$-P_{13}^{L_{max}} \leq B_{13}(\delta_{11} - \delta_{13}) \leq P_{13}^{L_{max}} \ldots\ldots\ldots\ldots\ldots\ldots 1^{st}\ hour\ time\ period$$
$$-P_{13}^{L_{max}} \leq B_{13}(\delta_{21} - \delta_{23}) \leq P_{13}^{L_{max}} \ldots\ldots\ldots\ldots\ldots\ldots 2^{nd}\ hour\ time\ period$$

Therefore,

$$-1 \leq B_{13}(\delta_{11}) \leq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1^{st}\ hour\ time\ period$$
$$-1 \leq B_{13}(\delta_{21}) \leq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2^{nd}\ hour\ time\ period$$

Transmission Line L_{23}

$$-P_{23}^{L_{max}} \leq B_{13}(\delta_{11} - \delta_{13}) \leq P_{23}^{L_{max}} \ldots\ldots\ldots\ldots\ldots\ldots 1^{st}\ hour\ time\ period$$
$$-P_{23}^{L_{max}} \leq B_{13}(\delta_{21} - \delta_{23}) \leq P_{23}^{L_{max}} \ldots\ldots\ldots\ldots\ldots\ldots 2^{nd}\ hour\ time\ period$$

Therefore,

$$-1 \leq B_{23}(\delta_{11}) \leq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1^{st}\ hour\ time\ period$$
$$-1 \leq B_{23}(\delta_{21}) \leq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2^{nd}\ hour\ time\ period$$

Voltage Angle at reference node-3

$$\delta_{13} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1^{st}\ hour\ time\ period$$
$$\delta_{23} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2^{nd}\ hour\ time\ period$$

(d) Implement your complete model of part 3 in a computer simulation code and find the optimal solution. No generic code. The code must be specific to the specific equations that you developed for this network in the previous part. List the accepted bids and offers, and rejected bids and offers, and the market clearing prices for each of the intervals at each node in separate well-organized/labelled tables. Do not forget the units. (40 marks)

```
#Pyomo objects exist within the pyomo.environ namespace
#Every Pyomo model starts with this; it tells Python to load the Pyomo Modeling Environment
from pyomo.environ import *
from pyomo.opt import SolverFactory
import cmath
import math

#Create an instance of a Concrete model
m = ConcreteModel("Question-(d)")

# In below equations, [i,j,k] indicates [time period, Generating unit number, block].

# Defining demand-1 for first hour
m.pd111 = Var(bounds=(0.0,0.4))     #--->Block 1
m.pd112 = Var(bounds=(0.0,0.2))     #--->Block 2
m.pd113 = Var(bounds=(0.0,0.1))     #--->Block 3
m.pd114 = Var(bounds=(0.0,0.1))     #--->Block 4

# Defining demand-2 for first hour
m.pd121 = Var(bounds=(0.0,0.6))     #--->Block 1
m.pd122 = Var(bounds=(0.0,0.3))     #--->Block 2
m.pd123 = Var(bounds=(0.0,0.1))     #--->Block 3
m.pd124 = Var(bounds=(0.0,0.1))     #--->Block 4

# Defining demand-1 for second hour
m.pd211 = Var(bounds=(0.0,0.7))     #--->Block 1
m.pd212 = Var(bounds=(0.0,0.3))     #--->Block 2
m.pd213 = Var(bounds=(0.0,0.2))     #--->Block 3
m.pd214 = Var(bounds=(0.0,0.1))     #--->Block 4
# Defining demand-2 for second hour
m.pd221 = Var(bounds=(0.0,0.8))     #--->Block 1
m.pd222 = Var(bounds=(0.0,0.4))     #--->Block 2
m.pd223 = Var(bounds=(0.0,0.2))     #--->Block 3
m.pd224 = Var(bounds=(0.0,0.1))     #--->Block 4

# Defining Unit-1 generation for first hour
m.pg111 = Var(bounds=(0.0,0.8))     #--->Block 1
m.pg112 = Var(bounds=(0.0,0.6))     #--->Block 2
m.pg113 = Var(bounds=(0.0,0.4))     #--->Block 3
m.pg114 = Var(bounds=(0.0,0.2))     #--->Block 4

# Defining Unit-2 generation for first hour
m.pg121 = Var(bounds=(0.0,0.5))     #--->Block 1
m.pg122 = Var(bounds=(0.0,0.5))     #--->Block 2
m.pg123 = Var(bounds=(0.0,0.5))     #--->Block 3
m.pg124 = Var(bounds=(0.0,0.5))     #--->Block 4

# Defining Unit-1 generation for second hour
m.pg211 = Var(bounds=(0.0,0.8))     #--->Block 1
m.pg212 = Var(bounds=(0.0,0.6))     #--->Block 2
m.pg213 = Var(bounds=(0.0,0.4))     #--->Block 3
m.pg214 = Var(bounds=(0.0,0.2))     #--->Block 4

# Defining Unit-2 generation for second hour
m.pg221 = Var(bounds=(0.0,0.5))     #--->Block 1
m.pg222 = Var(bounds=(0.0,0.5))     #--->Block 2
m.pg223 = Var(bounds=(0.0,0.5))     #--->Block 3
m.pg224 = Var(bounds=(0.0,0.5))     #--->Block 4
```

```
# Defining voltage angle for first hour.
m.δ11 = Var(bounds=(-math.pi,math.pi))    #--->Node 1
m.δ12 = Var(bounds=(-math.pi,math.pi))    #--->Node 2
m.δ13 = Var(bounds=(0,0))                 #--->Node 3 (Given=0 as reference node)

# Defining voltage angle for first hour.
m.δ21 = Var(bounds=(-math.pi,math.pi))    #--->Node 1
m.δ22 = Var(bounds=(-math.pi,math.pi))    #--->Node 2
m.δ23 = Var(bounds=(0,0))                 #--->Node 3 (Given=0 as reference node)

# Defining suseptance for transmission Line-13 and Line-23.
m.B13 = Var(bounds=(1,1))
m.B23 = Var(bounds=(1,1))


# Defining an objective function to minimize the total cost for Unit 1 and Unit 2.
m.objective = Objective(expr = 16.2*m.pd111 + 14.6*m.pd112 + 12.1*m.pd113 + 10*m.pd114 \
                        + 16.1*m.pd121 + 14.2*m.pd122 + 12.5*m.pd123 + 11*m.pd124\
                        + 16.2*m.pd211 + 14.6*m.pd212 + 12.1*m.pd213 + 10*m.pd214\
                        + 16.1*m.pd221 + 14.2*m.pd222 + 12.5*m.pd223 + 11*m.pd224\
                        - (11.8*m.pg111 + 12.6*m.pg112 + 13.5*m.pg113 + 14*m.pg114)\
                        - (10.1*m.pg121 + 11.2*m.pg122 + 12.3*m.pg123 + 13*m.pg124)\
                        - (11.8*m.pg211 + 12.6*m.pg212 + 13.5*m.pg213 + 14*m.pg214)\
                        - (10.1*m.pg221 + 11.2*m.pg222 + 12.3*m.pg223 + 13*m.pg224) , sense = maximize)

# Enegy balance equation for first hour.
m.constraint1 = Constraint(expr = m.pd111 + m.pd112 + m.pd113 + m.pd114 \
                        + m.pd121 + m.pd122 + m.pd123 + m.pd124\
                        - (m.pg111 + m.pg112 + m.pg113 + m.pg114)\
                        - (m.pg121 + m.pg122 + m.pg123 + m.pg124) == 0)
# Enegy balance equation for second hour.
m.constraint2 = Constraint(expr = m.pd211 + m.pd212 + m.pd213 + m.pd214\
                        + m.pd221 + m.pd222 + m.pd223 + m.pd224\
                        - (m.pg211 + m.pg212 + m.pg213 + m.pg214)\
                        - (m.pg221 + m.pg222 + m.pg223 + m.pg224) == 0)

# Energy balance at Node-1 for first hour.
m.constraint3 = Constraint(expr = m.pg111 + m.pg112 + m.pg113 + m.pg114 - m.pd111 - m.pd112\
                           - m.pd113 - m.pd114 - m.B13*(m.δ11-m.δ13) == 0)

# Energy balance at Node-1 for second hour.
m.constraint4 = Constraint(expr = m.pg211 + m.pg212 + m.pg213 + m.pg214== m.pd211 + m.pd212\
                           + m.pd213 + m.pd214 + m.B13*(m.δ21-m.δ23))

# Energy balance at Node-2 for first hour.
m.constraint5 = Constraint(expr = m.pg121 + m.pg122 + m.pg123 + m.pg124 ==  m.B23*(m.δ12-m.δ13))

# Energy balance at Node-2 for second hour.
m.constraint6 = Constraint(expr = m.pg221 + m.pg222 + m.pg223 + m.pg224 ==  m.B23*(m.δ22-m.δ23))

# Energy balance at Node-3 for first hour.
m.constraint7 = Constraint(expr = m.pd121 + m.pd122 + m.pd123 + m.pd124 == m.B13*(m.δ11-m.δ13) + m.B23*(m.δ12-m.δ13))

# Energy balance at Node-3 for second hour.
m.constraint8 = Constraint(expr = m.pd221 + m.pd222 + m.pd223 + m.pd224 == m.B13*(m.δ21-m.δ23) + m.B23*(m.δ22-m.δ23))

# Line limit loading constraint for Line 1-3 for first hour
m.constraint9 = Constraint(expr = -1 <= m.B13*(m.δ11-m.δ13))
m.constraint10 = Constraint(expr = 1 >= m.B13*(m.δ11-m.δ13))
# Line limit loading constraint for Line 1-3 for second hour
m.constraint11 = Constraint(expr = -1 <= m.B13*(m.δ21-m.δ23))
m.constraint12 = Constraint(expr = 1 >= m.B13*(m.δ21-m.δ23))

# Line limit loading constraint for Line 2-3 for first hour
m.constraint13 = Constraint(expr = -1 <= m.B23*(m.δ12-m.δ13))
m.constraint14 = Constraint(expr = 1 >= m.B23*(m.δ12-m.δ13))

# Line limit loading constraint for Line 2-3 for second hour
m.constraint15 = Constraint(expr = -1 <= m.B23*(m.δ22-m.δ23))
m.constraint16 = Constraint(expr = 1 >= m.B23*(m.δ22-m.δ23))
```

```
# To extract the dual variable.
m.dual = Suffix(direction=Suffix.IMPORT)

#Solving models
opt = SolverFactory('gurobi')
opt.solve(m)

#Display the result
m.display()

#Display the extracted multipliers
m.dual.pprint()
```

```
Model 'Question-(d)'

  Variables:
    pd111 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.3999999999999994 :   0.4 : False : False :  Reals
    pd112 : Size=1, Index=None
        Key  : Lower : Value             : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.199999999999999 :   0.2 : False : False :  Reals
    pd113 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.0999999999999987 :   0.1 : False : False :  Reals
    pd114 : Size=1, Index=None
        Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
        None :   0.0 : 2.6419270758218413e-15 :   0.1 : False : False :  Reals
    pd121 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.5999999999999993 :   0.6 : False : False :  Reals
    pd122 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.2999999999999988 :   0.3 : False : False :  Reals
    pd123 : Size=1, Index=None
        Key  : Lower : Value               : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.09999999999998785 :   0.1 : False : False :  Reals
    pd124 : Size=1, Index=None
        Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
        None :   0.0 : 1.3924621205762768e-14 :   0.1 : False : False :  Reals
    pd211 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.6999999999999992 :   0.7 : False : False :  Reals
    pd212 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.2999999999999983 :   0.3 : False : False :  Reals
    pd213 : Size=1, Index=None
        Key  : Lower : Value                 : Upper : Fixed : Stale : Domain
        None :   0.0 : 4.036707576942669e-15 :   0.2 : False : False :  Reals
    pd214 : Size=1, Index=None
        Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
        None :   0.0 : 1.1317903324177152e-15 :   0.1 : False : False :  Reals
    pd221 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.7999999999999993 :   0.8 : False : False :  Reals
    pd222 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.3999999999999981 :   0.4 : False : False :  Reals
    pd223 : Size=1, Index=None
        Key  : Lower : Value                 : Upper : Fixed : Stale : Domain
        None :   0.0 : 4.286232076714011e-14 :   0.2 : False : False :  Reals
    pd224 : Size=1, Index=None
        Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
        None :   0.0 : 1.7137489253976136e-15 :   0.1 : False : False :  Reals
    pg111 : Size=1, Index=None
        Key  : Lower : Value              : Upper : Fixed : Stale : Domain
        None :   0.0 : 0.7000000000000639 :   0.8 : False : False :  Reals
    pg112 : Size=1, Index=None
        Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
        None :   0.0 : 2.3310890063530397e-14 :   0.6 : False : False :  Reals
```

```
pg113 : Size=1, Index=None
    Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
    None :   0.0 : 2.1603816021007377e-15 :   0.4 : False : False :  Reals
pg114 : Size=1, Index=None
    Key  : Lower : Value                 : Upper : Fixed : Stale : Domain
    None :   0.0 : 1.328644865292083e-15 :   0.2 : False : False :  Reals
pg121 : Size=1, Index=None
    Key  : Lower : Value              : Upper : Fixed : Stale : Domain
    None :   0.0 : 0.4999999999999947 :   0.5 : False : False :  Reals
pg122 : Size=1, Index=None
    Key  : Lower : Value              : Upper : Fixed : Stale : Domain
    None :   0.0 : 0.4999999999999133 :   0.5 : False : False :  Reals
pg123 : Size=1, Index=None
    Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
    None :   0.0 : 1.1580511764588907e-16 :   0.5 : False : False :  Reals
pg124 : Size=1, Index=None
    Key  : Lower : Value                 : Upper : Fixed : Stale : Domain
    None :   0.0 : 9.034842334024899e-16 :   0.5 : False : False :  Reals
pg211 : Size=1, Index=None
    Key  : Lower : Value              : Upper : Fixed : Stale : Domain
    None :   0.0 : 0.7999999999999963 :   0.8 : False : False :  Reals
pg212 : Size=1, Index=None
    Key  : Lower : Value              : Upper : Fixed : Stale : Domain
    None :   0.0 : 0.4000000000000447 :   0.6 : False : False :  Reals
pg213 : Size=1, Index=None
    Key  : Lower : Value                 : Upper : Fixed : Stale : Domain
    None :   0.0 : 4.133516110684886e-15 :   0.4 : False : False :  Reals
pg214 : Size=1, Index=None
    Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
    None :   0.0 : 2.3980172016874665e-15 :   0.2 : False : False :  Reals

pg221 : Size=1, Index=None
    Key  : Lower : Value              : Upper : Fixed : Stale : Domain
    None :   0.0 : 0.4999999999999976 :   0.5 : False : False :  Reals
pg222 : Size=1, Index=None
    Key  : Lower : Value               : Upper : Fixed : Stale : Domain
    None :   0.0 : 0.49999999999999123 :   0.5 : False : False :  Reals
pg223 : Size=1, Index=None
    Key  : Lower : Value                  : Upper : Fixed : Stale : Domain
    None :   0.0 : 5.6829106406131916e-15 :   0.5 : False : False :  Reals
pg224 : Size=1, Index=None
    Key  : Lower : Value                 : Upper : Fixed : Stale : Domain
    None :   0.0 : 2.598038762213887e-15 :   0.5 : False : False :  Reals
δ11 : Size=1, Index=None
    Key  : Lower              : Value                  : Upper             : Fixed : Stale : Domain
    None : -3.141592653589793 : 8.837375276016246e-14 : 3.141592653589793 : False : False :  Reals
δ12 : Size=1, Index=None
    Key  : Lower              : Value             : Upper             : Fixed : Stale : Domain
    None : -3.141592653589793 : 0.999999999999905 : 3.141592653589793 : False : False :  Reals
δ13 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     0 : False : False :  Reals
δ21 : Size=1, Index=None
    Key  : Lower              : Value              : Upper             : Fixed : Stale : Domain
    None : -3.141592653589793 : 0.2000000000000437 : 3.141592653589793 : False : False :  Reals
δ22 : Size=1, Index=None
    Key  : Lower              : Value              : Upper             : Fixed : Stale : Domain
    None : -3.141592653589793 : 0.9999999999999956 : 3.141592653589793 : False : False :  Reals
δ23 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     0 :   0.0 :     0 : False : False :  Reals
B13 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     1 :   1.0 :     1 : False : False :  Reals
B23 : Size=1, Index=None
    Key  : Lower : Value : Upper : Fixed : Stale : Domain
    None :     1 :   1.0 :     1 : False : False :  Reals
```

```
  Objectives:
    objective : Size=1, Index=None, Active=True
        Key  : Active : Value
        None :   True : 16.01999999999983

  Constraints:
    constraint1 : Size=1
        Key  : Lower : Body : Upper
        None :   0.0 :  0.0 :   0.0
    constraint2 : Size=1
        Key  : Lower : Body                  : Upper
        None :   0.0 : 2.220446049250313e-16 :   0.0
    constraint3 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : 2.5206099886851367e-15 :   0.0
    constraint4 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : 1.3322676295501878e-15 :   0.0
    constraint5 : Size=1
        Key  : Lower : Body                  : Upper
        None :   0.0 : 4.107825191113079e-15 :   0.0
    constraint6 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : 1.5543122344752192e-15 :   0.0
    constraint7 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : 6.5503158452884236e-15 :   0.0
    constraint8 : Size=1
        Key  : Lower : Body                   : Upper
        None :   0.0 : 2.6645352591003757e-15 :   0.0
    constraint9 : Size=1
        Key  : Lower : Body                  : Upper
        None :  -1.0 : 8.837375276016246e-14 :  None
    constraint10 : Size=1
        Key  : Lower : Body                  : Upper
        None :  None : 8.837375276016246e-14 :   1.0
    constraint11 : Size=1
        Key  : Lower : Body               : Upper
        None :  -1.0 : 0.2000000000000437 :  None
    constraint12 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : 0.2000000000000437 :   1.0
    constraint13 : Size=1
        Key  : Lower : Body              : Upper
        None :  -1.0 : 0.999999999999905 :  None
    constraint14 : Size=1
        Key  : Lower : Body              : Upper
        None :  None : 0.999999999999905 :   1.0
    constraint15 : Size=1
        Key  : Lower : Body               : Upper
        None :  -1.0 : 0.9999999999999956 :  None
    constraint16 : Size=1
        Key  : Lower : Body               : Upper
        None :  None : 0.9999999999999956 :   1.0
dual : Direction=Suffix.IMPORT, Datatype=Suffix.FLOAT
    Key          : Value
     constraint1 :     11.800000000401287
    constraint10 :                    0.0
    constraint11 :                   -0.0
    constraint12 :                    0.0
    constraint13 :                   -0.0
    constraint14 :     0.3419329877011282
    constraint15 :                   -0.0
    constraint16 :     0.9630759067077719
     constraint2 :     12.600000194066551
     constraint3 : 2.0315288176716185e-10
     constraint4 :   9.69893018651401e-08
     constraint5 :     0.3419329875188512
     constraint6 :      0.963075906684621
     constraint7 :                    0.0
     constraint8 :                    0.0
     constraint9 :                   -0.0
```

Accepted production offers of the generating units

Accepted Offer	**Hour 1**								**Hour 2**							
	Unit $\mathbf{1}$				Unit $\mathbf{2}$				Unit $\mathbf{1}$				Unit $\mathbf{2}$			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(\boldsymbol{puMWh})$	0.7				0.5	0.5			0.8	0.4			0.5	0.5		

Table 9: Accepted production offers of the generating units

Accepted consumption bids of the demand

Accepted bids	**Hour 1**								**Hour 2**							
	Demand $\mathbf{1}$				Demand $\mathbf{2}$				Demand $\mathbf{1}$				Demand $\mathbf{2}$			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(\boldsymbol{puMWh})$	0.4	0.2	0.1		0.6	0.3	0.1		0.7	0.3			0.8	0.4		

Table 10: Accepted consumption bids of the demand

Rejected production offers of the generating units

Rejected Offer	**Hour 1**								**Hour 2**							
	Unit $\mathbf{1}$				Unit $\mathbf{2}$				Unit $\mathbf{1}$				Unit $\mathbf{2}$			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(\boldsymbol{puMWh})$	0.1	0.6	0.4	0.2			0.5	0.5		0.2	0.4	0.2			0.5	0.5

Table 11: Rejected production offers of the generating units

Rejected consumption bids of the demand

Rejected Bids	**Hour 1**								**Hour 2**							
	Demand $\mathbf{1}$				Demand $\mathbf{2}$				Demand $\mathbf{1}$				Demand $\mathbf{2}$			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(\boldsymbol{puMWh})$				0.1				0.1			0.2	0.1			0.2	0.1

Table 12: Rejected consumption bids of the demand

Market Clearing Price of the system and Social Welfare

	Hour 1	**Hour 2**
Market Clearing Price $\boldsymbol{\lambda}$	$11.8\ \$/puMWh$	$12.6\ \$/puMWh$
Social Welfare$(\$)$	*16.02 $*	

Table 13: Market Clearing Price and Social Welfare

This solution provides an optimal value of the objective function, i.e., the social welfare, of $16.02.

Market Clearing Price at each node and each time interval

	Hour 1			**Hour 2**		
	Node 1	**Node 2**	**Node 3**	**Node 1**	**Node 2**	**Node 3**
Market Clearing Price	$11.8\ \$/puMWh$	$11.8\ \$/puMWh$	$11.8\ \$/puMWh$	$12.6\ \$/puMWh$	$12.6\ \$/puMWh$	$11.8\ \$/puMWh$
Social Welfare$(\$)$	*16.02 $*					

Table 14: Market Clearing Price at each node for each time interval

The profit of each generating unit can be computed as follows:

Total profit of generating Unit-1

$$\pi_1 = (\lambda_1^* - C_{11}^G)\, p_{111}^G + (\lambda_1^* - C_{12}^G)\, p_{112}^G + (\lambda_1^* - C_{13}^G)\, p_{113}^G + (\lambda_1^* - C_{14}^G)\, p_{114}^G$$

$$+(\lambda_2^* - C_{11}^G)\, p_{211}^G + (\lambda_2^* - C_{12}^G)\, p_{212}^G + (\lambda_2^* - C_{13}^G)\, p_{213}^G + (\lambda_2^* - C_{14}^G)\, p_{214}^G$$

$$= (11.8 - 11.8) * 0.7 + (11.8 - 12.6) * 0.0 + (11.8 - 13.5) * 0.0 + (11.8 - 14.0) * 0.0$$

$$+ (12.6 - 11.8) * 0.8 + (12.6 - 12.6) * 0.4 + (12.6 - 13.5) * 0.0 + (12.6 - 14.0) * 0.0$$
$$= 0.64\ \$$$

Total profit of generating Unit-2

$$\pi_2 = (\lambda_1^* - C_{21}^G)\, p_{121}^G + (\lambda_1^* - C_{22}^G)\, p_{122}^G + (\lambda_1^* - C_{23}^G)\, p_{123}^G + (\lambda_1^* - C_{24}^G)\, p_{124}^G$$

$$+(\lambda_2^* - C_{21}^G)\, p_{221}^G + (\lambda_2^* - C_{22}^G)\, p_{222}^G + (\lambda_2^* - C_{23}^G)\, p_{223}^G + (\lambda_2^* - C_{24}^G)\, p_{224}^G$$

$$= (11.8 - 10.1) * 0.5 + (11.8 - 11.2) * 0.5 + (11.8 - 12.3) * 0.0 + (11.8 - 13.0) * 0.0$$

$$+ (11.8 - 10.1) * 0.5 + (11.8 - 11.2) * 0.5 + (11.8 - 12.3) * 0.0 + (11.8 - 13.0) * 0.0$$

$$= 0.85 + 0.3 + 0.85 + 0.3$$

$$= 2.3\ \$$$

Total Profit of generating Units

	Unit 1	Unit 2
Total profit	0.64 $	2.3 $

Table 15: Total Profit of generating Units

(e) Compare the optimal solutions from parts 2 and 4. If they are different, explain why. If they are not, explain why and speculate on conditions that would have made the solutions different. Apply your speculations in the code and verify. (40 marks)

	Accepted production offers for Part-2								Accepted production offers for Part-4							
Accepted Offer	Hour 1								Hour 1							
	Unit 1				Unit 2				Unit 1				Unit 2			
Block	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4
Energy $(puMWh)$	0.7				0.5	0.5			0.7				0.5	0.5		
Accepted Offer	Hour 2								Hour 2							
	Unit 1				Unit 2				Unit 1				Unit 2			
Block	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4
Energy $(puMWh)$	0.8				0.5	0.5	0.5		0.8	0.4			0.5	0.5		

Table 16: Comparison of Accepted production offers in Part-2 and Part-4

	Accepted Consumption bids for Part-2								Accepted Consumption bids for Part-4							
Accepted Bids	Hour 1								Hour 1							
	Demand 1				Demand 2				Demand 1				Demand 2			
Block	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4
Energy $(puMWh)$	0.4	0.2	0.1		0.6	0.3	0.1		0.4	0.2	0.1		0.6	0.3	0.1	
Accepted Bids	Hour 2								Hour 2							
	Demand 1				Demand 2				Demand 1				Demand 2			
Block	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4
Energy $(puMWh)$	0.7	0.3			0.8	0.4	0.1		0.7	0.3			0.8	0.4		

Table 17: Comparison of Accepted bids in Part-2 and Part-4

After analysing the impact of transmission capacity limits on the solution of a market clearing auction with transmission constraints, we get two different solutions.

- As shown in the *Table-16*, in the later case (especially for hour-2), not all the cheapest production blocks are used at capacity due to line limit loading constraints. i.e., Unit-2 has third block offer available to generate 0.5 $puMWh$ at 12.3 $\$/puMWh$; however, Unit-1's second block accepted with 12.6 $\$/puMWh$ over the Unit-2 due to the transmission line limit constraint.

	Market Clearing Price obtained in Part-2			Market Clearing Price obtained in Part-4		
	Hour 1			Hour 1		
	Node 1	Node 2	Node 3	Node 1	Node 2	Node 3
Market Clearing Price	11.8 $\$/puMWh$			11.8 $\$/puMWh$	11.8 $\$/puMWh$	11.8 $\$/puMWh$
	Hour 2			Hour 2		
	Node 1	Node 2	Node 3	Node 1	Node 2	Node 3
Market Clearing Price	12.5 $\$/puMWh$			12.6 $\$/puMWh$	12.6 $\$/puMWh$	11.8 $\$/puMWh$
Social Welfare ($\$$)	16.16 $\$$			16.02 $\$$		

Table 18: Comparison of Market clearing price obtained in Part-2 and Part-4

As seen in the *Table-18*, the market clearing price is same for all three nodes because we have ignored the transmission line loading constraints. On the other hand, for second hour, the market clearing price is different for both generating node. Here to serve the load, at node-1, last generation offer accepted at $12.6\ \$/puMWh$, while, at node-2, last generation offer accepted at $11.8\ \$/puMWh$ even though next generation offer available to dispatch at $12.3\ \$/puMWh$. For node-3, the market clearing price is same as the last demand bid accepted price which is the overall market clearing price $= 12.6\ \$/puMWh$.

To reiterate, the overall market clearing price in later case is comparatively higher due to the transmission line (L_{23}) loading constraint than the first case.

Profits Achieved by each generating unit

Generating Unit	Without transmission constraints	With transmission constraints
1	0.56 \$	0.64 \$
2	3.1 \$	2.30 \$

Table 19: Comparison of profit achieved by each generating unit

Note that due to the congestion in the transmission network, the profits of generating units 2 is reduced, while the profit of generating unit 1 increases. This is due to the changes in their power schedules and also to the price differences across the network.

We analyze the impact of transmission capacity limits on the solution of a market clearing auction with transmission constraints.

To do so, I solve again the previous example, but in this case, I relax the transmission capacity limits of all the lines and consider them equal to $1.5\ puMWh$.

In such a case, we obtain that the solution *(Table 19-23)* is the same than that obtained in the multi-period market clearing auction without transmission constraints (Part-2).

Accepted production offers of the generating units

Accepted Offer	Hour 1								Hour 2							
	Unit 1				Unit 2				Unit 1				Unit 2			
Block	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4
Energy $(puMWh)$	0.7				0.5	0.5			0.8				0.5	0.5	0.5	

Table 19: Accepted production offers of generation units

Accepted consumption bids of the demand

Accepted bids	Hour 1								Hour 2							
	Demand 1				Demand 2				Demand 1				Demand 2			
Block	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4	#1	#2	#3	#4
Energy $(puMWh)$	0.4	0.2	0.1		0.6	0.3	0.1		0.7	0.3			0.8	0.4	0.1	

Table 20: Accepted consumption bids of the demand

Rejected production offers of the generating units

Rejected Offer	**Hour 1**								**Hour 2**							
	Unit $\mathbf{1}$				Unit $\mathbf{2}$				Unit $\mathbf{1}$				Unit $\mathbf{2}$			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(\boldsymbol{puMWh})$	0.1	0.6	0.4	0.2			0.5	0.5		0.6	0.4	0.2				0.5

Table 21: Rejected production offers of the generating units

Rejected consumption bids of the demand

Rejected Bids	**Hour 1**								**Hour 2**							
	Demand $\mathbf{1}$				Demand $\mathbf{2}$				Demand $\mathbf{1}$				Demand $\mathbf{2}$			
Block	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$	$\#1$	$\#2$	$\#3$	$\#4$
Energy $(\boldsymbol{puMWh})$				0.1				0.1			0.2	0.1			0.1	0.1

Table 22: Rejected consumption bids of the demand

Market Clearing Price and Social Welfare

	Hour 1	**Hour 2**
Market Clearing Price	$11.8\ \$/puMWh$	$12.5\ \$/puMWh$
Social Welfare$(\$)$	*16.16 $*	

Table 23: Market Clearing Price and Social Welfare

Note: Above case shows the effect of transmission line loading limit on market. Here, transmission Line loading capacity is increased gradually and each time the result changes. To get the same result as question-b, I have taken $1.5\ puMWh$ line loading limit. Similarly, by changing Line loading capacity to lower side, we get the different solution. Please find an attachment of code and result herewith the report.